Passengers queue analysis in international airports terminals in Kerala using multiphase queuing system

Roshli Aniyeri*
Department of Mathematics,
MGGA College,
New Mahe, Thalassery,
Kerala 673311, India
Email: purerose2007@rediffmail.com
*Corresponding author

Ratnam Nadar
Noorul Islam University,
Kanyakumari District, India
Email: crnadar@gmail.com

Abstract: Waiting period is a global problem that almost everyone has to face, which causes a great waste of time for everyone. It is well known that all these waiting line problems critically restrict further development. The focus of this study is to deal with passengers’ queue issues of the international airport terminals of Kerala. Queuing theory is a mathematical approach to the study of waiting period in queues. This study evaluates the effectiveness of multi-server queuing model. The multi server approach of modelling was adopted in this cram to develop a mathematical model to solve problem of queuing of air transport passengers at the international airports in Kerala. The airport in the aviation industry of the country faces problems of many passengers queuing for boarding, departure with different arrival rate due to non availability of state of the art logistics management mechanisms for predicting the nature and service demands of travellers. The passengers’ average wait time for reaching the gate area measures system performance. A mathematical queuing model was developed in this study and comparisons are made using analysis of variance (ANOVA).

Keywords: multi server queuing modelling; airport passengers; queuing steady state condition; estimated queue length; utilisation factor; analysis of variance; ANOVA.


Biographical notes: Roshli Aniyeri is a PhD student in Queuing Theory at Noorul Islam University Kanyakumari, India. She holds a Master’s and a Bachelor’s degree in Mathematics at Calicut University. She has also done her MPhil in Mathematics at Bharathiyar University. Her research interests include queuing theory, simulation and Anova.

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1 Introduction

The Indian aviation industry is one of the fastest growing airline industries in the world. The history of Indian aviation industry started in December 1912 with its first domestic air route between Karachi and Delhi. The airport is the gateway of any country. A tree cannot make a forest, so says an adage. Thus the economy of a nation depends on some sectors like the agricultural sector, the health sector, the works and housing sector and most importantly, the airport, banking sector among others. All these sectors work together and integrate to make the life of a nation. A breakdown in any of these sectors will definitely have untold effects on the progress and success of the nation. The airline industry is important for the global economy. Airports, in particular, hub airports are the backbone of air transportation. Transportation plays a vital role in the changing global economy, linking people and places, facilitating trade and tourism, and encouraging economics competition and specialisation.

1.1 Problem definition

There are three international airports in Kerala. Therefore it is decided to collect data from three international airports in Kerala for this research and compare it. The main purpose of this paper is to review the application of queuing theory and to evaluate the parameters involved in the service unit of the airport terminals in Kerala. The broad objective of this study is to gain an understanding of the application of queuing theory to the problem of airport terminals in Kerala.

The ultimate aim is to build models of better quality and to understand the key concepts and to propose to develop a queuing model of the airports.

1.2 Problem statement

This study deals with the problem of finding the airport congestion problem in international airports in Kerala. For a better understanding of airport capacity and congestion problems, queuing theory gives great insights. As de Newville and Odoni (2003, pp.819–863) point out, all airport facilities can be described as systems of queuing systems, where arrivals at a service facility are randomly distributed, waiting lines form and users are therefore delayed and have to wait before being served. Queues develop at airports in numerous contexts (ticketing, baggage handling). The airport queuing system has several characteristics which make it unfit for most traditional analysis:

1. time variation in arrival rate
2. dependence of service times on weather
3. passenger dependence.
This study will focus on the M/M/C/∞ queuing system assume a Poisson arrival process. This assumption is a very good approximation for arrival process in real systems that meet the following rules:

1. the number of passengers in the system is very large
2. all passengers are independent, i.e., their decisions to use the system are independent of other users.

This probability density distribution equation for a Poisson process describes the probability of seeing \( n \) arrivals in a period from 0 to \( t \).

\[
P_n(t) = \frac{\left( \lambda t \right)^n}{n!} e^{-\lambda t}
\]

where

- \( t \) is used to define the interval 0 to \( t \)
- \( n \) is the total number of arrivals in the interval 0 to \( t \)
- \( \lambda \) is the total average arrival rate in arrivals/sec.

1.3 Parameters in queuing models (multiple servers, multiple queues model)

- \( n \) number of total passengers in the system (in queue plus in service)
- \( c \) number of parallel servers
- \( \lambda \) arrival rate (1/(average number of passengers arriving in each queue in a system))
- \( \mu \) serving rate (1/(average number of passengers being served at a server per hour))
- \( c \mu \) serving rate when \( c > 1 \) in a system
- \( \rho \) system intensity or load, utilisation factor ( = \( \frac{\lambda}{c \mu} \)) (the expected factor of time the server is busy that is, service capability being utilised on the average arriving passengers).

Departure and arrival rate are state dependent and are in steady-state (equilibrium between events) condition.

1.3.1 Objective of the study

In most of the airports, the major problem is waiting of passengers in the queue for more deviation. This study analyse the queuing problem using alternative approach of the multiserver method in the international departure of passengers in international airports in Kerala. There are three international airports in Kerala namely Trivandrum, Kochi, and Kozhikode. This study aims to study the existing queue pattern in these airports and to suggest if any possible alternatives. Therefore to main objectives of this study are

1. to study the existing queue pattern of passengers in these airports
2. to identify the suitable queuing model for this state of art to fit a model
to develop an integrated queuing model that models passenger flow in the airport terminal

to develop models with capability to determine resource utilisation at a high level of detail.

2 Literature review

This literature review reveals various decision support techniques for the analysis of an airport. The motivation for this study is to determine an appropriate model to solve the queuing problem in international airports in Kerala. Numerous studies and work have been done on the subject of airport. One of the studies carried out a simulation study of the passenger check-in system at Ottawa International Airport. They fixed their attention at the daily level and used simulation to determine: the minimal number of counters in order to meet a service level for each separate flight in terms of waiting times. Next, they provided some integer programming formulations to minimise the total number of counters and the total number of counter hours under the realistic constraint that counters for one and the same flight should be adjacent. Another studied specifically the case of the kiosks, or automated self-service check-in machines, and analysis has been done for the improving of service quality of self-service kiosks, but also assisted the industry in developing a SUSS (common-use self-service) standard that would enable airlines to share kiosks. de Barros et al. (2007) analysed transfer passengers’ views on the quality of services at the terminal building, using data collected at Bandaranaike International Airport in Sri Lanka. In which regression analysis was used to identify the transfer passenger facilities and services with the strongest effect on the overall perception of level of service.

Another study about the check-in process of the airport terminal has been modelled and examined. The goal of their study was to determine delays and solutions to improve efficiency. A great number of security measures were adopted to avoid new terrorist actions in airport terminals. Without proper security measures, people could consider the air transportation system as unsafe and could refrain from travelling by aircraft. The state of art overview highlights different research works in modelling airport terminal operations. There are so many studies about Airport terminal using different method. One study is about simulation model to analyse passenger and baggage flow in an airport terminal. Another analyse the bottlenecks in the passengers flow and provide integral solutions for supporting future airport developments. Another study is about baggage screening strategies using artificial intelligence techniques. Again, another analyse the problems related to the design and analysis of security screening and inspection system. They introduces a new baggage-tracking system for improving airport security. In this paper the authors propose a simulation model of the airport of Lamezia Terme (Calabria, Italy) for investigating system performance under the effects of different scenarios characterised by different resources allocation and availability. Many researchers have faced issues concerning the optimisation of an airport terminal The Nagoya University has conducted a simulation using the software Arena on departing flow passengers from the International Kansai airport in Japan, in order to reduce the number of passengers, because of long waiting times in peak periods and because of unavoidable delays, they lose their flights. NAIA, Abuja-Nnamdi Azikiwe International used the death and birth
rate approach to model to the waiting line at the airport and reported that more aircrafts are needed on daily basis both on the domestic and international routes for improved service. This study is a continuation of this past work aimed at developing a queuing model using alternative approach of the multi-server method to facilitate the prediction, processing of passengers at the airports in Kerala per operational period for effectiveness. There are so many studies in the literature analysing the problem of congestion in airport terminal. Guizzi et al. (2009) in their paper studied the analysis of passenger flow in the terminal airport, from entrance to boarding. In particular this study develops a simulation model based on the discrete event theory. A related study can be found in Thiagaraj (2014) in his paper about the analytical approach and shows how to build a simulation procedure. This study is to measure the performance of an airport runway used only for arrivals, with different traffic mixes and operational variables. Ashfaq (2006) in his paper studied analysis of the data at Birmingham Airport to provide effective and useful information about the dwell-time that passengers have between different points of their visit to the airport. The method he uses a genetic paradigm which is able to process that data using a compact and robust simulation model, so that the time spent by the visitors to the airport can be extracted from the raw data produced by the sensors. Barnhart (2003) in his paper studied several important areas of operations research applications in the air transport industry. Mehri et al. (2008) in their paper studied solving of waiting lines models in the airport using queuing theory model and linear programming. The model about airport for passengers on a level with reservation is the multiple-channel queuing model with Poisson arrival and exponential service times (M/M/S). Total expected costs are studied, total costs is the sum of the cost of providing service plus the cost of waiting time. Esteban and Pedro (2008) in their paper ‘Check in process at Lisbon Airport’ used two methods are implemented, event-based simulations and collaborative design.

2.1 Methodology

2.1.1 Data collection and sampling

There may be many possible extensions to this study. Therefore the data quality is crucial and important for this study. A better and larger set of data should be obtained from international airport in Kerala. Furthermore, an airport performance is crucial for applying this technique. Thus, to determine more appropriate airport performance variables like arrival of passengers and services in terminals using the different definitions and derivations of the variables. By using the key variables selected from the proposed methodologies, this paper reviews a queuing model for multiple servers. The average queue length can be estimated simply from raw data collected through the questionnaires, the number of passengers waiting in a queue in each minute. Then compare this average with that of queuing model.

In order to compare the deviations,

- The data was collected directly from three airports in Kerala, India for a period of one week by using the predetermined questionnaire.
- The observations were made for the number of passengers in a queue, their arrival-time and departure-time without distracting the employees.
The whole procedure of the service unit each day was observed and recorded using a time-watch during the same time period for each day. In these airport terminals there was no balk or renge.

2.1.2 Data analysis

The following assumptions were made for the analysis of the data.

1. The waiting line has two or more identical servers.
2. The arrivals follow a poison probability distribution with a mean arrivals rate of $\lambda$.
3. The service times follow an exponential probability distribution. The mean service rate, $\mu$, is the same for each server.
4. The arrivals wait in a single line and then move to the first open server for service in an orderly manner.
5. The queue discipline is first-come-first-serve (FCFS).
6. As the traffic intensity $\rho = \frac{\lambda}{c\mu}$ gets closer to one, there is a very rapid increase in the measures of congestion, $L_q$ (expected number of passengers in the queue) and $W_q$ (expected time a passenger in the system) gets closer to one.

The multi server approach of modelling was adopted in this study to develop a mathematical model to solve problem of queuing of air transport passengers at the international airports in Kerala. The elementary probability theory to predict average waiting time, average queue length, distribution of queue length, etc., on the basis of ‘arrival pattern of passengers’ to the resource, service pattern of the passengers’.

2.2 Components of queue models

Queues are not an unfamiliar phenomenon and to define it requires specification of the characteristics which describes the system such as the arrival pattern, the service pattern, the queue discipline and the queue capacity, Adedayo (2006) observed that there are many queuing models that can be formulated. According to them it is essential that the appropriate queuing model is used to analyse problems under study.

- **The arrival pattern**: This may be the arrival of an entity at a service point. This process involves a degree of uncertainty concerning the exact arrival times and the number of entities arriving. And to describe this process there are some important attributes such as the sources of the arrivals, the size of each arrivals, the grouping of such an arrival and the inter-arrival times.

- **The service pattern**: This may be any kind of service operation which processes the arriving entities. The major features which must be specified are the number of servers and the duration of the service.

- **The queue discipline**: This defines the rules of how the arrivals behave before service occurs.

- **The queue capacity**: The queue capacity may be finite or infinite.
2.3 Queuing model of international airports in Kerala

The system of operation at international airport terminals in Kerala can be modeled as a queuing process. There are three international airports in Kerala that have to collect data from three international airports in Kerala by questionnaire form for this research and compare it. An attempt is made in this study to find solution to the queue of airport terminal congestion in Kerala. The problem can be modeled as a multi-server queue problem with no system limit, arrival can be from a theoretically infinite source and the service is on first-come-first-serve priority rule.

2.4 The arrival process of passengers

The passengers arrival process may be described in two ways

- by characterising the number of arrivals per units time (the arrival rate)
- by characterising the time between the successive arrivals (inter arrival time).

If \( \lambda \) is defined as the no: of arrivals per unit time then \( 1/\lambda \) will be the successive arrival. If \( \mu \) is the rate of service then \( 1/\mu \) is the service time.

The average rate of passengers’ entering the queuing system as \( \lambda \) and the average rate of serving passengers’ as \( \mu \) a measure of traffic congestion for \( s \) server system is \( \rho = \lambda s / \mu \). When \( \rho >1 \), the average no: of arrivals in to the system exceeds the maximum average service rate of the system. To attain a steady state system \( \rho \) must less than one. When \( \rho = 1 \) arrivals and service are deterministic therefore the average arrival rate and average service rate guaranteed a steady state solution only if \( \rho < 1 \).

Our first goal is to study the property of a model of the arrival process to a system and to compare its features to the Poisson process.

In determining whether the Poisson process is a reasonable model for arrivals in a specific service system, it is useful to consider its three defining properties:

1. passengers arrive one at a time
2. the probability that a passengers arrives at any time is independent of when other passengers’ arrive
3. the probability that a passengers arrives at a given time is independent of the time.

The assumption of a Poisson process will generally be a good one when the three properties above are a reasonable description of the service system. However, it is possible to perform more rigorous tests to determine if it is a good fit. The simplest tests are based on the relationship of the standard deviation to the mean of the two distributions involved in the Poisson process. Since the variance (square of the standard deviation) of the Poisson distribution is equal to its mean, now examine the number of arrivals in each fixed interval of time, and determine whether the ratio of the mean to the variance is close to one. Alternatively, since the exponential distribution is characterised by its standard deviation being equal to its mean, then look at the inter-arrival times and compute the ratio of the standard deviation to the mean to see if it’s close to one.

The events occur successively in time, so that the intervals between successive events are independently and identically distributed according to an exponential distribution.
Consider an arrival counting process \{N(t) \mid t \geq 0\} denotes the total number of arrivals up to time \(t\), with \(N(0) = 0\) and which satisfies the following assumptions:

1. The probability that an arrival occurs between time \(t\) and time \(t + \Delta t\) is equal to \(\lambda \Delta t + o(\Delta t)\). We write this \(\Pr(\text{arrival occurs between } t \text{ and } t + \Delta t)\) where \(\lambda\) is a constant independent of \(N(t)\), \(\Delta t\) is an incremental element, and \(o(\Delta t)\) denotes a quantity that becomes negligible when compared to \(\Delta t\) as \(\Delta t \to 0\) that is
   \[
   \lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0.
   \]

2. \(\Pr(\text{more than one arrival between } t \text{ and } t + \Delta t) = o(\Delta t)\).

3. The number of arrivals in non-overlapping intervals is statistically independent; i.e., the process has independent increments.

Now to calculate \(P_n(t)\), the probability of \(n\) arrivals in a time interval of length \(t\), \(n\) being an integer \(\geq 0\). For \(n \geq 1\) we have

\[
P_n(t + \Delta t) = \Pr(n \text{ arrivals in } t \text{ and none in } \Delta t)
+ \Pr(n - 1 \text{ arrivals in } t \text{ and one in } \Delta t)
+ \Pr(n - 2 \text{ arrivals in } t \text{ and two in } \Delta t)
+ \ldots \ldots \ldots
+ \Pr(\text{no arrivals in } t \text{ and } n \text{ in } \Delta t)
\]

Using Assumptions 1 to 3 and equation (1) becomes

\[
P_n(t + \Delta t) = P_n(t)\left[1 - \lambda \Delta t - o(\Delta t)\right] + P_{n-1}(t)\left[\lambda \Delta t + o(\Delta t)\right] + o(\Delta t)
\]

where the last term, \(o(\Delta t)\), represents the term \(\Pr\{n - j \text{ arrivals in } t \text{ and } j \text{ in } (\Delta t); 2 \leq j \leq n\}\).

When \(n = 0\), we have

\[
P_0(t + \Delta t) = P_0(t) = \left[1 - \lambda \Delta t - o(\Delta t)\right]
\]

Rewriting equations (2) and (3) and combining all \(o(\Delta t)\) terms, we have

\[
P_n(t + \Delta t) - P_n(t) = -\lambda \Delta t P_n(t) + o(\Delta t)
\]

And

\[
P_n(t + \Delta t) - P_n(t) = -\lambda \Delta t P_n(t) + \lambda \Delta t P_{n-1}(t) + o(\Delta t) \quad (n \geq 1)
\]

Dividing equations (4) and (5) by \(\Delta t\), and take the limit as \(\Delta t \to 0\), and obtain the differential – difference equation

\[
\lim_{\Delta t \to 0} \left[\frac{P_n(t + \Delta t) - P_n(t)}{\Delta t}\right] = \lim_{\Delta t \to 0} -\lambda P_n(t) + \lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t}
\]

\[
\lim_{\Delta t \to 0} \left[\frac{P_n(t + \Delta t) - P_n(t)}{\Delta t}\right] = \lim_{\Delta t \to 0} \left\{-\lambda P_n(t) + \lambda P_{n-1}(t)\right\} + \lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t}
\]
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This reduces to
\[
\frac{dP_0(t)}{dt} = -\lambda P_0(t) 
\]
(6)

And
\[
\frac{dP_n(t)}{dt} = -\lambda P_n(t) + \lambda P_{n-1}(t) \quad (n \geq 1) 
\]
(7)

Equation (6) clearly has the general solution \( P_0(t) = Ce^{-\lambda t} \), where the constant \( S \) is equal to one, since \( P_0(0) = 1 \) let \( n = 1 \) in equation (7) then
\[
\frac{dP_1(t)}{dt} = -\lambda P_1(t) + \lambda P_0(t) 
\]

Or
\[
\frac{dP_1(t)}{dt} + \lambda P_1(t) = \lambda P_0(t) = \lambda e^{-\lambda t} 
\]
The solution to this equation is given by
\[
P_1(t) = Ce^{-\lambda t} + \lambda te^{-\lambda t} 
\]

Using the boundary condition \( P_n(0) = 0 \) for all \( n > 0 \) yields \( S = 0 \) and gives
\[
P_1(t) = \lambda te^{-\lambda t} 
\]
Continuing sequentially to \( n = 2, 3, 4, \ldots \) in equation (7) and proceeding similarly we get
\[
P_2(t) = \frac{(\lambda t)^2}{2} e^{-\lambda t}, \quad P_3(t) = \frac{(\lambda t)^3}{3!} e^{-\lambda t} 
\]
and in general
\[
P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t} 
\]
This is the formula for a Poisson distribution probability distribution with mean \( \lambda t \). Thus the independently identically distributed random variable defined as the number of arrivals to a queuing system by time \( t \), this random variable has the Poisson distribution with mean \( \lambda t \).

In order to show that if the arrival process is Poisson, an associated random variable defined as the time between successive arrivals follows exponential distribution. Let \( T \) be the random variable ‘time between successive arrivals’; then
\[
Pr(T \geq t) = Pr(\text{no arrivals in time } t) = P_0(t) = e^{-\lambda t} 
\]
Therefore the cumulative distribution function of \( T \) can be written as
\[
A(t) = P(T \leq t) = 1 - e^{-\lambda t} 
\]
With corresponding density function

$$a(t) = \frac{dA(t)}{dt} = \lambda e^{-\lambda t}$$

Thus $T$ has the exponential distribution with mean $1/\lambda$ if the inter arrival time are independent and have the same exponential distribution then the arrival follows the Poisson distribution. So in airport terminal the arrival of passengers are independent. Therefore, the arrival follows the Poisson distribution and service rate are exponentially distributed.

There are $s$ parallel service channels and a passengers can go to any of the free counter for his service, where the service time is identical and have the same probability density function $s(t)$ with mean service rate $\mu$ per unit of time per busy server. Thus, overall service rate when there are $n$ units in the system is given by:

1. If $n \leq s$, all the passengers’ may be served simultaneously and in such cases there will be no queue. ($s-n$) servers may remain idle and then mean service rate is $n\mu$, for $n = 0, 1, \ldots, s-1, s$.
2. If $n \geq s$, all the servers will remain busy, number of passengers’ waiting in the queue will be $(n-s)$ and then mean service rate is $s\mu$, i.e., the mean service time $\mu_n$ is given by

The mean service time $\mu_n$ is given time by

$$\mu_n = \begin{cases} 
n\mu & \text{if } 0 \leq n < s \\
\frac{s\mu}{n} & \text{if } n \geq s
\end{cases}$$

### 2.5 Performance measures – measures of effectiveness

When analysis a queuing system, this study craving to find
- number of passengers in the system
- waiting time for a passenger
- the length of a busy or idle period
- the current work backload (in units of time).

These are called measures of effectiveness. They are all random variable.

### 2.6 Number of passengers

Let $N$ denote the random variable that describes the number of passengers in the system at steady state. The probability that at steady state the number of passengers’ present in the system is $n$ denoted by $P_n$

$$P_n = \text{Prob}[N = n]$$

And average number in the system at steady state is
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\[ L = E[N] = \sum_{n=0}^{\infty} np_n \]

Within the queuing system, passenger may be present in the queue waiting for the turn to receive service or they may be receiving service. Let \( N_q \) be the random variables to describe the number of passengers’ waiting in the queue and its mean is denoted by \( L_q = E[N_q] \).

2.7 System time and queuing time

The time the passenger spends in the systems, from the instant of its arrival to the queue to the instant of its departure from the server, is called the response time or sojourn time. Let \( R \) denote the random variable that describes response time and its mean by \( E[R] \).

The response time composed of time the passenger spends waiting in the queue, called the waiting time, plus the passengers spends receiving service, called the service time. Let \( W_q \) be the random variables to describe the time the passengers spends waiting in the queue and its mean is denoted by \( E[W_q] \).

This study analyses the application of queuing theory in international airports passenger’s departures in the international airports in Kerala. In the airport terminals, there are several servers in several queues. Therefore the apt queuing model is the multiple servers in multiple queues of infinite capacity. Thus this study is a queuing models of (M/M/S): (∞/FIFO).

This model deals with a queuing system having several servers arranged in parallel, each of which has identically and independently distributed and service time is distributed exponentially. The arrivals are assumed to follow Poisson distribution. The system is assumed to have an infinite capacity and the passengers’ are served as first in first out basis.

Hence there are ‘S’ number of servers each with a mean service rate of \( \mu \) and let the mean arrival rate of passengers be \( \lambda \).

The steady state probabilities for a Poisson queue system are given by

\[ P_n = \frac{\lambda_0 \lambda_1 \lambda_2 \ldots \ldots \lambda_{n-1}}{\mu_1 \mu_2 \mu_3 \ldots \ldots \mu_n} P_0 \text{ where } n \geq 1 \quad \rightarrow 1 \]

Using the fact that

\[ \sum_{n=0}^{\infty} P_n = 1 \]

We can write

\[ P_0 + \sum_{n=1}^{\infty} P_n = 1 \]

Therefore
If there is a single server, the mean service rate $\mu_n = \mu$ for all $n$. But for the given model, there are $s$ servers working independently of each other. When there are $n$ passengers’ in the system, the mean service rate, $\mu_n$, can be calculated in two different situations:

1. If $n < s$, only $n$ of the $s$ servers will be busy and others will be idle. Hence, the mean service rate will be $n\mu$.

2. If $n \geq s$, all the servers will be busy. Hence, the mean service rate will be $s\mu$.

Thus, the following can be assumed.

1. the mean arrival time $\lambda_n = \lambda$ for all $n$

2. the mean service time $\mu_n$ is given by

$$
\mu_n = \begin{cases} 
  n\mu & \text{if } 0 \leq n < s \\
  s\mu & \text{if } n \geq s 
\end{cases}
$$

3. the mean arrival rate is less than $s\mu$, i.e., $\lambda < s\mu$

If $0 \leq n \leq s$, then substituting equation (3) in equation (1) we get

$$
P_0 = \frac{\lambda}{1.\mu.2.\mu.3.\mu\ldots\ldots n\mu} P_0 = \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n P_0
$$

If $n \geq s$, then substituting equation (3) in equation (1) we get

$$
P_0 = \frac{\lambda^n}{(s-1)!!.\mu^{s-1}(s\mu)s-s+1 (n-s+1) \times \ldots \times n} P_0 = \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^n P_0
$$

To find the value of $P_0$ we use the fact that

$$
\sum_{n=0}^{\infty} P_n = 1
$$

i.e.,

$$
\left[ \sum_{n=0}^{s-1} \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n \right] + \sum_{n=s}^{\infty} \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^n P_0 = 1
$$
i.e.,
\[
\left[ \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n + \sum_{n=1}^{\infty} \frac{s^n}{s!} \left( \frac{\lambda}{s \mu} \right)^n \right] P_0 = 1
\]
i.e.,
\[
\left[ \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n + \sum_{n=1}^{\infty} \frac{s^n}{s!} \left( \frac{\lambda}{s \mu} \right)^n \right] \frac{1}{1 - \frac{\lambda}{\mu s}} P_0 = 1
\]

using the Assumption 3 that \( \frac{\lambda}{s \mu} < 1 \) and the expansion is given by
\[
\sum_{n=0}^{\infty} \left( \frac{\lambda}{s \mu} \right)^n = \left( \frac{\lambda}{s \mu} \right) \left[ 1 + \frac{\lambda}{\mu s} + \left( \frac{\lambda}{\mu s} \right)^2 + \ldots \right] = \left( \frac{\lambda}{s \mu} \right) \frac{1}{1 - \frac{\lambda}{\mu s}}
\]

Thus it follows that
\[
\left[ \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n + \frac{1}{s!} \left( \frac{\lambda}{s \mu} \right)^s \right] P_0 = 1
\]

Hence
\[
P_0 = \frac{1}{\sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n + \frac{1}{s!} \left( \frac{\lambda}{s \mu} \right)^s} \rightarrow 6
\]

3 Characteristics of the model

Expected number of passengers’ in the queue or queue length \( (L_q) \). This model has a total of \( s \) servers. The expected queue length is given by
\[
L_q = E(N_q) = E(N - s) = \sum_{n=s}^{\infty} (n - s)P_n = \sum_{x=0}^{\infty} xP_{n+s}, \text{ putting } x = n - s
\]

Thus we have
\[
L_q = \sum_{x=0}^{\infty} x \frac{s^{x+1}}{s! s^x} \left( \frac{\lambda}{s \mu} \right)^x \quad P_0 = \frac{1}{s!} \left( \frac{\lambda}{s \mu} \right)^s \quad P_0 = \sum_{x=1}^{\infty} \frac{x}{s} \left( \frac{\lambda}{s \mu} \right)^x
\]
it follows that

\[ L_q = \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^x P_0 \frac{\lambda}{\mu s} \left( 1 - \frac{\lambda}{\mu s} \right)^2 \]

Using the fact that

\[ \sum_{x=1}^{\infty} \frac{\lambda^x}{x! s \mu} = \frac{\lambda}{s \mu} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{x! s \mu} = \frac{\lambda}{s \mu} \left[ 1 - \frac{\lambda}{s \mu} \right]^{-2} \]

Hence we deduce that

\[ L_q = \frac{1}{s s!} \left( \frac{\lambda}{\mu} \right)^{x+1} P_0 \]

3.1 Average or expected number of passengers’ in the system \( (L_s) \)

By Little’s formula we have

\[ L_s = L_q + \frac{\lambda}{\mu} = \frac{1}{s s!} \left( \frac{\lambda}{\mu} \right)^{x+1} P_0 + \frac{\lambda}{\mu} \]

where

\[ L_s = E[N] = \sum_{n=0}^{\infty} n P_n \]

3.2 Average waiting time of a passenger in the system \( (W_s) \) By Little’s formula

\[ W_s = \frac{L_s}{\lambda} = \frac{1}{\lambda} + \frac{1}{s s!} \left( \frac{\lambda}{\mu} \right)^{x+1} P_0 + \frac{1}{\mu} \]

i.e.,
3.3 The average waiting time of a passenger in the queue \( W_q \)

By Little’s formula

\[
W_q = \frac{L_q}{\lambda} = 1 + \frac{1}{\lambda} \frac{\left( \frac{\lambda}{\mu} \right)^{s+1}}{s!} P_0
\]

\[
W_q = \frac{1}{\mu \lambda} \frac{\left( \frac{\lambda}{\mu} \right)^{s+1}}{s!} P_0
\]

3.4 The probability that an arrival has to wait for service

The arrival has to wait for service if and only if \( T_s > 0 \) where \( T_s \) denote the waiting time of a passenger in the system, i.e., if and only if there are \( s \) or more passengers’ in the system. Thus the required probability is equal to

\[
P(T_s > 0) = P(N \geq s) = \sum_{n=s}^{\infty} P_n
\]

\[
= \sum_{n=s}^{\infty} \frac{1}{s! \lambda^{n-s}} \left( \frac{\lambda}{\mu} \right)^n P_0 = \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^n P_0 \sum_{n=s}^{\infty} \left( \frac{\lambda}{\mu s} \right)^{n-s}
\]

i.e.,

\[
P(T_s > 0) = P(N \geq s) = \frac{\left( \frac{\lambda}{\mu} \right)^s P_0}{s! \left( 1 - \frac{\lambda}{\mu s} \right)}
\]
3.5 The probability that an arrival enters the service without waiting

\[
1 - P(T_s > 0) = 1 - \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!} P_0 \left(1 - \frac{\lambda}{\mu s}\right)
\]

3.6 The mean waiting time in the queue for those who need to wait

\[
E[T_q | T_s > 0] = \frac{E[T_s]}{P[T_s > 0]}
= \frac{W_q}{P[T_s > 0]}
\]

Substituting the values of \( W_q \) and \( P[T_s > 0] \) in the above equation we get

\[
E[T_q | T_s > 0] = \frac{1}{\mu ss!} \left(\frac{\lambda}{\mu}\right)^s P_0 \times \left(1 - \frac{\lambda}{\mu s}\right)
\]

Simplifying we get

\[
E[T_q | T_s > 0] = \frac{1}{\mu s (1 - \frac{\lambda}{\mu s})}
= \frac{1}{\mu s - \lambda}
\]

3.7 The average or expected number of passengers’ in non empty queues (\( L_n \))

If \( N \) denote the number of passengers’ in the system and \( N_q \) denote the number of passengers’ in the queue, then \( L_n \) is the conditional expectation defined by

\[
L_n = E[N_q | N_q \geq 1]
= \frac{E[N_q]}{P[N \geq s]}
= \frac{L_q}{P[N \geq s]}
\]
Since there are $s$ servers, substituting the values of $L_q$ and $P[N \geq s]$ we get

$$L_o = \frac{1}{s} \left( \frac{\lambda}{\mu} \right)^{s+1} \frac{P_0}{1 - \frac{\lambda}{s\mu}} = \frac{s! \left( 1 - \frac{\lambda}{s\mu} \right)}{\left( \frac{\lambda}{\mu} \right)^{s+1}} \frac{P_0}{1 - \frac{\lambda}{s\mu}}$$

Now applying the above multiserver queuing formula from the data collected from the three airports in Kerala (see the Appendix).

4 Data collection at international airports in Kerala

- **Data collection at Kozhikode:** The path related to passengers departing from Terminal 1 consists of two floors: on the ground floor we have the check-in desks, and on the first floor, assessable by escalators, the security control facilities immigration, security and customs and some shops and food services are located. On the other side of the ground floor besides the path related to arriving passengers there are the ticket office, the two currency exchange offices, several car rentals, and so forth. Simulation studies require an exact description of processes and representative data. Therefore correct data is essential to get valid and valuable results about bottlenecks and to define relevant scenarios.

- **Data collection at Kochi:** The country’s first green field airport in the public private partnership model has integrated some unique and standout features in to its terminal design that marks the biggest expansion program since the airport turned operational in 1999. The present international terminal with a built-up space of 4.75 lakh sq.ft has a peak hour capacity to handle only 1,200 passengers each on the arrival and departure sides. The terminal’s elevation has been modelled along the State’s traditional and aesthetically-rich temple architecture. It would be a multi-level terminal complex with the ground level to feature arrivals, and departures in the first level.

- **Data collection at Trivandrum:** The path related to passengers departing from Terminal 1 consists of two floors: on the ground floor we have the check-in desks, and on the first floor, assessable by escalators, the security control facilities immigration, security and customs and some shops and food services are located. On the other side of the ground floor besides the path related to arriving passengers there are the ticket office, the two currency exchange offices, several car rentals, and so forth. Simulation studies require an exact description of processes and representative data. Therefore correct data is essential to get valid and valuable results about bottlenecks and to define relevant scenarios.
5 Data analysis and computation at international airports in Kerala

The most problematic phase of this study has been that on data acquisition. Sampling took place in the passenger terminal Karipur International airport. After a thorough inspection airport it came out that passenger traffic peaks during Wednesday. Therefore the focus data collection phase was on three days: Monday, Wednesday and Friday. Data was grouped in the spreadsheets that show some information such as basis information flight (the airline and destination), time of departure, time of opening and closing its check-in and finally the flight code. This information is important because from the flight code it is possible to know the type of the aircraft used for that flight and then the total number of aircraft available seats. In order to carry out a correct analysis of collected data it was necessary to make an appropriate data stratification for highlighting some key aspects. By observation the following data was collect at exactly 7.00 am to 1.00 pm on daily basis. Table shows the arrival, inter-arrival, service for a certain group of random passengers that used the facilities in airport terminal as at the time of observation and data collection.

Sampling took place in the passenger terminal Kochi International Airport. After a thorough inspection airport it came out that passenger traffic peaks during midnight. Data was grouped in the spreadsheets that show some information such as basis information flight (the airline and destination), time of departure, time of opening and closing its check-in and finally the flight code. This information is important because from the flight code it is possible to know the type of the aircraft used for that flight and then the total number of aircraft available seats. In order to carry out a correct analysis of collected data it was necessary to make an appropriate data stratification for highlighting some key aspects. By observation the following data was collect at exactly 7.00 am to 5.00 pm on daily basis. Table shows the arrival, inter-arrival, service for a certain group of random passengers that used the facilities in airport terminal as at the time of observation and data collection.

Sampling took place in the passenger terminal Trivandrum International Airport. After a thorough inspection airport it came out that passenger traffic peaks during Midnight. Data was grouped in the spreadsheets that show some information such as basis information flight (the airline and destination), time of departure, time of opening and closing its check-in and finally the flight code. This information is important because from the flight code it is possible to know the type of the aircraft used for that flight and then the total number of aircraft available seats. In order to carry out a correct analysis of collected data it was necessary to make an appropriate data stratification for highlighting some key aspects. By observation the following data was collect at exactly 12.00 am to 2.00 am on daily basis. Table shows the arrival, inter-arrival, service for a certain group of random passengers that used the facilities in airport terminal as at the time of observation and data collection.

5.1 Chart diagram of international airports in Kerala

In both the charts shows that in peak hours the passengers in the queue and waiting passengers in the queue is very high. This shows that the efficiency of servers in Dubai and Sharjah airlines has to increase.
Figure 1  Chart of Karipur Airport, (a) average number of passengers’ in the queue ($L_q$) (b) average waiting time of passengers in the queue (see online version for colours)

(a)

(b)
Cochin International Airport also known as Nedumbassery Airport and CIAL, is the largest and busiest airport in Kerala, India located at Nedumbassery. In India it is the fourth busiest in terms of international passenger traffic. Cochin International Airport (CIAL) is the first Greenfield airport setup in the public private partnership (PPP) model in civil aviation infrastructure sector in India. From the above graph shows that in every airlines average number of passengers in the queue and average number of passengers waiting in the queue is very high.
Figure 3 Chart of Trivandrum Airport, (a) this is the chart of average number of passengers’ in the queue ($L_q$) (b) this is the chart of average waiting time of passengers in the queue (see online version for colours)

From the above charts it shows that there will be a big queue in the airport in peak hours.

Now I switch on to scrutinising the above data’s used in multiserver queuing theory, by applying the analysis of variance.
6 Analysis of variance

Now this study moves on to the area of Analysis of variance using two way classifications. Analysis of variance, otherwise known as ANOVA, is a collection of statistical models used to analyse the differences among groups and their associated procedures (such as variation among and between groups), developed by statistician and evolutionary biologist Ronald Fisher test.

Two-way analysis of variance performs an analysis of variance for testing the equality of populations means when classification of treatments is by two variables or factors. In two-way ANOVA, the data must be balanced (all cells must have the same number of observations) and factors must be fixed. Two-way ANOVA model works in balanced case only. That is we need to have the same number of observations (replications) for each factor levels combinations.

7 Notations and computation

The data for two-factor ANOVA can be displayed in a two-way table

- no. of levels rows of factor = \( r \)
- no. of levels of columns factor = \( c \)
- total no. of observations = \( rc \)
- observation in \((ij)\)th cell of the table = \( x_{ij} \)
- \( i \)th level of row factor and \( i = 1, 2, \ldots, r \)
- \( j \)th level of column factor \( j = 1, 2, \ldots \)

Sum of \( c \) observation in the \( i \)th row
\[
T_{ri} = \sum_{i} x_{ij}
\]

Sum of \( r \) observation in the \( j \)th column
\[
T_{cj} = \sum_{i} x_{ij}
\]

Sum of all \( rc \) observations
\[
T = \sum_{j} \sum_{i} x_{ij} = \sum_{i} T_{ri} = \sum_{j} T_{cj}
\]

This lead to following computation

Total sum of squares
\[
SS_T = \sum_{i} \sum_{j} X_{ij}^2 - \frac{T^2}{rc}
\]
Between rows sum of squares

\[ SS_R = \sum_i \frac{T_{Ri}^2}{c} - \frac{T^2}{rc} \]

Between columns sum of squares

\[ SS_C = \sum_i \frac{T_{Ci}^2}{r} - \frac{T^2}{rc} \]

Error sum of squares

\[ SS_E = SS_T - SS_R - SS_C. \]

Table 1  
ANOVA table and hypothesis test

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of squares</th>
<th>Degrees of freedom</th>
<th>Mean squares</th>
<th>F ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between the rows</td>
<td>SSR</td>
<td>r - 1</td>
<td>MS_R</td>
<td>( \frac{MS_R}{MS_E} )</td>
</tr>
<tr>
<td>Between the columns</td>
<td>SSC</td>
<td>c - 1</td>
<td>MS_C</td>
<td>( \frac{MS_C}{MS_E} )</td>
</tr>
<tr>
<td>Error</td>
<td>SSE</td>
<td>(r - 1)(c - 1)</td>
<td>MS_E</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>SST</td>
<td>rc - 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7.1  ANOVA table and hypothesis test

Conditions

1. three sum of squares are \( SS_R, SS_C, SS_E \) are independently distributed
2. degree of freedom \( r - 1 + c - 1 + (r - 1)(c - 1) = rc - 1. \)

Using the F ratio tests for significant row effects and for significant column effects can be under taken.

- \( H_0: \) no effect due to row factor
- \( H_1: \) effect due to row factor
- Critical region
- \( F > F_{\alpha-1, (r-1)(c-1)} \)
- Test statistic

7.1.1  Data analysis by two ways classification in Karipur Airport

The null hypothesis is that there is no significant difference between the terminals as well as airlines. The level of significance is 5%. Here the table value of \( F(3,12) = 3.49 \) and \( F(4,12) = 3.26. \) Here \( F_T < F_\alpha \) (table value) and \( F_\lambda < F_\alpha \) (table value) means accept the hypothesis. Therefore concluded that lines in various points seems to be different. This may be due to the service time. If the service time decreases the length of the waiting line
or queue length can be minimised. These are all depending up on the efficiency of the service persons and infrastructure of air service.

Table 2  
Two-way classifications in $L_q$ of Karipur Airport

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of squares</th>
<th>Degree of freedom</th>
<th>Mean sum of squares</th>
<th>$F$ ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between the terminals</td>
<td>SST = 39.25511</td>
<td>$c - 1 = 3$</td>
<td>MST = SSC/D.F = 13.085</td>
<td>$F_T = 1.324$</td>
</tr>
<tr>
<td>Between the airlines</td>
<td>SSA = 75.7178</td>
<td>$r - 1 = 4$</td>
<td>MSA = SSR/D.F = 18.9294</td>
<td>$F_A = 1.916$</td>
</tr>
<tr>
<td>Error</td>
<td>SSE = 118.5731</td>
<td>12</td>
<td>MSE = SSE/D.F = 9.88109</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>233.54</td>
<td>19</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The null hypothesis is that there is no significant difference between the terminals as well as airlines. The level of significance is 5%. Here the table value of $F(3,12) = 3.49$ and $F(4,12) = 3.26$. Here $F_T < F_α$ (table value) accept the hypothesis and $F_A > F_α$ (table value) reject the hypothesis. Therefore, we concluded that lines in various points seem to be different. This may be due to the service time. If the service time decreases the length of the waiting line or queue length can be minimised. These are all depending up on the efficiency of the service persons and infrastructure of air service.

Table 3  
Two-way classification in $W_q$ of Karipur Airport

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of squares</th>
<th>Degree of freedom</th>
<th>Mean sum of squares</th>
<th>$F$ ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between the terminals</td>
<td>SST = 398.5486</td>
<td>$c - 1 = 3$</td>
<td>MST = SSC/D.F = 132.8495</td>
<td>$F_T = 1.015$</td>
</tr>
<tr>
<td>Between the airlines</td>
<td>SSA = 1,948.7612</td>
<td>$r - 1 = 4$</td>
<td>MSA = SSR/D.F = 487.194</td>
<td>$F_A = 3.612$</td>
</tr>
<tr>
<td>Error</td>
<td>SSE = 1,618.6976</td>
<td>12</td>
<td>MSE = SSE/D.F = 134.8925</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>3,952.8965</td>
<td>19</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7.1.2 Data analysis by two-way classification in Kochi Airport

The null hypothesis is that there is no significant difference between the terminals as well as airlines. The level of significance is 5%. Here the table value of $F(3,12) = 3.49$ and $F(4,12) = 3.26$. Here $F_T < F_α$ (table value) and $F_A < F_α$ (table value) means accept the hypothesis. Therefore, we concluded that lines in various points seem to be different. This may be due to the service time. If the service time decreases, the length of the waiting line or queue length can be minimised. These are all depending up on the efficiency of the service persons and infrastructure of air service.
Table 4  Two-way classification in $L_q$ of Kochi Airport

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of squares</th>
<th>Degree of freedom</th>
<th>Mean sum of squares</th>
<th>F ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between the terminals</td>
<td>SST = 8.829133</td>
<td>$c - 1 = 3$</td>
<td>MST = SSC/D.F = 2.943044</td>
<td>$F_T = 1.21$</td>
</tr>
<tr>
<td>Between the airlines</td>
<td>SSA = 21.474199</td>
<td>$r - 1 = 4$</td>
<td>MSA = SSR/D.F = 5.36854995</td>
<td>$F_A = 1.5068$</td>
</tr>
<tr>
<td>Error</td>
<td>SSE = 42.753625</td>
<td>12</td>
<td>MSE = SSE/D.F = 3.56280208</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>73.056958</td>
<td>19</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The null hypothesis is that there is no significant difference between the terminals as well as airlines. The level of significance is 5%. Here the table value of $F(3,12) = 3.49$ and $F(4,12) = 3.26$. Here $F_T < F_{0.05}$ (table value) and $F_A > F_{0.05}$ (table value) means accept the hypothesis. Therefore, we concluded that lines in various points seem to be different. This may be due to the service time. If the service time decreases, the length of the waiting line or queue length can be minimised. These are all depending up on the efficiency of the service persons and infrastructure of air service.

Table 5  Two-way classification in $W_q$ of Kochi Airport

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of squares</th>
<th>Degree of freedom</th>
<th>Mean sum of squares</th>
<th>F ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between the terminals</td>
<td>SST = 278.97521</td>
<td>$c - 1 = 3$</td>
<td>MST = SSC/D.F = 92.99173</td>
<td>$F_T = 1.7616$</td>
</tr>
<tr>
<td>Between the airlines</td>
<td>SSA = 1,911.17857</td>
<td>$r - 1 = 4$</td>
<td>MSA = SSR/D.F = 477.79565</td>
<td>$F_A = 2.9167$</td>
</tr>
<tr>
<td>Error</td>
<td>SSE = 1,965.79543</td>
<td>12</td>
<td>MSE = SSE/D.F = 163.81628</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>4,155.949212</td>
<td>19</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7.1.3 Data analysis by two-way classification in Trivandrum Airport

The null hypothesis is that there is no significant difference between the Terminals as well as airlines. The level of significance is 5%. Here the table value of $F(3,12) = 3.49$ and $F(4,12) = 3.26$. Here $F_T < F_{0.05}$ (table value) accept the hypothesis and $F_A > F_{0.05}$ (table value) reject the hypothesis. Therefore, we concluded that lines in various points seem to be different. This may be due to the service time. If the service time decreases the length of the waiting line or queue length can be minimised. These are all depending up on the efficiency of the service persons and infrastructure of air service.
Table 6  Two-way classification in $L_q$ of Trivandrum Airport

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of squares</th>
<th>Degree of freedom</th>
<th>Mean sum of squares</th>
<th>F ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between the terminals</td>
<td>SST = 46.12254</td>
<td>c – 1 = 3</td>
<td>MST = SSC/D.F = 15.37418</td>
<td>$F_T = 1.6466$</td>
</tr>
<tr>
<td>Between the airlines</td>
<td>SSA = 335.7423</td>
<td>r – 1 = 4</td>
<td>MSA = SSR/D.F = 83.93558</td>
<td>$F_A = 8.9895$</td>
</tr>
<tr>
<td>Error</td>
<td>SSE = 112.0447759</td>
<td>12</td>
<td>MSE = SSE/D.F = 9.33706</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>493.909617</td>
<td>19</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The null hypothesis is that there is no significant difference between the Terminals as well as airlines. The level of significance is 5%. Here the table value of $F(3,12) = 3.49$ and $F(4,12) = 3.26$. Here $F_T > F_o$ (table value) reject the hypothesis, and $F_A < F_o$ (table value) accept the hypothesis. Therefore, we concluded that lines in various points seem to be different. This may be due to the service time. If the service time decreases, the length of the waiting line or queue length can be minimised. These are all depending up on the efficiency of the service persons and infrastructure of air service.

Table 7  Two-way classification in $W_q$ of Trivandrum Airport

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of squares</th>
<th>Degree of freedom</th>
<th>Mean sum of squares</th>
<th>F ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between the terminals</td>
<td>SST = 1,437.88403</td>
<td>c – 1 = 3</td>
<td>MST = SSC/D.F = 479.1119</td>
<td>$F_T = 3.9185$</td>
</tr>
<tr>
<td>Between the airlines</td>
<td>SSA = 543.306329</td>
<td>r – 1 = 4</td>
<td>MSA = SSR/D.F = 135.8266</td>
<td>$F_A = 1.1108$</td>
</tr>
<tr>
<td>Error</td>
<td>SSE = 1,467.241942</td>
<td>12</td>
<td>MSE = SSE/D.F = 122.2702</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>3,447.8840314</td>
<td>19</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8 Results and discussion

A number of studies have performed passenger queue simulations in airport terminals for the purpose of analysing current and future levels of service. Traditionally these studies have focused on the mandatory processing facilities such as check-in, security, immigration, airside boarding lounge and the boarding gates themselves through the application of queueing theory.

The queue problem at international airports in Kerala was modelled as a multi-server queueing problem. Theoretically, arrival is from infinite source and the service pattern is on FCFS. One week data of departures of passengers at international airports in Kerala is considered. See Tables A1 to A3, Adelaboy (2008) stated that the closer the traffic intensity it to zero the more efficient the operations of the service facilities. According to Satty (1961), if $\lambda > 1$, the number of passengers would be infinite. In the International
Airport Karipur in peak hours the average no of customers in the queue is more than 20 per minute and in the non-peak hours is more than ten per minute. In the International Airport Karipur in peak hours the average no of customers waiting in the queue is more than 45 per minute and in the non-peak hours is more than 28 per minute. In the International Airport Kochi in peak hours the average no of customers in the queue is more than 25 per minute and in the non-peak hours is more than 15 per minute. In the International Airport Kochi in peak hours the average no of customers waiting in the queue is more than 1 1/2 hour and in the non-peak hours is more than one hour. In the International Airport Trivandrum in peak hours the average no of customers in the queue is more than 25 per minute and in the non-peak hours is more than ten per minute. In the International Airport Trivandrum in peak hours the average no of customers waiting in the queue is more than 1 and 1/2 hours and in the non-peak hours is more than 50-minute (Tables A1 to A3). This shows that that during weekdays prime hours there is heavy passengers in terminals.

9 Conclusions

In the case of Dubai Airlines, passengers take more time for baggage than other activities when compared to other air services. This may be due to their liberalism in baggage restriction.

This study has been done by observing the passengers arrival time, waiting time in the queue, different behaviour of passengers in the queue like jockeying and service time in airport terminals in Kerala. This study was made the information for one week duration in all the three airport during weekdays peak and non-peak hours. Generally, arrivals do not occur at fixed regular intervals of times but tend to be clustered for a duration of a week. The Poisson distribution involves the probability of occurrence of an arrival are random and independent of all other operating conditions. The inter arrival rate (i.e., the number of arrivals per unit of time) \( \lambda \) is calculated by considering arrival time of the customers to that of the number of customers. Service time is the time required for completion of a service, i.e., it is the time interval between beginning of a service from airport and its completion. In this study the researcher has calculated mean service time \( \mu \) of passengers by considering different service time of passengers. Based upon the tabulation and taking one day as a standard, the researcher inferred that during prime hours there is heavy passengers in terminals, which implies that the utilisation factor is 1. With the increasing number of customers’ coming to airport terminal, there must be trained employees serving at each service unit. Increasing more than sufficient number of servers may not be the solution to increase the efficiency of the server by each service unit.

References


Appendix

### Table A1  Data analysis of Karipur International Airport

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Interpretation of results for queuing model 1

The interpretation of the airport terminal analysis of Karipur International Airport from above table is that the probability for servers to be busy in Dubai flight-baggage screening is 0.1475, i.e., 14.75%. The average number of passengers waiting in a queue is $L_q = 5.16465$ passengers per minute. The waiting time in a queue per server is $W_q = 10.33$ minutes.

From the flight code, it is possible to know the type of the aircraft used for that flight and then the total number of aircraft available seats. In order to carry out a correct analysis of collected data it was necessary to make an appropriate data stratification for highlighting some key aspects.

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