Uncertainty quantification of thermal image-based concrete diagnosis

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Abstract: This paper investigates uncertainty quantification in internal damage diagnosis in concrete based on thermal imaging. Thermal imaging can be used for detection, localisation, as well as quantification of the damage. The proposed methodology for uncertainty quantification aggregates various sources of uncertainty introduced at each step of image processing. Further, global sensitivity analysis is applied to identify the dominant contributors to diagnosis uncertainty. Based on the results of uncertainty quantification and global sensitivity analysis, a Bayesian technique is formulated to identify the optimal values of parameters to be selected at each step of the image processing, in order to minimise the uncertainty in diagnosis. An illustrative example of damage diagnosis of a concrete slab is used to examine the effectiveness of the proposed uncertainty quantification and parameter selection methods.

Keywords: image processing; concrete; damage diagnosis; uncertainty; Bayesian statistics; thermography.


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1 Introduction

Image processing has been studied for several decades in many different engineering applications. With the rapid development in computing speed and storage capabilities in recent years, image processing has rapidly advanced in several directions, such as image compression, enhancement and restoration. Image compression focuses on the balance between fewer bits for image representation vs. deterioration in quality. Image enhancement and image restoration techniques have been studied to improve the quality of an image, usually through raising the contrast of the image from the background either for better visualisation by the human eye or for better detection by the computing algorithm (Petrou and Petrou, 2010). With respect to image enhancement, researchers have studied several techniques such as threshold transformation (Haralick and Shapiro, 1991), logarithmic transformation (Jain et al., 1995), histogram equalisation (Russ, 2011; Hummel, 1975), and local enhancement (Umbaugh, 1997).

Digital image processing techniques have also been studied in the context of non-destructive testing (NDT) of structures and materials in recent years. Researchers have tried to apply image processing to monitor and evaluate the condition of concrete structures using different type of imaging systems. Ito et al. proposed an automatic detection technique for cracks in a concrete block by utilising image-processing techniques on raw images acquired by a high-resolution camera. Fujita et al. implemented a line-filter and threshold processing to improve the robustness of the above method (Ito et al., 2002; Fujita et al., 2006). Edge-detection algorithms and statistical methods (Shahid Kabir, 2009; van de Wouwer et al., 1999; Foucher et al., 2001) have been advocated and evaluated for assessing images obtained by acoustic televiewers and high-resolution cameras. An automatic detection and quantification technique for micro-cracks and other micro-defects was introduced based on scanning electron microscope (SEM) and optical microscope images (Ammouche et al., 2000).

Image processing using high-resolution camera, acoustic televiewers, SEM, and optical microscopes are rather effective in identifying micro-cracks or largely propagated cracks on the surface of concrete. But none of them are helpful in internal structure inspection, since internal damage may not cause detectable external change until the damage has progressed to a considerable extent. X-ray tomography has been studied and applied in internal structure diagnosis (Masad et al., 1999). However, as a highly radioactive and hazardous technique, X-ray tomography is not suitable for frequent inspections. Therefore, an automated, less hazardous internal damage detection method is attractive, considering both efficiency and objectivity of internal damage assessment.

In this paper, we investigate the processing of thermal images for internal defect diagnosis (detection, localisation and quantification) in concrete. We use holes drilled in a concrete slab to represent internal damage for the sake of illustration; however, the proposed methodology in this paper is quite general and can also be applied to other types of damage. Thermographic cameras have been used to detect the differences of temperature in the material in the health monitoring of metallic components such as
turbine blades (Rumsey and Paquette, 2008; Doliński and Krawczuk, 2009). The idea behind this is that defects or irregularity in a material might cause differences in heat conductivity and temperature diffusivity, when compared with intact material (Stanley, 1997; Milovanović and Pečur, 2013). Thermographic damage diagnosis has been applied locally and globally (Dutton, 2004; Hameed et al., 2009; Del Grande and Durbin, 1996; Pollock et al., 2008). One influential factor is the resolution of the thermographic camera (Paynter and Dutton, 2003).

Typically, in image processing, the parameters for the different steps in the image analysis are selected based on the analyst’s experience. Often, the parameter selection is by trial and error, and the effect of the parameter choice on the diagnosis result is assessed in a subjective manner. This paper proposes a systematic, quantitative approach for parameter selection using a Bayesian perspective. The image processing technique consists of four steps: cropping, smoothing, feature identification, and decision making. Each of these steps requires the selection of values for the processing parameters, which will affect the diagnosis result in turn. We will elaborate the parameters in our proposed method in Section 3. In reality, however, it is hard to determine which values should be chosen for these parameters. In order to study the uncertainty in the diagnosis result, we first perform Monte Carlo simulation of the image processing system by randomly choosing values of the image processing parameters. Next, the contributions of various image processing parameters to the uncertainty in the diagnosis result are analysed using the global sensitivity analysis (GSA) technique. Finally, a Bayesian approach is pursued to select the optimal values of parameters for the image processing system. The obtained optimal parameters can provide guidance in the selection of parameters for damage diagnosis of similar structures. Thus the novel contribution of this paper is in developing a four-step parameter selection methodology:

1. thermal image processing with multiple parameter values
2. uncertainty quantification
3. sensitivity analysis
4. optimum parameter selections using a Bayesian approach.

The remainder of the paper is organised as follows. Section 2 provides background information on basic filtering techniques used in this paper. Section 3 describes the proposed methodology for damage diagnosis in concrete, using thermal image processing, uncertainty quantification, sensitivity analysis, and parameter selection. Section 4 illustrates the implementation of the proposed methodology for a concrete slab. Finally, Section 5 provides concluding remarks and future work.

2 Background

This section briefly reviews the main filtering techniques used in this paper for image processing. Figure 1 shows the general procedure of structural damage diagnosis using image processing. Images of the surface are first acquired. Second, different kinds of filters are applied for noise cancellation in the raw images to remove the environmental and operational effects. Third, based on the image content or objective of the monitoring
process, different kinds of algorithms are applied to calculate the features of the image. Lastly, damage diagnosis decision making is accomplished based on appropriate criteria.

### Figure 1
Steps in damage diagnosis using image processing
![Diagram of image processing steps](image)

#### 2.1.1 Two-dimensional simple moving average filter

In the noise cancellation step in Figure 1, a moving average filter tends to smooth out short-term variations and leaves in the long-term changes, by taking an average value of the data within a fixed-size window; the window shifts from one side of the data series to the other, thus covering the entire dataset (Farrar and Worden, 2012). Moving average has been used for time series analysis in many fields such as financial analysis, signal processing, etc. There are usually three types of moving average approaches: simple moving average (SMA), cumulative moving average, and weighted moving average (Chou, 1963). The thermal images appear to have relatively long-term variation. Considering the balance between effectiveness and computational expense, the SMA filter is adequate (Akaike, 1974), and is described as

\[
\hat{P} = \frac{P_d + P_{d-1} + \ldots + P_{d-(n-1)}}{n}
\]

where \( \hat{P} \) is the new value at the middle point of vector: \( [P_d, P_{d-1}, \ldots, P_{d-(n-1)}] \), \( P_i \) is the original value at position \( i \), and \( n \) is the length of the vector.

#### 2.1.2 Sobel filter

Image processing typical has two approaches to extract features: colour-based and texture-based. In our system, the thermal images have differences in contours which reflect the heat conductivity difference. Therefore, a texture filter, such as Sobel filter, is more appropriate in our case. Sobel filter, also called Sobel operator, is one of the most commonly applied in image processing for edge detection. Sobel filter contains two 3 by 3 convolution masks as follows (Chaudhuri et al., 1989; Pratt, 2001).

\[
S_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}, \quad S_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & 2 & -1 \end{bmatrix}
\]

Given an image \( A \), to apply the Sobel filter, we usually convolve the mask separately with the input image to obtain the corresponding gradient component (\( g_x \) and \( g_y \)) along each direction as follows (Chaudhuri et al., 1989).

\[
g_x = S_x \ast A
\]
\[
g_y = S_y \ast A
\]
Typically, based on the result from the two formulae above, the gradient magnitude, $G$, and the angle of the image orientation, $\phi$, could be calculated (Pratt, 2001).

$$G = \sqrt{g_x^2 + g_y^2}$$  \hspace{0.5cm} (5)

$$\phi = \arctan \left( \frac{g_x}{g_y} \right)$$  \hspace{0.5cm} (6)

As one of the most commonly applied methods in the gradient-based filter family, Sobel filter uses the property that an edge is usually characterised by a threshold value of the gradient. Since the pixel intensity value at the edges will be usually higher than their surrounding pixels, a threshold value is commonly set (Litwinowicz, 1997; Franke, 1979). And an edge will be declared when the calculated gradient at a pixel crosses the threshold.

3 Internal damage diagnosis and uncertainty quantification

In this section, we will first discuss how thermal image processing can be implemented for internal damage diagnosis. Following that, we will quantify the uncertainty in the diagnosis results and develop the method for selecting the optimal values of image processing parameters.

3.1 Damage diagnosis using thermal image processing

The main idea of using thermal image processing for damage detection is that an internal damage beneath the surface will cause a discontinuity in the thermal conductivity of the material. Therefore, the captured images will be different from those of the original, intact material in heating cycles. As long as differences in images exist, we can distinguish them by calculating features with image processing algorithms.

**Figure 2** Flowchart of internal damage diagnosis of concrete
Figure 2 provides a flowchart of the thermal image processing system for internal damage diagnosis. The inputs of the system are raw images captured from thermographic camera. Due to the limitations of the camera and experimental setup, some portion of an image may capture regions that are not related to the studied specimen. Therefore, the first step of our image processing system is to crop the raw image to remove the irrelevant parts. Secondly, with the cropped image, a same size cropped image from an intact structure is subtracted from the acquired one as baseline removal (Farrar and Worden, 2012). After that, a smoothing filter (namely, SMA filter) mentioned in Section 2 is employed for noise-cancellation (Farrar and Worden, 2012). With the smoothed image, the feature extraction filter (namely, Sobel filter) as reviewed in Section 2 is used for feature calculation. With the calculated features, we can apply threshold settings for pixel-wise decision making. After scanning through the entire image, a pixel-wise damage detection and decision matrix is produced and can be shown as a black-and-white image (suspected damage shown as white spots in Figure 6). Based on the aforementioned matrix, the amount of damage is estimated. In order to determine the location of the damage, a pixel-to-length unit convertor first needs be calculated based on photogrammetry and the relative positions of the camera and the specimen. An illustrative example will be provided in Section 4.3.

In Figure 2, each of the dashed boxes contains the parameters that need to be determined for the corresponding image processing step. In the cropping step, we have $x$ and $y$, representing the $x$ and $y$ coordinates of the starting point; $h$ and $w$, representing the height and weight of the cropped images; and $\beta$, representing the potential rotation of the studied specimen. In the noise cancellation step, we have $win$, which is the window size of the selected smoothing filter. In the pixel-wise decision making step, we have $xThr$ and $yThr$, which are the threshold values for the $x$ gradient and $y$ gradient respectively. Even if some empirical ranges can be provided for these parameters, determining the optimal values of the parameters is not straightforward.

In order to quantify the effects of the image processing parameters on the damage diagnosis results, the next step is to perform uncertainty quantification of the damage diagnosis, and assess the relative contributions of different parameters to the diagnosis uncertainty through sensitivity analysis.

### 3.2 Uncertainty quantification and sensitivity analysis

As mentioned in Section 3.1, ranges of values may be available, based on previous experience, for the parameters in the image processing system. Based on the intervals, we first use Monte Carlo sampling to quantify the uncertainty in the diagnosis results. We then use GSA to identify important parameters. The noises and errors in the measurement and numerical method are also considered during this process.

The Monte Carlo simulation is performed by generating random realisations of the image processing parameters. Uniform distributions over the aforementioned ranges are used for the random sampling of the parameter values, in the absence of any information about preferred values. It should be noted here that, even though we use Monte Carlo simulation, the image processing parameters are not random variables. Their values are chosen by the analyst, and the chosen value affects the diagnosis result. The purpose of using Monte Carlo simulation and sensitivity analysis is to observe the effect of the choices on the diagnosis result in a quantitative manner. In addition, this exercise can also shed light on the stability of the image processing procedure, by observing whether the
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diagnosis result changes slightly or drastically for a small perturbation of the parameter values.

The purpose of sensitivity analysis is to analyse how the output of a given model is affected by its inputs. There are two types of sensitivity analysis in practice, local sensitivity analysis and GSA. Local sensitivity analysis is performed at a specific given point in the region of inputs. GSA focuses on the study of the output uncertainty by considering the variation of the input parameters over their entire range (Friedman et al., 1997; Wang and Vassileva, 2003). As a form of GSA, variance-based sensitivity analysis decomposes the variance of the output into contributions from each input and quantifies the sensitivity based on the percentage of variance corresponding to each input. Mathematically, variance-based GSA can be expressed as follows (Farrar and Worden, 2012).

Given a model

\[ M = g(X) \]  \hspace{1cm} (7)

where \( M \) is the model output and \( X \) denotes a vector of model inputs, i.e., \( \{x_1, x_2, \ldots, x_n\} \).

The decomposition of variance is expressed as

\[ \text{Var}(M) = \sum_{i=1}^{n} V_i + \sum_{i<j} V_{ij} + \ldots + \sum_{i=1}^{n} V_{i,2,\ldots,n} \]  \hspace{1cm} (8)

\[ V_i = \text{Var}_{X_i} \left( E_{X_i} \left( M \left| X_i \right. \right) \right) \]  \hspace{1cm} (9)

\[ V_{ij} = \text{Var}_{X_{ij}} \left( E_{X_{ij}} \left( M \left| X_i, X_j \right. \right) \right) \]  \hspace{1cm} (10)

where \( X_i \) denotes the \( i^{th} \) variables in vector \( X \). \( X_{ij} \) in equation (9) denotes all other input variables except \( X_i \). \( X_{ij} \) denotes all other input variables except \( X_i \) and \( X_j \).

Based on the above decomposition, the first order sensitivity index is expressed as (Saltelli et al., 1999; Homma and Saltelli, 1996)

\[ S_i = \frac{V_i}{\text{Var}(M)} \]  \hspace{1cm} (11)

This first order sensitivity index measures the individual contribution of \( X_i \) to the output variance. The index is computed by calculating the multivariate integrals in equations (12) and 13 through Monte Carlo sampling (Sobol, 1966). By investigating the global sensitivity results, we can distinguish the parameters of more significant impact on the desired output from those with less significant impact. This information can be used to narrow the study to the most important parameters for more detailed investigation (McRae et al., 1982; Saltelli et al., 1993).

3.3 Selection of optimal image processing parameter values

Define the input parameters of the damage diagnosis system as \( \alpha = [x, y, \text{win}, x\text{Thr}, y\text{Thr}] \), and expressing the damage diagnosis result mathematically as \( r = g(\alpha) + e_r \), where \( r \) is the estimated amount of damage, \( g(\alpha) \) is the internal damage diagnosis system, and \( e_r \) is the error in the diagnosis system. Then the optimisation problem to identify the optimal values of the image processing parameters \( \alpha \) is formulated as \( \text{argmin}|r - r_{\text{true}}| \) where
$r = g(\alpha) + e$, $r_{\text{true}}$ is the true damage and $\alpha \in \Omega$, where $\Omega$ stands for the random domain of the input variables.

Due to the fact that $r_{\text{true}}$ is unknown during actual diagnosis and the input parameters can be chose as any value in the domain $\Omega$, solving the optimisation model given above is impossible during actual diagnosis. However, if the damage diagnosis system is to be applied to a class of similar structures, we may able to calibrate the optimal parameters for the internal damage diagnosis system based on experimental observation of $r_{\text{true}}$ for a given structure.

Three approaches commonly available for parameter estimation can also be used to solve the optimisation problem. The least squares (LS) approach attempts to minimise the square of the sum of differences between the observed data and the model prediction. The maximum likelihood estimator (MLE) obtains the value of $\alpha$ that maximises the likelihood $f(r_{\text{true}}|\alpha)$. A third approach is to use Bayesian estimation, and the parameter values could be selected as maximum a posteriori (MAP) estimates, thus including any prior information about the parameters in the estimation (Fräke, 1979). If a uniform prior is assumed, then the MAP estimate is the same as the MLE estimate. The Bayesian approach is used in this paper.

Thus the problem of image processing parameter value selection is solved through a parameter estimation approach in this paper. Note that the selection of the processing parameter values is actually a design problem; the analyst gets to select these values. However, using a parameter estimation approach is a convenient way to solve the design problem and is mathematically correct.

Using Bayes’ theorem, the posterior distribution of the image processing system parameters are computed as

$$f(\alpha|r_{\text{true}}) = \frac{f(r_{\text{true}}|\alpha)f(\alpha)}{\int f(r_{\text{true}}|\alpha)f(\alpha)d\alpha}$$

where $r_{\text{true}}$ is the true damage extent, $\alpha$ represents the parameters selected in the image processing system. $f(\alpha)$ represents the prior distributions of the parameters. $f(r_{\text{true}}|\alpha)$ represents the likelihood of the true damage extent of the particular parameter selections. $f(\alpha|r_{\text{true}})$ represents the posterior of the distributions of the image processing parameter given the true damage extent.

Markov Chain Monte Carlo (MCMC) simulation is commonly used to estimate the posterior distribution based on the following proportionality relationship:

$$P(\alpha | r_{\text{true}}) \propto P(r_{\text{true}} | \alpha) P(\alpha)$$

In this paper, MCMC is implemented using the Python package PyMC, and the Metropolis-Hastings sampling method is used (Bishop, 2006; Friedman et al., 1997; Wang and Vassileva, 2003).

Thus we obtain the optimal parameters for the image processing system. It should be noted that the optimal parameter values may vary with structure and damage geometry. The obtained optimal parameters are only applicable to structures with similar features and damage characteristics as the structure used for the parameter optimisation. An illustrative example is given in the next section to explain the implementation procedure of the proposed framework.
4 Illustrative example

Damage detection in a concrete slab is used to illustrate the proposed methodology. The experimental setup is first described, followed by description of the diagnosis implementation and parameter estimation.

4.1 Experimental setup

The thermal imaging experiment was carried out on concrete slabs with dimensions (in inches) $15.5 \times 15.5 \times 1.75$. The slab was placed on a thermal blanket for uniform heating at the bottom, and a thermographic camera was placed to take pictures from over the top of the slab. To simulate internal damage, three parallel holes with different sizes were drilled from one side of the concrete slab. The dimensions and positions of the holes are shown in Figure 3. Experiments were carried out both before and after drilling. The images were taken while the slab was subjected to heating.

**Figure 3** Dimensions and locations of the holes in the concrete slab (cm) (see online version for colours)

4.2 Data collection

The images were obtained using an FLIR® Infrared (IR) camera from the top of the slab every two minutes during the experiment. The relative positions of the slab and the thermographic camera are shown in Figure 4.

The temperature profile of the heating blanket is shown in Figure 5. And the image selected for parameter optimisation is at 18 min. The reason for choosing that particular time instant is that at the beginning of the heating cycle, initial conditions dominate and it may take a while for the heating blanket and other equipment to function smoothly. Since the internal damage in the concrete will have effect on the heat conductivity in the corresponding area, the images that are taken while the temperature of the slab is increasing should reflect the heat conductivity of the internal condition better than during steady temperature. Thus we selected 18 min which is immediately after the temperature of the heating blanket reaches its maximum value.
Figure 4  Positions of the thermographic camera and concrete slab, (a) concrete slab (b) experimental setup (see online version for colours)

Figure 5  Temperature profile in the thermal blanket (see online version for colours)

Figure 6  Raw image from thermographic camera (see online version for colours)
The images were taken for both the intact concrete slab and the drilled slab. Figure 6 shows one collected raw image of the scaled temperature on the surface of the slab with the holes.

4.3 Image processing methodology

As shown in Figure 5, due to the limitation of the camera, there are segments on both edges, which are due to the metal frame and unrelated to the concrete slab. Therefore, the first step is to crop the image into purely concrete part. Next, a cropped image of the same size from an intact slab is subtracted from the acquired one for baseline removal. After that, a two-dimensional SMA filter, with a window-size $win$, is applied for noise-cancellation (Kay, 1993) as described in Section 2. With the smoothed image, a Sobel filter is employed to calculate the gradients (features) $g_x$ and $g_y$ of the image as introduced in Section 2.1.2. A threshold value needs to be selected for each feature; in each pixel, if both $x_{Thr}$ and $y_{Thr}$ are below the corresponding threshold; damage is declared at that pixel. After scanning through the entire image, a pixel-wise damage detection and decision matrix is produced and can be shown as a black-and-white image (suspected damage shown as white spots in Figure 7).

Figure 7  Pixel-wise damage detection and decision matrix

Notes: Parameter values: $[x = 74, y = 16, win = 25, x_{Thr} = 0.08, y_{Thr} = -0.08]$. The location of damage can be quantified based on the pixel-wise damage matrix. In this calculation, the length-to-pixel convertor is as follows.

$$\text{1 pixel } \approx 0.0777 \text{ cm, or } 12.86 \text{ pixel/cm}$$

The extent of damage is first provided by the pixel counts in terms of the damage area-ratio defined as

$$\text{area} - \text{ratio} = \frac{N_d}{N_t}$$

where $N_d$ is the number of suspected internal damage pixels and $N_t$ is the total number of pixels within the cropped image.
We make an assumption that the area ratio calculated in equation (15) is equal to the actual area ratio in the concrete slab. With the calculated area ratio from equation (15) and knowing the area of the slab, we build the following relationship:

\[
\text{estimated}_\text{ damage}_\text{ area} = \text{area}_\text{ ratio} \times \text{slab}_\text{ area}
\]

(16)

The function above is deterministic, which means that given a combination of all the five parameters and the measurement of the concrete slab dimensions, \( \text{estimated}_\text{ damage}_\text{ area} \) is a fixed number. However, the analyst is uncertain about the values to be selected for these parameters; each combination of the parameter values will give a different result. As the three parallel holes are not extremely close to each other, we can assume that they are independent. Therefore, we can further predict the damage area individually (Table 1). Based on the formulae (14) to (16), we can calculate the damage area for upper hole, 16.324 cm\(^2\), the damage area for middle hole, 17.075 cm\(^2\), and the damage area for lower hole, 12.436 cm\(^2\). Compared with the true areas, 15.494 cm\(^2\), 11.590 cm\(^2\), and 9.638 cm\(^2\), though the middle hole area seems relatively overestimated, the estimated results are close to the true values. We are able to estimate the diameters of the drilled holes from Figure 7, upper hole, 2.33 cm, middle hole, 1.55 cm, and lower hole, 1.32 cm. Compared with the true diameters, 1.27 cm, 0.95 cm, 0.79 cm, the estimations are also very close to the truth.

<table>
<thead>
<tr>
<th>Upper hole</th>
<th>Middle hole</th>
<th>Lower hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>True area (cm(^2))</td>
<td>15.494</td>
<td>11.590</td>
</tr>
<tr>
<td>Predicted area (cm(^2))</td>
<td>16.324</td>
<td>17.075</td>
</tr>
<tr>
<td>True diameter (cm)</td>
<td>1.27</td>
<td>0.95</td>
</tr>
<tr>
<td>Predicted diameter (cm)</td>
<td>2.33</td>
<td>1.55</td>
</tr>
</tbody>
</table>

Next, we will investigate how to quantify the uncertainty in the diagnosis result due to the uncertainty regarding parameter selection.

4.4 Uncertainty quantification and sensitivity analysis

As mentioned in Section 3.2, we assume that the parameter settings in the image processing steps are independent from each other (Ishigami and Homma, 1990). The selected range of \(x\) and \(y\) are based on direct observation of the raw images. The range of window size \(\text{win}\) is assumed to be 15 to 35 pixels. The selected ranges of \(x_\text{Thr}\) and \(y_\text{Thr}\) are based on trial and error. The trial and error process simply chooses different values for the parameters in an arbitrary manner and examines their effect on the diagnosis result; there is no systematic procedure or quantitative guidance, except the analyst’s experience.

For the purpose of uncertainty quantification using Monte Carlo simulation, all five parameters are sampled from uniform distributions, since there is no information regarding preferred values for the parameters. Table 2 summarises the five variables in the image processing system. Note that \(x\), \(y\) and \(\text{win}\) refer to the coordinate and smoothing window sizes which are measured in pixels, whereas, \(x_\text{Thr}\) and \(y_\text{Thr}\) are threshold values of gradients. Therefore, we sample \(x\), \(y\) and \(\text{win}\) from discrete uniform distributions and
$x_{Thr}$ and $y_{Thr}$ from continuous uniform distributions. The lower and upper bounds of the uniform distributions are also given in Table 2.

Table 2 Five variables of the image processing system and their empirical intervals

<table>
<thead>
<tr>
<th>Variable</th>
<th>$x$</th>
<th>$y$</th>
<th>win</th>
<th>$x_{Thr}$</th>
<th>$y_{Thr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distribution</td>
<td>Discrete-uniform</td>
<td>Discrete-uniform</td>
<td>Discrete-uniform</td>
<td>Uniform</td>
<td>Uniform</td>
</tr>
<tr>
<td>Parameters</td>
<td>(64, 84)</td>
<td>(6, 26)</td>
<td>(15, 35)</td>
<td>(0, 0.1)</td>
<td>(-0.1, 0)</td>
</tr>
</tbody>
</table>

Notes: ‘Discrete uniform’ means the variable follows uniform distribution but can only have integer values.

From each of the distributions in Table 2, we randomly draw 30,000 samples. With each realisation of the parameter values, the four image processing steps are applied according to the procedure given in Figure 2 in order to calculate the estimated damage area. With the 30,000 values of $estimated\_damage\_area$, the histogram is plotted as in Figure 8. The total computation time for the Monte Carlo simulation is about 31.3 hours on a desktop personal computer.

Figure 8 Histogram of $estimated\_damage\_area$ obtained from MCS (see online version for colours)

The true damage area (known because we drilled the holes to a specified size) is 36.008 cm$^2$. From the MCS results in Figure 8, we can observe that the peak of the histogram of estimated damage area is at about 60 cm$^2$ assuming that the area ratio in the cropped image carries over to the full slab. If we directly compute the damage area from the pixels and the convertor in equation (14) without carrying the area ratio to the full slab, the peak of the histogram is still at about 56 cm$^2$. Thus there is a conservative overestimation of damaged area with the thermal imaging technique in this example. This is expected, because the thermal conductivity of the areas adjacent to the holes is reduced by the presence of the holes, and therefore these adjacent areas are also identified as damaged through the image processing analysis. This is also visually seen in Figure 7, where the white pixels cover widths larger than the true widths of the three holes.
Next we perform GSA of the damage diagnosis result, and Table 3 gives the first-order indices (i.e., individual effects).

Table 3 indicates that $x_{Thr}$ has the highest first-order sensitivity index, followed by $y_{Thr}$. It implies that the selection of $x_{Thr}$ and $y_{Thr}$ is critical for the accuracy of the diagnosis result. The reason for the dominant sensitivity of $x_{Thr}$ is due to the geometry of the drilled holes in the concrete slab. As shown in Figure 6, due to the specific geometry of the holes, the temperature field in the slab varies less horizontally (x-direction) than vertically (y-direction). Therefore, a small change in the threshold $x_{Thr}$ will make a significant difference in the decision making result, i.e., $x_{Thr}$ should have a relatively high sensitivity. In the next section, we select optimal values of these parameters based on our knowledge of the true damage.

### 4.5 Selection of optimal parameters

Figure 9 shows a simple Bayesian network connecting all the variables, output and observation. The priors for the five variables $x$, $y$, $win$, $x_{Thr}$, and $y_{Thr}$ are chosen as shown in Table 2. In this example, we assume that the error $e_r$ in the system follows normal distribution. The mean of the error is the assumed to be zero, and the variance is unknown and needs to be estimated along with the parameters. To consider the uncertainty during manufacture of concrete specimen and the uncertainty of damage estimation framework, we define a uniform prior for the precision [i.e., $\frac{1}{\sigma^2}$ ~ Uniform (6,250, 10,000), $\sigma$ represents the standard deviation of the assumed normal distribution for damage area].
Figure 10  Posterior distribution of $x$ (see online version for colours)

Figure 11  Posterior distribution of $y$ (see online version for colours)

Figure 12  Posterior distribution of $\text{win}$ (see online version for colours)
We estimate the posterior distributions of the processing parameters based on the Bayesian updating mentioned in Section 4.3. Figures 10–14 give the obtained posterior distributions of the image processing parameters. We ran 10,000 samples for MCMC using Metropolis-Hastings algorithm. Since the algorithm takes many iterations to reach a steady state (Hastings, 1970), we discarded the first 1,000 samples. The total computational time is about 26.9 hours on a desktop personal computer.

The posterior distributions of $x$, $y$ and $\text{win}$ do not provide much guidance for selecting their optimal values. This result is as expected, since these three parameters were observed to have very low sensitivity indices (Table 3). In the case of significant parameters $x\text{Thr}$ and $y\text{Thr}$ the MAP estimates are found to be $x\text{Thr} = 0.09$ and $y\text{Thr} = -0.09$; these estimates are similar to the result from trial and error $x\text{Thr} = 0.08$ and $y\text{Thr} = -0.08$. Using equations (14) to (16), we calculate the damage area using
parameters from Bayesian updating, 26.706 cm² and the parameters from trial and error, 45.835 cm². Compared with the actual area, 36.008 cm², the result from Bayesian updating of the parameters is more aggressive and the result from trial and error seems more conservative.

5 Conclusions

This paper investigated thermal image processing for internal damage detection, localisation and quantification in concrete. From the Monte Carlo simulation, we are able to gain insight into the uncertainty of the diagnosis result based on ranges of the values for the image processing parameters. The sensitivity analysis identifies the significant parameters that affect the diagnosis uncertainty. The Bayesian approach is able to identify optimum values of the significant parameters. The proposed damage detection and quantification framework is illustrated using drilled holes to represent ‘damage’, due to the fact that concrete specimen casting and curing consume weeks. Specimens containing other kinds of realistic damage may be tested in future work to study the generality of the proposed damage diagnosis framework.

Future work can investigate the performance of the thermal image processing for more complicated and realistic damage scenarios in concrete. Concrete is a heterogeneous material consisting of aggregates, reinforcement and voids; thus damage diagnosis using thermal images might pose significant challenges for realistic damage scenarios caused by mechanisms such as alkali silica reaction, chloride diffusion etc. The thickness of the slab is another challenge; the example here used a relatively thin slab with drilled holes. Future work needs to investigate the effectiveness of damage diagnosis for realistic structural sizes and damage geometries. However, the proposed methodology for uncertainty quantification, sensitivity analysis, and parameter value selection is general and can be applied to a variety of image processing-based damage diagnosis techniques.

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References


