Development of Inventory model for inventory induced demand and time-dependent holding cost for deteriorating items under inflation

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Abstract: In the last few years, researchers have established their economic order quantity (EOQ) models considering both demand rate and selling price as constant. In actual practice, it is only possible in a growth stage. Demand is a function of stock-level or time for high-tech products. The research proposed here has considered a model of deteriorating items with inventory induced demand and inflation. Holding cost is assumed to be time induced and selling price is exponential time sensitive. We then developed a mathematical model to obtain total profit. Approximate optimal solution is also discussed. Numerical example and sensitivity analysis are presented indicating effects of change in several elements. Second order approximations are used for exponential terms in case of low deterioration rates.

Keywords: selling price; inflation; holding cost; stock-sensitive demand; profit.

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1 Introduction

Inventory control policy is mainly dependent on several companies which are the backbone for developing a country. Most of the industries found that they can obtain more advantages by running long-term relationship between vendor and buyers. The coordination between supplier and retailer is a promotional tool to achieve maximum profit of the system. Large number of inventory modellers considered that the demand is always constant in all circumstances. In real world, demand rate fluctuates from place to place and situation to situation. The advertisement of any type of commodity motivates customers to buy more. Levin et al. (1972) established an economic order quantity (EOQ) model stating that large piles of customer items displayed in a supermarket attract the customer to purchase more. Silver and Peterson (1978) considered that sales at the retail level are proportional to stock-displayed. Giri and Chaudhuri (1998) proposed a generalised EOQ – model for a damageable commodities that the demand rate is a function of the on-hand inventory. Min et al. (2010) considered a lot-sizing model for spoiling items with a current – stock dependent demand and trade credits. Liao (2000) proposed EOQ models for stock-induced demand. Sana and Chaudhuri (2008) considered EOQ model with present-inventory-dependent demand rate in which a supplier provides a buyer's permissible delay time and price discount on the purchase of merchandise. Soni and Shah (2008) designed optimal ordering policy when demand is stock-sensitive. Goyal and Chang (2009) provided an ordering-transfer inventory model to determine the retailer's optimal order quantity and the number of transfer per order from the warehouse to the display space. Goyal and Chang (2009) also discussed an inventory model in which the amount of display space is limited for which demand rate depends on the current stock-level. Mandal and Phaujdar (1989) established a production inventory model for deteriorating items with linearly stock-dependent demand under uniform rate of production. Bar-Lev et al. (1994) established an EOQ model for stock induced demand. Other related articles based on stock-induced demand are: Gerchak and Wang (1994), Ray and Chaudhuri (1997), Pal and Chandra (2014), Padmanabhan and Vrat (1995), Datta and Pal (1990), Roy and Maiti (1998), Chang and Feng (2010), Tripathi and Kumar (2014), Soni (2013), Chang et al. (2010) and Teng et al. (2011).

Large number of research papers has been published on inventory models of deteriorating products such as fashion goods, volatile liquids, vegetables, electronic parts. Ghare and Schrader (1963) considered an EOQ model for exponentially decaying item. Misra (1975) pointed out an EOQ model with Weibull deterioration for perishable items. Dave and Patel (1981) provided an inventory model for deteriorating items with time-induced demand. Sicilia et al. (2014) provided a deterministic model for commodities with a constant deterioration rate and time induced demand. Wang et al. (2014) pointed out "an EOQ model for a seller by incorporating the following facts deteriorating products deteriorate continuously and have their maximum lifetime and credit period increases not only demand but also default risk." Several related research papers of deteriorating products can be found in Atici et al. (2013), Liao et al. (2008), Yang et al. (2010), Skouri et al. (2012), Bakker et al. (2013) and others.

Various inventory models have established that inflation induces in most of the businesses. Buzacott (1975) was first who presented an EOQ model for inflationary effects. Tripathi et al. (2011) generalised an EOQ model for non-deteriorating items under inflation. Jaggi and Aggarwal (1994) considered a model on credit financing in economic ordering policies of deteriorating items using discounted cash flow (DCF) approach. Tripathi (2013) and Shukla et al. (2015) considered effect of inflation in their inventory models. Chang (2004) established an EOQ model for deteriorating items with inflation in which seller provides the customer a trade credits if the buyer orders a large quantity.

2 Assumption and notations

- The demand rate is a function of inventory level, i.e., $D\{I(t)\} = \alpha + \beta I(t)$, $\alpha > 0$, $0 < \beta < 1$.
- $P(t)$ = pert is selling price/unit at time t.
- r is inflation rate.
- $h_1(t) = h.t$, is time dependent holding cost, where h is holding cost parameter and is greater than zero.
- A and C are ordering and unit purchase costs respectively.
- T is cycle time.
- θ is deterioration rate, $0 < \theta < 1$.
- Q is lot size.
- $TP(T)$ is total profit/cycle time.

Under these assumptions, we first formulate a mathematical model with optimal solution. Next, some important managerial implications were derived to distinguish the optimal solution. Sensitivity analysis and their inferences were also made.

3 Model development

Inventory model is developed considering stock-dependent demand for deteriorating items. Inventory level gradually goes down due to demand and partial due to deterioration. The inventory equation of state in [0, T] is

$$\frac{dI(t)}{dt} = -\theta I(t) - \alpha - \beta I(t) \tag{1}$$

with the condition

$$I(T) = 0. \tag{2}$$

Solution of equation (1) is discussed in Appendix B.

Total cost contains the following components:

1 Ordering cost

$$OC = A \tag{3}$$

2 The sales revenue

$$SR = \int_0^T pe^{rt}\{\alpha + \beta I(t)\}\,dt = \frac{p\alpha}{(\theta+\beta)}\left\{\frac{\theta\left(e^{rT}-1\right)}{r} + \frac{\beta\left(e^{rT}-e^{(\theta+\beta)T}\right)}{r-\theta-\beta}\right\} \tag{4}$$

3 The deterioration cost

$$CD = C\left[Q - \int_0^T \{\alpha + \beta I(t)\}\,dt\right] = \frac{C\theta\alpha}{(\theta+\beta)}\left\{\frac{e^{(\theta+\beta)T}-1}{\theta+\beta} - T\right\} \tag{5}$$

4 The holding cost

$$HC = \int_0^T h.t.I(t)dt = \frac{h\alpha}{\theta+\beta}\left\{-\frac{T}{\theta+\beta} + \frac{e^{(\theta+\beta)T}-1}{(\theta+\beta)^2} - \frac{T^2}{2}\right\} \tag{6}$$

$$TP(T) = \frac{1}{T}\{SR - (OC + CD + HC)\} \tag{7}$$

Differentiating equation (7) w.r.t. *T*, we get

$$\begin{aligned}\frac{dTP(T)}{dT} = &\frac{1}{T}\left[\frac{p\alpha}{\theta+\beta}\left\{\theta e^{rT} + \frac{\beta}{r-\theta-\beta}\left(re^{rT} - (\theta+\beta)e^{(\theta+\beta)T}\right)\right\}\right.\\ &\left.-\frac{C\alpha\theta}{\theta+\beta}\left(e^{(\theta+\beta)T}-1\right) - \frac{h\alpha}{\theta+\beta}\left\{-\frac{1}{\theta+\beta} + \frac{e^{(\theta+\beta)T}}{\theta+\beta} - T\right\}\right]\\ &-\frac{1}{T^2}\{SR - (OC + CD + HC)\}\end{aligned} \tag{8}$$

The maximum value of *TP*(*T*) is maximum is obtained by solving
$\frac{dTP(T)}{dT} = 0,$ provided $\frac{d^2TP(T)}{dT^2} < 0$ (see Appendix A)

$$\begin{aligned}\frac{d^2TP(T)}{dT^2} = &\frac{1}{T}\frac{p\alpha}{\theta+\beta}\left[\theta re^{rT} + \frac{\beta}{r-\theta-\beta}\left\{r^2e^{rT} - (\theta+\beta)^2 e^{(\theta+\beta)T}\right\}\right]\\ &-\frac{1}{T}\left\{C\alpha\theta e^{(\theta+\beta)T} + \frac{h\alpha}{\theta+\beta}\left(e^{(\theta+\beta)T}-1\right)\right\}\end{aligned} \tag{9}$$

From equation (9) $\frac{d^2TP(T)}{dT^2} < 0,$ if,

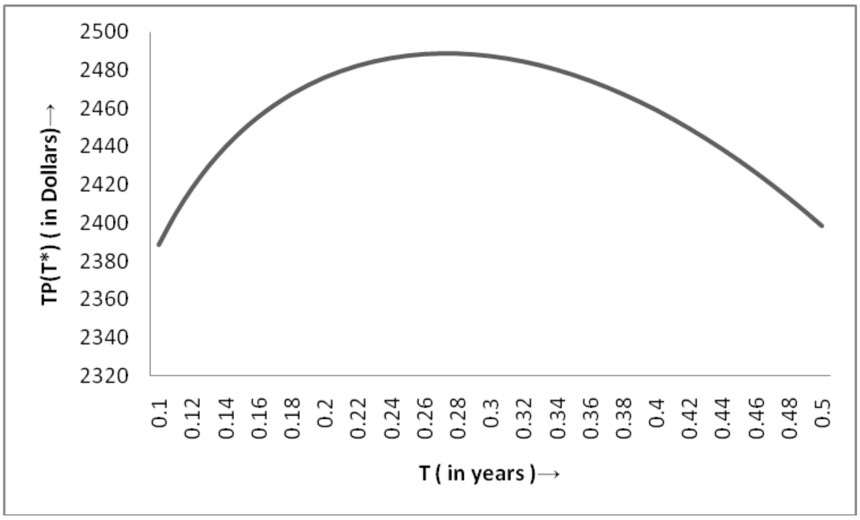
$$\begin{aligned}&p\alpha\left[\theta re^{rT} + \frac{\beta}{r-\theta-\beta}\left\{r^2e^{rT} - (\theta+\beta)^2 e^{(\theta+\beta)T}\right\}\right]\\ &< \left\{C\alpha\theta(\theta+\beta)e^{(\theta+\beta)T} + h\alpha\left(e^{(\theta+\beta)T}-1\right)\right\}\end{aligned}$$

This shows that total profit is concave function with respect to time which can be easily seen by the following graph:

Putting $\frac{dTP(T)}{dT} = 0,$ we get

$$\frac{p\alpha T}{\theta+\beta}\left\{\theta e^{rT}+\frac{\beta}{r-\theta-\beta}\left(re^{rT}-(\theta+\beta)e^{(\theta+\beta)T}\right)\right\}-\frac{C\alpha\theta T}{\theta+\beta}\left(e^{(\theta+\beta)T}-1\right)$$
$$-\left\{SR-(OC+CD+HC)\right\}=0 \tag{10}$$

Figure 1 *TP*(*T**) (in dollars) and cycle time *T* (in years)



For low deterioration rates $\theta << 1$, therefore truncated Taylor's series approximations are used for exponential terms for finding closed form optimal solution. The exponential terms can be approximated as $e^{(\theta+\beta)T}=1+(\theta+\beta)T+\frac{(\theta+\beta)^2T^2}{2}$ etc. equation (10) reduces to

$$p\alpha T\left\{1+(r+\beta)T+\frac{r(r+\beta)T^2}{2}+\frac{\beta(\theta+\beta)T^2}{2}\right\}-C\alpha\theta T^2\left\{1+\frac{(\theta+\beta)T}{2}\right\}$$
$$-\frac{h\alpha T^3}{2}\left\{1+\frac{(\theta+\beta)T}{3}\right\}-p\alpha T\left\{1+\frac{(\beta+r)T}{2}+\frac{r(\beta+r)T^2}{6}+\frac{\beta(\beta+\theta)T^2}{6}\right\}$$
$$+A+\frac{C\theta\alpha T^2}{2}\left\{1+\frac{(\theta+\beta)T}{3}\right\}+\frac{h\alpha T^3}{6}\left\{1+\frac{(\theta+\beta)T}{4}\right\}=0$$

or

$$p\left\{\frac{(r+\beta)}{2}+\frac{(r^2+r\beta+\beta\theta+\beta^2)T}{3}\right\}-C\theta\left\{\frac{1}{2}+\frac{(\theta+\beta)T}{3}\right\}$$
$$-\frac{hT}{2}\left\{\frac{2}{3}+\frac{(\theta+\beta)T}{4}\right\}+\frac{A}{\alpha T^2}=0 \tag{11}$$

From the above discussion we find the following theorems:

Theorem 1: The optimal cycle time T^* increases with setting price p, if

$$24A+\alpha T^{*3}\left[h\left\{4+3(\theta+\beta)T\right\}+4C\theta(\theta+\beta)-4p\left(r^2+r\beta+\beta\theta+\beta^2\right)\right]>0.$$

Proof: Taking the first order derivative of equation (11) w.r.t. p, after little simplification, we get

$$\frac{dT^*}{dp}=\frac{2\alpha T^3\left\{3(r+\beta)+2\left(r^2+r\beta+\beta\theta+\beta^2\right)T\right\}}{24A+\alpha T^{*3}\begin{bmatrix}h\left\{4+3(\theta+\beta)T\right\}+4C\theta(\theta+\beta)\\-4p\left(r^2+r\beta+\beta\theta+\beta^2\right)\end{bmatrix}}>0 \tag{12}$$

Since $\frac{dT^*}{dp}>0,$ thus the optimal T^* increases with selling price p.

Theorem 2: The optimal cycle time T^* is increasing function of unit purchase cost C, if

$$24A+\alpha T^{*3}\left[h\left\{4+3(\theta+\beta)T\right\}+4C\theta(\theta+\beta)-4p\left(r^2+r\beta+\beta\theta+\beta^2\right)\right]>0.$$

$$\frac{dT^*}{dC}=-\frac{2\alpha CT^3\left\{3+2(\theta+\beta)T\right\}}{24A+\alpha T^{*3}\begin{bmatrix}h\left\{4+3(\theta+\beta)T\right\}+4C\theta(\theta+\beta)\\-4p\left(r^2+r\beta+\beta\theta+\beta^2\right)\end{bmatrix}}<0 \tag{13}$$

Since $\frac{dT^*}{dC}<0,$ therefore optimal T^* decreases with respect to unit purchase cost C.

Theorem 3: The optimal cycle time T^* decreases with unit holding cost h, if

$$48A+\alpha T^{*3}\left[h\left\{4+3(\theta+\beta)T\right\}+4C\theta(\theta+\beta)-4p\left(r^2+r\beta+\beta\theta+\beta^2\right)\right]>0.$$

Proof: Differentiating equation (11) w.r.t. C, and simplifying, we have

$$\frac{dT^*}{dC}=-\frac{\alpha T^4\left\{8+3(\theta+\beta)T\right\}}{48A+2\alpha T^{*3}\begin{bmatrix}h\left\{4+3(\theta+\beta)T\right\}+4\theta(\theta+\beta)\\-4p\left(r^2+r\beta+\beta\theta+\beta^2\right)\end{bmatrix}}<0 \tag{14}$$

Since $\frac{dT^*}{dh}<0,$ therefore optimal T^* is decreasing function of holding cost h.

4 Numerical example

The following numerical examples are given to illustrate the validity of the proposed problem.

Let us take α = 100 units/year, β = 0.2, θ = 0.05, C = \$20 units/year, h = \$80/year, p = \$25/unit. Putting these values in equation (11), we get $T = T^*$ = 0.181327 year the corresponding maximum order quantity is Q^* = 18.5437 units and $TP(T) = TP(T^*)$ = \$2,467.96.

5 Sensitivity analysis

We now discuss the effects of variations in α, β, θ, r, C, h and p on the T^*, Q^* and $TP(T^*)$. The sensitivity analysis is shown in Table 1.

Table 1 Variation of T^*, Q^* and $TP(T)^*$ w.r.t. α, β, θ, r, C, h, A and p

Parameter	*Change*	*T**	*Q**	*TP(T*)*
α	105	0.178445	19.1547	2,594.01
	110	0.175740	19.7561	2,720.08
	115	0.173194	20.3485	2,846.17
	120	0.170791	20.9325	2,972.26
	125	0.168517	21.5083	3,098.37
β	0.22	0.184838	18.9450	2,474.44
	0.24	0.188505	19.3657	2,481.11
	0.26	0.192334	19.8068	2,487.96
	0.28	0.196333	20.2693	2,495.03
	0.30	0.200512	20.7548	2,502.30
θ	0.06	0.183251	18.7617	2,467.05
	0.07	0.185209	18.9840	2,466.08
	0.08	0.187202	19.2108	2,465.06
	0.09	0.189230	19.4422	2,463.98
	0.10	0.191292	19.6781	2,462.83
r	0.26	0.179846	18.3889	2,469.55
	0.27	0.178391	18.2369	2,471.11
	0.28	0.176960	18.0874	2,472.62
	0.29	0.175554	17.9406	2,474.10
	0.30	0.174172	17.7964	2,475.54
C	15	0.183023	18.7210	2,471.12
	16	0.182681	18.6853	2,470.48
	17	0.182341	18.6497	2469.85
	18	0.182001	18.6142	2,469.22
	19	0.181663	18.5788	2,468.59
h	85	0.177484	18.1422	2,463.33
	90	0.173947	17.7729	2,458.96
	95	0.170676	17.4317	2,454.82
	100	0.167637	17.1150	2,450.89
	105	0.164803	16.8198	2,447.15
A	16	0.185210	18.9498	2,464.44
	17	0.188935	19.3397	2,460.96
	18	0.192515	19.7148	2,457.57
	19	0.195964	20.0764	2,454.09
	20	0.199293	20.4258	2,450.69

Table 1 Variation of T^*, Q^* and $TP(T)^*$ w.r.t. α, β, θ, r, C, h, A and p (continued)

Parameter	*Change*	*T**	*Q**	*TP(T*)*
p	21	0.179531	18.3560	2,050.54
	22	0.179976	18.4025	2,154.87
	23	0.180423	18.4492	2,258.09
	24	0.180874	18.4965	2,363.57
	26	0.181782	18.5913	2,572.37

The following points are observed:

1. T^* decreases while Q^* and $TP(T^*)$ increase with α.
2. T^*, Q^* and $TP(T^*)$ increase with β.
3. T^* and Q^* increase while $TP(\mathrm{T}^*)$ decreases with θ and C.
4. T^* and Q^* decrease while $TP(T^*)$ increases with r.
5. T^* and Q^* and $TP(T^*)$ decrease with h. Both T^* and Q^* are moderately sensitive with h, while $TP(T^*)$ is sensitive in value of parameter h.
6. T^* and Q^* and $TP(T^*)$ increase with the increase with p. T^* and Q^* are less sensitive to changes in p and $TP(T^*)$ is sensitive to variations in p.

6 Conclusions

The main purpose of this study is to simulate the market competition scenario, and to relax the earlier consider research direction. This model may have a strong use for determining an optimal inventory policy in situations such as stationary stores, designable products and super-market bakeries, which could unveil the characteristics modelled.

At present inflation plays a crucial role in all kind of business transactions. Inflation is more realistic in rare type of commodities. We have developed an inventory model for inventory dependent demand. It is also shown that total profit is concave with cycle time. Based on the theoretical results three important theorems have been also discussed. From managerial point of view, the following inferences can be made:

1. total profit increases with α and β
2. $TP(T)$ decreases with θ and C
3. inflation rate, selling price and total profit are proportional to each other
4. holding cost and total profit are inversely proportional to each other.

The model discussed in the work can be generalised in different ways. We may further generalise the model to allow shortages, freight charges and others.

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Appendix A

We prove the following Lemma to Proof Appendix A:

Lemma: Let us consider $\psi(T) = \frac{\phi(T)}{T}$ where $\phi(T)$ twice differentiable function of T. Then maximum value of $\psi(T)$ exists at $T = T^*$, if $\frac{1}{T}\frac{d^2\phi(T)}{dT^2} < 0$ or $\frac{d^2\phi(T)}{dT^2} < 0$ at $T = T^*$.

Proof: We have

$$\psi(T) = \frac{\phi(T)}{T} \tag{A1}$$

For extremum, the necessary condition is $\frac{d\psi(T)}{dT} = 0$

Differentiating equation (A1) w.r.t T, we get

$$\frac{d\psi(T)}{dT} = \frac{1}{T}\frac{d\phi(T)}{dT} - \frac{1}{T^2}\phi(T) \tag{A2}$$

$\frac{dH(T)}{dT} = 0,$ gives $\frac{1}{T}\frac{d\phi(T)}{dT} - \frac{1}{T^2}\phi(T) = 0$ or

$$T\frac{d\phi(T)}{dT} - \phi(T) = 0 \tag{A3}$$

Let equation (A3) be satisfied at $T = T^*$

Again differentiating equation (A2), we get

$$\frac{d^2\psi(T)}{dT^2} = \frac{1}{T}\frac{d^2\phi(T)}{dT^2} - \frac{2}{T^3}\left\{T\frac{d\phi(T)}{dT} - \phi(T)\right\} \tag{A4}$$

at $T = T^*$, $\frac{d^2\psi(T)}{dT^2} = \frac{1}{T}\frac{d^2\phi(T)}{dT^2}$ using equation (A3)

$H(T)$ is maximum $\frac{d^2\psi(T)}{dT^2} < 0,$ or $\frac{d^2\phi(T)}{dT^2} < 0.$

This proves the Lemma A.

Therefore $TP(T) = \frac{1}{T}\{SR - (A + CD + HC)\}.$

For extremum of $TP(T)$, the necessary condition is $\frac{dTP(T)}{dT} = 0$.

If $T = T^*$ is a extremum value of $TP(T)$, then at T^*, we have

$$\frac{d^2TP(T)}{dT^2} = \frac{1}{T}\left\{\frac{d^2SR}{dT^2} - \frac{d^2A}{dT^2} - \frac{d^2CD}{dT^2} - \frac{d^2HC}{dT^2}\right\}$$, by Lemma A [using equation (1)]

At $T = T^*$

$$\frac{d^2TP(T)}{dT^2} = \frac{1}{T}\left[\frac{p\alpha}{\theta+\beta}\left\{\theta re^{rT} + \frac{\beta}{r-\theta-\beta}\left(r^2e^{rT} - (\theta+\beta)^2 e^{(\theta+\beta)T}\right)\right\} - C\theta\alpha e^{(\theta+\beta)T} - \frac{h\alpha}{\theta+\beta}\left\{e^{(\theta+\beta)T} - 1\right\}\right]$$

If

$$C\theta(\theta+\beta)e^{(\theta+\beta)T} + h\left\{e^{(\theta+\beta)T} - 1\right\} > p\left\{\theta re^{rT} + \frac{\beta}{r-\theta-\beta}\left(r^2e^{rT} - (\theta+\beta)^2 e^{(\theta+\beta)T}\right)\right\}$$

$$\frac{d^2TP(T)}{dT^2} < 0$$

Hence, the proof of Appendix A.

Appendix B

The solution of equation (1) with condition (2) is

$$I(t) = \frac{\alpha}{\theta+\beta}\left\{e^{(\theta+\beta)(T-t)} - 1\right\} \qquad \text{(B1)}$$

and

$$Q = \frac{\alpha}{\theta+\beta}\left\{e^{(\theta+\beta)T} - 1\right\} \qquad \text{(B2)}$$