Hydraulic analogy and visualisation of two-dimensional compressible fluid flows: part 2: water table experiments

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Abstract: In part 1, the theoretical aspects of hydraulic analogy are given in detail. In this part, the water table experiments are reported. Pictures of the flow patterns over different models show the formations of analogous shock waves and expansion fans. Pointed airfoils are tested first. Both biconvex and diamond airfoils are used. Busemann biplane model is included. The formation of a shock over a sharp pointed concave body is also tested. For blunt bodies, flow over a cylinder with a detached bow shock is studied. A sharp and not-so-sharp-spike added to the cylinder and the flow patterns are compared. Shocks inside and outside convergent-divergent nozzles are shown. Also, some results are reported for an experiment with a very simple arrangement of a plate in a plastic container, where the model is fixed, and the water is poured on the plate. Such an arrangement can be used for classroom demonstrations. Issues of manufacturing the models, nozzles and airfoils, are studied as well. Measurement techniques for obtaining quantitative results are briefly mentioned. Finally, concluding remarks about the educational use of this experiment are discussed.

Keywords: water tables; flow patterns of wedges; biconvex airfoils; Busemann biplane; cylinders and convergent/divergent nozzles.

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1 Introduction

Compressible flows are very important for many applications. The governing equations are, in general, nonlinear in particular in high speed transonic and supersonic regimes. Wind tunnel experiments for compressible flow applications are expensive. On the other hand, numerical simulations are complicated due to the presence of shock waves and/or mixed subsonic/supersonic regions. To gain some insight about the physical phenomena in these regimes, at least from an educational point of view, water table is an ideal solution. It is an affordable tool to visualise the flow patterns and study their features, thanks to the hydraulic analogy. The theoretical aspect of this theory has been discussed in Part 1 of this work. The simplest case of steady flow in a nozzle is summarised in Table 1, where the analogy is established between surface waves of shallow water layer on a flat surface in a channel and quasi one dimensional isentropic flow in the same geometry namely the cross-section is the same as the channel width. The analogy is not restricted to one-dimensional flows. Indeed, steady and unsteady two dimensional compressible flows can be simulated with shallow water surface waves.

The analogy between sound waves in air and surface waves on shallow water layer on a flat plate is well known see Kuperus (1971, Lamb (1945) and Rayleigh (1914).

Indeed, the analogy has been used early in the forties and until today in both academia and industry, see Avendze (2006), Barclay et al. (1963), Becker (1953), Black and Mediratta (1951), Bombelberg (1955), Born and Wolf (2003), Bruman (1947), Carbonaro (1994), Catchpole (1940), Crossley and Harleman (1952), Fledderman and Stancil (1955), Flowers (1951), Garg (2012), Gilmore et al. (1949), Hafez (2017), Hambrice and Hopper (2004), Hatch (1949), Hedgecock (1950), Jacobs (1947), Johnson et al. (1947), Kuperus (1971), Laitone (1952, 1950, 1961), Laitone and Helmer (1954), Lamb (1945), Lavicka et al. (2007), Liepmann and Toshko (1957), Orlin et al. (1947), Otto and Orvold (1975), Oswatitsch (1956), Pal and Bose (1993, 1995), Preiswerk

(1940), Rani (1998), Rani and Wooldridege (2000), Rao et al. (1983), Rayleigh (1914), Rovenskaya (2013), Shams et al. (2009) and Shapiro (1954).

Table 1 The formulas for hydraulic analogy

2D compressible fluid flow	*Shallow water surface wave*
$\rho u A = \rho_r u_r A_r = \text{constant}$	$\rho_w h u A = \rho_w h_r u_r A_r$
$\frac{\rho}{\rho_r}\cdot\frac{u}{u_r}\cdot\frac{A}{A_r}=1$	$\frac{h}{h_r}\cdot\frac{u}{u_r}\cdot\frac{A}{A_r}=1$
$\rho u\frac{du}{dx}=-\frac{dP}{dx}=\frac{dP}{d\rho}\cdot\frac{d\rho}{dx}$	$\rho_w u\frac{du}{dx}=-\frac{dP}{dx}$
$\frac{dP}{d\rho}=a^2\rightarrow u\frac{du}{dx}=-\frac{a^2}{\rho}\cdot\frac{d\rho}{dx}$	Hydrostatoc equation $P=P_{atm}+\rho_w g(h-z)$
$\frac{P}{P_r}=\left(\frac{\rho}{\rho_r}\right)^{\gamma}=\left(\frac{T}{T_r}\right)^{\frac{\gamma}{\gamma-1}}$	$\rho_w u\frac{du}{dx}=-\rho_w g\frac{dh}{dx}$
Poisson's adiabatic $\frac{a^2}{a_\infty^2}=\frac{\gamma RT}{\gamma RT_\infty}=\frac{T}{T_\infty}$	$u\frac{du}{dx}=-\frac{gh\frac{dh}{dx}}{h}$
$\frac{u}{u_\infty}\cdot\frac{d\left(\frac{u}{u_\infty}\right)}{dx}=-\frac{a^2}{a_\infty^2}\cdot\frac{a_\infty^2}{u_\infty^2}\cdot\frac{\frac{d(\rho/\rho_\infty)}{dx}}{h/h_\infty}$	Speed of surface wave $c=\sqrt{gh}$ $\frac{u}{u_\infty}\cdot\frac{d\left(\frac{u}{u_\infty}\right)}{dx}=-\frac{gh}{gh_\infty}\cdot\frac{gh_\infty}{u_\infty^2}\cdot\frac{\frac{d(h/h_\infty)}{dx}}{h/h_\infty}$
$\frac{\rho}{\rho_\infty}=\frac{h}{h_\infty},\ \frac{a^2}{a_\infty^2}=\frac{gh}{gh_\infty}=\frac{T}{T_\infty}$ $\frac{P}{P_\infty}=\frac{\rho RT}{\rho_\infty RT_\infty}=\frac{\rho}{\rho_\infty}\cdot\frac{T}{T_\infty}=\left(\frac{h}{h_\infty}\right)^2\quad \gamma=2$	Mach number vs. Froud number $M_a^2=\frac{u_\infty^2}{a_\infty^2}\qquad F_r^2=\frac{u_\infty^2}{gh}$

There are two types of water tables; the model can be towed or the model is fixed in a stream of water. The latter case can be easily realised via an inclined plate, with water running on it, under the influence of gravity. This way a continuous test is possible. Results from both types will be reported here.

For a demonstration of hydraulic jumps, one can use the faucet in the kitchen sink as a source of a water jet. When the jet hits the flat surface, a thin layer of water is formed and a noticeable jump in the layer thickness is observed. This picture can be seen also in any water fountain. Students can easily run this experiment without the need of any apparatus. In Figure 1, a picture of the hydraulic jump in a kitchen sink is shown.

Most of the students are excited to see the analogy between hydraulic jumps and shock waves and the compare Froude numbers versus Mach number.

The water tables at UC Davis were built by undergraduate students from scratch, as part of a capstone design project. A course on compressible flow simulations based on hydraulic analogy and using water tables was offered for undergraduate students at UC Davis and they were able to run the experiments themselves to take pictures for flow patterns around different models.

In the following, the apparatus is described together with the models. Pointed airfoils were also manufactured by the students including biconvex and diamond shapes. Busemann biplane model as well as a sharp pointed concave body was also tested.

The results of these tests are shown in the figures followed by a discussion of the limitation of the analogy. Next, the alternative simple arrangement of the water table is described together with the results of tests using this apparatus.

As discussed in Part 1, the hydraulic analogy is valid for gases with specific heat ratio, $\gamma = 1.4$, however results can be calibrated using the concept of transonic similitude. Such a procedure is explained here in detail.

Next, a section on manufacturing the models is included, together with a discussion of the measurement techniques of the height of the water layer; see Carbonaro (1994), GArg (2012), Pal ad Bose (1993, 1995) and Rani (1998).

Finally, some concluding remarks about the educational use of water tables are mentioned.

2 Discussion of the results of the experiments

2.1 Description of the apparatus

The water table, see Figure 2, consists of a large rectangular tank made of acrylic, with rails of T-slotted framing running above the container. A motorised belt drive moves a car along the length of the rail. The models are attached to the car. The rectangular tank is filled with water with the desired height. The model moves in both directions. A controller is used to adjust the speed. Given the speed and the height of the water layer, the Froude number can be calculated.

2.2 Flow patterns around pointed airfoils

The flow pattern around biconvex airfoil is shown in Figure 3, and the corresponding pattern around a diamond shape in Figure 4. For comparison see Hatch (1949).

2.3 Flow pattern around Busemann biplane model

Figure 5 shows the flow pattern around a Busemann biplane model. Notice the reflected waves from the inside surface. For more details about Busemann biplane model see Ashley and Landhal and Liepmann and Roshks. For water table experiment, see von Karman Institute report (Carbonaro, 1994).

2.4 Flow pattern around pointed concave body

Figure 6 shows the flow pattern around a concave body. Notice the interaction of weak oblique shock originated at the leading edge and the bow shock generated from the curved surface. The latter can be explained as the merging of Mach lines with different inclinations. See Rayleigh (1914) and Thompson (1972).

2.5 Flow pattern around cylinders with and without spikes

Flow pattern around a cylinder with a bow shock, detached from the body, is shown in Figure 7.

The flow pattern around a cylinder with a sharp spike is shown in Figure 8, the case with not so sharp spike is shown in Figure 9, for comparison see Liepmann and Toshko (1957) and Orlin et al. (1947).

2.6 Flow patterns inside and outside convergent/divergent nozzles

In Figure 10, a normal shock is observed in the divergent part. Shocks outside the nozzle are shown for different conditions in Figure 11.

Performance of nozzles at design and off design conditions are discussed in Liepmann and Toshko (1957) and in Oswatitsch (1956). Results of water table experiments are shown in Carbonaro (1994).

2.7 Experiments using simple arrangement

A simple arrangement to do the experiment is shown in Figure 12.

The apparatus consists of a plastic container and flat plate. The model is fixed on the plate. Water is poured on the plate to generate a stream around the model. (A pump can be used to have a continuous operation where the water can be pumped from the lower container to another smaller container connected to the plate to generate a continuous stream of water around the fixed model).

Figure 1 Hydraulic jump forming from water stream dropping from a kitchen faucet (see online version for colours)

In Figure 13, the flow pattern around a diamond airfoil, using this simple arrangement, is shown. Notice both compression and expansion waves. Controlling the speed of the water stream and the height of the water layer allows the variation of the Froude number.

Of course, more sophisticated designs are possible. The point we want to make here that the students can build this apparatus, do the test and see the flow pattern with different model with minimal cost and effort. They can easily investigate several phenomena at their convenience without the need of supersonic wind tunnels!

Figure 2 The water table at UC Davis (see online version for colours)

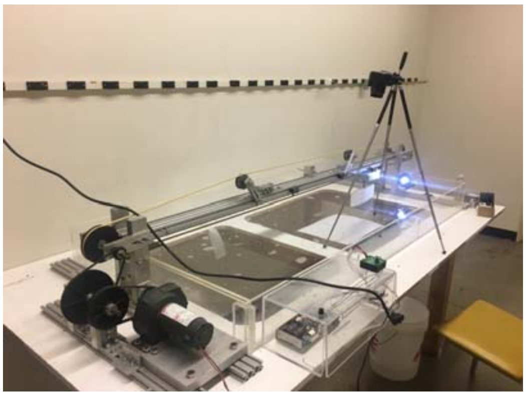


Figure 3 The symmetric biconvex model showing both leading edge and trailing edge shocks (see online version for colours)

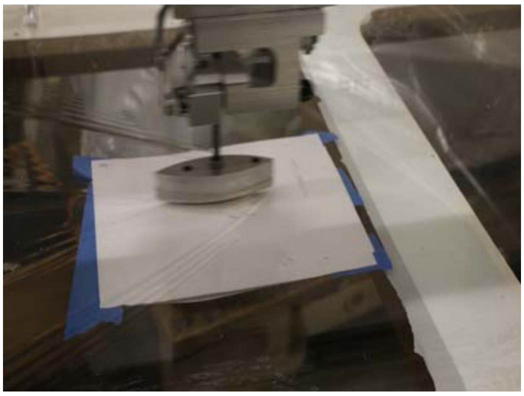


Figure 4 Flow around a diamond airfoil (wedge angle 15°) (see online version for colours)

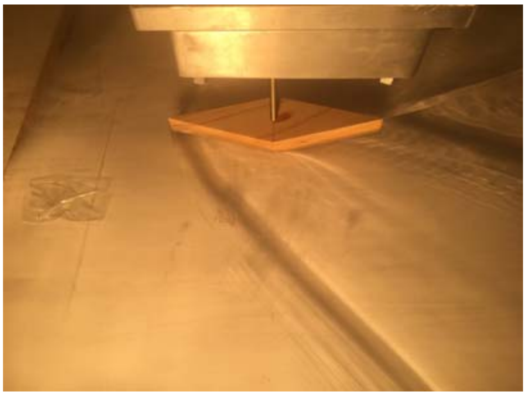


Figure 5 A picture of the Busemann's biplane with the hydraulic analogy (see online version for colours)

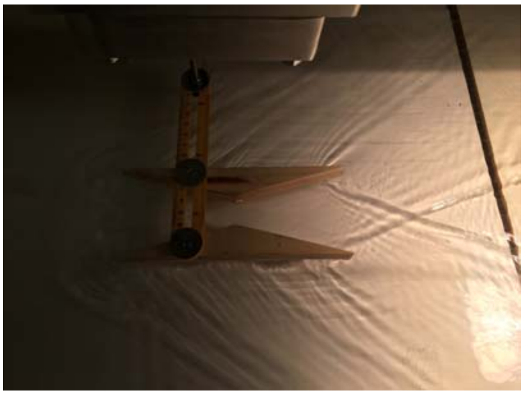


Figure 6 Shock formation around a concave body shape (see online version for colours)

Figure 7 A bow shock is formed in front of a moving cylinder (see online version for colours)

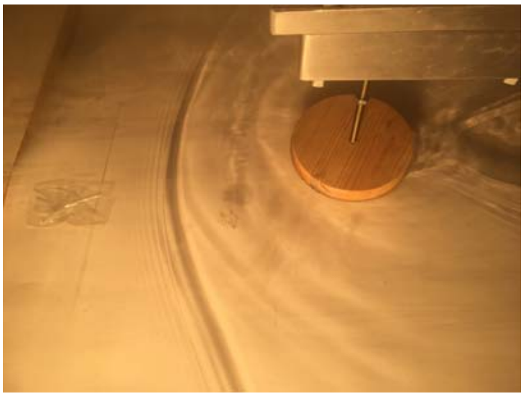


Figure 8 Flow over a cylinder with a sharp spike (see online version for colours)

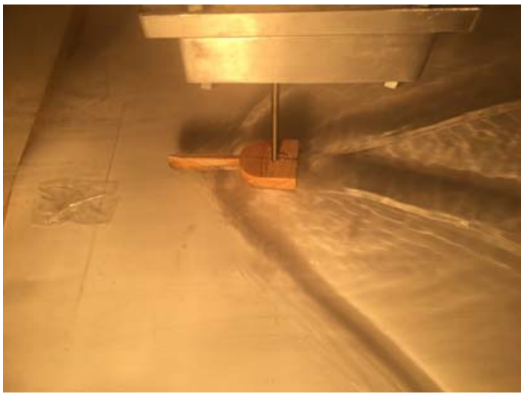


Figure 9 Shock pattern for flow around a cylinder with a spike (not so sharp) (see online version for colours)

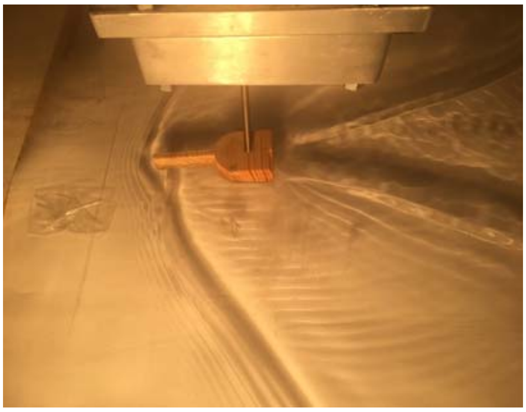
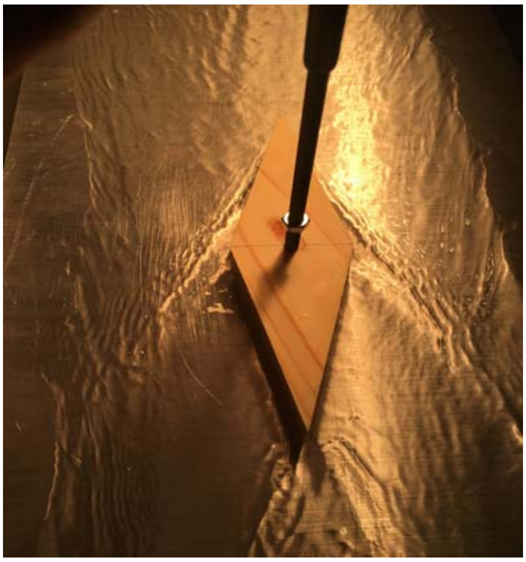


Figure 10 Shock formations inside the convergent/divergent nozzle (see online version for colours)

Figure 11 Flow pattern at the exit of the nozzle (see online version for colours)

Figure 12 A simple arrangements to demonstrate shallow water surface waves (see online version for colours)

Figure 13 Pictures for waves around a diamond airfoil on an inclined plate in a plastic container (see online version for colours)



2.8 Calibration of experimental results using transonic similitude

The transonic small disturbance equation can be written in terms of potential function, ϕ, in the form:

$$\left((1-M_\infty^2)-(\gamma+1)M_\infty^2\phi_x\right)+\phi_{xx}+\phi_{yy}=0$$

with the boundary condition at a solid surface as

$$\frac{\partial\phi}{\partial y}=\tau\frac{d\overline{y}}{dx}-\alpha$$

where τ is the thickness ratio and α is the angle of attack. Introducing scalded variables, the governing equation and boundary conditions can be written in a form admitting a similarity solution. Following Ashley and Landhal (1965), for example, let

$$\varphi=\varepsilon\overline{\varphi} \text{ and } y=\frac{\overline{y}}{\delta}.$$

Hence, in the nonlinear transonic regime, we must have

$$1-M_\infty^2=0(\varepsilon).$$

Moreover, let

$$\delta^2=M^2(\gamma+1)\varepsilon$$

Therefore, the scaled equation reads,

$$\left(K-\bar{\phi}_x\right)\bar{\phi}_{xx}+\bar{\phi}_{\bar{y}\bar{y}}=0.$$

where

$$K=\frac{1-M_\infty^2}{\varepsilon M_\infty^2(\gamma+1)}.$$

From the boundary condition, we have

$$\bar{\phi}_{\bar{y}}=\frac{\tau}{\varepsilon\delta}\left(\frac{d\bar{y}}{dx}-\frac{\alpha}{\tau}\right)$$

For,

$$\frac{\alpha}{\tau}=0(1).$$

$$\varepsilon=\left(\frac{\tau}{M_\infty}\right)^{\frac{2}{3}}(\gamma+1)^{-\frac{1}{3}}$$

and

$$K=\frac{1-M_\infty^2}{\left(\tau(\gamma+1)M_\infty^2\right)^{\frac{2}{3}}}.$$

Notice, the reduced pressure coefficient becomes:

$$\bar{C}_p=\frac{\left((\gamma+1)M_\infty^2\right)^{1/3}}{\tau^{2/3}}C_p.$$

Based on the above analysis, the reduced equation and boundary conditions depend only on K, the similarity parameter, which depends on a combination of Mach numbers, thickness ratio (or angle of attack) and γ. For K = constant $\frac{\Delta\tau}{\tau}$, $\frac{\Delta M^2}{M^2}$, $\frac{\Delta\gamma}{\gamma}$ are related, which means that an increment change of γ is related to incremental changes of τ and M_∞^2, to have the same phenomena.

So if the result of the experiment is analogous to compressible flow of $\gamma = 2$ the results corresponding to flow of $\gamma = 1.4$ can be obtained with an adjusted M_∞ and/or τ such that K remains the same. Similar procedure can be adopted to calculate the pressure coefficient such that the reduced pressure coefficient remains the same.

We like to mention that the students wrote their own computer code to simulate transonic flows based on the small disturbance equation, for different values of γ and verified the above calibration procedure. The details of the numerical solution are a subject of a separate publication.

2.9 Other tests

The water table can be used to simulate supersonic wind tunnel using double throat nozzles as well as supersonic intake and diffusers. Mach reflection is another application. The analogy is valid also for unsteady two dimensional flows.

Flows around cascades and turbine blades can be simulated using water tables.

Propagation of sound waves and problems of acoustics can be simulated as well.

Other applications in astrophysics, see Thompson (1972), are possible, for example, solar wind and bow socks around planets (assuming two dimensional models).

3 Airfoil and nozzle models

The airfoil and nozzle models in the water table experiment are prisms with a constant cross-section and a height that needs to exceed the water height and expected waves.

Surface roughness affects the fluid flow. For example, Wang et al. analysed the influence of roughness on discharge coefficient in critical flow Venturi nozzles and proved the importance of relative roughness Wang et al. (2014). Shams et al. (2009) showed for microchannels that roughness distribution and shape have a high influence on both compressible and incompressible flows. Rovenskaya (2013) found that the surface roughness of microchannels has an important influence on friction factor in compressible gas flow.

Feasible airfoil and nozzle model materials are aluminium, balsa wood, polyethylene, and construction paper. Aluminium nozzles can be manufactured by water-jet cutting or milling. Polyethylene material can also be water-jet cut, milled or laser-cut depending on thickness. Balsa wood models can be made by laser-cutting or cutting with a wood router table. The construction paper is folded and glued in place. Additive manufacturing, for example with a wire extrusion 3D printer, is an option to make nozzle models from polymers from CAD files. All materials and processing technologies have individual advantages and disadvantages with regard to costs, embodied energy, environmental impact, etc.

4 Water height measurement

The goal of the measurement system for a water table is twofold. First, to measure the change in water height for the wave caused by the airfoil model. And second, to measure the angle at which the wave forms off the side of the airfoil model. Measuring liquid height is possible through a number of measurement schemes; and most schemes are used to determine the volume of liquid present in a tank. Hambrice and Hopper (2004), identified twelve such methods. Most of these methods are either restricted to measurements of uniform fluid level, or easily exceed preventative costs on a low budget project. In conjunction, a majority of the methods described are only capable of measuring the height, and would not be able to measure the wave angle generated by the model.

A photographic measurement method is utilised to measure both the wave height and the wave angle, see Figure 14, under a low cost budget. A high watt lamp (500 W) is used to illuminate the measurement area. A microscope camera is used to capture the area of interest as seen on the computer in the figure.

Figure 14 Photographic measurement system (see online version for colours)

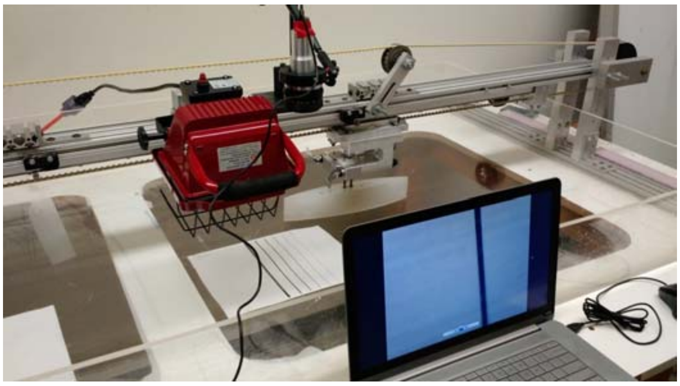
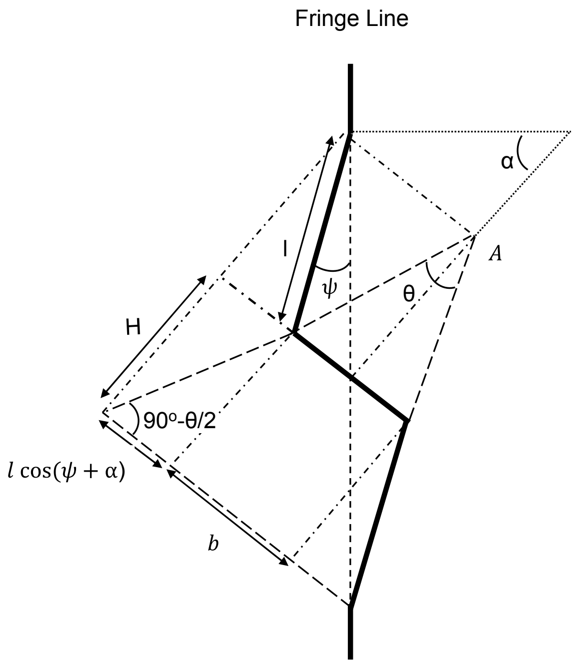


Figure 15 Wave measurement



Source: adapted from Rani and Wooldrige (2000)

A prismatic analogy is utilised to determine the water height. It is assumed that the wave generated from the model acts like a prism in function and shape (Rani and Wooldrige, 2000). The height of a prism can be determined from the deviation in a straight line (termed fringe line) caused by the prism. Thus, by measuring the deviation of a straight line caused by the water wave, the height of that wave can be calculate, as seen in

Figure 15. Both the round-top and flat-top prims models are utilised from Rani and Wooldrige (2000), the flat top model will be described here.

The prismatic analogous model for wave height calculates height (H) as a function of prism orientation angle (α), fringe deviation angle (ψ), the prism apex angle (θ), and the fringe deviation length (l). Assuming that the fringe line is orientated perpendicular to the airfoil travel direction, then the prism orientation angle (α) is equivalent to the wave angle.

The deviation angle of the prism can be found using equation (2), from (Rani and Wooldrige, 2000).

$$\psi = \sin^{-1}\left(f\sin(\alpha)\right) / \sqrt{f + \sec^2(a) - 2f} \tag{1}$$

Where f is a value defined using equation (3), from (Rani and Wooldrige, 2000). And μ is the prism refractive index of the material, a value of 1.330 is used for water (Born and Wolf, 2003).

$$f = \frac{\cot\left(\frac{\theta}{2}\right)}{\cot\left(\frac{\theta}{2}\right) + \tan\left(\left(\frac{\theta}{2}\right) + \sin^{-1}\left(\frac{\cot\left(\frac{\theta}{2}\right)}{\mu}\right)\right)} \tag{2}$$

Assuming the prism has a flat top, and the wave sides are symmetric and form an isosceles triangle if extended to a meeting point A, then the wave height (H) is calculated using equation (4) from (Rani and Wooldrige, 2000).

$$H = l \cdot \cos(\psi + \alpha) \cdot \cot\left(\frac{\theta}{2}\right) \tag{3}$$

5 Concluding remarks

Pictures of flow patterns around airfoils and in convergent/divergent nozzles confirm qualitatively the theory of hydraulic analogy. Attached oblique shocks and detached bow shocks are observed as well as expansion fans. Normal shocks in convergent/divergent nozzles and oblique shocks at the exit are also easily identified.

The results of these experiments can be obtained using simple arrangements by undergraduate students without special experiences. On the other hand, experiments are needed to obtain quantitative results.

Despite the limitations of the hydraulic analogy, water table experiments can be used to study compressible flows in schools and in industry. These inexpensive tools are useful to demonstrate many subtle nonlinear phenomena.

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