On extending transitions logic in hybrid dynamic systems based on bond graph and Petri nets combination

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Abstract: In order to better take into account complexity of the heterogeneous dynamics in hybrid dynamic systems (HDS); we propose an unified and integrated fuzzy logic hybrid approach extending the concept of a classical switch bond graph modelling to a logical transition that takes into account a gradation instead of abrupt behavioural changes. It allows extension of the reachable states space where we propose real physical intermediate states (fluidification) that can be maintained with a sense. We also modify the mathematical structure of ideal primitive switching element (PSTS) by proposing a formula based on predictive and effective notions that fully comply with standard bond graph syntax and semantics as well as the Petri Nets, grants a freedom of the sampling instants according to the solver. We demonstrate the effectiveness of our approach on a flow control bi-tank and its usefulness both in academia and industrial field. Finally, we discuss some ideas about discrepancy between the level of mathematical abstraction and that of the physical level.

Keywords: hybrid dynamic systems; HDS; STS; primitive switch transition system; PSTS; differential-algebraic equation; DAE; time Petri nets; TPN; bond graph; Petri nets.

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1 Introduction

The increasingly important interaction between phenomena of a continuous nature and discrete nature makes hybrid dynamic systems (HDS) more complex. HDS have dynamical evolutions where continuous and discrete dynamics interact and influence each other; The continuous behaviour is due to the natural evolution of the physical process while discrete behaviour can be due to the presence of switches. The transition problematic remains the most crucial issue in this context since abrupt behavioural changes are very ubiquitous in engineering. The expressiveness in the determination of an explicit dynamic model that simultaneously and explicitly shows these interactions is the most significant reason. So many systems involve mixed continuous behaviour and discrete, such as embedded systems. Several approaches have been proposed for modelling, analysis, verification, control, diagnosis and supervisory HDS (Lin and Antsaklis, 2014; Wu et al., 2005; Sekhri and Haffaf, 2014).

Bond graph tool is a unified and multidisciplinary graphic language for all areas of engineering sciences. Bond graph modelling does not require the writing of general conservation laws; it is essentially based on the characterisation of energy exchange within the system (Borutzky, 2011; Karnopp et al., 2012). Bond graph is formally powerful and makes it possible to explicitly describe the physical causality in its graphical representation as well as the state model. Nevertheless, it does not allow the logical transitions to be taken into account as well as the conditions that govern these transitions. The difficulty to obtain such a model of system with switches lies in the systematic taking into account all dynamics, especially discrete whether autonomous or external which can occur during the evolution of the system. Thus, discrete event formalism such as the Petri net (David and Alla, 2010) taking efficiently into account a complete logical transition is needed. Moreover, state space construction is often needed to capture the overall discrete states of the system dynamic behaviour (Dal Zilio et al.,

2014) through existing tools like for example time Petri net analyser (TINA) (Berthomieu et al., 2015).

In many engineering applications, transition problem lies mainly to the command and control part. In fact, Petri net is the most suitable tool for modelling and studying the properties of such systems with a powerful event aspect. Within this context, our contribution is an extended approach under the same assumption that operating modes are determined by switching phenomena. In general, discontinuities are related to two types:

- external control
- internal control with autonomous switching and state jumps.

Several approaches exist in literature to represent the behaviour of such systems; for example in Ghomri and Alla (2008, 2012) and Taleb et al. (2015) but Boolean firing transitions condition in the discrete part can only block or allow the firing of the entire quantity of flow of a continuous transition, i.e., the discrete part cannot influence the instantaneous rate of the amount of flow even if the continuous part influences marking of the discrete places. In the case of D-enabling degree (particularly useful when duplicating sources or servers): at the level of each continuous transition, if U is the flow rate, the D-enabling degree can only ensure the instantaneous firing speeds: U, $2U$, $3U$ … nU with n integer $\left(n = \min_{p_i \in {}^{o}T_j \cap P^D} \left\lfloor \frac{m_i}{pre(P_i, T_j)} \right\rfloor \right)$ or conversely when n decreases; but cannot be for example any rational number; it cannot reach $(3/4)U$. Also for a system of tanks, the level of the height of the liquid is proportional to the firing speed, so for real implementation, in such nominal case the use of sensors is unavoidable. In the HBG (hybrid bond graph) area, for example in Abdallah et al. (2016) and Triki et al. (2014), dynamic behaviours are variables but transitions are of type of 'all or nothing' by controlled junctions. This situation leads us to combine two formalisms (Bouhalouane et al., 2015) trying to show the expressive power of the obtained hybrid model both in academia and industrial fields through didactical use case of bi-tank system.

Firstly, we will deal with fast instantaneous transitional change as mythical states (Bouhalouane et al., 2015; Mosterman and Biswas, 1998) by taking into account real intermediate states introducing fuzzy logic. Fluidification enable us to extend the STS-based classical ideal primitive switch transition system (PSTS) element of a valve that can still be generalised to other interconnecting subsystems. To this end a quantisation using a proposed formula will be taken, adopting two notions of predictive values and effective values within a state model of differential-algebraic equations (DAEs); that fully comply with syntax and semantics of the bond graph theory as well as that of the Petri Net. These states are real manifestation in the system. Then we will propose two variants time Petri nets (TPN): firstly, a p-temporal Petri Net for the nominal case where a token of each marked place is characterised by a switched bond graph in order to subsume the class of standard PN. Secondly, we motivate our extension to a T-temporal Petri net that can be useful for diagnosis activities to provide relevant alarms; where structural and causal properties of bond graph can be exploited (see Borutzky, 2018; Abdesselam et al., 2016; Ould Bouamama et al., 2014). To this end we will give a brief demonstration about constraint on the quantity of flow and the corresponding time interval constraint. Before concluding, we will go through the state space model by applying reachability and structural analysis. We give also an idea on prediction instants to support DAE solvers.

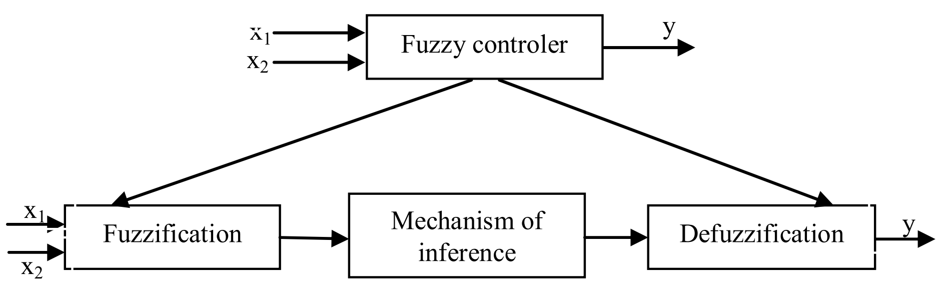
We recognise that it is the states of the switch that determine system configurations (modes of operation). The approach we are undertaking concerns the logic of transitions, in other words the control structure. Indeed, classical concept of switching behaviours is of binary control logic 'all or nothing (on/off)'. However, fuzzy logic is an extension of classical logic and makes also it possible to effectively deal with imperfect information, incompleteness and inaccuracies (Bělohlávek et al., 2017); this transition logic we are going to extend while preserving the control structure and the switched bond graph modelling theory; i.e., more than the On and Off switching states we will consider gradual states to deal with abrupt changes, like the case of some switch elements such as a hydraulic valve (fully open or fully closed, but also barely open and barely closed) controlling a flow, where we will demonstrate on the basis of its opening/closing angle the possibility of the existence of a cylindrical disk equivalent to the area section of this angle (in the case of a cylindrical type of pipe). Against this background, in our approach we will make the choice of semantics that conforms to the definitions in the literature of bond graph based on (Edstrom et al., 1996) works and we conjecture that most of the development can be transposed to other fields of application.

2 Fuzzy logic

The fuzzy set theory has used to formalise and process approximate or imprecise knowledge in complex systems. Generally a fuzzy controller is an inference system used in the automation technique (see Figure 1) (De Silva, 2018). Fuzzy logic has several advantages to:

- formalise and simulate the expertise of an operator or a designer in driving and adjusting a process
- give a simple answer for processes whose modelling is difficult
- to take into account, cases or exceptions of different natures, and to integrate them progressively in the expertise
- take into account several variables and carry out the 'weighted fusion' of influence quantities.

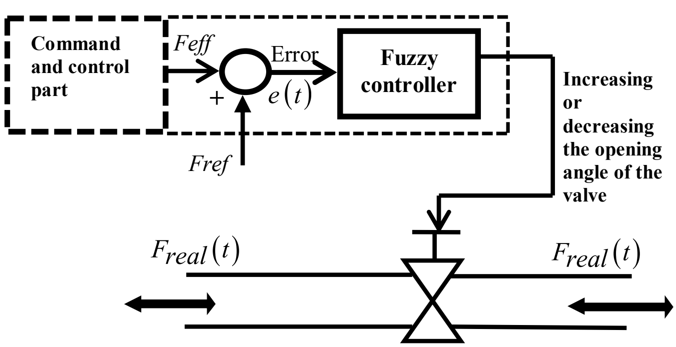
Figure 1 Fuzzy inference system



3 Mathematical vs. physical models and hybrid modelling approach

Generally, in modelling traditions, a valve may be in one of two on/off states (open or closed), but in reality it can also be half open or half closed, that is to say with varying degrees of opening or closing, which consequently influences the quantities of material flow. These states that we will call completely open/completely closed were basically derived from the binary logic and any change in the section requires reuse of the same model with new parameters of the new dimension on/off. This work consists in proposing within the same model an induction of several cases that could be treated separately by the previous classical approaches. The concept of fuzzy subset will be introduced to avoid the abrupt change in the transitions logic, while trying to extend this logic, by taking into account a hybrid gradation as dictated by the nature of certain switch elements such as the hydraulic valve. On the one hand; the combination of the two formalisms bond graph and Petri net allows the support of both quantitative and qualitative aspects of HDS. On the other hand, the pragmatic aspect of fuzzy logic makes it possible to remedy the problems of ideal and theoretical order inherent to formal modelling; Such as the mythical states engendered by the model and which had no real physical interpretation. In fact, the logical conditions are the invariants that represent the switching actions at this level. Otherwise, in the engineering domain, it is very common that a physically-based conception of the observed phenomena is necessary to derive mathematical representation. As a matter of fact, this conceptual level that is closer to the observed phenomena than the abstract mathematical description is henceforth relevant to the purpose of modelling. For this reason, we chose semantics of conformity to deal with The problem due to discrepancy between of mathematical level of abstraction and that of the physical level; where the question is not in 'how does it work?' or 'as long as it serves' but rather in the shaping and the conduct of the reasoning. It is straightforward to find that these intermediate states between On/Off are not just an artefact to highlight the detail of an abrupt passage but they can also be maintained having a sense with respect to discrete event systems modelling theory; it is therefore of concretising these states in Petri net modelling. A bi-tank model is used as a demonstration use-case.

Figure 2 Generalised system flow



We need in general to study the controlling amount of material flow in the establishment and breaking of connection by means of a transfer channel between two physical subsystems. This phenomenon is governed by boundary conditions relating to a switch element which will be assumed here as a valve for reasons of problem specification relating to the conceptual relevance of the switching element. This facilitates the characterisation of energy transfer at the physical level. We have a system for controlling the flow of a liquid F_{real}, whose objective is to decide the angle position θ of a valve which controls it according to a reference flow rate F_{ref}. The actual flow is predicted in the command and control part as shown in Figure 2.

The expression of the errors given by:

$$\begin{aligned} e(t) &= \text{reference flow (set-point)} - \text{effective flow} \\ &= F_{ref} - F_{eff}(t) \\ &\cong F_{ref} - F_{real}(t) \end{aligned}$$

We will also need another size, the variation of the rate error with respect to the time, because combined with the error variable makes it possible to know the convergence and the divergence of the real flow; i.e., if the real flow moves away or approaches the reference flow rate.

Note that the effective flow is absolutely equal to the actual flow when the valve is fully open or completely closed, the other states are relatives related to the scales of values and the control. It is up to manufacturers to define the proportionality according to the characteristics of the valve and the duct section.

$$\Delta e(t) = e(t+T) - e(t) = F_{effective}(t) - F_{effective}(t+T)$$

With T is any instant of prediction.

The convergence and the divergence of the volume flow depend on the signs of the two variables for the construction of the rules and to take action; for example if $e(t) < 0$ then the effective flow is greater than the reference flow and it is decreasing or increasing, it approaches if $\Delta e(t) < 0$.

Let the fuzzy subsets of the three linguistic variables be: PB – positive big, EZ – close to zero and NB – negative big. Let the discourse universe of the error be [–0.1 0.1], the discourse universe of the variation of the error is [–0.2 0.2] and that of the output $\Delta\theta$ is [–90 90]. Let the controller output the linguistic variable $\Delta\theta$ with the following properties:

Output: the variation of the rotation angle of the standard valve on [–90 90].

$$\Delta\theta > 0 \text{ then } F_{Real}(t) \text{ must increase}$$
$$\Delta\theta < 0 \text{ then } F_{Real}(t) \text{ must decrease}$$

The fuzzy subsets are:

PG positive big (large increase in the opening angle of the valve)

EZ close to zero (no change of angle)

NG negative big (large decrease in the opening angle of the valve)

The linguistic variables e and Δe controller inputs with the following properties:

- input1: the error e normalised on [–0.1 0.1]

  PB positive big (effective flow far below the reference flow)

  EZ close to zero (effective flow very close to reference flow)

  NB negative big (effective flow far up the reference flow)

- input2: variation of the error Δe normalised on [–0.2 0.2]

  PB positive big (effective flow goes down quickly)

  EZ close to zero (effective flow is practically constant)

  NB negative big (effective flow goes up quickly).

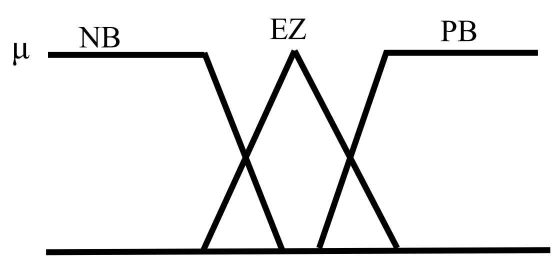
All parameters used to control flow of liquid through the valve are now determined. We give the different inference rules that serve to the construction of the decision matrix.

Table 1 Fuzzy decision matrix

Δe \ *e*	*NB*	*EZ*	*PB*
NB	NB	NB	EZ
EZ	NB	EZ	PB
PB	EZ	PB	PB

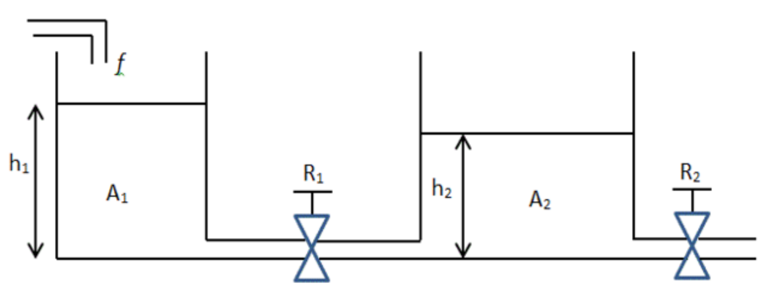
The membership functions of the three linguistic variables are trigonometric and trapezoid having the same form:

Figure 3 General form of the membership function



We will apply this approach on a didactic example given in Fichou (2004) illustrated in Figure 4, more precisely on the valve 1. The gateway between the two tanks, on the time interval [0 200] corresponds to the first configuration ON-ON. The system is composed by a first tank with a section A1 supplied with a volume flow ***f***, at the outlet of this tank a valve to let and prevent the passage of fluid to a second tank of section A2. Another valve is placed at the outlet of this second tank to empty it.

Figure 4 Use case of bi-tank system (see online version for colours)



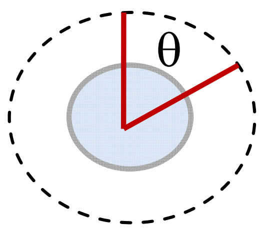
The parameters used in this example are: R1 = 10, R2 = 20, C1 = 1, C2 = 2, f = 0.1.

To be able to calculate the new effective flow rate after defuzzification, we propose the following formula, which is meant to be proportional, and the instants of prediction depend on the instants of integration, so variable. This depends on the characteristics of the valve as well as the duct section; i.e., objectives and specifications of manufacturers. We assume that this type of valve changes from the on state to the off state by a full rotation; in other words to 360° and vice versa. We will also assume that the duct is of cylindrical type. The given formula can also be used for other purposes. According to the prediction time *T*, the new real volume flow is given as follows:

$$flow(t+T) = f_3(t+T) - \frac{\left|\sum_0^i \theta_i\right|}{360} f_3(t+T)$$

With $\left|\sum \theta i\right|$ the closing angle and *i* iterations related to prediction instants; and f_3 is the predictive volume flow rate.

Figure 5 Real disc, imaginary disc and rotation angle (see online version for colours)



To explain, In that case, which mainly depends on the specifications of the manufacturers, the same angle of rotation is made between the disc of the supposed section of cylindrical type and an imaginary disk made by the rotation of the valve that makes at each moment of time the proportionality $k = \frac{f_{eff}}{f_{pred}}$ where $k_i = 1 - \frac{\left|\sum_0^i \theta_i\right|}{360}$ is always verified, with respect to the scales of values (note that 360° is equivalent to 2π in radian).

We demonstrate for each opening/closing angle, that there is a disk (imaginary) equivalent to the area section created by this angle:

Let *d* be the diameter of the duct section. Let $\theta = \left|\sum_0^i \theta_i\right|$ be the closing angle, the opening angle θ' is dual: $\theta' = 360 - \theta$.

1 Closing angle:

$$A = \frac{\frac{\pi\theta}{180}\left(\frac{d}{2}\right)^2}{2} = \pi\left(\frac{d_1}{2}\right)^2 \Rightarrow \pi d_1^2 = \frac{\theta}{360} d^2$$

$$\Rightarrow d_1 = \sqrt{\frac{\theta}{360}} d \in \mathbb{R}^+ \ldots (\alpha)$$

Thus: $\forall t, \forall \theta \in [0\ 360], \exists d_1 \in \mathbb{R}^+ : d_1 = \sqrt{\frac{\theta}{360}} d$ a disk.

2 Opening angle:

$$A' = \frac{\left(2\pi - \frac{\pi\theta}{180}\right)\left(\frac{d}{2}\right)^2}{2} = \pi\left(\frac{d_2}{2}\right)^2 \Rightarrow \pi d_2^2 = \left(1 - \frac{\theta}{360}\right) d^2$$

$$\Rightarrow d_2 = \sqrt{1 - \frac{\theta}{360}} d \in \mathbb{R}^+ \ldots (\beta)$$

cause $0 \le 1 - \frac{\theta}{360} \le 1 \quad \forall \theta : 0 \le \theta \le 360.$

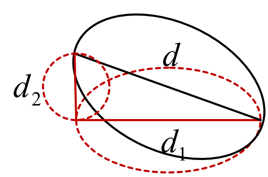
Thus: $\forall t, \forall \theta \in [0 \quad 360], \exists d_2 \in \mathbb{R}^+ : d_2 = \sqrt{1 - \frac{\theta}{360}} d$ a disk.

From the two relationships (α)and (β)we conclude:

$$\forall t, \forall \theta \in [0 \quad 360], \forall d \in \mathbb{R}^+, \exists d_1, d_2 \in \mathbb{R}^+ : d^2 = d_1^2 + d_2^2$$

It Is the Pythagorean relationship in a right triangle. d_1, d_2 are flexible and express relativity with respect to d. We schematise this relationship as shown in Figure 6.

Figure 6 Duality and relationship between real and possible disks (see online version for colours)



Where the solid line represents the real dimension and the dotted line represents the possible dimensions (imaginary discs) in duality.

For the extended model, we give a new definition of the actual value model that uses the original whose values are trivial. As a matter of fact, a case of specific individual switches from witch the composed mode transition system (MTS) is to be derived is needed for generalising of such concept, we now formalise the new concept of the hydraulic valve on the PSTS proposed by Edstrom et al. (1996):

A PSTS is a three-tuple <*M*, *T*, *Q*> Where:

If the initial state is ON then θ_0: = 0; If the initial state is off then θ_0: = –360.

1 Q is a hybrid model structure ($l, Z, U, \mathcal{M}, G, \mathcal{A}$) where:

- $\{l_\varepsilon, l_{F1}, l_{F2}, l_{F3}, \ldots, l_{Fn}\}$ is a set of states where lF_i is determined by the closing angle which is also determined by θ_i the angle of rotation
- U is a set of external real-valued (independent) input variables
- $Z = \{e, f\} \cup U$ where the variables e and f are the port variables of the primitive switch itself

- $\mathcal{M} = \left\{ \{e := 0\}, \left\{ f_i := \left(1 - \frac{\left|\sum_0^i \theta_i\right|}{360}\right) f_i' \right\} \right\}$ where f_i is the effective value, f_i' is the predictive value and θ_i the rotation angle $\left(\left|\sum \theta_i\right| \text{ the closing angle}\right)$ with $i = 0, 1, \ldots n$.
- $G = \{g_{\varepsilon F_n}(Z), g_{F_i F_{i+1}}(Z'), g_{F_n \varepsilon}(Z'')\}$ where $Z = (z_1, \ldots, z_k)^T$, $Z' = (Z_1', \ldots, Z_k')^T$, $z'' = (z_1'', \ldots, z_k'')^T$ are vectors composed by $z_1, \ldots, z_k, z_1', \ldots, z_k', z_1'', \ldots, z_k'' \in Z$.
- $\mathcal{A}$ is a set of initialisation values of a new state are the final values of the last states

2 $M = \{m_\varepsilon, m_{F1}, m_{F2}, \ldots, m_{Fn}\}$ where:

$$m_\varepsilon = (l_\varepsilon, \{e := 0\})$$

$$m_{Fi} = \left(l_{Fi}, \left\{ f_i := \left(1 - \frac{\left|\sum_0^i \theta_i\right|}{360}\right) f_i' \right\} \right)$$

with $i = 0, 1, \ldots, n$

3 $T = \{T_{\varepsilon F_n}, T_{F_i F_{i+1}}, T_{F_n \varepsilon}\}$ where:

$$T_{\varepsilon F_n} = (m_\varepsilon, m_{Fn}), g_{\varepsilon F_n}(Z)$$
$$T_{F_i F_{i+1}} = (m_{Fi}, m_{Fi+1}), g_{F_i F_{i+1}}(Z')$$
$$T_{F_n \varepsilon} = (m_{Fn}, m_\varepsilon), g_{F_n \varepsilon}(Z'')$$

We can do the same reasoning on the On state instead of the Off state. In order to illustrate the expression of closing/opening angle sampled by the controller and the relationship between them, the following algorithm presented in pseudo-code gives an idea about this calculation with the actual rotation angle where the instant of prediction is an array of increasing values:

```
Begin
if state = 'Off' then
    opening-angle = 0
else
    opening-angle = 360
endif
i = 1
test = prediction-instant (i)
while (test< = max(prediction-instant)) repeat
    closing-angle (i + 1) = opening-angle (i) + actual-angle (i)
    if opening-angle> = 360 then
        state = 'On'
        opening-angle = 360
```

```
        else
            if opening-angle< = 0 then
                state = 'Off'
                opening-angle = 0
            endif
        endif
        i = i+1
        test = prediction-instant(i)
    endwhile
    closing-angle = 360-closing-angle
    end
```

The new PSTS schema becomes as Figure 7 with the causal assignment where μ is a Boolean state variable and α an arbitrary label.

Figure 7 New states of SW-element

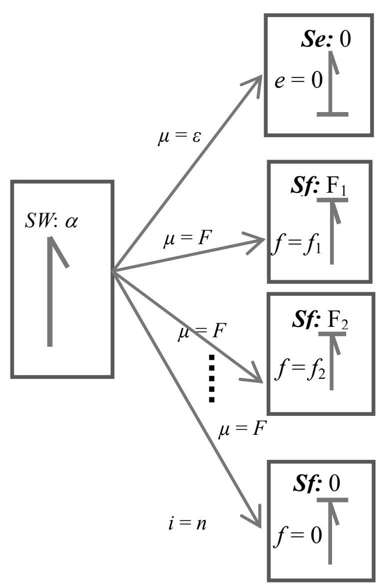
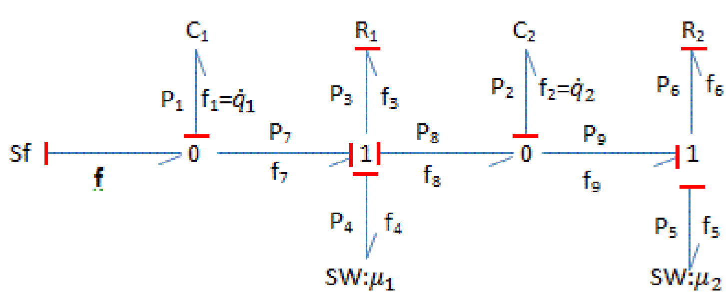


Figure 8 Bond graph model of the system (see online version for colours)



The changes that are related to switch state changes are external controls and correspond to changes in the dynamics of the state variables. To highlight these phenomena we are going to assume that at time t = 200, the valve 2 will be closed completely, at time t = 400 the valve1 will be closed completely, at time t = 600 the valve2 will be completely opened. In this example, we have assumed that both tanks are of unlimited heights, we have not considered the overflow effect. The bond graph representation of the first configuration of the On-On system (both valves are completely open) of the example (Fichou, 2004) is:

Note that the presence of the two switches μ_1 and μ_2 as physical elements is due to the ideal switch approach that we adopt. We denote by f_i the flow variable, and by p_i the pressure variable. After development, we get the following state-space model that will serve as a prediction model:

$$\begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1 C_1} & \frac{1}{R_1 C_2} \\ \frac{1}{R_1 C_1} & -\frac{R_1 + R_2}{R_1 R_2 C_2} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} + \begin{bmatrix} f \\ 0 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} f_1 \\ f_2 \\ f_4 \\ f_5 \\ f_7 \\ f_8 \\ f_9 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_3 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \end{bmatrix} + \begin{bmatrix} f \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} p_3 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_7 \\ p_8 \\ p_9 \end{bmatrix} \tag{3}$$

A state-space construction is needed to capture the overall discrete dynamic behaviour states of the system. Thus, the model of this system can be viewed as a P-temporal Petri net. At the level of the command and control part; in addition to the external control, the set-point can also control an autonomous switching.

The transitions t_4, t_5, t_6, t_7, t_8, t_9 and t_{10} correspond to the prediction instants (integration). In this study, we did not focus on the systematic composition of the whole model since we rely only on external switching; otherwise there could have been direct transitions between for example the places p_0, p_2 and between p_1, p_3 if we change configurations.

Figure 9 P-temporel Petri net of the system (see online version for colours)

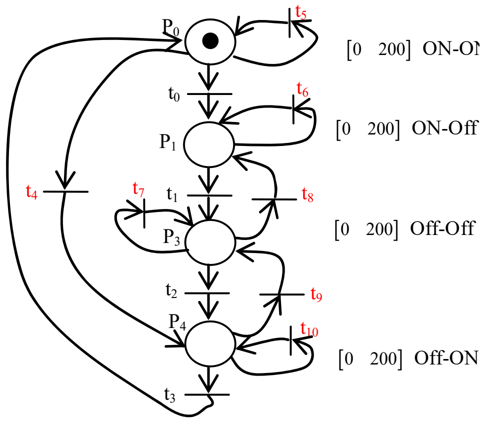
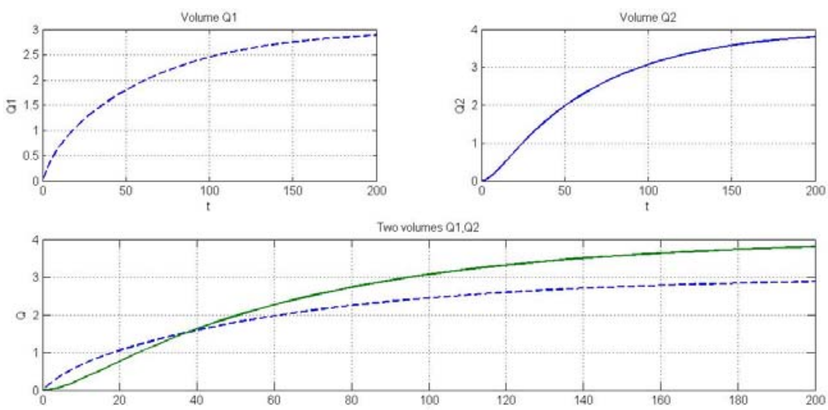
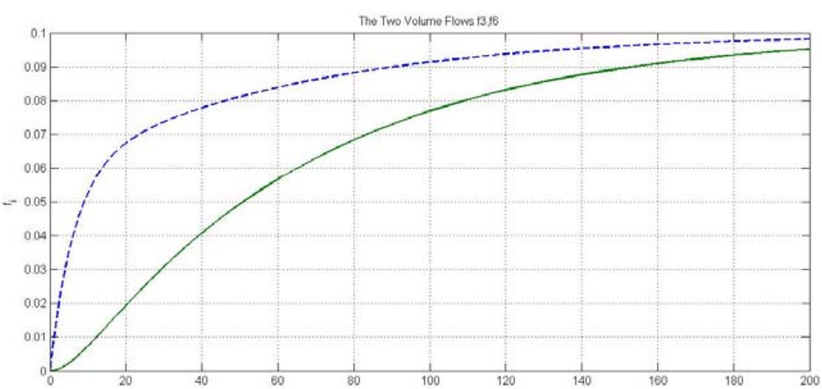


Figure 10 (a) Volumes (heights) in both tanks when valve1 is fully open (b) Volume flow f3 (--) when valve1 is fully open (see online version for colours)



(a)



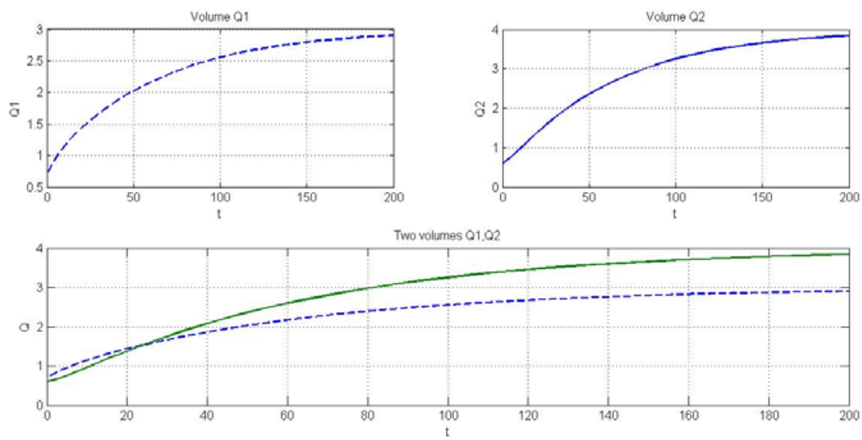
(b)

3.1 Real time simulation results

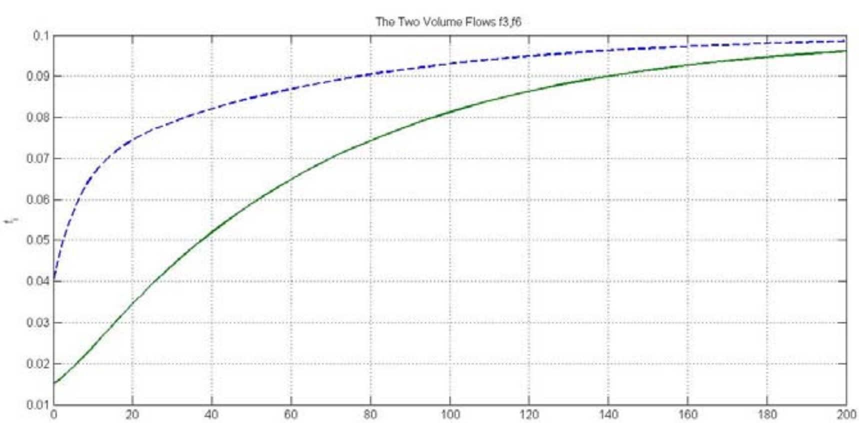
Aim to achieve the subject of our study which consists to highlight the intermediate states as well as the simulation of its dynamics, based on the system of state model equations (1), (2) and (3) presented above. First, we will first make sure of the straightness and performance of our development and simulation (on Matlab software) compared to the results provided in Fichou (2004). For the same initial values, i.e., the two tanks initially assumed to be empty $h_1 = h_2 = 0$ we obtain:

So it is clear graphically that we got the same results. To be able to highlight the efficiency of simulation, let's now suppose that initially the two tanks are at respective heights h_1 = 70 cm, h_2 = 30 cm. Let us take also an example of set-point at the value cons = 0.062339147815353 m^3/s. We give the new volume flow results that are close to the set-point as they appear in Figure 11(c) with the black colour.

Figure 11 (a) Predictive volumes (heights) in both tanks: Tank1(--) (b) The predictive volume flow f3 (--) of valve1 (c) Old rate (–), new rate (--) and the set-point (in green) (see online version for colours)

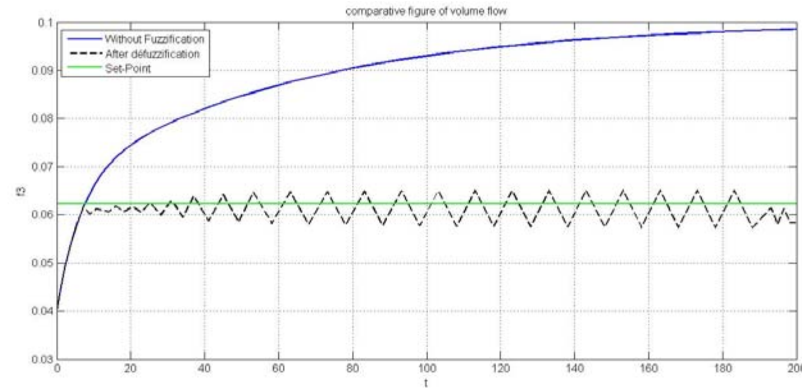


(a)



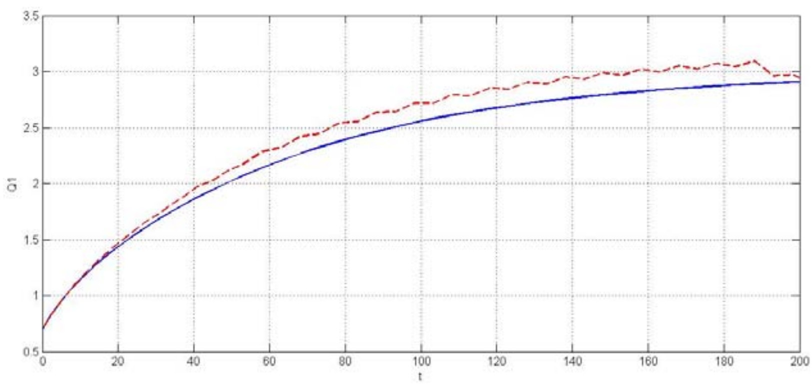
(b)

Figure 11 (a) Predictive volumes (heights) in both tanks: Tank1(--) (b) The predictive volume flow f3 (--) of valve1 (c) Old rate (–), new rate (--) and the set-point (in green) (continued) (see online version for colours)

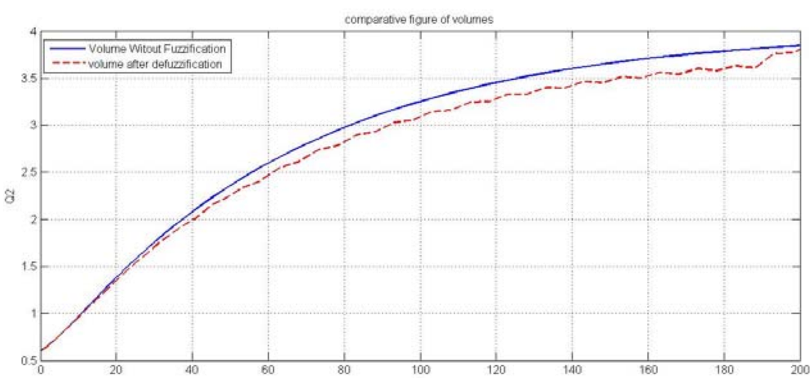


(c)

Figure 12 (a) Tank 1: old volume (–), new volume (--) (b) Tank 2: old volume (–), new volume (--) (see online version for colours)



(a)



(b)

The effective values obtained after deffuzzification in comparison with the predictive values whose t prediction times are given in the following table, as well as the values of the effective volume flow and the rotation angles in degrees. Table 2 shows some examples only.

Table 2 Some values of rotation angle and volume flow with their integration instants

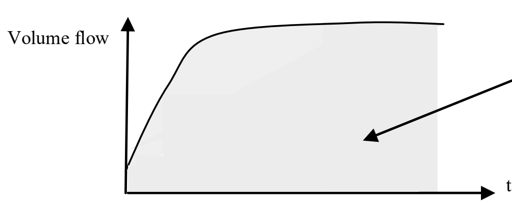
*t = 1.0e+002**	*Volume flow =*	*Angle =*
0.0175832050	0.0473897833	0
0.0721563574	0.0617077625	0
0.1416893779	0.0606509376	–8.8440453880
0.2053612896	0.0616872338	–0.2583725173
0.2512679815	0.0625273853	5.5957650009
0.2809269398	0.0599212016	–16.5988470678
0.4102480090	0.0587043704	–26.8150700052
0.5312805918	0.0646554563	23.2010211977
0.6312805918	0.0647498317	24.1844534941
1.3812805918	0.0574460602	–29.3499964929
1.9312805918	0.0614319403	14.8413896207
1.9828201479	0.0583711039	–10.7365217374

We can give also the curves of the two effective volumes after defuzzification in comparison with the predictive volumes without fuzzification-based bond graph. For the first tank we give exact results in Figure 12(a) but those about the second tank [Figure 12(b)] are just estimates.

3.2 Discussion and analysis

Note that this approach is able of reaching the entire region between the abscissa axis and the predictive volume flow curve if the set-point value is varied between min (volume flow) and max (volume flow). This gives a space of real reachable consistent states for the specific case of a valve, as illustrated in Figure 13.

Figure 13 Space of possible reachable consistent states



We have:

$$\forall i \in \mathbb{N} : 0 \leq \frac{\left|\sum_0^i \theta_i\right|}{360} \leq 1 \Rightarrow -1 \leq -\frac{\left|\sum_0^i \theta_i\right|}{360} \leq 0 \Rightarrow \leq 1 - \frac{\left|\sum_0^i \theta_i\right|}{360} \leq 1$$

We have also for the predictive value:

$$\forall t \in \mathbb{R}^+ : f_3(t) \geq 0 \Rightarrow 0 \leq \left(1 - \frac{\sum_0^i \theta_i}{360}\right) f_3(t) \leq f_3(t)$$

With respect to the demonstration given in the previous subsection (Figure 5); for each angle position (open/close); there is an equivalent disk whose area section is exactly the one created by this angle. This approach has also given good results in the case of autonomous switching that can occur in the system.

4 Reachable state space

Another important advantage of the presented approach compared to existing approaches is in avoiding the use of sensors using two predictive/effective notions, thus allowing a gain of cost and energy. What this means for large-scale facility is not difficult to imagine; but this advantage remains dependent on certain industrial characteristics of the physical switch element and the objectives of the industrial manufacturers where the proposed formula can always be modified. The problem presented may serve as a typical problem to generalise on others much more complex. It should work perfectly for a functional modelling of dynamics in nominal cases.

Typically, a P-temporal Petri net is thus obtained by associating at each place a minimum and maximum residency time constraint for the tokens. While t-temporal Petri nets are particularly well suited to describe mechanisms triggering when time constraints are violated, like mechanisms triggered by watchdogs. P-temporal Petri nets are well suited to describe a set of events that must absolutely check a set of temporal constraints (Cardoso et al., 2005). This work can therefore be used primarily as model-based diagnosis to automatically diagnose faults in the case of leaks or any other anomaly relating to volume flow. Indeed, depending on the purpose of modelling it is also interesting to model the system as a t-temporal Petri net. We demonstrate this possibility.

In general, a constraint on a quantity of material flow (in this specific case relating to the hydraulic domain is the volume) noted $c(q_i)$ can be expressed by a time constraint over a time interval i of length T; since they are related by mathematical equation via the predictive model. We have:

$$q(t) = q(0) + \int_0^t f_3(t)dt \quad f_3$$

The new effective volume flow

$$q(t+T) = q(0) + \int_0^{t+T} f_3(t)dt = q(0) + \int_0^t f_3(t)dt + \int_t^{i+T} f_3(t)dt$$

We obtain so: $q(t+T) - q(t) = \int_t^{t+T} f_3(t)dt$

Let: $q_i = q(t + T) - q(t)$.

We have also: $\forall t \in \mathbb{R}^+ : f_3(t) \geq 0 \Rightarrow q(t) = \int f_3(t)dt$ is an increasing function on $\mathbb{R}^+$.

We avoid the case where $f_3(t) = 0$, this is when the valve is completely closed (let the set of instants is R_1) or the two tanks become empty (let the set of instants is R_2) or during an autonomous switching (let the set of instants is R_3). Out of these instants the function $q(t)$ is strictly increasing. That is to say:

$\forall t \in \mathbb{R}^+ - \{R_1 \cup R_2 \cup R_3\} : f_3(t) \succ 0$ and the function $q(t)$ is strictly increasing.

$$\forall T', T \in \mathbb{R}^+ - \{R_1 \cup R_2 \cup R_3\} : T' \neq T :$$

1 $T' \succ T : q(t+T') \succ q(t+T) \Rightarrow q(t+T') - q(t) \succ q_i$

2 $T' \prec T : q(t+T') \prec q(t+T) \Rightarrow q(t+T') - q(t) \prec q_i$.

Hence $c(i) \Rightarrow c(q_i)$, this explains that for each interval i there is exactly one quantity q_i. By the same reasoning if we fix the starting instant t_i for each interval:

$$\forall q_i, q_i' \in \mathbb{R}^+ : q_i \neq q_i$$

1 $$q_i' \prec q_i : q(t_i + T') - q(t_i) \prec q(t_i + T) - q(t_i) \Rightarrow q(t_i + T') \prec q(t_i + T) \Rightarrow T' \prec T$$

2 $$q_i' \succ q_i : q(t_i + T') - q(t_i) \succ q(t_i + T) - q(t_i) \Rightarrow q(t_i + T') \succ q(t_i + T) \Rightarrow T' \succ T.$$

So we get $c(q_i) \Rightarrow c(i)$ the implication in the opposite direction, hence the equivalence. That is to say for each quantity of flow related to a fixed starting instant t_i of an interval there is one and only one length of interval T that satisfy constraint on this quantity of flow. For example, if we wait for a quantity of flow q_i then we detect a different value q_j, this necessarily corresponds to a time constraint violation. In the case where the state is not feared one can envisage a temporary solution:

- if $q_j \succ q_i$, if possible, decrease the angle of rotation
- if $q_j \prec q_i$, if possible, increase the angle of rotation.

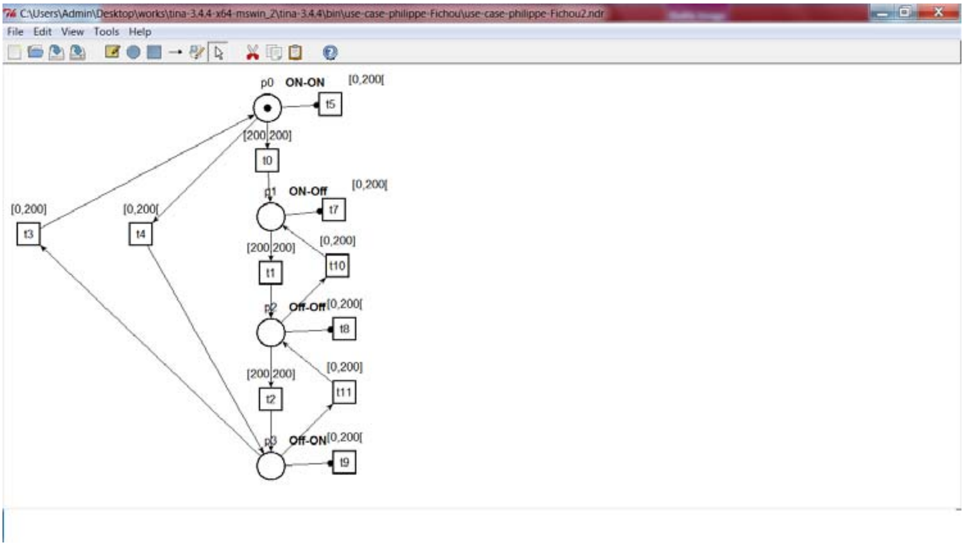
The following TPN of the system is given in Figure 14(a) using time net analyser (TiNA) tool in order to apply an automatic analysis and verification; such as the reachability analysis given in Figures 14(b) and 14(c); structural analysis as shown in Figure 14(d).

The reachability analysis in the figure shows 9 states and 20 transitions. If we apply it with essential states without delay it gives 803 states and 162,400 transitions. But if we choose coverability graph or partial graph with covering steps; or marking graph we get 4 states and 11 transitions with bounded, live and reversible TPN.

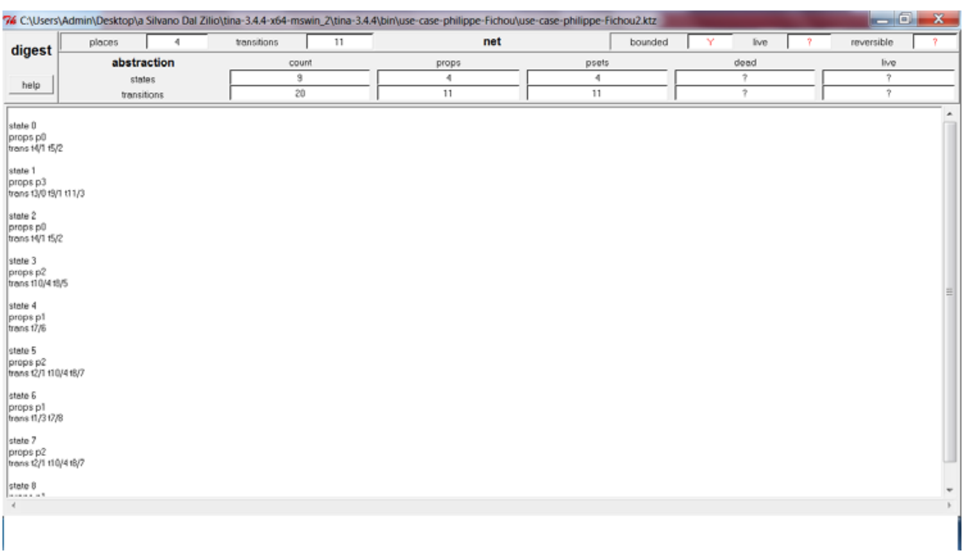
In the case of the essential states, the results shown do not correspond only to the real states but to the possible states according to the temporal constraints; which will make it possible to grant the solvers flexibility to choose any possible prediction instants inside the transitions time intervals.

The structural analysis shows an invariant of places: $p_0 + p_1 + p_2 + p_3 = 1$ and a set of consistent transitions: t_9, t_8, t_7, t_5, t_3 t_4, t_1 t_{10}, t_0 t_1, t_2, t_3, t_{11}, t_2.

Figure 14 (a) T-temporal Petri net of the bi-tank system (b) Reachability analysis with state class preserving (c) Reachability analysis with essential states (d) Structural analysis with semi-flows (see online version for colours)

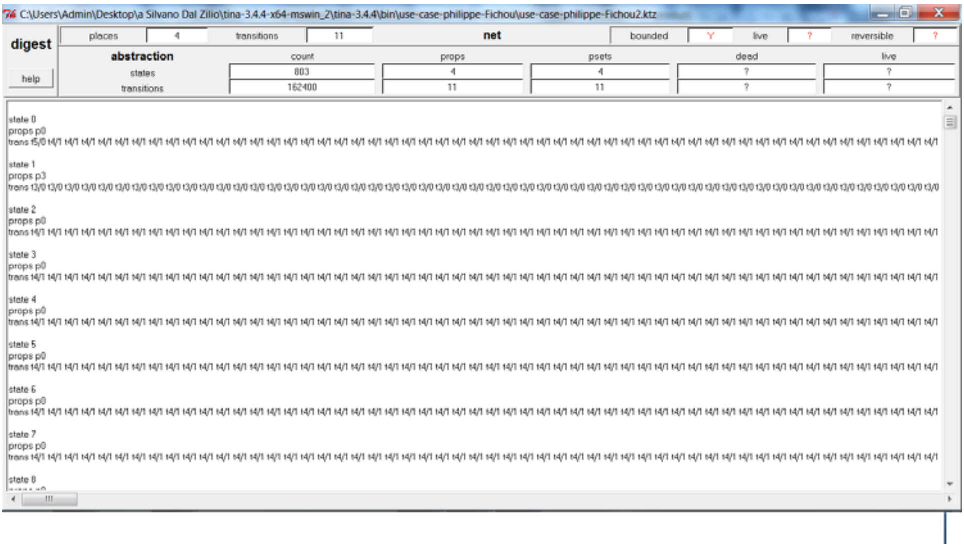


(a)

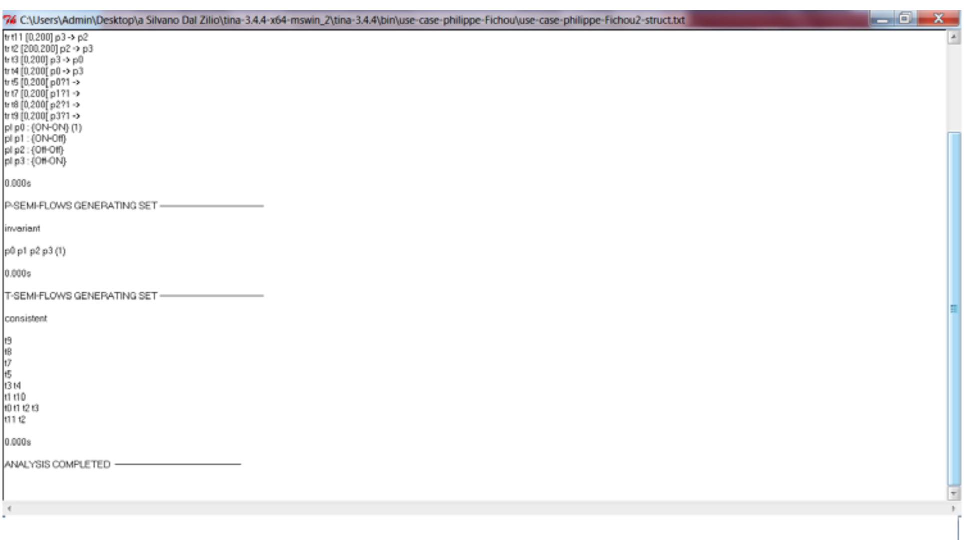


(b)

Figure 14 (a) T-temporal Petri net of the bi-tank system (b) Reachability analysis with state class preserving (c) Reachability analysis with essential states (d) Structural analysis with semi-flows (continued) (see online version for colours)



(c)



(d)

4.1 Discussion and analysis

Let Δq be a quantity of flow such that:

$$\Delta q = |q_{eff} - q_{real}|$$

In the case of abnormality, this difference is non-zero. Referring to the discussion in the previous section of Figure 13, there is still f_3 which satisfies this quantity, where the region between the abscise axis and its plot corresponds precisely to this difference. This is interpreted by a difference in the angle of rotation with respect to its nominal case:

Let f_3 the predictive volume flow

$$\Delta q = \left| q_{eff} - q_{real} \right| \neq 0 \Rightarrow \left| \int_0^t \left(1 - \frac{\sum_0^i \theta}{360} \right) f_3(t)dt - \int_0^t \left(1 - \frac{\sum_0^i \theta'}{360} \right) f_3(t)dt \right| \neq 0$$

$$\Rightarrow \left| \int_0^t \left(\frac{\sum_0^i \theta' - \sum_0^i \theta}{360} \right) f_3(t)dt \right| \neq 0$$

must be necessarily different where f_3 is the predictive volume flow.

Thus, there are always rotation angles (events) which can fill this difference in quantity, in other words, there is always a succession of angles (open/close) equivalents in the nominal case that satisfy this difference (where in the case of an excess of quantity of flows $q_{eff} \prec q_{real}$ we take its absolute value and we realise the inverse action; increase instead of decreasing and vice versa). The question that arises: over a time interval, [0 200] for example in this case study: how many successions of possible events exist that can give this quantity. In other words; graphically, how many f_3 function plots are possible whose quantity in the region between the abscissa axis and f_3 is the same? According to the discussion in the previous section, there are more than f_3 possible functions.

Finally, we think that the problem of discrepancy between the level of mathematical abstraction and that of the physical level is recursive and goes back to its origins; not historical but rather the metamodelling level is of paramount importance. Philosophy becomes quite clear when considering ideal and practical approaches in the literature where the concepts overlap each other. Consequently it makes them with time disparate and often conflicting; also the poorly known areas make the classification very hard to grasp. This is not to be confused with autonomy but if this reciprocity is well defined and carried out, it can be very useful for designers but not a panacea. In some context; we need to reconcile the two trends.

5 Conclusions

We have extended the STS of the classical bond graph switch concept (in specific case of valve) by the fluidification of its on/off states; real states which is exhibited in the model by a quantisation using the proposed formula, while adopting two notions of predictive values and effective values; we did not violate the syntax and semantics of the bond graph theory as well as that of the Petri net theory. We have also presented in pseudo-code an algorithm giving an idea about the calculation of the opening/closing angle. The resulting states are intermediate between on and off (completely on and completely off) states without the use of sensors and can still be maintained with sense in Discrete event systems modelling theory as well or in HDS; in the contrary, they are consistent transient reachable states. We have demonstrated the effectiveness of this approach on a didactic simulation example avoiding the use of sensors for the nominal case compared to the only one Petri net tool-based approaches. Although these results remain valid in the specific case of the switching valve; the approach can nevertheless be generalised for other application fields.

To subsume the class of standard PN, We proposed two variants of TPN. First, a p-temporal Petri net for the nominal case. Second, a T-temporel Petri net intended to diagnosis task where we gave a brief demonstration on the correspondence between a constraint on the quantity of flow and a constraint on the time interval.

The expressive power of the obtained hybrid model shows the usefulness of the approach both in academia and in industry that we have discussed the model complexity from different angles. The approach can serve to design engineering systems that will exhibit high degree of autonomy in performing useful tasks. As a perspective, we shall apply this method for more complex heterogeneous systems, and for large-scale diagnosis.

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