A new method for distinct inversions and isomorphism detection in kinematic chains

Syed Shane Haider Rizvi* and Ali Hasan
Mechanical Engineering Department,
Jamia Millia Islamia (Central University),
New Delhi 110025, India
Email: amsalrizvi@gmail.com
Email: alihasan786@rediffmail.com
*Corresponding author

Rasheed Ahmad Khan
Mechanical Engineering Department,
Galgotias University,
Gautam Buddha Nagar, Greater Noida 201308,
India
Email: rasheed_jmi@hotmail.com

Abstract: In this paper, a new method for obtaining the number of distinct mechanisms from a kinematic chain based on a unique matrix representation of the links of a kinematic chain termed as link identity matrix (LI) is presented and a new invariant link signature (LS) is introduced, which is the sum of absolute value of the characteristics polynomial coefficient of the LI matrix for the representation of a distinct link. The similar values of the LS represent equivalent links further the LS values of a chain are used to determine the isomorphism among the kinematic chains and also assigns a signature to every chain known as chain signature (CS) obtained by summing all LS values of that chain and it is a unique identity assigned to every non-isomorphic chain.

Keywords: chain signature; distinct mechanisms; kinematic chains; link identity matrix; link signature.


Bibliographic notes: Syed Shane Haider Rizvi is the research scholar in the Mechanical Engineering Department at Jamia Millia Islamia (Central University) New Delhi, India. His areas of research interests include kinematics of machines, automation and optimisation of design.

Ali Hasan is an Assistant Professor in the Mechanical Engineering Department at Jamia Millia Islamia (Central University) New Delhi. He has 20 years of teaching experience and 5 years of industrial experience. His areas of research interests include theory of machines, computational mechanics and mechatronics. He has published 30 research papers in the national and international journals.
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Rasheed Ahmad Khan has 45 years of teaching experience; he serves various prestigious government institutions such as NIT Kurukshetra, Al-Anbar University Iraq, Jamia Millia Islamia New Delhi at various positions as Professor, Dean and Head. He is now giving his services as a Professor to the Galgotias University, Gautam Budh Nagar, India. His areas of research interests include design of mechanisms and computational mechanics. He has published more than 140 research papers in the national and international journals. He is the founder member of Association of Machines and Mechanisms (AMM) and Indian Management Science Association (IMSA).

1 Introduction

The search for effective mechanisms starts long back in the past with the starting of making simple machines, but now these mechanisms plays an important role everywhere from miniaturisation (i.e., NEMS and MEMS) to automation (i.e., NC, CNC and ROBOTS). Hence, there is a growing demand for new and effective mechanisms. Initially at the conceptual phase of the design, some of the structural requirements of the mechanisms are taken from the functional requirements.

Synthesis and analysis are the important steps of the structural studies of kinematic chains. Synthesis involves degrees of freedom, number and types of links, the types of joints between the links of all possible kinematic chains and develops several kinematic chains, which may or may not be redundant and follows the isomorphism checking.

The most sensitive issue while enumerating kinematic chains is to prevent the duplication of the chains and develop all the inversions during initial stage of design, inversions of a kinematic chain are obtained by fixing the links of the kinematic chain, one at a time so as many inversions are obtained from a kinematic chain as the number of links in it, but there is a chance that two or more inversions can perform same task, so it is necessary to determine the exact number of possible distinct inversions that can be obtained from a particular kinematic chain.

The two kinematic chains are isomorphic, if there is a one-to-one mapping between the links of both the chains. Checking isomorphism in kinematic chains with same link assortment becomes necessary to avoid duplication. The researchers have spent a lot of time and effort for developing efficient and reliable methods for isomorphism detection in kinematic chains; therefore, a cluster of literature related to this topic is available. But, there is a further scope for an efficient, simple and reliable method, and this paper is an attempt in this direction.

Davies and Crossley’s (1966) methods use the expertise of the designer hence can be applied to the chains with few links. Uicker and Raicu (1975) give an approach based on characteristic polynomial of the adjacency matrix of the kinematic chains but Mruthyunjaya and Balasubramanian (1987) concluded that this method besides its lengthy calculations also fails in determining the isomorphism in two pairs of 10-link kinematic chains, which are non-isomorphic and are having same characteristic polynomials, which was the condition for isomorphism laid down by Uicker and Raicu. Mruthyunjaya (1984) proposed a method of binary coding of chains. Rao and Raju
(1991) introduced the hamming number technique, which works well and does not reveal any counter example; however in some cases when the primary hamming string fails, it requires the comparison of the secondary hamming string. Chu and Cao (1994) present a method of link adjacent table for isomorphism detection and inversions of kinematic chains. Rao (2000) proposed a method based on genetic algorithm, which identifies the isomorphism and also the number of possible distinct mechanisms in a kinematic chain, but like the hamming string test it also requires the fitness of the first, second and third generation strings for the comparison of the chains. Sarkar and Khare (2004) proposed an approach that works on water flow analogy to detect isomorphism in 10-link kinematic chains. Cubillo and Wan (2005) presented a method based on the comparison of eigenvalue and eigenvectors of the adjacency matrices of the corresponding kinematic chains. Ding and Huang (2007) proposed the method of perimeter topological graph and forms the rules for relabelling its vertices canonically and writing a canonical adjacency matrix set of kinematic chains to detect isomorphism but the method is not so simple to understand. Yang et al. (2012) present incident matrices approach to check the isomorphism, but forming the incidence matrix is a complicated task. Sunkari and Schmidt (2006) challenged the reliability of the existing spectral techniques for isomorphism detection. Dargar, Hasan and Khan (2013) present some new codes for isomorphic detection and distinct inversions. Rizvi, Hasan and Khan (2014a) proposed a method based on fuzzy similarity index and an another method was also devised by Rizvi, Hasan and Khan (2014b) based on the instantaneous centres of the kinematic chains. Chiu and Yan (2015) presented an algorithm for automatic sketching of chains. Ding, Huang and Kecksméthy (2015) present a method of enumeration of valid non-fractionated kinematic chains. The methods developed to detect isomorphism in the past generally uses the graphs of the kinematic chain or their adjacency matrices, some of the methods are simple but they fail in some cases, other requires lengthy calculations or difficult to understand and some are restricted only up to 10-link chains. Hence, this paper attempts to give a simple and reliable method that is based on link identity matrix.

2 Inversions and isomorphism detection

Different inversions are obtained for a kinematic chain by fixing the links turn wise but it is possible for a chain that two or more inversions can do same type of work, so it is necessary to know the number of possible distinct mechanisms obtained from the selected kinematic chain. Two chains are said to be isomorphic if there is a one-to-one relation among the links of one chain and that of the other chain.

2.1 Link identity matrix

The link identity matrix (LI) is a unique representation of a fixed link of the kinematic chain, which represents an inversion. Inversions are identified by summing the absolute characteristics polynomial coefficient of the link identity matrices, which gives the value of the invariant LS that represents the link, same value of LS for the links indicates equivalent links. For isomorphism detection, the LS values of the two chains are compared if they are in one-to-one correspondence between LS values of the chains are
isomorphic or otherwise. The chain signature ‘CS’ which is a unique identity of a
kinematic chain obtained by summing the entire LS values of that chain.
LI matrix for the selected link of a chain is obtained as under
\[ LI(i,j) = 1 \] if \( i^{th} \) link is connected to \( j^{th} \) link
\[ LI(i,j) = n, \] where ‘\( n \)’ is the number of the links commonly connected to both \( i^{th} \) and \( j^{th} \) link
\[ LI(i,j) = \text{degree of the link}, \] if \( i = j \) and the link is not fixed a fixed link
\[ LI(i,j) = 0, \] if \( i = j \) and the link is a fixed link

2.2 Algorithm

Step 1: Obtain the LI matrices by fixing the links alternatively for each link of the chain
as explained above.
Step 2: Determine the characteristics polynomial coefficient values of each LI matrix
obtained in Step 1.
Step 3: Obtain the LS values for each link of the chain by summing the absolute values
characteristics polynomial coefficient of the LI matrix of the corresponding link.
Step 4: Now compare LS values of the links, if the two or more LS values are same the
corresponding links of the chain identical links.
Step 5: Now all the LS values of the two chains are compared if there is one-to-one
correspondence between the LS values of the chains then the two chains are isomorphic
or otherwise.
Step 6: The CS value is obtained by summing all the LS values of a chain; the two
isomorphic chains will have the same value of CS.

The aforementioned steps are converted into a MATLAB code for the determination of
distinct mechanisms and isomorphism detection.

3 Test examples

Example 1: A pair six-link non-isomorphic chains shown in Figure 1(a) and (b) are tested
for distinct inversions and isomorphism.

**Figure 1** Watt’s and Stephenson’s chain
The LI matrices for Watt’s chain can be written as under

\[
\begin{align*}
\text{LI}_1 &= \begin{bmatrix} 0 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 2 & 1 & 1 & 0 \\ 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 0 & 2 & 1 & 2 \end{bmatrix} \\
\text{LI}_2 &= \begin{bmatrix} 3 & 1 & 2 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 & 0 & 1 \\ 2 & 1 & 2 & 1 & 1 & 0 \\ 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 0 & 2 & 1 & 2 \\ 3 & 1 & 2 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 & 0 & 1 \\ 2 & 1 & 2 & 1 & 1 & 0 \\ 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 0 & 2 & 1 & 2 \end{bmatrix} \\
\text{LI}_3 &= \begin{bmatrix} 3 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 & 0 \\ 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 0 & 2 & 1 & 2 \\ 3 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 & 0 \\ 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 0 & 2 & 1 & 2 \end{bmatrix} \\
\text{LI}_4 &= \begin{bmatrix} 3 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 2 & 1 & 1 & 0 \\ 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 0 & 2 & 1 & 0 \\ 3 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 2 & 1 & 1 & 0 \\ 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 0 & 2 & 1 & 0 \end{bmatrix} \\
\text{LI}_5 &= \begin{bmatrix} 3 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 2 & 1 & 1 & 0 \\ 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 0 & 2 & 1 & 2 \\ 3 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 2 & 1 & 1 & 0 \\ 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 0 & 2 & 1 & 2 \end{bmatrix} \\
\text{LI}_6 &= \begin{bmatrix} 3 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 2 & 1 & 1 & 0 \\ 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 0 & 2 & 1 & 0 \end{bmatrix}
\end{align*}
\]

From the aforementioned six matrices, the LS values for the links of Watt’s chain are obtained by summing the absolute characteristics polynomial coefficient of the aforementioned LI matrices using MATLAB.

\[
\begin{align*}
\text{LS}_1 &= 175, \quad \text{LS}_2 = 133, \quad \text{LS}_3 = 133, \\
\text{LS}_4 &= 175, \quad \text{LS}_5 = 133, \quad \text{LS}_6 = 133.
\end{align*}
\]

It is observed from the aforementioned values that there are two sets of equivalent links, i.e., (1, 4) and (2, 3, 5, 6). Hence, only two distinct mechanisms are possible from Watt’s chain. The chain signature for Watt’s chain is written as under

\[
\text{CS}_{1a} = 822
\]

Similarly, the LI matrices for Stephenson’s chain can be written and the LS values are calculated as:

\[
\begin{align*}
\text{LI}_1 &= 108, \quad \text{LI}_2 = 222, \quad \text{LI}_3 = 108, \\
\text{LI}_4 &= 222, \quad \text{LI}_5 = 106, \quad \text{LI}_6 = 106.
\end{align*}
\]

From the aforementioned LS values, it can be seen that there are three sets of equivalent links, i.e., (1, 3), (2, 4) and (5, 6). Hence, three distinct mechanisms are possible from Stephenson’s chain and the chain signature is as follows:
Inversions and isomorphism detection in kinematic chains

CS\textsuperscript{ib} = 872

As the link values of the two chains are not in one-to-one correspondence, the two chains are non-isomorphic and hence represented by two different chain signatures CS\textsuperscript{ia} and CS\textsuperscript{ib}.

Example 2: The pair of two 10-link isomorphic single degree of freedom chains shown in Figure 2(a) and (b).

**Figure 2** A pair of 10-link isomorphic chains

The chains shown in Figure 2 are isomorphic according to the available literature studies (Kong, Li and Zhang, 1999; Rizvi, Hasan and Khan, 2014b), and now these chains are tested by the method suggested in this paper. LI matrices of both the chains are obtained and the LS values, depicted in Table 1, are calculated.

<table>
<thead>
<tr>
<th>Link</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chain 2(a)</td>
<td>8221</td>
<td>6693</td>
<td>7998</td>
<td>7452</td>
<td>7195</td>
<td>7071</td>
<td>7935</td>
<td>7935</td>
<td>8379</td>
<td>6481</td>
</tr>
<tr>
<td>Chain 2(b)</td>
<td>8211</td>
<td>8379</td>
<td>6481</td>
<td>7452</td>
<td>7195</td>
<td>7071</td>
<td>7935</td>
<td>7935</td>
<td>6693</td>
<td>7998</td>
</tr>
</tbody>
</table>

Table 1 shows that the links of chain-2(a) and 2(b) are in one-to-one correspondence, i.e., link 1→1, 2→9, 3→10, 4→4, 5→5, 6→6, 7→7, 8→8, 9→2, 10→3 and hence the two chains are isomorphic.

It is can also be seen from Table 1 that the link numbers 7 and 8 are having the same LS values in both the chains, i.e., 7935 hence link 7 and 8 are identical links, only nine distinct mechanisms are possible.

The chain signature for both the chains is

CS\textsuperscript{2a} = CS\textsuperscript{2b} = 75350

Example 3: A pair of non-isomorphic, three degree of freedom, 10-link chains having same characteristics polynomial for their adjacency matrices is shown in Figure 3(a) and (b).
LI matrices of both the chains shown in Figure 3(a) and (b) are obtained, and the LS values, depicted in Table 2, are calculated.

Table 2  LS values of chains-3(a) and 3(b)

<table>
<thead>
<tr>
<th>Link→</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chain 3(a)</td>
<td>7910</td>
<td>7088</td>
<td>7177</td>
<td>7088</td>
<td>5395</td>
<td>5625</td>
<td>4754</td>
<td>5625</td>
<td>5395</td>
<td>5941</td>
</tr>
<tr>
<td>Chain 3(b)</td>
<td>7177</td>
<td>7088</td>
<td>7177</td>
<td>7088</td>
<td>4811</td>
<td>5625</td>
<td>5265</td>
<td>5625</td>
<td>4811</td>
<td>5941</td>
</tr>
</tbody>
</table>

Table 2 shows that the links of chain-3(a) and 3(b) are not in one-to-one correspondence, i.e., link 2→2, 3→3, 4→4, 6→6, 8→8, 10→10 but the links 1, 5, 7 and 9 does not have any correspondence hence the two chains are non-isomorphic.

The chains shown in Figure 3 are co-spectral and non-isomorphic, which was also concluded by Mruthyunjaya and Balasubramanian (1987).

It is also seen from Table 2 that the chain-3(a) is having the following pairs of identical links (2,4), (5,9) and (6,8) hence from chain-3(a) only seven distinct mechanisms are possible.

The chain-3(b) is having the following pairs of identical links (1,3), (2,4), (5,9) and (6,8) as the link values are same hence from chain-3(b), six distinct mechanisms are possible.

The chain signature for the chains 3(a) and 3(b) is as follows:

CS$_{3a}$ = 61998 and CS$_{3b}$ = 60608

Example 4: A pair of non-isomorphic, single degree of freedom 10-link chains is shown in Figure 4(a) and (b) (Ding and Huang 2007).

Figure 4  A pair of non-isomorphic, single degree of freedom, 10-link chains (see online version for colours)
LI matrices of both the chains shown in Figure 4(a) and (b) are obtained, and the LS values, depicted in Table 3, are calculated.

Table 3 LS values of chains-4(a) and 4(b)

<table>
<thead>
<tr>
<th>Link</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chain 4(a)</td>
<td>11434</td>
<td>7889</td>
<td>7889</td>
<td>11434</td>
<td>11434</td>
<td>7889</td>
<td>7889</td>
<td>11434</td>
<td>14389</td>
<td>14389</td>
</tr>
<tr>
<td>Chain 4(b)</td>
<td>14879</td>
<td>10430</td>
<td>8575</td>
<td>8575</td>
<td>8575</td>
<td>8575</td>
<td>10430</td>
<td>14879</td>
<td>10430</td>
<td>10430</td>
</tr>
</tbody>
</table>

Table 3 shows that the links of chain-4(a) and 4(b) are not in one-to-one correspondence as their LS values are different; hence, the two chains are non-isomorphic.

It is also seen from Table 3 that the chain-4(a) is having the following pairs of identical links (1, 4, 5 and 8), (2, 3, 6 and 7) and (9, 10) hence from chain-4(a) only three distinct mechanisms are possible.

The chain-4(b) is having the following pairs of identical links (1, 8), (2, 7, 9 and 10) and (3–6) as the link values are same hence from chain-4(b), three distinct mechanisms are possible.

The chain signature for the chains 4(a) and 4(b) is as follows:

CS_{4a} = 106030 and CS_{4b} = 105780

Example 5: A pair of non-isomorphic, 15 node graphs having same characteristic polynomial for their adjacency matrix is shown in Figure 5(a) and (b) (Chu and Cao, 1994).

Figure 5 A pair of non-isomorphic, 15 node graphs

Table 4 shows that the nodes of graphs-5(a) and 5(b) are not in one-to-one correspondence as their NS values are different hence the two graphs are non-isomorphic.
Table 4  NS values of graphs-5(a) and 5(b)

<table>
<thead>
<tr>
<th>Nodes→</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tbody>
<tr>
<td>Graphs↓</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Graph 5(a)</td>
<td>2160400</td>
<td>2137800</td>
<td>2148400</td>
<td>296700</td>
<td>2945400</td>
<td>3137600</td>
<td>1582400</td>
<td>1577400</td>
</tr>
<tr>
<td>Graph 5(b)</td>
<td>3705800</td>
<td>3705800</td>
<td>3705800</td>
<td>5585700</td>
<td>5585700</td>
<td>5585700</td>
<td>2602400</td>
<td>2602400</td>
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<table>
<thead>
<tr>
<th>Nodes→</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
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</tr>
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<tbody>
<tr>
<td>Graphs</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Graph 5(a)</td>
<td>1568000</td>
<td>2159700</td>
<td>1935900</td>
<td>2239800</td>
<td>1445900</td>
<td>1445600</td>
<td>1568000</td>
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<tr>
<td>Graph 5(b)</td>
<td>2602400</td>
<td>3705800</td>
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<td>3705800</td>
<td>2602400</td>
<td>2602400</td>
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</tbody>
</table>

Note: For the graphs in place of LS the term NS (node signature) and in place CS the term GS (graph signature) is used.

It is also seen from Table 3 that the graph-5(b) is having the following pairs of identical nodes (1,3), (4,6), (7,9), (10,12) and (13,15).

The graph signature for the graphs 5(a) and 5(b) is as follows:

\[ GS^{5a} = 31059662 \] and \[ GS^{5b} = 54606000. \]

Now considering two degree of freedom 9-links chains, the total number of inversions comes out to be 254 which is supported by the literature (Rao and Pathapati, 2000) but Dargar, Hasan and Khan (2013) reported 267 inversions by his method, which means that there is some fault in the method reported by Dargar. Table 5 shows the discrepancies in result obtained by Dargar.

Table 5  Discrepancies in the result of Dargar, Hasan and Khan (2013)

<table>
<thead>
<tr>
<th>Chain no.</th>
<th>13</th>
<th>21</th>
<th>22</th>
<th>24</th>
<th>30</th>
<th>31</th>
<th>32</th>
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<th>37</th>
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<tbody>
<tr>
<td>No of inversions</td>
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<tr>
<td>By Rao (2000)</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>7</td>
<td>5</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>By Dargar, Hasan and Khan (2013)</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>8</td>
<td>4</td>
<td>9</td>
<td>9</td>
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<tr>
<td>By the method suggested in this paper</td>
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</table>

Table 6 shows the LS values of the chains reported in Table 5, and the figures of these chains are shown in Table 7.

Table 6  LS values of chains reported in Table 5

<table>
<thead>
<tr>
<th>Link→</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<td>Chain↓</td>
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<tr>
<td>13</td>
<td>2539</td>
<td>2539</td>
<td>1937</td>
<td>2042</td>
<td>2414</td>
<td>2009</td>
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<td>2205</td>
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</table>
Table 6  LS values of chains reported in Table 5 (continued)

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<th>7</th>
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Table 7  Figures of the chains reported in Table 5

4 Conclusion

The algorithm presented in this paper is a powerful tool not only for kinematic chains but also for graph isomorphism problem as it is tested for several examples, which were challenged in the literature and with the help of the MATLAB code it becomes very easy and less time-consuming task to determine isomorphism and also the number of possible distinct mechanisms that can be obtained from a kinematic chain. The method has been found to be successful in distinguishing all known 16 kinematic chains of 8-links, 230 kinematic chains of 10-links having 1-F, 40 kinematic chains of 9-links having 2-F and
98 kinematic chains of 10-links having 3-F and also the number of distinct mechanisms obtained from 1-F; 6-link chains are 5, 1-F; 8-link chains are 71 and 2-F; 9-link chains are 254 1-F; 10-link chains are 1834, and 3-F; 10-link chains are 684. These results agree those reported already in the literature. The link identity matrices can be written with very little effort, even by mere inspection of the chain. The proposed test is quite general in nature and can be used to detect isomorphism of not only planar kinematic chains of one degree of freedom, but also kinematic chains of multi degree of freedom. Further, the method can also be extended to determine the distinct inversions in multiple joint kinematic chains.

References


Inversions and isomorphism detection in kinematic chains


