Matching and optimising analysis of multi-axle steering vehicle steering system

Yunchao Wang and Chengzhi Wang*

College of Mechanical and Energy Engineering,
Jimei University,
Xiamen 361021, China
Email: ychaowang@jmu.edu.cn
Email: wcz3@tom.com
*Corresponding author

Abstract: The analysis of the multi-axle steering link mechanism (MASLM) link forces provides the foundation for the mechanism’s design. The matching level between the steering cylinder driving torque and the tyre pivot steering resistance torque can dramatically influence the link forces. In this study, the accuracy of the tyre resistance torque formula reached 90.7%. A steady kinematic-mechanical coupling model of the MASLM was built modularly, and a method of calculating the link forces and steering hydraulic pressure is proposed. To verify the coupling models, an all-terrain QAY130 crane was chosen for simulation and testing. Two vehicle models were built by using the Adams and Matlab software, respectively. The discrepancies were less than 23.5% between the test values and the predicted values for the previous two models, but exhibited similar variation trends. The installation site and diameter parameters of the steering cylinders were optimised to minimise the link forces.

Keywords: multi-axle steering vehicle; steering link mechanism; matching design; optimal design; tyre resistance torque.


Biographical notes: Yunchao Wang received his PhD in School of Mechanical Science and Engineering from Jilin University. He is currently working as a Professor of Vehicle Engineering in School of Mechanical and energy Engineering from Jimei University. His interests include multi-axle steering system matching design and optimisation, multi-axle steering vehicle dynamics.

Chengzhi Wang received his MS in graduate school of Northeastern University, China. He is a Professor of Mechanical and Energy Engineering at Jimei University, China. His research interests include the synthesis and analysis of mechanisms, visualisation in scientific computing, and optical measuring technique.
1 Introduction

With the widespread application of multi-axle steering technology to heavy engineering vehicles, various events such as steering link breaking, cracking, and deformation usually occur in these multi-axle steering vehicles with a linkage hydraulic-assist steering system. This indicates that there exist unreasonable design problems in these systems.

In recent years, most researchers have mainly focused on the dynamical multi-axle steering vehicle models and their handling stability. For example, in the early days of dynamical investigation, various scholars extended the vehicle-handling concepts and traditional conventions commonly found in literature relevant to two-axle vehicles, to the three-axle models. Then, some of them developed steering control strategies or controllers based on these three-axle models, such as the linear quadratic regulator technique with integral control (Chen et al., 2007), desired yaw rate (An et al., 2008), and the model-following variable structure (Qu et al., 2008), were analogous to the techniques of the two-axle vehicle. To further improve vehicle stability and manoeuvrability, Shen et al. (2016) developed a direct yaw moment control method based on an 8-axle vehicle with 16-independent driving wheels. Moreover, various scholars have constructed various other general models for multi-axle vehicles and analysed their dynamic characteristics. Watanabe et al. (2007) established a general simulation model based on the tyre brush model in order to evaluate the basic turning characteristics of multi-axle steering vehicles on level ground. Bayar and Unlusoy (2008) and Aoki et al. (2013) developed a zero vehicle sideslip angle control strategy and introduced the lateral dynamics of a multiple-axle vehicle. Furthermore, William (2011) presented the concepts of equivalent wheelbase and the understeer coefficient, and extended them to vehicles (2012) with an arbitrary number of steerable axles to develop a generalised dynamic system of equations. However, William did not consider the vehicle roll dynamics. Ding et al. (2016) provided a twice equivalent approach to calculate the wheelbase and the steady factor, and discussed both the effect of vehicle body and \( n \)-axle handling on vehicle dynamics. Zhang et al. (2018) proposed an equivalent modelling method for a multi-axle vehicle based on the dynamic equivalence of force/moment at the centre of gravity (CG). However, the latter two methods were only applicable to vehicles with one steering front-axle and \( n-1 \) driving rear-axles. Although these studies have benefited the improvement of multi-axle steering vehicle dynamic stability, an attempt to propose solutions to the abovementioned problems has not been made.

To improve the MASLM design, various researchers have paid close attention to MASLM’s kinematic optimisation, link force analysis, and robust design. Wang et al. (2014) presented a dual limit objective optimisation method for a dual steering linkage system, which consisted of a linkage hydraulic-assisted steering system for the front axles, and an electro-hydraulic active steering system for the rear axles in order to reduce the tyre wear by the interference between the different steering systems. Lu et al. (2015) recommended a response surface optimisation method for the vehicle steering system in order to reduce the mass of the system, increase the first order natural frequency, and avoid resonance at idling speed. To make the design results less sensitive to uncertainties in the design process, Zhang et al. (2014) proposed a robust optimisation design method for a double front axle steering system in heavy trucks. The steering system in a multi-axle steering vehicle is such a complicated design problem that many design factors, including mechanical, electrical, and hydraulic elements, are coupled to each other. Although Kim et al. (2013) and Du et al. (2016) focused their concern on the
electro-hydraulic power steering (EHPS) models, those models were not applicable to multi-axle steering vehicles with a linkage hydraulically-assisted steering system. Taking the minimum force of the steering linkage as the optimisation objective, Wang et al. (2013) promoted the matching level between the driving torque of the hydraulic steering cylinder and the tyre steering resistance moment by optimising the steering cylinder diameters. The study found that the non-matching between the driving torques of the steering cylinders and the tyre pivot resistance torques is the root cause of the above-mentioned problems. However, few studies have been conducted on the matching design of an MASLM, and a suitable matching design theory for a multi-axle steering system has not yet been proposed.

Therefore, this study attempted to develop the matching design model of a multi-axle steering system and analyse the influencing factors on the link forces in the model. The rest of this paper is organised as follows. Section 2 improves the formula of the tyre pivot resistance steering torques. Section 3 develops the coupling model of steering link mechanism. Section 4 takes the all-terrain QAY130 crane as an example to discuss the main influence factors and optimise these factors. The conclusions drawn from the investigation are presented in Section 5.

2 Tyre pivot resistance steering torque

The theoretical calculation of a tyre pivot steering resistance torque is the foundation of building an MASLM mechanical equilibrium model. It is clear that whether the predicted magnitudes of the link forces in the MASLM tally with the actual situation depends on the correctness of the tyre torque’s theoretical formula. In this study, we considered a predicted value to be the theoretical resistance torque calculated with a tyre torque formula, and considered the test value to be the experimental resistance torque measured through the actual tyres in a multi-axle steering vehicle under the same conditions.

Based on our previous experimental and theoretical study, two formulae of the tyre pivot resistance steering torque, which consist of an experimental (Wang et al., 2010) and a theoretical formula (Wang et al., 2014), were deduced. These formulae can predict the tyre torques with large steering angle better than those with a small steering angle. Figure 1 shows the comparison between the predicted and test values, and presents the variation graphs of these two resistance torques with steering angle. These graphs were plotted with thick black and thin blue curves, respectively. These two types of curves corresponding to the same workloads indicate that some discrepancies exist in these predicted and test values when the wheel steering angle is less than five degrees.

According to the Ackerman principle, for all steering axles in a multi-axle steering vehicle, the fact of the wheel steering angle for each axle being different, i.e., the shorter the distance from a steering axle to the turning centre is, the smaller is the wheel steering angle for the axle, can be reasoned out. Moreover, the smaller the wheel steering angle is, the smaller is the tyre resistance torque from Figure 1. For some wheels in the multi-axle steering vehicle, their steering angles often operate with small angles. Thus, the predicted values of the tyre resistance torques are important in the link force analysis of an MASLM when the wheel steering angle is small. It can be seen that there existed various significant initial values in the test values because the wheel steering angles were less than five degrees. However, there was no initial value in the predicted values. Therefore, for those axles whose steering angles were less than five degrees, the predicted values of
the tyre pivot resistance steering torques may result in great errors between the calculated and tested link forces.

Figure 1  Comparisons between test and predicted values of tyre pivot steering resistance torques (see online version for colours)

With regard to the theoretical formula mentioned above, recent research has found that the two influencing factors, namely, the viscous delay of the tyre rubber and the friction between the kingpin and its bearing, were not considered. Therefore, by introducing the tyre torsion viscous damping coefficient around the kingpin and the friction coefficient between the kingpin and its bearing to the formula, we obtained an improved formula for the tyre pivot resistance steering torques, as follows:

\[
M = M_\gamma + M_\mu + M_\alpha + M_\beta = F_s (R \sin \gamma \sin \alpha + \mu) + \xi \dot{\beta} + \frac{2 \mu F_s}{3 r_0^2 (k \beta)} e^{-0.4 \frac{R}{r_0}} \left( \left(1 + (kr_0 \beta)^2 \right)^{1.5} - 3 \left(1 + (kr_0 \beta)^2 \right)^{0.5} + 2 \right)
\]  

(1)

Figure 2 shows the relationship between the improved predicted values calculated by equation (1) and the test values of the tyre pivot resistance steering torques. The comparison shown in Figure 2 indicates that the error between these two values obviously decreased, particularly in the initial phase, and that the error was less than 9.3%. Therefore, the predicted values of the link forces and hydraulic pressures in the MASLM based on equation (1) are in very good agreement with the actual practical cases, and we used them to calculate the theoretical values of the tyre pivot resistance steering torque, as will be described in the following sections.

3  Steady-state kinematic-mechanical MASLM model

3.1  MASLM kinematic model

The main function of an MASLM is to restrain the steering angle relationship among all of the wheels, and to maintain the pure rotation of all wheels around its turning centre. The MASLM can be considered to be constructed with several sets of RSSR (Revolute spherical spherical revolute) spatial four-bar mechanisms (Wang et al., 2013).
In Figure 3, the relationship between angle $\theta_0$ and $\theta_3$ can be expressed as follows (Tanik and Parlaktas, 2010):

$$\theta_3 = 2 \arctan \frac{A \pm \sqrt{A^2 + B^2 - C^2}}{B - C}, \quad (2)$$

where

$$A = a_1 (s_0 \sin \alpha_0 - a_1 \sin \theta_0 \cos \alpha_0),$$

$$B = a_1 (a_0 - a_1 \cos \theta_0),$$

$$C = 0.5 \left( a_0^2 + a_1^2 + a_0^2 - a_2^2 + s_1^2 \right) + a_1 \left( a_0 \cos \theta_0 + s_1 \sin \theta_0 \sin \alpha_0 \right) + s_0 s_1 \cos \alpha_0.$$

Here, the choice of a coordinate system for every joint, and the definitions of the mechanism’s various structural parameters, are described below.
The common perpendicular line of the $z_{i-1}$ and $z_i$ axes is regarded as the $x_i$-axis of the $i$th joint, and the $x_i$-axis direction can be arbitrary. The common perpendicular line segment from $z_{i-1}$ to the $z_i$ axis is named for $a_i$ and is positive in the $x_i$-direction. The common perpendicular line segment from $x_i$ to the $x_{i+1}$ axis is named for $s_i$ and is positive in the $z_i$-direction. The angle around $x_i$ from $z_{i-1}$ to the $z_i$ axis is defined as $\alpha_i$ and is positive in the anti-clockwise direction. The angle around $z_i$ from $x_i$ to the $x_{i+1}$ axis is named for $\theta_i$ and is positive in the anti-clockwise direction. The structural parameter values in equation (2) were determined by the above definitions and the actual structural dimensions.

Because an MASLM can be considered to consist of several sets of RSSR mechanisms, the output angle of the $i$th set of the RSSR mechanism $\theta_{3i}$ is equal to the input angle of the $(i+1)$th set of the RSSR mechanism $\theta_{0(i+1)}$. Therefore, every rocker angle in the MASLM can be calculated by the steering gear angle.

### 3.2 Mechanical model of RSSR mechanism

A mechanical model of the RSSR mechanism was developed using the kinematic MASLM model, Newton’s Second Law, and vector space theory. To simplify the model, the following assumptions were made:

- the friction coefficient of every joint was ignored; therefore, the coupler link in the RSSR mechanism could be regarded as an axial force link.
- the RSSR mechanism was only considered to be in a steady-state equilibrium of forces; hence, the inertia force of each link was disregarded.

The objective of the MASLM mechanical analysis is to obtain the force acting on the coupler link in every RSSR mechanism. The force on link $B_iC_i$ can be obtained according to the input torque of the RSSR mechanism, $M_{di}$, and the perpendicular distance from link $B_iC_i$ to the $z_i$ axis of the revolute joint $A_i$; namely, the moment arm of link $B_iC_i$ with respect to the $z_i$ axis of joint $A_i$, which is shown in Figure 4.

#### 3.2.1 Moment arm of link $B_iC_i$ with respect to $z_i$ axis of joint $A_i$

To simplify the expression and calculation, the coordinate system was fixed at point $A_i$, and $A_i$ is the origin of the system.
The common perpendicular vector of vectors \( \mathbf{n}_i \) and \( \mathbf{m}_i \) can be expressed as follows:

\[
\mathbf{l}_i = \mathbf{n}_i \times \mathbf{m}_i
\]  

Point \( B_i \) can be presented by \((x_{Bi}, y_{Bi}, z_{Bi})\), and vector \( \mathbf{f}_i \) of link \( A_i B_i \) can be described as follows:

\[
\mathbf{f}_i = [x_{Bi}, y_{Bi}, z_{Bi}]
\]

Therefore, the moment arm of link \( B_i C_i \) around the \( z_i \)-axis of joint \( A_i \) can be written as follows:

\[
d_i = \frac{[f_i \cdot f_i]}{\|f_i\|}
\]  

### 3.2.2 Angle between vector \( \mathbf{n}_i \) and \( \mathbf{m}_i \)

The torque of link \( B_i C_i \) with respect to the \( z_i \)-axis of joint \( A_i \) is equal to the product of the component force of link \( B_i C_i \) in the perpendicular direction of the \( z_i \)-axis and its moment arm. Therefore, the angle of vectors \( \mathbf{n}_i \) and \( \mathbf{m}_i \) must be calculated as follows:

\[
\theta_i = \arccos \left( \frac{\mathbf{n}_i \cdot \mathbf{m}_i}{\|\mathbf{n}_i\| \|\mathbf{m}_i\|} \right)
\]  

### 3.2.3 Force of link \( B_i C_i \)

\[
F_{Bi} = \frac{T_i}{d_i \sin \theta_i}
\]

### 3.2.4 Resistance torque \( m_{zi} \)

To calculate the force of the next set of the RSSR mechanism, the moment arm of link \( B_i C_i \) needs to be determined with respect to the axis of the revolute joint \( D_i \). The moment arm was derived similarly to equation (4) as follows:

\[
d_i' = \frac{[f_i' \cdot f_i']}{\|f_i'\|}
\]

Thus, the resistance torque \( M_{zi} \) can be expressed as follows:

\[
M_{zi} = F_{Bi} \cdot d_i' \cdot \sin \theta'_i
\]

where the value of \( \theta'_i \) can be calculated by equation (5).

### 3.3 Mechanical model of steering cylinder

The structure of a steering trapezoid is shown in Figure 5. A local coordinate frame was fixed to the axle centre. The steering trapezoid consists of two knuckle arms, a tie rod, and left and right steering cylinders. In addition to driving the corresponding wheel, the
steering cylinder can also coordinate the other steering axles to be steered by the steering link mechanism between the axles.

**Figure 5** Structure of a steering trapezoid (see online version for colours)

A mechanical model of the steering trapezoid can be established according to the method for the RSSR mechanism. The objective of the main investigation described below was to introduce the driving torque model of the steering cylinder.

### 3.3.1 Vector of steering cylinder BD

The coordinates of points A, B and D, are defined as \((0, y_A, 0)\), \((x_B, y_B, z_B)\), \((x_D, y_D, z_D)\), respectively, with consideration to the coordinate frame at the axle centre. Here, point B is a point fixed on the axle, or its coordinates are three constant numbers. Unlike point B, point D is a moving point, and its coordinates vary with the angle position of the knuckle arm, which rotates around the kingpin. The instantaneous coordinates of point D are determined as follows.

The turning radius of point D around the kingpin can be written as follows:

\[
r = \sqrt{\left(z_{D0} + \frac{y_{D0} - y_{A0}}{\tan \gamma}\right)^2 \sin^2 \gamma + x_{D0}^2}
\]  

(9)

The acute angle between \(x\) direction of the vehicle and line AD can be expressed as follows:

\[
\theta_a = \arctan\left|\frac{z_{D0} + \frac{y_{D0} - y_{A0}}{\tan \gamma}}{x_{D0}}\sin \gamma\right|
\]

(10)

The instantaneous coordinates of point D in the left of the axle can be described as follows:

\[
x_D = \text{sign}(x_{D0})r \cos (\theta_a + \theta)
\]
where the clockwise direction is the positive direction of $\theta$.

The instantaneous coordinates of point $D'$ in the right of the axle can be described as follows:

\[
\begin{align*}
x_{D'} &= \text{sign}(x_{D_0})r \cos (\theta_0 + \theta) \\
y_{D'} &= y_{D_0} - \text{sign}(x_{D_0})\left[z_{D_0} + \text{sign}(z_{D_0})r \sin (\theta_0) \sin \gamma \right] \tan \gamma + \text{sign}(x_{D_0})r \sin (\theta_0 + \theta) \cos \gamma \\
z_{D'} &= z_{D_0} - \text{sign}(x_{D_0})\left[r \sin (\theta_0) - r \sin (\theta_0 + \theta) \right] \sin \gamma 
\end{align*}
\]

Vector $BD$ can be written as follows:

\[
g = [x_{D_0} - x_b, y_{D_0} - y_b, z_{D_0} - z_b]
\]

Vector $AD$ can be written as follows:

\[
h = [x_D - x_b, y_D - y_b, z_D - z_b]
\]

### 3.3.2 Moment arm of steering cylinder with respect to kingpin

\[
d_{g_i} = \frac{|g_i \cdot h_i|}{|g_i|} 
\]

The driving torque $M_{g_i}$ can be expressed as follows:

\[
M_{g_i} = F_{g_i} \cdot d_{g_i} \sin \phi_i 
\]

### 3.4 MASLM coupling model

#### 3.4.1 MASLM coupling analysis

In a multi-axle steering system, the driving torques are mainly produced by the left and right steering cylinders and the resistance torques originate briefly from the corresponding left and right tyres. Therefore, a multi-axle steering system is a multi-output/multi-input and mechanical-hydraulic coupling system. During the steady-state cornering, the magnitudes and directions of the link forces in the MASLM maintain a transient state. A multi-input or multi-output state may occur in some RSSR mechanisms in the MASLM. Specifically, an RSSR mechanism may be subjected to the multi-inputs torques from its connected multiple links or transmit the driving torques to its connected multiple links.
In theory, in a hydraulic assisted steering system, the equilibrium state between the driving torque of every steering cylinder and the corresponding tyre pivot resistance steering torque cannot be maintained at any moment. Therefore, the steering link force varies with the working loads, and its magnitude indirectly reflects the matching level between the steering cylinder’s driving torque and the corresponding tyre pivot resistance steering torque. The smaller the magnitude is, the higher is the matching level.

Presently, only a few researchers (Wang et al., 2013) have investigated a matching design method for an MASLM. However, in practice, link bending or breakage occur frequently in the MASLM, and may result in abnormal tyre wear or serious safety incidents, as shown in Figures 6 and 7.

Figure 6 Abnormal tyre wear (see online version for colours)

Figure 7 Link breakage (see online version for colours)

3.4.2 Coupling model of steering link mechanism

As shown in Figure 8, the steering axle with a double-loop hydraulically-assisted steering system is generally adopted by most multi-axle steering vehicles. By considering this type of steering axle as an example, the authors deduced its coupling equilibrium force model. For the \( i \)th steering axle, the equilibrium force model can be written as follows:

\[
\frac{M_u + T_i}{d_\theta \sin(\theta_i)} + \frac{M_d}{d_\psi \sin(\theta_i)'} = \frac{F_x d_\psi \sin(\phi_i)}{d_\psi \sin(\theta_i)} + \frac{F_y d_\psi \sin(\phi_i')}{d_\psi \sin(\theta_i)'}
\]  

(13)
Since torque $T_i$ is the inner torque of the MASLM, the following equation can be established:

$$\sum_{i=1}^{n} \xi_i T_i = 0$$  \hspace{1cm} (14)

When a steering axle is the input axle, then, $\xi_i = -1$.

**Figure 8** Steering axle with a double-loop hydraulically-assisted steering system (see online version for colours)

Since the output of the steering cylinder is the pushing force, it can be written as follows:

$$F_{sp} = P_a \cdot \pi R_i^2$$  \hspace{1cm} (15)

Since the output of the steering cylinder is the dragging force, it can be written as follows:

$$F_{sp} = P_a \cdot \pi (R_i^2 - r_i^2)$$  \hspace{1cm} (16)

if

$$A_i = \frac{\pi (R_i^2 - r_i^2) d_{mi} \sin(\phi_i)}{d_{mi} \sin(\theta_i)} + \frac{\pi R_i^2 d_{mi} \sin(\phi_i')} {d_{mi} \sin(\theta_i')}$$  \hspace{1cm} (17)

and

$$M_{iu} = \frac{M_{iu}} {d_{mi} \sin(\theta_i')} + \frac{M_{iu}} {d_{mi} \sin(\theta_i')}$$  \hspace{1cm} (18)

Substituting equations (15)–(18) into equation (13) gives the following equation:

$$P_a = \frac{\tau_i}{d_{mi} \sin(\theta_i')} + \frac{M_{iu}} {A_i}$$  \hspace{1cm} (19)

Substituting equations (17) and (18) into equation (13), and allying equation (14), an n-dimension equation set can be written as follows:
Matching and optimising analysis of multi-axle steering vehicle steering system

\[
\begin{align*}
\frac{A_1}{d_{11} \sin(\theta_1)} T_1 + \frac{A_1}{d_{12} \sin(\theta_2)} T_2 &= M_{a1} A_2 + M_{a2} A_1 \\
&\vdots \\
\frac{A_{(i+1)}}{d_{ii} \sin(\theta_i)} T_i + \frac{A_{(i+1)}}{d_{i(i+1)} \sin(\theta_{i+1})} T_{i+1} &= M_{a(i+1)} A_{(i+1)} + M_{a(i+1)} A_{i} \\
&\vdots \\
\frac{A_{(n-1)}}{d_{(n-1)n} \sin(\theta_{n-1})} T_{(n-1)} + \frac{A_{(n-1)}}{d_{(n-1)n} \sin(\theta_{n})} T_n &= M_{a(n-1)} A_{n-1} + M_{a(n-1)} A_{(n-1)} \\
\sum_{i=1}^{n} \xi_i T_i &= 0
\end{align*}
\]  

From equation (20), the external input or output torque \( T_i \) can be derived, and the substitution of \( T_i \) into equation (19) yields \( P_a \), which is the pressure in the hydraulic steering system. Substituting \( T_i \) into equation (6) can solve each link force in the MASLM.

4 Numerical simulation and application evaluation

4.1 Model validation

To validate the correctness of the above theoretical MASLM model, and by considering the all-terrain QAY130 crane as an example, we developed a 5-axle vehicle model by using the Adams software, as shown in Figure 9, and conducted various experiments on the MASLM. The front three axles (1st ~ 3rd axles) of the vehicle steered by a set of steering link mechanisms, while the rear two axles (4th-5th axles) steered by electrical-hydraulic steering systems. The set of steering link mechanism consisted of six sets of RSSR spatial four-bar mechanisms (numbers 1–6 in Figure 9) in order to connect the front three axles and the five sets of the steering trapezoids (numbers 7–11 in Figure 9) in order to coordinate the left and right steering wheels on every axle. Its structural parameters are listed in Table 1.

Figure 9 5-axle vehicle model (see online version for colours)
Table 1  Structural parameters of vehicle

<table>
<thead>
<tr>
<th>Item</th>
<th>1-2 wheel base</th>
<th>2-3 wheel base</th>
<th>3-4 wheel base</th>
<th>4-5 wheel base</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values (m)</td>
<td>2.75</td>
<td>1.62</td>
<td>2.0</td>
<td>1.62</td>
</tr>
<tr>
<td>Item</td>
<td>Mass of vehicle</td>
<td>Cylinder diameters</td>
<td>Tyre</td>
<td>Track</td>
</tr>
<tr>
<td>Value</td>
<td>57.98t</td>
<td>φ 60 × 28 mm</td>
<td>Michelin14.00R25</td>
<td>2.59 m</td>
</tr>
</tbody>
</table>

4.1.1 ADAMS model

By using the above theoretical models, a simulation model was developed with the Matlab software. To compare the simulation results of the Adams model to those of the Matlab model, the following preparations had to be made:

- The two models had to have the same datum, all of which were from the datum of the all-terrain QAY130 crane.
- The modified tyre model pac2002_315_80R22_5 in the ADAMS software was used for the prototype. The purpose of modifying the tyre model file was to maintain the consistency between the structural size and the vertical stiffness characteristics of the tyre model with those of the actual tyre, and to eliminate the factors influencing the steering resistance torque of the tyre, which was simulated by applying torque to the kingpin. Equation (2) was used to establish the expression of the stimulation torque.
- A step function was applied to the Adams model in order to simulate the variation of the hydraulic pressure with the wheel angle during cornering. The output force of the steering cylinder could be calculated according to the hydraulic pressure and cylinder motion. In this study, the original diameters of the cylinder barrel and rod were 65 mm and 32 mm, respectively.

4.1.2 Measurement of link forces

The experiment conditions for measuring the link forces consisted of placing the vehicle on concrete ground and a travelling velocity of zero. The strain gauge was mounted in the middle of each tie rod, as shown in Figure 10, and recorded the force values of the rods as the vehicle turned left.

**Figure 10** Part picture of the actual test field (see online version for colours)
Test results show that the coupler links in the 1st and 2nd sets of the RSSR mechanisms in the MASLM were subjected to very small forces, and their magnitudes were only related to the characteristics of the steering gear. The connecting wire of the strain gauges on link 7 and 8 broke during the test; therefore, its results were not obtained.

4.1.3 Analysis of results

The link forces of the 3rd–6th sets of the RSSR mechanisms and the tie rod forces of the front-three-axle steering trapezoids were analysed as follows. Note that every axle load was 1.2 × 10^5 N.

In the legend of Figures 10 and 11, the numbers represent the tie rods or coupler links corresponding to the labels of the RSSR mechanisms or the steering trapezoids in Figure 9. These figures show the comparisons between the tie rod or coupler link forces that resulted from the test, Adams model, and Matlab model, respectively. The results indicate that the force distributions in the two models were in very good agreement. However, there were acceptable discrepancies between the measured values from the test and the predicted values from the previous two models. The reason is that the axle loads varied as the wheel steering angle increased during the cornering test, as shown in Figure 12. The previous two models failed to consider the variation effect of the tyre vertical loads. The ratio of the discrepancies to the corresponding test values was less than 23.5%. The variation trends of the measured values from the test and those of the predicted values from the previous two models were almost the same. Hence, the maximum discrepancies between these values were generally acceptable. The above comparisons confirmed that the Matlab model was developed correctly. Thus, the theoretical MASLM model, namely, equation set (20), had high reliability and could be implemented with confidence.

4.2 Design analysis

The main work in matching the MASLM design consists of matching the steering cylinder output torque with the tyre pivot resistance steering torque. The influencing factors of the matching relationship will be discussed first in order to further explain how the matching design of the MASLM was realised.

4.2.1 Analysis of influencing factors

In the investigation presented above, we determined that the range of each wheel steering angle was different, and that the maximum of the tyre pivot resistance steering torque was also different. Therefore, the output torques of the steering cylinders on each axle should be different in order to match the corresponding tyre torques, which results in the corresponding cylinder dimension also being different. However, all steering cylinders have the same dimensions for most all-terrain cranes. Therefore, for some axles such as those shown in Figure 13, non-matching may occur between the output torque of the steering cylinder and the tyre pivot resistance steering torque.
Figure 11 Comparison of the link forces: (a) tie rod forces of the front three axles steering trapezoids and (b) coupler link forces of the 3rd–6th sets of the RSSR mechanisms (see online version for colours)

Figure 12 Tyre vertical load vs. wheel steering angle (see online version for colours)

Figure 13(a) shows that the driving torque of the right steering cylinder matched the corresponding tyre resistance torque well, but the driving torque of the left steering cylinder on the 1st axle was obviously less than its corresponding tyre resistance torque. When the vehicle turned left or right (Figure 13 shows the variation graph of the driving/resistance torques with the wheel angle, when the vehicle turned left), the steering cylinder diameters of the 1st axle, generally speaking, were too small to match the tyre resistance torque. In Figure 13(b), the driving torque of the left steering cylinder on the 2nd axle was slightly greater than its corresponding tyre resistance torque, and the driving
torque of the right steering cylinder matched the corresponding tyre resistance torque well. That is, the steering cylinder diameters of the 2nd axle were a better match to the tyre resistance torques. In Figure 13(c), the driving torque of the left steering cylinder of the 3rd axle matched the corresponding tyre resistance torque. However, the driving torque of the right steering cylinder was obviously greater than its corresponding tyre resistance torque. For the same reason, the steering cylinder diameters of the 3rd axle were too large to match the tyre resistance torques.

**Figure 13** Comparison between output torque of each-axle cylinder and tyre pivot resistance steering torque: (a) comparison between output torque of 1st axle cylinder and tyre pivot resistance steering torque; (b) comparison between output torque of 2nd-axle cylinder and tyre pivot resistance steering torque and (c) comparison between output torque of 3rd-axle cylinder and tyre pivot resistance steering torque (see online version for colours)
By the above investigation, we found that the steering cylinder diameters may influence the matching level between the driving and resistance torques.

Moreover, we found that the steering cylinders mounted at the front of the 2nd axle produced a larger driving torque or the left cylinder output torque, the magnitude of which only balanced with the bigger tyre resistance torque. However, the other axles had an inverse relationship. Therefore, the installation site of the steering cylinders (right and left) may also be a key influencing factor that involves the matching level between the driving and resistance torques.

4.2.2 Influence of cylinder dimension

Based on the above analysis, the cylinder diameters were modified as $\phi 70 \times 32$ mm for the 1st axle, $\phi 60 \times 28$ mm for the 2nd axle, and $\phi 60 \times 32$ mm for the 3rd axle. The MASLM simulation results were compared to the earlier results. The results of the comparison are shown in Figures 14–16.

**Figure 14** Hydraulic pressure (see online version for colours)

![Hydraulic pressure graph](image)

**Figure 15** Coupler link forces of every set of the RSSR mechanism (see online version for colours)

![Coupler link forces graph](image)
Figure 14 shows that the maximum hydraulic pressure of the steering system increased by 10%, from 14.4 MPa to 15.66 MPa.

In Figures 15 and 16, the coupler link or tie rod forces of the original model are denoted by $F^*$, and the coupler link or tie rod forces of the steering system with the modified cylinder diameters are denoted by $F^{*dm}$. The symbol * in these and following figures indicates the tie rods or coupler links corresponding to the labels of the RSSR mechanisms or steering trapezoids shown in Figure 9. Figure 15 shows that the coupler link forces of the modified steering system decreased more than 40%, in comparison to those of the original system. Therefore, the optimisation of the steering cylinder diameter can improve the matching level between the steering cylinder output torques and the tyre pivot resistance steering torques, and thereby dramatically reduce the coupler link force.

In Figure 16, the trapezoid tie rod force reflects the matching level between the steering cylinder output torques and the pivot resistance steering torques of its driving tyre. The smaller the tie rod force is, the higher is the matching level. Therefore, the matching level of the 2nd axle was higher than the others. The matching level of the 1st axle was the most unsatisfactory one. Because the driving torque of the steering cylinders on the 3rd axle was greater than the corresponding tyre pivot resistance steering torque, the matching level of the 3rd axle deteriorated. The surplus torque was transformed from the tie rod to the steering link mechanism between the axles in order to help steer the other axles.

4.2.3 Influence of steering cylinders installation site

The installation sites of the steering cylinders on all axles were modified from the back to the front of the axles. The results simulated with the original and modified installation site models are compared as below.

As shown in Figure 17, the maximum hydraulic pressure of the steering system decreased from 14.4 MPa to 13.88 MPa after the steering cylinder was mounted onto the front of the axle. In Figures 18 and 19, symbol $F^*$ in the legend indicates the results of the original model, while symbol $F^{*pm}$ indicates the results of the model with the installation site modified. In Figure 18, the coupler link forces declined to a certain extent, with the exception of the fifth set of the RSSR mechanism. As shown in Figure 19, the trapezoid tie rod forces obviously decreased. In particular, this trend was
most pronounced for the 3rd axle. This demonstrates that the matching level between the steering cylinder output torques and the tyre pivot resistance steering torques improved dramatically.

**Figure 17** Hydraulic pressure (see online version for colours)

![Hydraulic pressure graph](image)

**Figure 18** Comparison between link forces of each set of steering link mechanism (see online version for colours)

![Link forces graph](image)

The above investigation proves that the installation site adjustment of the steering cylinders can improve the matching level of the steering system. In particular, the matching level between the steering cylinders’ driving torques and the corresponding tyre pivot resistance steering torques improved.

### 4.2.4 Comprehensive effect of cylinder diameters and installation sites

In this case, the comprehensive modified model, the installation sites, and steering cylinder diameters, were modified simultaneously. The comparisons between the simulation that resulted from the model with the comprehensive modified and original models are presented in Figures 20–22.
Matching and optimising analysis of multi-axle steering vehicle steering system

Figure 19  Comparison between trapezoid tie rod forces (see online version for colours)

Figure 20  Steering hydraulic pressure (see online version for colours)

Figure 21  Comparison between coupler link forces (see online version for colours)
As shown in Figure 20, the maximum hydraulic pressure of the steering system increased from 14.4 MPa to 15.06 MPa after the installation site and diameter of the steering cylinder were modified. In Figures 21 and 22, the symbol $F^*m$ in the legend denotes the results of the original model, while symbol $F^*cm$ denotes the results of the comprehensive modified model. As shown in Figure 21, the coupler link forces of the steering link mechanism obviously decreased after the steering cylinders were suitably adjusted.

As can be seen in Figure 22, the trapezoid tie rod forces declined further. This result indicates that the driving torque of the steering cylinder was a good match to the tyre pivot resistance steering torque.

4.3 Optimisation

To further improve the matching level of the steering system, we optimised the installation sites of its steering cylinders.

4.3.1 Selection of optimisation variables

The radius of pivot D around the kingpin shown in Figure 5, the initial z-coordinate of point D ($z_{D0}$), and the angle between the x-axis and line AD were chosen as the optimal variables. Because each axle had three variables, there were nine optimal variables, and they are presented in Table 2.

<table>
<thead>
<tr>
<th>Item</th>
<th>$z_{D10}$</th>
<th>$z_{D20}$</th>
<th>$z_{D30}$</th>
<th>$R_1$</th>
<th>$R_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial value(m)</td>
<td>−0.116</td>
<td>0.101</td>
<td>−0.171</td>
<td>0.193</td>
<td>0.203</td>
</tr>
<tr>
<td>Variable range</td>
<td>−0.3~−0.4</td>
<td>−0.15~−0.4</td>
<td>−0.3~−0.4</td>
<td>0.15~0.4</td>
<td>0.15~0.4</td>
</tr>
<tr>
<td>Item</td>
<td>$R_3$</td>
<td>$a_1$</td>
<td>$a_2$</td>
<td>$a_3$</td>
<td></td>
</tr>
<tr>
<td>Initial value(m)</td>
<td>0.193</td>
<td>0.159</td>
<td>0.333</td>
<td>0.308</td>
<td></td>
</tr>
<tr>
<td>Variable range</td>
<td>0.15~0.4</td>
<td>−0.5~−0.5</td>
<td>−0.5~−0.5</td>
<td>−0.5~−0.5</td>
<td></td>
</tr>
</tbody>
</table>
Matching and optimising analysis of multi-axle steering vehicle steering system

4.3.2 Optimisation objective

The sum of each maximum absolute magnitude of the link forces, such as the coupler link and tie rod forces, was chosen as the optimisation objective to minimise.

4.3.3 Optimisation method

There are a lot of optimisation tools and functions in Matlab. The function Fmincon(x), which searches for the minimum of a problem, was applied to solve the minimisation of the optimisation objective defined by equation (21).

The option of the Interior Point Algorithm (default) was set, i.e., the interior-point approach to the constrained minimisation was used to solve a sequence of approximate minimisation problems. The original problem is expressed as follows:

$$\min_x f(x), \text{subject to } h(x) = 0 \text{ and } g(x) \leq 0.$$  

(21)

where \(h(x), g(x)\) are vector functions representing all of the inequality and equality constraints (meaning bound, linear, and nonlinear constraints), respectively.

For each \(\mu > 0\), the approximate problem is expressed as follows:

$$\min_{x,s} f(x,s) = \min_{x,s} f(x) - \mu \sum_i \ln(s_i).$$  

(22)

subject to \(h(x) = 0\) and \(g(x) + s = 0\).

There are as many slack variables \((s_i)\) as there are inequality constraints \((g)\). The \(s_i\) are restricted to be positive in order to keep \(\ln(s_i)\) bounded. As \(\mu\) decreases to zero, the minimum of \(f_\mu\) should approach the minimum of \(f\). The added logarithmic term is termed the barrier function.

The approximate problem expressed by equation (22) is a sequence of equality constrained problems. These are easier to solve than the original inequality-constrained problem expressed by equation (21).

To solve the approximate problem, the algorithm uses one of two main types of the steps at each iteration:

(1) A direct step in \((x, s)\).

To solve equation (23) for direct step \((\Delta x, \Delta s)\), the algorithm carried out LDL factorisation (Factor square Hermitian positive definite matrices into lower, upper, and diagonal components) of the matrix. One result of this factorisation is the determination of whether the projected Hessian was positively definite or not. If not, the algorithm used a conjugate gradient step.

$$H = \nabla^2 f_\mu, \sum_i 1/2s_i^2, \sum_j 1/2s_j^2$$

(23)
where

\( J_g \): Jacobian of the constraint function \( g \).

\( J_h \): Jacobian of the constraint function \( h \).

\( S \): \( \text{diag}(s) \).

\( \lambda \): Lagrange multiplier vector associated with constraints \( g \).

\( \Lambda \): \( \text{diag}(\lambda) \).

\( y \): Lagrange multiplier vector associated with \( h \).

\( e \): vector of ones that is the same size as \( g \).

(2) A CG (conjugate gradient) step, using a trust region.

The conjugate gradient approach to solving the approximate problem expressed by equation (22) is similar to other conjugate gradient calculation. In this case, the algorithm adjusts both \( x \) and \( s \), while keeping the slack \( s \) positive. The approach consists of minimising the quadratic approximation to the approximate problem in a trust region subject to the linearised constraints.

Specifically, let \( R \) denote the radius of the trust region, and let the other variables be defined as in Direct Step. The algorithm obtains Lagrange multipliers by approximately solving the Karush-Kuhn-Tucker (KKT) equations

\[

\nabla_x L = \nabla_x f(x) + \sum_i \lambda_i \nabla g_i(x) + \sum_j \lambda_j \nabla h_j(x)

\]

in terms of least-squares, and subject to \( \lambda \) being positive. Then, the algorithm takes a step \((\Delta x, \Delta s)\) to approximately solve:

\[

\min_{\Delta x, \Delta s} \frac{1}{2} \Delta x^T J_g^T J_g \Delta x + \frac{1}{2} \Delta s^T J_h^T J_h \Delta s

\]

\[

\nabla_x \nabla_s \frac{1}{2} \Delta x^T J_g^T J_g \Delta x + \lambda^T S^{-1} \Delta s + \frac{1}{2} \Delta s^T S^{-1} \lambda \Delta s

\]

subject to the following linearised constraints:

\[

g(x) + J_x \Delta x + \Delta s = 0, \quad h(x) + J_x \Delta x = 0

\]

To solve equation (25), the algorithm attempts to minimise the norm of the linearised constraints inside a region with radius scaled by \( R \). Then, equation (24) is solved by the constraints matching the residual from solving equation (25), while keeping within the trust region of radius \( R \), and keeping \( s \) strictly positive.

4.3.4 Optimisation result

The optimisation results are listed in Table 3.

<table>
<thead>
<tr>
<th>Item</th>
<th>Optimisation value(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_3 )</td>
<td>0.198</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>0.248</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>0.470</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>0.468</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Item</th>
<th>( z_{D10} )</th>
<th>( z_{D20} )</th>
<th>( z_{D30} )</th>
<th>( R_1 )</th>
<th>( R_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimisation value(m)</td>
<td>–0.0</td>
<td>0.397</td>
<td>0.206</td>
<td>0.259</td>
<td>0.248</td>
</tr>
<tr>
<td>Item</td>
<td>( R_1 )</td>
<td>( a_1 )</td>
<td>( a_2 )</td>
<td>( a_3 )</td>
<td></td>
</tr>
<tr>
<td>Optimisation value(m)</td>
<td>0.198</td>
<td>0.248</td>
<td>0.470</td>
<td>0.468</td>
<td></td>
</tr>
</tbody>
</table>
In Figures 24 and 25, symbol $F^{\text{cm}}$ in the legend indicates the results of the comprehensively modified model, while symbol $F^{\text{O}}$ indicates the results of the optimised model. From Figure 23, it can be seen that the maximum hydraulic pressure obviously decreased from 15.06 MPa to 12.2 MPa after the optimisation of the steering cylinder’s installation site was carried out. Additionally, the hydraulic pressure declined by approximately 19%. As shown in Figure 24, although the trapezoid tie rod forces of the 1st and 2nd axles increased, the trapezoid tie rod force of the 3rd axle obviously declined. Moreover, all of the force curves fluctuated symmetrically in the vicinity of the zero line along the $x$-axis, which indicates that the output torque of the steering cylinder matched the tyre steering resistance torque perfectly, and that the coupler link forces between the axles dramatically declined, as shown in Figure 25. The results shown in Figure 25 indicate that the matching level of the steering system greatly improved.

**Figure 23** Hydraulic pressure (see online version for colours)

**Figure 24** Comparison between coupler link forces (see online version for colours)
Figure 25  Comparison between trapezoid tie rod forces (see online version for colours)

5 Conclusions
From the discussion regarding the matching design and multi-axle steering system optimisation, the following conclusions can be drawn:

- By investigating the initial characteristics of the tyre pivot resistance steering torque, it was found that the friction coefficient and viscous torsion damping coefficient around the kingpin were the major influencing factors of the tyre pivot resistance steering torque’s initial characteristics. Based on this, the tyre pivot resistance steering torque formula was improved further and the errors between the formula and the test values of the tyre torques were less than 9.3%.

- We built a kinematic-mechanical model based on the RSSR type kinematic model of the spatial four-bar mechanisms.

- This study developed a kinetic-mechanical coupling equilibrium model of a multi-axle steering system.

- The key tasks with regard to the matching design of a steering link mechanism in multi-axle vehicles consisted of improving the matching level of the steering cylinder driving torque and the tyre pivot resistance steering torque. The main influencing factors were the installation site and dimension of the steering cylinders.

- A simulation investigation revealed that the steering cylinder installed on the front of the axle was more favourable to driving the corresponding tyre. Additionally, the range of the wheel steer angle was different and its steering cylinder should be designed with different dimensions. Namely, the smaller the wheel steer angle was, the smaller was the steering cylinder diameter.
Matching and optimising analysis of multi-axle steering vehicle steering system

Acknowledgement

The work described in this paper was supported by National Natural Science Foundation of China under Project No. 51575233.

References


Nomenclature

\[ M_z \] Self-aligning torque produced around kingpin
\[ M_s \] Tyre pivot steering resistance torque in the tyres/ground contact region
\[ F_z \] Vertical load of tyre
\[ R \] Turning radius of a tyre around kingpin
\[ \gamma \] Angle of the kingpin inclination
\[ \beta \] Steering angle
\[ \mu_0 \] Coefficient of sliding friction between tyre and ground
\[ \mu \] Friction coefficients between its bearing and kingpin
\[ r_0 \] Radius of contact region
\[ k \] Normalization torsion deformation coefficient of stiffness
\[ M_p \] Friction resistance torque produced around its kingpin
\[ M_d \] Torsional viscous damping torque produced around its kingpin
\[ M_{di} \] Input torque of RSSR mechanism
\[ \zeta \] Tyre torsional viscous damping coefficient produced around its kingpin
\[ n_i \] z-axis vector of joint \( A_i \)
\[ n_i' \] z-axis vector of joint \( D_i \)
\[ m_i \] z-axis vector of link \( B_iC_i \)
\[ f_i \] Vector of link \( A_iB_i \)
\[ f_i' \] Vector of link \( C_iD_i \)
\[ l_i \] Common perpendicular vector of vectors \( n_i \) and \( m_i \)
\[ l_i' \] Common perpendicular vector of vectors \( n_i' \) and \( m_i \)
\[ \theta_i \] Angle between vector \( n_i \) and \( m_i \)
\[ \theta_i' \] Angle between vector \( n_i' \) and \( m_i \)
\[ d_i \] Moment arm of link \( B_iC_i \) around the z-axis of joint \( A_i \)
\[ d_i' \] Moment arm of link \( B_iC_i \) with respect to the z-axis of joint \( D_i \)
\[ F_{li} \] Force of link \( B_iC_i \)
\[ M_{zi} \] Resistance torque
\[ r \] Turning radius of point d around the kingpin
\[ \theta_0 \] Acute angle between x direction of vehicle and line \( AD \)
\[ \theta \] Wheel steering angle
\[ g_i \] Vector of BD
\[ h_i \] Vector of AD
\[ F_{gi} \] Output force of steering cylinder
\[ d_{gi} \] Moment arm of the steering cylinder around the corresponding kingpin
Matching and optimising analysis of multi-axle steering vehicle steering system

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_l, \phi_r$</td>
<td>Angle between the axis of the left, right steering cylinder and the axis of the corresponding kingpin, respectively</td>
</tr>
<tr>
<td>$M_{li}, M_{ri}$</td>
<td>Left, right tyre pivot steering resistance torque, respectively</td>
</tr>
<tr>
<td>$T_i$</td>
<td>External input, output torque of the $i$th steering axle</td>
</tr>
<tr>
<td>$F_{gli}, F_{gri}$</td>
<td>Left, right steering cylinder forces</td>
</tr>
<tr>
<td>$d_{li}, d_{tri}$</td>
<td>Left, right moment arm of the tie rod around the corresponding kingpin</td>
</tr>
<tr>
<td>$d_{gli}, d_{gri}$</td>
<td>Moment arm of the left, right steering cylinder around the corresponding kingpin, respectively</td>
</tr>
<tr>
<td>$\zeta_i$</td>
<td>Amplification factor of a torque from an output axle to an input axle</td>
</tr>
<tr>
<td>$P_a$</td>
<td>Pressure of the steering hydraulic system</td>
</tr>
<tr>
<td>$R_i, r_i$</td>
<td>Diameters of the steering cylinder barrel and rod, respectively</td>
</tr>
</tbody>
</table>