Required strength estimation of a cemented backfill with the front wall exposed and back wall pressured

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Abstract: Determining the required strength is a critical task in backfilled stopes design. Over the years, several solutions have been proposed by considering a backfill with one face exposed and confined by three rock walls. In practice, a cemented backfill may be in contact with an uncemented backfill and exposed on the opposite side. The uncemented backfill can apply a pressure on the exposed cemented backfill and affects its stability. In this study, a lateral pressure equal to the isostatic overburden pressure is considered for the uncemented backfill. An analytical solution is proposed to evaluate the minimum required strength of the cemented backfill exposed on one side, pressured by the uncemented backfill on the opposite side, and confined by two rock walls. The proposed analytical solution is validated by numerical simulations with FLAC3D.
1 Introduction

Open stoping with delayed backfill is increasingly utilised world-wide in underground mines. Long holes drilling and blasting considerably improve the mining production, whereas the application of delayed backfill improves the ground stability, maximises ore recovery, and improves the energy efficiency of ventilation (Hartman, 1992; Hassani and Archibald, 1998; Darling, 2011; Potvin et al., 2015). The reduction in ground subsidence associated with mining activities and the reduction in environmental impacts from mining operations are additional advantages to fill the mine voids with backfill made of mine wastes (tailings or waste rocks) (Aubertin et al., 2002; Bussière, 2007; Simms et al., 2007; Benzaazoua et al., 2008; Yang et al., 2015; Zhang et al., 2011, 2015).

In open stoping with delayed backfill, the ore body is usually divided into a series of stopes, named as primary and secondary stopes. The primary stopes are firstly mined out and filled with cemented backfill to form vertical man-made pillars, which must remain stable during the mining operation of the adjacent secondary stope(s). The cementation of this backfill is mainly achieved by the addition of Ordinary Portland cement or other binders (fly ash, slag, pozzolans, etc.) to the fill materials (tailings or waste rocks). Backfill strength increases with binder consumption. Therefore, it is a critical concern for
a mining engineer to have a good estimate of the minimum required strength of the cemented backfill exposed on one side owing to the excavation of an adjacent secondary stope.

Before the early 1980s, two methods were mainly used to determine the required backfill strength for primary stopes. One method was to compare the vertical stress ($\sigma_v$) with the unconfined compressive strength ($\sigma_c$) of the backfill. The vertical stress ($\sigma_v$) was estimated based on the overburden of the backfill ($\sigma_v = \gamma h$; where $\gamma$ is the bulk unit weight of the cemented backfill and $h$ is the depth from the top surface of the backfill). This led to a minimum required unconfined compressive strength ($\sigma_c$) varying from zero at the top surface of the backfill to a maximum value of $\gamma H$ ($H$ is the overall height of the exposed backfill) at the base of the stope (Mitchell et al., 1982). The second method was to consider the cemented backfill as a vertical 2D (plane strain) slope made of cohesive and frictionless material. This resulted in a strength requirement as $\sigma_c \geq \gamma H/2$ (Duncan and Wright, 2005). These two approaches neglected the confining effects of the side walls, leading to uneconomical and overly conservative backfill strength design.

Later, the confining effects of the two side walls were taken into account by Mitchell et al. (1982). Their solution proposed for assessing the minimum required strength of the exposed cemented backfill has been largely accepted in academia (Arioglu, 1984; Chen and Jiao, 1991; Zou and Nadarajah, 2006; Dirige et al., 2009). The application of this solution has led to important economic benefits for the mining industry.

Recently, Li and coworkers revisited the Mitchell et al. (1982) model. Several updates have been made after accounting for the stope geometry, internal friction angle of the backfill, shear strengths along the three confining walls and failure mechanism of the exposed backfill (Li, 2014a, 2014b; Li and Aubertin, 2012, 2014).

Figure 1 A cemented backfill in the primary stope ⊙ exposed on one side owing to the excavation of the second secondary stope ⊙ and pressured on the opposite side by the uncedmented backfill placed in the first secondary stope ⊙ (see online version for colours)
These analytical solutions contribute to better understanding and evaluation of the backfill strength requirements. However, all these solutions were developed by considering the cemented backfill with one open face associated with the excavation of one adjacent secondary stope. In practice, it is quite often that this (first) secondary stope is filled with an uncemented backfill and the cemented backfill must be exposed on the opposite side owing to the excavation of a second secondary stope (Belem and Benzaazoua, 2008; Darling, 2011; Villaescusa, 2014). This case, as shown in Figure 1, cannot be treated by the original and modified Mitchell et al. (1982) solutions, as they do not take into account the pressure exerted by the uncemented backfill in the first secondary stope.

In this study, the stability of the cemented backfill exposed on one side and subjected to an isostatic overburden pressure on the opposite side will be analysed. Analytical solutions are formulated and compared with numerical simulations performed with FLAC3D.

2 Solution development

2.1 Model consideration

Figure 2 presents an isolated presentation of the primary stope shown in Figure 1 with the various forces and pressures acting on the cemented backfill body. A potential sliding plane is considered to pass through the toe of the exposed face. In Figure 2, \( H, B \) and \( L \) (m) are the height, width and length of the cemented backfill, respectively; \( p_0 \) (kPa) is the vertical pressure on the top surface of the cemented backfill owing to possible loads from mining equipment or newly deposited backfill slurry; \( \alpha \) (°) is the angle of the sliding plane to the horizontal; \( N \) (kN) and \( S \) (kN) are the normal and shear forces on the sliding plane, respectively; \( S_s \) (kN) is the shear resistant forces along the two side walls; \( \beta \) (°) is the angle between the side shear resistance forces \( S_s \) and the horizontal; \( W \) (kN) is the weight of the sliding wedge; \( P_b \) (kN) is the resulting force exerted by the uncemented backfill on the back wall of the sliding wedge.

In this study, the primary stope has a high height-to-width aspect ratio because the sliding plane intercepts the back wall \( (H \geq B \tan \alpha) \). The uncemented backfill slurry placed in the secondary stope (see Figure 1) is considered as a liquid (no shear strength). The pressure exerted by the uncemented backfill slurry on the back wall of the cemented backfill is then equal to the isostatic overburden pressure \( (\gamma_u h; \gamma_u \text{ (kN/m)}^3 \text{ is the unit weight of the uncemented backfill; } h \text{ (m) is the depth). The resulting force } P_b \text{ (kN) on the back wall of the sliding wedge can be expressed as follows:}

\[
P_b = \int_0^{H'} \gamma_u h L \, dh = \frac{1}{2} \gamma_u L (H - B \tan \alpha)^2
\]

where \( H' = H - B \tan \alpha \) (m) is the height of the sliding wedge on the back wall.

The effective weight of the sliding wedge \( W' \) (kN) is obtained by considering the weight \( (W) \) of the sliding wedge and the load on the top surface of the backfill as follows:

\[
W' = W + p_0 L B = (\gamma H' + p_0)LB
\]
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where \( \gamma \) (kN/m\(^3\)) is the bulk unit weight of the cemented backfill in the primary stope; \( H' \) (m) is the equivalent height of the sliding cemented backfill wedge:

\[
H' = H - \frac{B \tan \alpha}{2}
\]  

Figure 2 An isolated presentation of the cemented backfill of the primary stope \( \odot \) shown in Figure 1 with the various acting forces and pressures (see online version for colours)

The shear resistant force, \( S_s \) (kN) along the side rock walls can be expressed as:

\[
S_s = \int_0^h \tau_s B \, dh + \int_0^H \tau_s \frac{H - h}{\tan \alpha} \, dh
\]  

where \( h \) (m) is the depth from the top surface of the backfill; \( \tau_s \) (kPa) is the shear strength along the interfaces between the cemented backfill and side rock walls in the primary stope:

\[
\tau_s = c_s + \sigma_t \tan \delta_s
\]  

where \( c_s \) (kPa) and \( \delta_s \) (°) are the cohesion and friction angle of the interfaces along the side walls, respectively. They can be expressed as a proportion of the shear strength of the cemented backfill:

\[
c_s = r_s c
\]  

\[
\delta_s = r_s \phi
\]

where \( c \) (kPa) and \( \phi \) (°) are the cohesion and friction angle of the cemented backfill, respectively; \( r_s (= c_s/c; [0, 1]) \) and \( r_s (= \delta_s/\phi; [0, 1]) \) are the adherence and friction angle
ratios of the fill-wall interfaces, respectively. The two shear strength ratios of the fill-wall interfaces \( r_s \) and \( r_i \) depend on the roughness and weathering conditions of the rock walls. The recent results obtained by laboratory tests show that \( r_s \) typically varies around 0.25–1, while \( r_i \) typically takes the unity (Manaras, 2009; Fall and Nasir, 2010; Koupouli et al., 2016).

In equation (5), \( \sigma_h \) (kPa) is the horizontal stress normal to the interfaces between the cemented backfill and side rock walls at a depth \( h \). It can be obtained using a typical two-dimensional arching solution (Li et al., 2003, 2005; Li and Aubertin, 2009a; Li, 2014a):

\[
\sigma_h = \frac{\gamma L}{2 \tan \delta} \left( 1 - e^{-2K \tan \delta} \right) + K p_o e^{-\frac{2K \tan \delta p_o}{L}}
\]

where \( K \) is an earth pressure coefficient. For most cases within backfilled stopes, it takes a value close to the Rankine active earth pressure coefficient (e.g., Sobhi et al., 2017):

\[
K = K_a = \tan^2 \left( 45^\circ - \frac{\phi}{2} \right)
\]

Introducing equations (3), (5)–(7) to equation (4) leads to the following equation for the shear force \( S_s \):

\[
S_s = B H^* r_s c + X
\]

where

\[
X = \frac{L B}{2} \left[ \gamma H^* - \left( \frac{\gamma L}{2 K \tan \delta} - p_o \right) \right] + \frac{L^2}{4 K \tan \alpha \tan \delta} \left( \frac{\gamma L}{2 K \tan \delta} - p_o \right) \left( e^{-\frac{2 K \tan \delta p_o}{L}} - e^{-\frac{2 K \tan \delta p_o}{L}} \right)
\]

Considering the equilibrium of the sliding wedge leads to the following expressions for the normal (\( N \)) and shear (\( S \)) forces on the sliding plane:

\[
N = Y - 2S_s \sin(\beta - \alpha)
\]

\[
S = Z - 2S_s \cos(\beta - \alpha)
\]

where

\[
Y = W' \cos \alpha - p_s \sin \alpha
\]

\[
Z = W' \sin \alpha + p_s \cos \alpha
\]

The factor of safety (\( FS \)) of the sliding wedge can then be expressed as follows:

\[
FS = \frac{\text{Resisting forces}}{\text{Driving forces}} = \frac{c \times (\text{area of the sliding plane}) + N \tan \phi}{S} = \frac{c \times L(B / \cos \alpha) + [Y - 2S_s \sin(\beta - \alpha)] \tan \phi}{Z - 2S_s \cos(\beta - \alpha)}
\]
This is a general solution for describing the stability of the cemented backfill in the primary stope. It is noted that $FS$ depends on the angle values $\alpha$ and $\beta$.

### 2.2 Formulation

In previous solutions (Mitchell et al., 1982; Arioglu, 1984; Chen and Jiao, 1991; Zou and Nadarajah, 2006; Dirige et al., 2009; Li, 2014a, 2014b; Li and Aubertin, 2012, 2014), the angle $\alpha$ was determined as $\alpha = 45^\circ + \phi/2$ by considering the backfill in an active state. This has been partly verified by physical model tests (Mitchell et al., 1982) and numerical simulations (Falaknaz, 2014; Liu, 2017; Liu et al., 2016a). For the angle $\beta$ of the shear force $S_s$, it has been postulated to be $\beta = 90^\circ$ in the Mitchell et al. (1982) and extension models (Arioglu, 1984; Chen and Jiao, 1991; Zou and Nadarajah, 2006; Dirige et al., 2009; Li, 2014a, 2014b; Li and Aubertin, 2012). Li and Aubertin (2014) performed preliminary numerical analyses on the movement of the exposed backfill. They found that the sliding wedge could be divided into two zones: the upper zone with $\beta = 90^\circ$ and the lower zone with $\beta \approx 45^\circ + \phi/2$.

The above-mentioned analyses were made for the exposed cemented backfill confined by three rock walls. With a lateral pressure applied on the back wall of the cemented backfill, the two angles $\alpha$ and $\beta$ can be expected to change more or less compared with the Mitchell et al. (1982) and extension models. Hereafter, four possible cases are considered.

**Model 1, $\alpha = 45^\circ + \phi/2$ and $\beta = 90^\circ$**

This model considers that the lateral pressure does not have any influence on the two angles $\alpha$ and $\beta$. The Mitchell et al. (1982) model applies with respect to the mobilisation of the sliding wedge. Introducing $\alpha = 45^\circ + \phi/2$ and $\beta = 90^\circ$ into equation (15) leads to the factor of safety ($FS$) expressed as follows:

$$FS = \frac{cLB / \cos \alpha + (Y - 2S \cos \alpha) \tan \phi}{Z - 2S \sin \alpha}$$

(16)

Introducing equations (9) and (10) into equation (16) leads to the following expression for the required backfill cohesion $c$:

$$c = \frac{FS \cdot Z - Y \tan \phi - 2X \left( FS \cdot \sin \alpha - \cos \alpha \tan \phi \right)}{LB / \cos \alpha + 2BH \cdot r_i \left( FS \cdot \sin \alpha - \cos \alpha \tan \phi \right)}$$

(17)

**Model 2, $\alpha = \beta = 45^\circ + \phi/2$**

This model considers that the sliding plane is not affected by the lateral pressure and the wedge slides in a direction parallel to the sliding plane. Introducing $\alpha = \beta = 45^\circ + \phi/2$ leads to an expression of $FS$ as follows:

$$FS = \frac{cLB / \cos \alpha + Y \tan \phi}{Z - 2S_i}$$

(18)

The required cohesion $c$ is given as:

$$c = \frac{FS \cdot (Z - 2X) - Y \tan \phi}{LB / \cos \alpha + 2 \cdot FS \cdot BH \cdot r_i}$$

(19)
Model 3, $\alpha = \beta$

This model considers that the sliding plane and direction of the sliding wedge are parallel but in any direction ($90^\circ \geq \alpha = \beta \geq 0^\circ$). $FS$ has the same expression as equation (18), while the required cohesion $c$ has the same expression as equation (19).

To obtain the critical angle $\alpha (= \beta)$, one performs partial derivative to the function $FS$ with respect to the variable $\alpha$ and one obtains the following expression:

$$
\frac{\partial FS}{\partial \alpha} = \frac{c + \gamma H' \tan \phi - \gamma B \tan \phi / (2 \tan \alpha) \{ LB \sin \alpha / \cos ^2 \alpha - Z \tan \phi \}}{Z - 2S_y} - \frac{cLB / \cos \alpha + Y \tan \phi}{(Z - 2S_y)^2} 
$$

$$
\times \left\{ \frac{B}{2} \left[ 2 \gamma cB - 2 \gamma L'H' \cos \alpha + \gamma LB(1 - \sin \alpha) \right] + Y 
+ \frac{L^2}{2K \tan \delta_y} \left[ \frac{\gamma L}{2K \tan \delta_y} - p_o \right] \left[ \left( \frac{1}{\sin ^2 \alpha} - \frac{4KB \tan \delta}{L \sin 2\alpha} \right) e^{\frac{2K \tan \delta_y \delta}{L} - \frac{1}{\sin ^2 \alpha}} - \frac{2K \tan \delta_y \delta}{L} \right] \right\} 
$$

(20)

By imposing $\partial FS / \partial \alpha = 0$, the critical angle $\alpha (= \beta)$ can readily be calculated with Microsoft Excel®. $FS$ and the required cohesion can then be calculated with equations (18) and (19), respectively.

Model 4 (proposed solution), $\alpha = 45^\circ + \phi/2, 90^\circ \geq \beta \geq 0^\circ$

This model considers that the sliding plane is not affected by the lateral pressure ($\alpha = 45^\circ + \phi/2$) but the sliding tendency can be in any direction ($90^\circ \geq \beta \geq 0^\circ$). $FS$ is expressed by equation (15).

Performing partial derivative to the function $FS$ with respect to the variable $\beta$ and solving the equation $\partial FS / \partial \beta = 0$ lead to the following equation for the critical angle $\beta$:

$$
\beta = \alpha + \arcsin \frac{2SU - Z \sqrt{U^2 - 4S_y^2 + Z^2}}{Z^2 + U^2} 
$$

(21)

where

$$
U = Y + \frac{cLB}{\cos \alpha \tan \phi} 
$$

(22)

As the critical angle $\beta$ given by equation (21) involves the cohesion $c$, equations (15), (21) and (22) have to be applied by iterative process to obtain the minimum required backfill cohesion. Calculations with typical stope geometry and material properties suggest that the critical angle $\beta$ takes an expression as follows:

$$
\beta = 45^\circ - \frac{\phi}{2} 
$$

(23)

or

$$
\alpha - \beta = \phi 
$$

(24)

Applying equation (24) in equation (15) leads to the following expression for the stability of the cemented backfill in the primary stope Ω:
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\[ FS = \frac{cLB / \cos \alpha + (Y + 2S_y \sin \phi) \tan \phi}{Z - 2S_z \cos \phi} \]  
(25)

Considering \( FS = 1 \) in equation (25) leads to the minimum required cohesion \( c \) as follows:

\[ c = \frac{Z \cos \phi - Y \sin \phi - 2X}{LB \cos \phi + 2BH \cos \alpha} \]  
(26)

Equations (25) and (26) constitute the proposed solution to evaluate the stability and required cohesion of the exposed backfill based on the numerical modelling presented here.

3 Numerical simulations and analysis

3.1 Numerical models

Previous studies have shown that the numerical analyses are very helpful to understand the mechanical response of backfilled stopes (Rankine, 2004; Pirapakaran and Sivakugan, 2007; Li and Aubertin, 2009b; Veenstra, 2013; Falaknaz, 2014; Falaknaz et al., 2015a, 2015b, 2015c; Li and Aubertin, 2014; Liu, 2017; Liu et al., 2016a, 2016b, 2017; Yang et al., 2017a). In this study, FLAC\(^{3D}\) is used to analyse the stability of the exposed backfill.

Figure 3(a) illustrates half of a physical model of the cemented backfill (Stope 1) with its front wall exposed (Stope 3) and back wall in contact with an uncemented backfill slurry (Stope 2). Figure 3(b) shows the numerical model constructed with FLAC\(^{3D}\). The rock walls have a thickness of 2 m. The effect of the uncemented backfill slurry in Stope 2 is represented by a lateral pressure equal to the isostatic overburden pressure. The displacements along the plane \( Y = 0 \) m are prohibited in \( Y \) direction but allowed in \( X \) and \( Z \) directions to simulate this symmetry plane. The bottom boundary of the model is fixed in all directions, whereas the external boundaries of the rock mass are fixed in the horizontal directions normal to the external boundary planes but allowed to freely move in all directions parallel to the planes. Table 1 shows a summary of the boundary conditions of the numerical models.

By observing the sequence of excavation and backfilling in two-step open stoping with delayed backfill, the numerical modelling with FLAC\(^{3D}\) has been processed in five stages as follows:

1. Construction of the model before any excavation and backfilling.
2. Excavation of Stope 1 in one step. The displacement field was reset to zero after the system reached an equilibrium state.
3. Backfilling of Stope 1 in 16 layers to obtain numerical results stable and close to static state.
Excavation of Stope 2 in one step. When the system reached an equilibrium state, a lateral pressure equal to the isostatic overburden pressure was applied on the back wall of the cemented backfill to simulate the effect of the uncemented backfill slurry.

Excavation of Stope 3 in one step.

It should be noted that the number of filling layers in Stage 3 and the number of excavation steps in Stages 4 and 5 were determined after several sensitive analyses. More details were given in Liu et al. (2016a, 2016b) and Liu (2017).

Figure 3  (a) A physical model of the exposed cemented backfill in a primary stope and (b) the corresponding numerical model constructed with FLAC\(^3\text{D}\) accounting for the symmetry plane \(Y = 0\) m (see online version for colours)

Table 1  Summary of the boundary conditions of the numerical models

<table>
<thead>
<tr>
<th>Boundary</th>
<th>(X) direction</th>
<th>(Y) direction</th>
<th>(Z) direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetry plane ((Y = 0) m)</td>
<td>Free</td>
<td>Fixed</td>
<td>Free</td>
</tr>
<tr>
<td>Bottom of the model ((R_b; Z = -2) m)</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
</tr>
<tr>
<td>External rock (R_1) ((X = -2) m)</td>
<td>Fixed</td>
<td>Free</td>
<td>Free</td>
</tr>
<tr>
<td>External rock (R_2) ((Y = L/2 + 2) m)</td>
<td>Free</td>
<td>Fixed</td>
<td>Free</td>
</tr>
<tr>
<td>External rock (R_3) ((X = 3B + 2) m)</td>
<td>Fixed</td>
<td>Free</td>
<td>Free</td>
</tr>
<tr>
<td>Contact between Stopes 1 and 2</td>
<td>Pressured</td>
<td>Free</td>
<td>Free</td>
</tr>
<tr>
<td>Contact between Stopes 1 and 3</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
</tr>
</tbody>
</table>

The rock mass is considered to be homogeneous, isotropic and linearly elastic characterised by \(\gamma_r = 27\) kN/m\(^3\) (unit weight), \(E_r = 40\) GPa (Young’s modulus) and \(\mu_r = 0.2\) (Poisson’s ratio). These parameters were taken, based on typical values given in the literature on rock masses in underground metal mines (e.g., Hoek, 2001; Brady and
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Brown, 2004; Gercek, 2007; Zhang and Mitri, 2008; Walton et al., 2015). The cemented backfill in the primary Stope 1 (Figure 3(b)) is modelled as an elasto-plastic material obeying the Mohr–Coulomb criterion. It is characterised by $\gamma = 21$ kN/m$^3$ (unit weight), $E = 700$ MPa (Young’s modulus), $\mu = 0.3$ (Poisson’s ratio), $c$ (cohesion), $\phi$ (internal friction angle), $\psi = 0^\circ$ (dilation angle) and $\sigma_t$ (tensile strength $= \sigma/10 = c/20$). The parameters for the cemented backfill used here were taken, mainly based on the typical values shown in the literature (e.g., Mitchell and Wong, 1982; Hassani and Archibald, 1998; Belem et al., 2000; Potvin et al., 2005; Yilmaz et al., 2009; Galaa et al., 2011; Dehghan et al., 2013; Kumar et al., 2016), also partly based on the authors’ experience gained during the realisation of several projects with different types of mine backfill.

The interfaces between the cemented backfill and side rock walls are characterised by interface cohesion $c_s (= r_s c)$, frictional angle $\delta_s (= r_s \phi)$, normal ($k_n$) and shear ($k_s$) stiffness. The normal ($k_n$) and shear ($k_s$) stiffness values of the interfaces were determined by an equation recommended in the FLAC 3D manual (Itasca, 2012) for slip and separation fill-wall interfaces.

To ensure stable numerical results, a series of mesh sensitivity analyses have been made for typical metal mine stopes ($L = 10–30$ m, $B = 5–20$ m and $H = 20–60$ m). The stresses and displacements along the vertical central line $A_1A'_1$ were monitored during the excavation and backfilling operations in the stopes. The optimal mesh size for the numerical model was found to be 0.5 m.

3.2 Critical strength of the exposed backfill

Upon exposure of the front wall, the cemented backfill in the primary stope can stay stable or fall, depending on its strength. A strength will be called critical if the exposed cemented backfill changes from a stable state to an unstable state when the strength of the backfill decreases from a value slightly higher to a value slightly lower than this value. For a given problem, a number of numerical simulations have to be made to find the critical strength.

Figure 4 illustrates the variation of the total displacements at three depths $h = 10, 20, 30$ m, respectively, along the monitoring line $A_2A'_2$ with different backfill cohesion after the excavation of Stope 3 (Figure 3(b)). The simulations were made with $L = 10$ m, $B = 5$ m, $H = 40$ m, $\phi = \delta_s = 33^\circ$ and $r_s (= c/c_s) = 0.5$. At $c = 300$ kPa, the displacement values are everywhere very small, indicating a stable state of the exposed backfill. When the backfill cohesion decreases from 300 kPa to 230 kPa, the displacements at the three monitoring points show very small and progressive increments. When the backfill cohesion further decreases from 230 kPa to 200 kPa, a jump of the displacements takes place at $c = 228$ kPa. The large displacements at $c < 228$ kPa indicate the collapse of the exposed backfill. Therefore, the critical cohesion of the exposed cemented backfill is determined as $c = 228$ kPa.

This procedure based on displacement monitoring for determining the critical strength of the exposed backfill is inspired from the instability criterion proposed by Yang et al. (2017b) for barricades constructed with waste rock.

Figure 5 further illustrates the stable state of the exposed cemented backfill at $c = 228$ kPa (Figure 5(a)) and the collapse state at $c = 225$ kPa (Figure 5(b)), based on the iso-contours of the total displacement and strength–stress ratio (equivalent to $FS$).
Figure 4  Total displacement variations at three depths (h = 10, 20, 30 m) along the monitoring line $A_2A'_2$ with different backfill cohesion (see online version for colours)

Figure 5  Iso-contours of the total displacement and strength–stress ratio of the exposed cemented backfill in Stope 1 (see Figure 3); (a) stable at $c = 228$ kPa; (b) collapsed at $c = 225$ kPa (see online version for colours)

4  Comparison between analytical and numerical results

The procedure for determining the critical strength of the exposed backfill has been repeated for other stope geometries and backfill properties.

Figure 6 shows the variation of the critical cohesions, obtained by numerical modelling and predicted by the four analytical solutions presented in Section 2 with different stope width $B$ (Figure 6(a), $L = 10$ m, $H = 40$ m), length $L$ (Figure 6(b), $B = 10$ m, $H = 40$ m), height $H$ (Figure 6(c), $L = 10$ m, $B = 8$ m) and backfill
friction angle $\phi$ (Figure 6(d), $L = 20$ m, $B = 10$ m, $H = 40$ m). Other parameters are $\gamma = \gamma_u = 21$ kN/m$^3$, $\phi = 33^\circ$, $r_s = 0.5$, $r_i = 1$ and $p_0 = 0$ kPa. In all cases, it is seen that the minimum required strengths (i.e., $FS = 1$) predicted by the solution of Model 4 are the closest to the critical cohesions obtained by the numerical modelling. The equations given in Model 4 (equation (26)) are recommended as the proposed analytical solution for evaluating the minimum required cohesion of the vertically exposed cemented backfill pressured on the back wall.

![Figure 6](attachment:figure6.png)

**Figure 6** Variation of the critical cohesions obtained by numerical modelling and of the minimum required cohesion predicted by the four analytical solutions with different stope width $B$ (a), length $L$ (b), height $H$ (c) and friction angle of the cemented backfill $\phi$ (d) (see online version for colours)

5 Discussion

5.1 Comparison with the existing solution

Recently, a similar model was reported by Yang et al. (2015). Arching effect has been neglected in the primary and secondary stopes. The stresses in the cemented (primary stope) and uncemented (secondary stope) backfills were all calculated by the traditional overburden solution. The vertical stress was expressed as $\sigma_v = \gamma h$ and the horizontal stress as $\sigma_h = K \gamma h$. Numerous recent studies have shown that this solution can be valid when the
stopes are very large in plane and small in height. For most cases, the arching effect has to be considered for estimating the stresses in backfilled stopes (Aubertin et al., 2003; Li et al., 2003, 2005; Li and Aubertin, 2009a, 2009b, 2010; Thompson et al., 2012; Falaknaz et al., 2015a, 2015b, 2015c; Liu et al., 2016b, 2017).

As the arching effect was neglected in the calculation of the lateral pressure along the two side walls and stability analysis of the exposed cemented backfill, the solution of Yang et al. (2015) can be rewritten as follows for estimating the minimum required cohesion ($FS = 1$) of the cemented backfill:

$$\frac{(\gamma H' + p_o) \sin \alpha + \frac{\gamma_s H'^2}{2} - K \cos \alpha - \frac{H'}{L}(2c_s + Kp_o \tan \delta_s) - \frac{c}{\cos \alpha}}{K \left( \frac{H'}{L} \tan \delta + \tan \phi \right)} = \gamma H' + p_o \quad (27a)$$

or

$$c = \frac{2p_o H' + \frac{L}{2} - p_o H' \tan (r \phi)}{2r H' + \frac{L}{\cos \alpha}} \quad (27b)$$

It should be noted that equation (27) was obtained after several modifications to the original solution of Yang et al. (2015). These include the removal of an earth pressure coefficient from the equation for calculating the resisting forces along the sliding plane in cemented backfill. The lateral pressure exerted by the uncemented backfill was taken into account both in the driving force and in the resisting force.

Figure 7 shows the variation of the critical cohesions obtained by numerical modelling and of the minimum required cohesions predicted by the Yang et al. (2015) solution with different stope width $B$ (Figure 7(a), $L = 10$ m) and length $L$ (b) (see online version for colours).
(Figure 7(b), $B = 10$ m); other parameters are $H = 40$ m, $\gamma = \gamma_u = 21$ kN/m$^3$, $\phi = 33^\circ$, $r_s = 0.5$, $r_i = 1$ and $p_0 = 0$ kPa. The minimum required cohesions predicted by the proposed solution (Model 4) are also plotted in the figure. It can be seen that the minimum required cohesions are considerably underestimated by the Yang et al. (2015) solution. The backfill strength design based on this solution may be non-conservative.

5.2 Limitations and future development

In this study, an isostatic overburden pressure (i.e., $\sigma_h = \sigma_v = \gamma_u h$) has been considered in the first secondary stope for the uncemented backfill slurry (Stope 2, Figure 3). This pressure state is achieved in paste backfill shortly after the fast backfilling of the stopes (Thompson et al., 2012; Li et al., 2013; El Mkadmi et al., 2014). For most cases in practice, drainage and consolidation can take place during the filling of the first secondary stope, waiting period and excavation of the second secondary stope, resulting in a lateral pressure smaller than the isostatic overburden pressure. This indicates that the proposed solution (equation (26)) provides an upper bound estimation of the minimum required backfill cohesion. More work is required and ongoing to develop a more realistic solution.

Another limitation of the proposed solution is related to the estimation of the horizontal stress normal to the interfaces between the cemented backfill and side rock walls. Two-dimensional arching solutions have been used (Li et al., 2003, 2005; Li, 2014a; Li and Aubertin, 2009a). More work is needed to develop a 3D solution for estimating the stresses in the primary stope by considering the exposure of the cemented backfill.

Finally, it should be pointed out that the primary stopes are commonly filled in two stages with the plug pour and final pour. The backfill for the plug pour usually contains more cement than the final pour (Li, 2014b). More work is required to take into account the stabilisation effect of the plug pour.

6 Conclusions

The stability of a cemented backfill with its front wall exposed and back wall pressured has been investigated. An isostatic overburden pressure has been considered for the lateral pressure exerted by the uncemented backfill on the back wall of the cemented backfill. Four models have been considered in relation with the sliding plane and sliding direction of the potential sliding wedge. The comparison between the analytical and numerical solutions tends to indicate that the model with the sliding plane making an angle $\alpha = 45^\circ + \phi/2$ and sliding direction making an angle $\beta = 45^\circ - \phi/2$ is the most appropriate. The good agreement between the minimum required cohesion predicted by the proposed solution and the critical cohesion obtained by numerical modelling indicates that the proposed (recommended) analytical solution has been validated. It can be used for cemented backfill design for estimating the minimum required strength of vertically exposed cemented backfill next to a secondary stope filled with an uncemented (or low cemented) paste fill.
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References


Required strength estimation of a cemented backfill


**List of symbols**

- $\alpha$: Angle of the sliding plane of the cemented backfill wedge to the horizontal (°)
- $\beta$: Angle of the shear resistant force $S_s$ to horizontal along the interfaces between the cemented backfill and side rock walls (°)
- $B$: Width of the cemented backfill (m)
- $c$: Cohesion of the cemented backfill (kPa)
- $c_s$: Cohesion along the interfaces between the cemented backfill and side rock walls (kPa)
- $\delta$: Friction angle along the interfaces between the cemented backfill and side rock walls (°)
- $E$: Young’s modulus of the cemented backfill (MPa)
- $E_r$: Young’s modulus of the rock mass (GPa)
- $\phi$: Friction angle of the cemented backfill (°)
- $\psi$: Dilation angle of the cemented backfill (°)
- $FS$: Factor of safety of the cemented backfill sliding wedge
- $h$: Depth below the top surface of the cemented backfill (m)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>Overall height of the cemented backfill (m)</td>
</tr>
<tr>
<td>$H'$</td>
<td>Equivalent height of the sliding wedge (m), see equation (3)</td>
</tr>
<tr>
<td>$H''$</td>
<td>Height of the sliding wedge on the back wall of the cemented backfill (m)</td>
</tr>
<tr>
<td>$K$</td>
<td>Earth pressure coefficient</td>
</tr>
<tr>
<td>$K_a$</td>
<td>Rankine’s active earth pressure coefficient, see equation (8)</td>
</tr>
<tr>
<td>$k_n$</td>
<td>Normal stiffness of interface elements in FLAC3D models (Pa/m)</td>
</tr>
<tr>
<td>$k_s$</td>
<td>Shear stiffness of interface elements in FLAC3D models (Pa/m)</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of the cemented backfill (m)</td>
</tr>
<tr>
<td>$p_0$</td>
<td>Pressure on the top surface of the cemented backfill (kPa)</td>
</tr>
<tr>
<td>$P_b$</td>
<td>Equivalent lateral force exerting on the back wall of the cemented backfill by the isostatic overburden pressure of the uncemented backfill (kN), see equation (1)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Bulk unit weight of the cemented backfill (kN/m$^3$)</td>
</tr>
<tr>
<td>$\gamma_u$</td>
<td>Bulk unit weight of the uncemented backfill (kN/m$^3$)</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>Bulk unit weight of the rock mass (kN/m$^3$)</td>
</tr>
<tr>
<td>$r_f$</td>
<td>Friction angle ratio along the interfaces between cemented backfill and side rock walls ($= \delta_s/\phi; [0, 1]$)</td>
</tr>
<tr>
<td>$r_s$</td>
<td>Adherence (cohesion) ratio along the interfaces between the cemented backfill and side rock walls ($= c_s/c; [0, 1]$)</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Unconfined compressive strength of the cemented backfill (kPa)</td>
</tr>
<tr>
<td>$\sigma_h$</td>
<td>Horizontal stress within backfill at a depth $h$ (kPa), see equation (7)</td>
</tr>
<tr>
<td>$\sigma_t$</td>
<td>Tensile strength of the cemented backfill (kPa)</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>Vertical stress within backfill at a depth $h$ (kPa)</td>
</tr>
<tr>
<td>$S_s$</td>
<td>Equivalent shear resistant force along the interfaces between the cemented backfill and side rock walls (kN), see equations (4) and (9)</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>Shear strength along the interfaces between the cemented backfill and side rock walls (kPa), see equation (5)</td>
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<tr>
<td>$\mu$</td>
<td>Poisson’s ratio of the cemented backfill</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>Poisson’s ratio of the rock mass</td>
</tr>
<tr>
<td>$U$</td>
<td>A common symbol to simplify formulation see equation (22)</td>
</tr>
<tr>
<td>$W''$</td>
<td>A sum of the self-weight of the cemented backfill sliding wedge and vertical load on the top surface of the cemented backfill (kN), see equation (2)</td>
</tr>
<tr>
<td>$X$</td>
<td>A variable, see equation (10)</td>
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<tr>
<td>$Y$</td>
<td>A variable, see equation (13)</td>
</tr>
<tr>
<td>$Z$</td>
<td>A variable, see equation (14)</td>
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