The structural synthesis of planar 10-link, 3-DOF simple and multiple joint kinematic chains

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Abstract: A method has been proposed for structural synthesis of the planar 10-link, 3-DOF simple and multiple joint kinematic chains in this paper. The procedure for structural synthesis of the kinematic chains is obtained by applying the single-kinematic-chain (SKC) adding theory with the help of double-colour-topological-graph (DCTG) model representation. The mean feature of this method is that a multiple joint kinematic chain with L independent loops can be formed by subsequently adding single-open-chain (SOC) on a single-closed-chain (SCC) with three adding styles. By this method, the structures of all the 10-link, 3-DOF simple and multiple joint kinematic chains have been synthesised by computer.

Keywords: structural synthesis; planar simple and multiple joint kinematic chain; single-kinematic-chain adding theory; double-colour-topological-graph; DCTG; model representation; independent loop; single-open-chain; SOC; single-closed-chain; SCC; adding style; procedure.

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1 Introduction

The structural synthesis of planar kinematic chain is one of the most creative and important stages in the mechanical design, so attracted many researchers to study on it. Since the 1960s, many systematic methods for the synthesis of kinematic chains have been proposed, especially for the different kinds of simple joint kinematic chains with up to 16 links and one or more DOF.

As we know, the kinematic chains studied the most in structural synthesis are those with simple joints, and the synthesis technique for this kind of kinematic chains is comparatively mature, such as: the method based on Franke’s notation (Davies and Crossley, 1996); the dual graph-based approach (Soni, 1971); the approach relied on the finite group theory (Tuttle et al., 1989a, 1989b); the permutation group approach (Yan and Hwang, 1990; Yan, 1992); the Hamming number technique (Rao, 1997a, 1997b) and so on. Until now, the structural synthesis results of 16 types 8-links 1-DOF kinematic chains, 40 types 9-links 2-DOF kinematic chains and 230 types 10-links 1-DOF kinematic chains are supposed to be right in the world. Besides the simple joint kinematic chains, multiple joint kinematic chains also occupy an important position in planar kinematic chains; however, the structural synthesis for this kind of kinematic chain is still an issue not well studied. The ‘single-opened-chain (SOC)’ method (Yang, 1998, Yang and Yao, 1988) was developed for structural synthesis of simple joint kinematic chains with planar and
non-planar graphs. It is an effective method for structural synthesis of mechanism. A unified topological model called double-color-graph (DCG) was presented to synthesis the 157 Baranov Trusses with multiple joint (Chu and Cao, 1989; Chu, 1992), but this process is not suitable for the synthesis of other special kinematic chains. A double-coloured contracted graph (DCCG) and weighted double-coloured contracted graph (WDCCG) method to synthesis the 44 8-bar one-DOF and 2239 10-bar one-DOF multiple joint kinematic chain also proposed in the author’s other paper (Chu and Cao, 1998a, 1998b; Cao and Chu, 1994). Song and Yang (2001) present a combination method of type synthesis for planar kinematic chain under 10-bars with multiple joints, but it is not given the number of structural types about certain multiple joint factor about a special kinematic chain. Ding et al. (2008a, 2008b) on basis of the topological model DCCG to proposed a structural synthesis theory of multiple joint kinematic chains method, but this paper have not given all the possible types of the specific multiple joint kinematic chains about 8-links, 1-DOF and 9-links, 2-DOF kinematic chains, what is more, it is difficult to obtain the kinematic chains with more DOF and more multiple joints by this method. Therefore, developing a more effective synthesis method for synthesis all the possible types of the multiple joint kinematic chains is a difficult and important problem still not well resolved now.

In this paper, a method based on the SOC adding theory for structural synthesis of the planar 10-bar, 3-degree simple and multiple joint kinematic chains has been presented. The main feature of this proposed method is that it applied the SOC adding theory with the help of DCG topological model to synthesis the kinematic chains. As a result, we have enumerated all the 95 types of 10-bar, 3-DOF simple joint kinematic chains and all the 497 types of 10-bar, 3-DOF multiple joint kinematic chains.

2 The DCG topological model

The planar kinematic chain studied the most so far are the kinematic chain with simple joints. Their topological graphs are generally represented by the common topological graphs. The graph are established as follows: vertices of the graph denote the links of the kinematic chain and edges of the graph denote the joints. And it is very convenient to use this common topological graphs, but it can not be applied directly to the study about the multiple joint kinematic chains for the reason that the resulting topological graphs contain rigid polygons.

So a unified representation is needed for the structural synthesis of the planar simple and multiple joint kinematic chains. In this paper, uses the DCG topological models to represent the simple and multiple joint kinematic chains. They are established as follows: the solid vertices (●) denote the links of the kinematic chain and the hollow vertices (○) denote the joints, and connect the corresponding solid and hollow vertex with an edge when a link is connected with a joint. The DCG is a non-weight graph and thus relatively easier to be handled with the graph theory, the most important is that it can well facilitate the SKC adding theory to structural analysis of the simple and multiple joint kinematic chains. For example, the simple and multiple joint kinematic chain as shown in Figures 1(a) and 1(c), their DCG topological model are corresponding to Figures 1(b) and 1(d).

Figure 1 The simple and multiple joint kinematic chain and their DCG, (a) a simple joint kinematic chain (b) the DCG of (a) (c) a multiple joint kinematic chain (d) the DCG of (c)

A DCG of the kinematic chain can be represented by its incident matrix and adjacency matrix. The elements of the incident matrix are defined:

\[ A = \begin{bmatrix} a_{ij} \end{bmatrix}_{nm} \]

where \( n \) is the number of the links, \( m \) is the number of the joints.

3 The SKC adding theory for structural synthesis

A kinematic chain with \( L \) independent loops can be formed by subsequently adding SOC on a single-closed-chain (SCC). The composition method are established as follows in Figure 2. The first is to choose a SCC1 be a foundation to form a kinematic chain \( F_1 \) with one independent loop; then the second SOC2 added on to the kinematic chain \( F_1 \) to form a kinematic chain \( F_2 \) with two independent loops; \( \ldots \); generally the SOC \( j \) added on to the kinematic chain \( F_{j-1} \) to form a kinematic chain \( F_j \) with \( j \) independent loops; \( \ldots \); and so on to form the kinematic chain \( F_L \) with \( L \) independent loops.
In the adding process, when a SOC is added on to a link, then this link becomes a multiple joint link; otherwise, a multiple joint is obtained when a SOC is added on to a joint. So the SKC adding theory can be convenient used to synthesis of the simple and multiple joint kinematic chains.

**Figure 2** The process to form a kinematic chain

![Kinematic Chain Process](image)

3.1 The properties of SKC represented by DCG

As we know, the simple and multiple joint kinematic chains can be represented by DCG, and the kinematic chain can be formed by subsequently adding SOCs on a SCC1, the structure of SCC1 can be represented by DCG as shown in Figures 3(a) and 3(b), the length $Q_1$ of SCC1 (defined as the number of edges in DCG) is equal to the number of joints plus the number of links, which are shown as follow:

$$Q_1 = 2m_1$$

**Figure 3** The SKC represented by DCG

![Simple Kinematic Chain](image)

The structure of SOCs can be represented by DCG as shown in Figures 3(c) and 3(d), and three properties are identified as follow:

1. When both ends of a SOC are added to link vertices, connecting edges would be produced. The situations for such connecting edges are shown in Figure 4. In this case, this adding pattern $t_j = 1$ cause the number of multiple joint factor $V_{t_j} = 0$ (a 4 degree joint is regarded as $V_{t_j} = 2$, 5 degree joint is regarded as $V_{t_j} = 3$, and so on); the length $Q_j$ of SOC is equal to one plus the number of joints and the number of links, which are shown as follow:

$$Q_j = 2m_j$$

2. When one end of a SOC is added to a link and the other to a joint, the end connected to the link would produce a connecting edge. The situations for such a connecting edge are shown in Figure 5. In this case, the adding pattern $t_j = 2$ cause the number of multiple joint factor $V_{t_j} = 1$, and the length $Q_j$ of the SOC is equal to the number links plus the number of joints, which are shown as follow:

$$Q_j = 2m_j - 1$$

3. When both ends of a SOC are added to joints, connecting edges would be developed as shown in Figure 6. In this case, the adding pattern $t_j = 3$ cause the number of multiple joint factor $V_{t_j} = 2$, and the length $Q_j$ of the SOC is equal to the number of links plus the number of joints minus one, which are shown as follow:

$$Q_j = 2m_j - 2$$

**Figure 4** Adding of SOC in DCG: both ends of the SOC to link vertices

![Adding of SOC](image)

**Figure 5** Adding of SOC in DCG: one end to link vertex and the other to joint vertex

![Adding of SOC](image)

**Figure 6** Adding of SOC in DCG: both ends to joint vertices

![Adding of SOC](image)

3.2 An algorithm for determining the length of each SKC

Supposed that a kinematic chain has $L$ independent loops, the length $Q_j$ of SKCs is determined with the following manner. At the beginning, in all the loops of the kinematic chains, choose one with the minimal length as the first
independent loop, so the length $Q_1$ for the SCC_1 is determined. Then from those remaining SOCs that construct the second independent loop, choose the one with minimal length; the length $Q_2$ for the first SOC_2 is then obtained. This procedure will proceed until L loops then the length $(Q_1, Q_2, ..., Q_L)$ of the (SCC_1, SOC_2, ..., SOC_L) are constructed.

### 4 Algorithm for the structural synthesis

For a kinematic chain, the degrees-of-freedom $F$:

$$F = 3(n-1) - 2m$$

So if know the $F$ and $n$, it is easy to get the independent loops $L$ as follow:

$$L = m - n + 1$$

The total length $Q$ of the SKCs:

$$Q = Q_1 + Q_2 + ... + Q_L = 2m - V$$

where $V = V_1 + ... + V_t$ is the total number of the multiple joint factor in a kinematic chain, and the max value of the multiple joint factor is defined as follow:

$$V_{max} = 2(L - 1)$$

#### 4.1 Distribution the length and adding patter of the SKC

In the process of the structural synthesis for the kinematic chain, it is very important to determine the distribution of the length and adding patter for SKCs by know the degrees-of-freedom $F$, the number of links $n$, the total of the multiple factor $V$.

1. **Determination of the values of $Q_1$**

   For a kinematic chain, notice that the length of SCC_1 must not be smaller than a four links single kinematic chain, and the first independent loop must be the shortest one, so we have the following equation:

   $$8 \leq Q_1 \leq \text{Int}\left\{\left(12L - 4\right) / L\right\}$$

2. **Determination of the values of $(Q_2, t_2)$**

   According to the earlier discussion on the algorithm to determine the length of each independent loop, the addition of the second SOC_2 to the first loop should not produce a loop smaller than the first loop, therefore we have:

   $$Q_2 \geq \text{Int}\left(Q_1 / 2\right)$$

   $$Q_1 + Q_2 \leq Q$$

   The $t_2$ must ensure that its adding patter cause the number of multiple joint factor $V_{t_2}$ meet the following condition:

   $$V_{t_2} \leq V$$

3. **Determination of the values of $(Q_j, t_j)\ (j = 3, 4, ..., L)$**

   Following the same argument in the determination of the values for SOC_3, the addition of the SOC_j to an existing loop kinematic chain should not produce a loop smaller than the first loop and the second loop, the length $Q_j$ must be satisfied as following:

   $$Q_j \geq \text{Max}\left\{\text{Int}\left((Q_1+1) / 2\right), \text{Int}\left((Q_2+1) / 2\right)\right\}$$

   $$Q_j \leq Q_{(j-1)} \leq \cdots \leq Q_2$$

   $$Q_1 + Q_2 + \cdots + Q_j \leq Q$$

   The $t_j$ must ensure that its adding patter cause the number of multiple joint factor $V_{t_j}$ meet the following condition:

   $$V_{t_j} \leq V$$

4. **At last, determination the values of $(Q_L, t_L)$**

   Following the same argument in the determination of the values for SOC_L, the length $Q_L$ must be satisfied as following:

   $$Q_L \geq \text{Max}\left\{\text{Int}\left((Q_1+1) / 2\right), \text{Int}\left((Q_2+1) / 2\right)\right\}$$

   $$Q_L \leq Q_{(L-1)} \leq \cdots \leq Q_2$$

   $$Q_1 + Q_2 + \cdots + Q_L \leq Q$$

   The $t_L$ must ensure that its adding patter cause the number of multiple joint factor $V_{t_L}$ meet the following condition:

   $$V_{t_L} \leq V$$

So calculating the $Q, L$ by the input parameter $F, n, V$, it is easy to obtain all the possible distribution sets of values $[(Q_1); (Q_2, t_2); \cdots; (Q_L, t_L)]_w$ (w denotes the total number of these possibilities).

#### 4.2 The number of links and joints in the distribution set

For a distribution set $[(Q_1); (Q_2, t_2); \cdots; (Q_L, t_L)]$, the number of links and joints can be obtain by the following way:

1. when $j = 1$, the length of the first SCC_1 $(Q_1)$:
   
   $$m_1 = n_1 = \text{Int}\left(Q_1 / 2\right)$$

2. when $j > 1$, three situations need to be considered according to the adding patter of every SOCs.

   Firstly, when the adding patter $t_j = 2$, the value of $Q_j$:

   $$m_j = n_j = \text{Int}\left(Q_j / 2\right)$$

   $$m_j = n_j + 1$$
Secondly, when the adding pattern $t_j = 1$, the value of $Q_j$:

$$m_j = \text{Int} \left( \frac{Q_j}{2} \right)$$

$$n_j = m_j - 1$$

Thirdly, when the adding pattern $t_j = 3$, the value of $Q_j$:

$$n_j = \text{Int} \left( \frac{Q_j}{2} \right)$$

$$m_j = n_j + 1$$

Use this method to get all the possible distribution sets $\{(Q_1), (Q_2, t_2); \ldots; (Q_t, t_t)\}_n = [(m_1, n_1); (t_2, m_2, n_2); \ldots; (t_t, m_t, n_t)]_n$.

4.3 Determination the incident matrix for each independent loop

The corresponding incident matrixes for a given distribution set $\{(m_1, n_1); (t_2, m_2, n_2); \ldots; (t_t, m_t, n_t)\}$ can be obtain as follow:

**Step 1**: Develop the incident matrix for the first independent loop. This is done by the first SCC; according to the values of $(m_1, n_1)$:

$$A(n_i, m_i) = 1, A(n_i, m_i) = 1, A(i, i) = 1, A(i + 1, i) = 1$$

where $1 \leq i \leq (n_1 - 1), 1 < j \leq (m_1 - 1)$.

**Step 2**: Develop the incident matrix for the second independent loop by $(t_2, m_2, n_2)$. This is done by sketching the second SOC 2 to any two vertices $v_a$ and $v_b$ of the first loop. Only those vertices $v_a$ and $v_b$ can be chosen that the adding of the second loop on them does not produce a loop smaller than the first loop. This condition required that the lengths $q_{q_2}$ of two such vertices should meet the following condition:

- when $(Q_1 - Q_2) \leq 0$:
  $$\text{Int} \left( \frac{Q_1}{2} \right) \geq q_{q_2} \geq 0$$

- when $(Q_1 - Q_2) > 0$:
  $$\text{Int} \left( \frac{Q_1}{2} \right) \geq q_{q_2} \geq (Q_1 - Q_2)$$

According to the SOC adding pattern $t_2$, the number of links $m_2$ and joints $n_2$, there are three situations need to be considered:

1. when both ends of SOC 2 are added to link vertices ($t_2 = 1$), so that:
   $$v_a = 1, v_b = 1 + \left( \frac{qq_2}{2} \right)$$
   $$n_2 = 0, A(v_a, m_1 + 1) = 1, A(v_b, m_1 + 1) = 1$$
   $$n_2 \geq 1, A(v_a, m_1 + 1) = 1, A(v_b, k_2 + 1) = 1, A(i, i) = 1, A(i, i + 1) = 1$$
   $$k_i = n_i + n_2, k_2 = m_1 + m_2 - 2$$

2. when one end of SOC 2 is added to a link and the other to a joint ($t_2 = 2$), so that:
   $$v_a = 1, v_b = 1 + \left( \frac{qq_2}{2} \right)$$
   $$n_2 = 0, \text{no exist}$$
   $$n_2 = 1, A(v_a, m_1 + 1) = 1, A(n_1 + 1, v_b) = 1$$
   $$A(m_1 + 1, n_1 + 1) = 1, n_2 > 1, A(v_a, m_1 + 1) = 1, A(k_1 + 1, v_b) = 1$$
   $$A(k_1 + 1, k_2 + 1) = 1, A(i, i) = 1, A(i, i + 1) = 1$$
   $$n_1 + 1 \leq i \leq k_1, (m_2 + 1) \leq i \leq k_2, n_1 + n_2 - 1$$

3. when both ends of SOC 2 are added to joints ($t_2 = 3$), so that:
   $$v_a = 1, v_b = 1 + \left( \frac{qq_2}{2} \right)$$
   $$n_2 = 0, \text{no exist}$$
   $$n_2 = 1, A(v_a, n_1 + 1) = 1, A(n_1 + 1, v_b) = 1$$
   $$n_2 > 1, A(v_a, n_1 + 1) = 1, A(i, i) = 1, A(i + 1, i) = 1$$
   $$A(k_1 + 1, v_b) = 1$$
   $$\text{where}$$
   $$(n_1 + 1) \leq i \leq (k_2 - 1), (m_2 + 1) \leq i \leq k_2, k_i = n_i + n_2, k_2 = m_1 + m_2 - 2.$$
1 when both end of SOC$_j$ are added to link vertices ($t_j = 1$), so that:

\[ n_j = 0, A(v_u, P_{j2} + 1) = 1, A(v_b, P_{j2} + 1) = 1 \]

\[ n_j \geq 1, A(v_u, P_{j2} + 1) = 1, A(v_b, k_2 + 1) = 1, \]

\[ A(i,i) = 1, A(i,i+1) = 1 \]

where

\[ (P_{ij} + 1) \leq i \leq k_1, (P_{j2} + 1) \leq i \leq k_2, \]

\[ k_1 = P_{ij} + n_j + 1, k_2 = P_{j2} + m_j - 2 \]

$P_{ij}$ represented the number of links in the $(j-1)$th independent loop

$P_{j2}$ represented the number of joints in the $(j-1)$th independent loop.

2 When one end of SOC$_j$ is added to a link and the other to a joint ($t_j = 2$), so that:

\[ n_j = 0, \text{no exist} \]

\[ n_j = 1, A(v_u, P_{j2} + 1) = 1, A(P_{ij} + 1, v_b) = 1, \]

\[ A(P_{ij} + 1, P_{j2} + 1) = 1 \]

\[ n_j \geq 1, A(v_u, P_{j2} + 1) = 1, A(k_i + 1, k_2) = 1, \]

\[ A(k_i + 1, k_2 + 1) = 1, A(i,i) = 1, A(i,i+1) = 1 \]

where

\[ (P_{ij} + 1) \leq i \leq k_1, (P_{j2} + 1) \leq i \leq k_2, \]

\[ k_1 = P_{ij} + n_j - 1, k_2 = P_{j2} + m_j - 2 \]

3 When both ends of SOC$_j$ are added to joints ($t_j = 3$), so that:

\[ n_j = 0, \text{no exist} \]

\[ n_j = 1, A(P_{ij} + 1, v_u) = 1, A(P_{ij} + 1, v_b) = 1 \]

\[ n_j \geq 1, A(P_{ij} + 1, v_u) = 1, A(i,i) = 1, \]

\[ A(i+1,i) = 1, A(k_i + 1, v_b) = 1 \]

where

\[ (P_{ij} + 1) \leq i \leq (k_i - 1), (P_{j2} + 1) \leq i \leq k_2, \]

\[ k_1 = P_{ij} + n_j, k_2 = P_{j2} + m_j - 2. \]

At last, get all the incident matrix $(A_1, A_2, A_3, ..., A_e)$ for a special distribution set $\{(m_1, n_1); (m_2, n_2); \ldots; (m_e, n_e)\}$.

### 4.4 A general algorithm for the structural synthesis

A general algorithm is given for the structural synthesis of planar simple and multiple joint kinematic chains. If the number of links $n$, degrees-of-freedom $F$, and the multiple joint factor $V$ are specified, all possible kinematic chains can be synthesised by this method:

**Step 1:** Input the value $n$, $F$, $V$ to obtained the value of independent loops $L$ and the total length $Q$ of the SKCs; then according to the previous discussion in Section 4.1, calculating the distribution of the SKCs, denoted by $\{(Q_1); (Q_2, t_2); \ldots; (Q_e, t_e)\}$.

**Step 2:** For the first distribution $\{(Q_1); (Q_2, t_2); \ldots; (Q_e, t_e)\}$, determine the distribution for the number of joints and the number of links, denoted by $\{(m_1, n_1); (m_2, n_2); \ldots; (m_e, n_e)\}$.

**Step 3:** For the first $\{(m_1, n_1); (m_2, n_2); \ldots; (m_e, n_e)\}$, get all the incident matrix $(A_1, A_2, A_3, ..., A_e)$ for a corresponding distribution set.

**Step 4:** Determine the isomorphic kinematic chains and immovable chains resulted from Step 3 by the method provided by the other papers (Chu and Cao, 1994; Song and Yang, 2004) that get $(A_1, A_2, A_3, ..., A_e)(p < e)$.

**Step 5:** Increment $w$ by 1 and repeat Steps 2, 3, 4, and 5, until the last distribution of $w$ is processed and that get all the kinematic chains expressed by incident matrix $(A_1, A_2, A_3, ..., A_e)(h > = p)$.

**Step 6:** Use the incident matrix $(A_1, A_2, A_3, ..., A_e)$ to draw the corresponding DCG and then get the corresponding schematic diagram of mechanism for each kinematic chain.

In this paper, the structural synthesis procedure for the 10-bar, 3-DOF simple and multiple joint kinematic chains are programming by computer in the MATLAB 7.0.

### 5 The structural synthesis result

The 10-links, 3-DOF kinematic chains have $L = 3$ independent loops and the max multiple joint factor $V_{\text{max}} = 4$. The structural synthesis results as shown in Table 1, there are 95 simple joint kinematic chains are obtained, and there are 208 $V = 1$, 188 $V = 2$, 80 $V = 3$ and 21 $V = 4$ multiple joint kinematic chains are obtained.

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<td>208</td>
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<td>21</td>
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<td>Appendix B</td>
<td>Appendix C</td>
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6 Conclusions

In this paper, a method has been presented for structural synthesis of the simple and multiple joint kinematic chains. The main feature of this proposed method is to apply the SKC adding theory to structural synthesis the multiple joint kinematic chain. In this way, the structures of 10-link, 3-DOF simple and multiple joint kinematic chains have been synthesised by computer.

In this way, the structures of kinematic chains with other specified DOF, number of links and multiple joint factor can be synthesised.

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References


Appendix A

The 208 10-link, 3-DOF, $V = 1$ multiple joint kinematic chains
Appendix B

The structural synthesis of planar 10-link, 3-DOF simple and multiple joint kinematic chains

The 18810-link, 3-DOF, $V = 12$ multiple joint kinematic chains
Appendix C

The 80 10-link, 3-DOF, $V = 3$ multiple joint kinematic chains
Appendix D

The 21 10-link, 3-DOF, $V = 4$ multiple joint kinematic chains