Forecasting the yield curve with macroeconomic information – evidence from European markets

Isabel Maldonado*

Research on Economics, Management and
Information Technologies (REMIT),
Portucalense University,
Porto, Portugal
and
Governance Competitiveness and Public Policies (GOVCOPP),
University of Aveiro,
Aveiro, Portugal
Email: ianm@upt.pt
*Corresponding author

Carlos Pinho

Governance Competitiveness and Public Policies (GOVCOPP),
Department of Economics Management and Industrial Engineering,
University of Aveiro,
3810-193 Aveiro, Portugal
Email: cpinho@ua.pt

Francisco Rodríguez de Prado

Department of Financial Economics and Accounting,
University of Vigo,
Lagoas Marcosende, 36310 Vigo, Spain
Email: fprado@uvigo.es

Carla Azevedo Lobo

Research on Economics, Management and
Information Technologies (REMIT),
Department of Economics and Management,
University Portucalense,
Porto, Portugal
Email: cadsa@upt.pt

Abstract: In this paper we analyse the predictive content of the introduction of macroeconomic variables in term structure dynamic models. We tested the dynamic models using data from the public debt, inflation rate and annual variation of the industrial production index for four European countries: Portugal, Spain, the UK and Germany. Results obtained for the period from January 1990 to December 2012 indicate that considering macroeconomic factors makes a positive contribution to the improvement of forecasts for different countries and maturities. However, the paper presents evidence of time-varying forecast accuracy, not only across yield maturities and forecast horizons, but also over data sub-periods.

Keywords: yield curve; dynamic factor models; forecasting; out-of-sample forecasting evaluations.

Reference to this paper should be made as follows: Maldonado, I., Pinho, C., de Prado, F.R. and Lobo, C.A. (2021) 'Forecasting the yield curve with macroeconomic information – evidence from European markets', *Int. J. Banking, Accounting and Finance*, Vol. 12, No. 2, pp.177–200.

Biographical notes: Isabel Maldonado received her PhD in Financial Economics and Accountancy from the University of Vigo, Spain, an MSc in Finance from the Portucalense University, Porto, and BSc in Management from Portucalense University. Currently, she is a Professor in Portucalense University and in Porto Accounting and Business School (ISCAP) and lectures under graduated and graduated courses of General and Financial Accounting, Cost, Analytical and Management Accounting. She is also a Program Coordinator of the MSc in Management at Portucalense University.

Carlos Pinho received his PhD in Applied Economics from the University of Santiago de Compostela, an MSc in Finance and BSc in Management from the Portucalense University. He is an Associate Professor in the University of Aveiro, at the Department of Economics, Management, Industrial Engineering and Tourism. He lectures undergraduate and graduate courses of finances and economics. Also, he is a member of research teams of the University of Aveiro, participating in several national sponsored projects in the fields of economics and finance, regional development, decision support tools development. He is an author and co-author of more than 100 papers presented in national and international conferences or published in scientific journals and books.

Francisco Rodríguez de Prado has received his PhD in Economics and Business Sciences from the University of Vigo and is a Professor of Financial Economics and Accounting at the University of Vigo. He was the Dean of the Faculty of Economic and Business Sciences, University of Vigo and Director of the Department of Financial Economy and Accounting at the University of Vigo.

Carla Azevedo Lobo has receiver her PhD in Management from Portucalense University in Oporto, an MSc in Industrial Economics from the Minho University, Braga, and a degree in Economics from Portucalense University. Currently, she is a Professor in the Portucalense University and lectures under graduated and graduated courses of industrial economics, financial and project analysis, management techniques and international economics. Currently, she is also a Program Coordinator of the BSc in Management at the Portucalense University.

1 Introduction

Modelling and forecasting the term structure of interest rates has seen significant progress in recent decades, both in dynamic models (equilibrium and non-arbitrage) as well as in statistical models.

The models of Merton (1973), Vasicek (1977) and Cox et al. (1985), among others, trigger the development of a series of models, mostly related models that are either unifactorial or multifactorial: Duffee and Kan (1996), Dai and Singleton (2000), Duffee (2002) and Ang and Piazzesi (2003). These models were characterised by incorporating the evolution of interest rates over time, assuming that their behaviour is defined by stochastic processes.

The statistical or econometric models estimate the term structure of interest rates on the basis of existing information about the cash flows generated by the bonds and their market prices. These are based on the calculation of the spot rates that, at the time of estimation, are adjusted to the prices contracted in the equilibrium market. They use historical market data to estimate the discount function – the representative curve of bond rates. A large part of the models found in the literature are based on the methodologies originally developed by McCulloch (1971, 1975), Carleton and Cooper (1976), Vasicek and Fong (1982), Nelson and Siegel (1987) and Svensson (1994), among others. However, the Nelson and Siegel model stands out as one of the most popular models for modelling and forecasting the term structure of interest rates.

Nelson and Siegel (1987) use a statistical model to capture the movement of the yield curve, represented by a three-factor model. According to the authors, these factors explain most of the variation in interest rates for different maturities. Recognising the good performance of the model, Diebold and Li (2006) adapted the Nelson and Siegel model and transformed it into a dynamic model of latent factors in which the parameters vary over time. The tests carried out concluded that the dynamic model presents out-of-sample performance superior to Nelson and Siegel's original model.

In order to improve the predictive capacity of the models, advances have been made that combine the dynamics of the term structure of interest rates with fundamental economic variables. Among the works of reference related to arbitrage free models are those of Ang and Piazzesi (2003), Kim and Wright (2005) and Rudebusch and Wu (2008). Ang and Piazzesi (2003) apply an affine model with latent factors and two groups of macroeconomic variables associated with real activity and inflation. With a simple three-factor arbitrage-free term structure model, a model incorporating expected future inflation, real term premium and inflation risk premium was developed. Kim and Wright (2005) studied the specific role of inflation for explaining yield dynamics. Rudebusch and Wu (2008) developed and estimated a macro-finance model that combines an affine no-arbitrage model with macroeconomic variables associated with output and inflation. Using a statistical model, Diebold et al. (2006) depart from the dynamic version of the Nelson and Siegel model proposed by Diebold and Li (2006) and expand the scope by introducing macroeconomic variables as explanatory factors for the term structure of interest rates.

More recent studies, such as Kaya (2013) for the Turkish economy, and Favero et al. (2012), Bikbov and Chernovb (2013), Coroneo et al. (2016), Paccagnini (2016), Rubaszek (2016) and Vieira et al. (2017) for the USA, indicate that introducing macroeconomic variables improves the forecasting performance of the term structure.

However, not all studies point to the same conclusions. Among others, Matsumura et al. (2011) in a study for Brazil conclude that including macroeconomic variables does not improve the forecasting capacity of the models, but the inclusion of a financial variable (a stock index) contributes positively to the forecast performance. None of the models tested was generally able to perform better than the random walk model.

Studies such as those of Ang and Piazzesi (2003), Diebold et al. (2006) and Diebold and Li (2006) try to assess the predictive ability of the term structure of interest rates using either information from the term structure of interest rates and macroeconomic information, or information from the term structure only. These models used a small set of macroeconomic variables: output and inflation and latent factors of the term structure [such as Ang and Piazzesi (2003) and Diebold et al. (2006)], or only latent factors of the term structure of interest rates (Diebold and Li, 2006). This option was based on the model estimation problem: the higher the number of parameters to estimate the greater the difficulty in working with the models.

Based on a different perspective, another set of studies uses of a broad set of macroeconomic series from which macroeconomic factors are extracted while maintaining a relatively low number of factors to be estimated [e.g., Hördahl et al., 2006; Favero et al., 2012; Coroneo et al., 2016, among others]. Hördahl et al. (2006) use a model with a macroeconomic structure for modelling and predicting interest rates, and comparing them with the random walk model, a VAR model of unrestricted interest rates, an essentially affine three-factor model and a model based on that of Ang and Piazzesi (2003). The results indicate better performance of the VAR model compared to the other models tested, although, for longer forecast horizons and longer time frames, the macroeconomic model used by the authors outperforms the VAR. Therefore, for Hördahl et al. (2006) the inclusion of macroeconomic variables generally contributes to improving the predictive capacity of models.

Moench (2008) also compares the predictive performance of a non-arbitrage model with a non-arbitrage VAR macro model (FAVAR), a VAR model of unrestricted interest rates, two three-factor models based on different specifications of the Nelson and Siegel model and an essentially affine model based on the one proposed in Duffee (2002), a simple AR model of interest rates and the random walk model. It concludes that the non-arbitrage model presents good out-of-sample forecasting capacity, especially in medium and long-term horizons and in periods when interest rates present a more unstable dynamic. However, for short horizons, the forecast quality of the random walk model is clearly higher.

Similar conclusions are presented by De Pooter et al. (2010) by testing models with different levels of complexity, ranging from simple AR and VAR models to Nelson and Siegel class models and affine models, taking as a reference the random walk model. They conclude that the predictive ability of the models varies over time. Models that include macroeconomic information seem to be more accurate in periods of greater uncertainty about future interest rates, while models with only latent factors perform well in periods where the dynamics of interest rates is more stable. However, for short and medium-term forecast horizons, several studies find that the random walk presents better results [see Duffee, 2002; Ang and Piazzesi, 2003; Diebold et al., 2006; Moench, 2008; Orphanides and Wei, 2012; Favero et al. 2012; Diebold and Rudebusch, 2013; Van Dijk et al., 2014, among others].

Favero et al. (2012) also compare the predictive capacity of a set of models including Nelson and Siegel class models and related models. They conclude that the inclusion of macroeconomic factors improves the forecasting ability of the models for longer horizons and for interest rates with short and medium term maturities. However, for shorter horizons, the models tested cannot achieve better results than the random walk model.

Rubaszek (2016) also compares the accuracy of interest rates forecasts of dynamic and dynamic affine yield curve models for the USA, including the yield curve latent factors and macroeconomic variables. Rubaszek points out "that in order to increase the accuracy of forecasts for yields it is essential to have a good forecasting framework for macroeconomic time series" (p.11). His results also suggest that affine models are better at explaining future movements in interest rates than the arbitrage-free models.

Still in pursuit of the contribution of macroeconomic factors to the yield curve estimation, Coroneo et al. (2016) use a data set of bond yields and a macroeconomic dataset. They conclude that two macroeconomic factors characterising economic growth and real interest have significant predictive power for the bond yields.

Our study based on previous findings examines the performance of term structure dynamic models in four distinct European markets, using only latent factors as well as macroeconomic information. The estimated models and their performance in and out-of-sample are examined and the temporal evolution of relative performance of the different models is assessed.

The paper is structured as follows. The next section presents the various models considered in the subsequent analysis. In Section 3, we describe the dataset and in Section 4 we discuss the estimated models. In Section 5, we assess the out-of-sample forecasting relative performance of models for the whole sample and their temporal evolution. Finally, Section 6 presents the conclusion and summarises our findings.

2 Methodology

2.1 Diebold and Li model

Diebold and Li (2006) proposed analysing and estimating the time structure of interest rates using the model proposed by Nelson and Siegel (1987), assuming that the parameters vary over time. They propose an exponential approximation of three components:

$$f_t(\tau) = \beta_{1t} + \beta_{2t} e^{(-\lambda_t \tau)} + \beta_{3t} \lambda_t e^{(-\lambda_t \tau)} \tag{1}$$

where $f_t(\tau)$ is the forward rate, β_{1t} is constant and $\beta_{2t} e^{(-\lambda_t \tau)} + \beta_{3t} \lambda_t e^{(-\lambda_t \tau)}$ is a Laguerre function.

The dynamics of the term structure would be described by the following equation:

$$Y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{(-\lambda_t \tau)}}{\lambda_t \tau}\right) + \beta_{3t}\left(\frac{1-e^{(\lambda_t \tau)}}{\lambda_t \tau} - e^{(-\lambda_t \tau)}\right) \tag{2}$$

β_{1t}, β_{2t}, β_{3t} and λ_t are the parameters to be estimated. $Y_t(\tau)$ represent the interest rates for different maturities τ, β_{1t}, β_{2t}, β_{3t} correspond to three dynamic latent factors, λ_t corresponds to the exponential decrease rate of the second and third components, τ is the rate maturity, with $\tau = 1, ..., N$ and t is the moment of analysis, $t = 1, ..., T$. According to Diebold and Li (2006), these time-varying latent factors can be interpreted as level (L_t), slope (S_t) and curvature (C_t) of the yield curve, respectively, which leads to:

$$Y_t(\tau) = L_t + S_t\left(\frac{1-e^{(-\lambda_t \tau)}}{\lambda_t \tau}\right) + C_t\left(\frac{1-e^{(-\lambda_t \tau)}}{\lambda_t \tau} - e^{(-\lambda_t \tau)}\right) \tag{3}$$

This measurement equation can be translated by:

$$y_t(\tau) = \Lambda f_t + \varepsilon_t \tag{4}$$

where each rate $Y_t(\tau)$ is guided in part by common latent factors (L_t S_t C_t) and in part by the idiosyncratic factor ε_t. $Y_t(\tau)$ corresponds to the vector of rates with different maturities, Λ represents the factor weight matrix and f_t the factor vector.

2.2 Dynamic Nelson and Siegel model with macroeconomic variables

According to Diebold et al. (2006) Nelson and Siegel's dynamic model can be expanded by incorporating macroeconomic variables to analyse the interaction between the latent factors that determine the shape of the yield curve and the macroeconomic variables. The measurement equation can be represented by:

$$y_t(\tau) = \Lambda f_t' + \varepsilon_t \tag{5}$$

The factor vector f_t' contains both the three latent factors and observable factors. Our study adds to the previous model the macroeconomic variables: inflation rate and annual variation of the industrial production index, represented by π_t and ΔIPI_t, respectively. That way:

$$f_t' = \left(L_t, S_t, C_t, \pi_t, \Delta IPI_t\right) \tag{6}$$

2.3 Affine multifactor term structure models

Duffee and Kan (1996) describe a class of arbitrage-free term structure multi-factor models in which zero-coupon yields are affine functions of a vector of state variables. In this multifactor affine models' class, the interest rate of a zero-coupon bond with maturity T is defined as an affine function of a set of state variables X_t of the following form:

$$R(t,T) = a(t,T) + b(t,T)X_t \tag{7}$$

where a and b depend on the time to maturity and vector X_t may be composed of observable or latent state variables. Thus, we have a model where the price of zero coupon bonds is an exponential function in order of the state variables $X_t = (x_1, x_2, \ldots, x_n)$ with $i = 1, \ldots, n$. The price of a zero-coupon bond at time t with maturity T, such that T: $\tau = T - t$ it would be given by:

$$P\left(\tau, X_t\right) = e^{A(\tau)+B_i'(\tau)X_t} \tag{8}$$

where $A(\tau)$ it is a scalar vector and $B_i(\tau)$ is the vector of coefficients of state variables X_t. $A(\tau)$ and $B(\tau)$ are τ mature functions. A key point in the development of such models was the generalisation proposed by Duffie and Kan (1996) followed by the development of numerous affine multi-factorial models, such as Dai and Singleton (2000), Duffee (2002), among others, or models in which macroeconomic variables were included, for example, Ang and Piazzesi (2003), Ang et al. (2006), Duffee (2007), Carriero (2011), among many others. Duffie and Kan (1996) propose an affine model based on the multi-factorial model of Cox et al. (1985) that would serve as a basis for the development of several other models, particularly ones that include the influence of macroeconomic variables, such as that developed by Ang and Piazzesi (2003). In the proposed multi-factorial model, the factors translate into observable variables corresponding to the zero-coupon bond interest rates with different maturities, assuming the existence of a market price risk associated with shock in the state variables given by:

$$\lambda_t = \lambda_0 + \lambda_1 X_t \tag{9}$$

The authors assume that the instantaneous interest rate is a function of a vector of N state variables X_t, such that:

$$r(t) = \delta_0 + \delta_1' X_t \tag{10}$$

where δ_0 corresponds to a scalar and δ_1 is a vector of dimension $N \times 1$.

Duffee and Kan (1996) show that the price of a zero coupon bond at time t with maturity T, such that T: $\tau = T - t$ is an exponential function in order such that:

$$P(t,T) = e^{A(\tau)+B_i'(\tau)X_t} = E_t^Q\left[e^{-\int_t^T r(s)ds}\right] \tag{11}$$

2.4 No-arbitrage Nelson and Siegel dynamic model

Looking to combine the performance of Nelson and Siegel's dynamic model as developed by Diebold and Li (2006) with the restrictions necessary to eliminate arbitrage opportunities underlying the affine models, Christensen et al. (2011) set the Nelson and Siegel dynamic model by introducing the non-arbitrage restriction through a rate adjustment term. Based on the multi-factorial affine model proposed in Duffie and Kan (1996) and Christensen et al. (2011) consider the existence of a three-factor model with a constant volatility matrix, and that the instantaneous interest rate is an affine function such that:

$$r(t) = X_t^1 + X_t^2 \tag{12}$$

where the state variables $X_t = (X_t^1, X_t^2, X_t^2)$ are described by the following system of stochastic differential measurement equations with the risk neutrality Q:

$$\begin{pmatrix} dX_t^1 \\ dX_t^2 \\ dX_t^3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda & \lambda \\ 0 & 0 & \lambda \end{pmatrix} \times \left[\begin{pmatrix} \theta_t^1 \\ \theta_t^2 \\ \theta_t^3 \end{pmatrix} - \begin{pmatrix} X_t^1 \\ X_t^2 \\ X_t^3 \end{pmatrix}\right] dt - \sum \begin{pmatrix} dw_t^{1,Q} \\ dw_t^{2,Q} \\ dw_t^{3,Q} \end{pmatrix}, \lambda > 0 \tag{13}$$

where λ represents the exponentially decreasing rate, θ_t^j is a vector of constants, Σ is a constant matrix and $dw_t^{j,Q}$ an independent Brownian movements vector in the probability measure Q.

The price of a zero coupon bond will be achieved through:

$$P(t,T) = E_t^Q\left[e^{-\int_t^T r(u)du}\right] = e^{\left[B^1(t,T)X_t^1 + B^2(t,T)X_t^2 + B^3(t,T)X_t^3 + A(t,T)\right]} \tag{14}$$

where $B^1(t, T)$, $B^2(t, T)$, $B^3(t, T)$ and $A(t, T)$ represent the weights of the factors, K^Q is the mean reversion matrix, Σ is the constant volatility matrix, θ^Q represents the average of the latent factors and τ the time to maturity ($\tau = T - t$).

The zero coupon bonds rates will be determined by:

$$y(t,T) = X_t^1 + \left(\frac{1-e^{-\lambda(T-t)}}{\lambda(T-t)}\right)X_t^2 + \left(\frac{1-e^{-\lambda(T-t)}}{\lambda(T-t)} - e^{-\lambda(T-t)}\right) - \frac{A(t,T)}{T-t} \tag{15}$$

The equation obtained is close to that proposed by Diebold and Li (2006):

$$Y(\tau) = L_t + S_t\left(\frac{1-e^{(-\lambda_t\tau)}}{\lambda_t\tau}\right) + C_t\left(\frac{1-e^{(-\lambda_t\tau)}}{\lambda_t\tau} - e^{(-\lambda_t\tau)}\right) \tag{16}$$

only distinguished by the rate adjustment term: $-\frac{A(t,T)}{T-t}$.

3 Data

To assess the predictive ability of the Nelson and Siegel dynamic model we use public debt interest rates from four European countries: Germany, UK, Spain and Portugal for the period from January 1990 to December 2012. The countries selected share some common economic structures and have been affected differently by a recent major crisis, namely the sovereign debt crisis. For the construction of the term structure of interest rates, monthly observations of interest rates of the respective public debt with maturities of 3, 6, 9, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months were used.

For all the countries, the data was collected directly from the respective central banks. For some maturities where data was not available, we used the non-parametric interpolation procedure proposed by McCulloch (1971, 1975). Data on macroeconomic variables was obtained through the Organization for Economic Cooperation and Development (OECD): EO = economic outlook and MEI = main economic indicators. For the inflation rate, we used the consumer price index (CPI), and for the annual change in industrial production index, data relating to the industrial production index, total industry excluding construction (ICI) was used.

4 Estimation and adjustment quality

In order to assess the quality of the adjustment of dynamic Nelson and Siegel models (DNS), we proceeded with the estimation, assuming autoregressive specifications of first order, AR(1), and vector autoregressive specifications of first order, VAR(1). For comparative purposes, we also estimated the dynamic version of the affine Nelson and Siegel model, ADNS, which will be used in the analysis of the root mean square error (RMSE). The estimation was carried out by maximum likelihood, and the results for the DNS dynamic model with AR(1) and VAR(1) specifications, as well as for the affine dynamic model ADNS, show that the level factor is highly persistent, followed by the factors of slope and curvature (Table 1).

Table 1 RMSE for Germany, UK, Portugal and Spain

GERMANY	*3*	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
DNS-AR(1)	0.2863	0.179	0.1277	0.1087	0.0784	0.0991	0.1187	0.1141	0.0905	0.057	0.0288	0.0504	0.0884
DNS-VAR(1)	0.0672	0.0812	0.1144	0.1287	0.0825	0.0255	0.0471	0.0534	0.0394	0.023	0.0451	0.0809	0.1215
ADNS-AR(1)	0.0908	0.0428	0.0755	0.0914	0.0641	0.0553	0.08	0.0806	0.0613	0.0342	0.0367	0.0699	0.1086
ADNS-VAR(1)	0.0619	0.0535	0.0878	0.1037	0.0631	0.0321	0.0614	0.0653	0.0476	0.023	0.0369	0.0739	0.1152
UK	*3*	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
DNS-AR(1)	0.2578	0.1508	0.0929	0.0863	0.0749	0.0975	0.1491	0.1831	0.1906	0.1387	0.0686	0.1173	0.2201
DNS-VAR(1)	0.12	0.0824	0.12	0.142	0.1109	0.0577	0.1073	0.1477	0.1604	0.1135	0.0648	0.1374	0.2388
ADNS-AR(1)	0.1597	0.0784	0.0987	0.1252	0.1044	0.08	0.1276	0.1646	0.1746	0.1275	0.0766	0.1343	0.2335
ADNS-VAR(1)	0.1223	0.0957	0.133	0.1539	0.1222	0.0633	0.1067	0.1453	0.1571	0.11	0.0674	0.1437	0.2448
PORTUGAL	*3*	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
DNS-AR(1)	0.2247	0.1596	0.1302	0.111	0.1497	0.1753	0.2393	0.2819	0.2555	0.2772	0.1704	0.0964	0.2453
DNS-VAR(1)	0.2556	0.2036	0.1098	0.089	0.2081	0.1411	0.1409	0.1689	0.1514	0.1837	0.165	0.1983	0.3251
ADNS-AR(1)	0.1576	0.035	0.0853	0.1174	0.183	0.1195	0.1689	0.2045	0.1959	0.2282	0.1447	0.1288	0.3017
ADNS-VAR(1)	0.1296	0.0436	0.1221	0.1577	0.2021	0.1243	0.1735	0.2224	0.1919	0.2031	0.1739	0.2026	0.365
SPAIN	*3*	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
DNS-AR(1)	3217	0.1985	0.1968	0.2154	0.2511	0.3442	0.4378	0.3623	0.2464	0.2084	0.1535	0.115	0.3567
DNS-VAR(1)	0.1388	0.0796	0.1423	0.1744	0.116	0.2507	0.385	0.3307	0.2465	0.2232	0.1659	0.1056	0.3379
ADNS-AR(1)	0.1385	0.0794	0.1424	0.1745	0.1159	0.2505	0.3848	0.3306	0.2465	0.2233	0.1659	0.1057	0.3379
ADNS-VAR(1)	0.1895	0.0796	0.1296	0.1716	0.1054	0.2119	0.3558	0.3128	0.2407	0.2214	0.1672	0.1103	0.3437

Notes: Table 1 presents the in-sample RMSE for the estimation of Diebold-Li models with autoregressive first-order process DNS-AR(1), with vector autoregressive first-order process, DNS-VAR(1) and for the affine models ADNS-AR(1) and ADNS-VAR(1).

The degree of adjustment between the observed rates and estimated rates, measured by the RMSE, is generally high for the different models and the models are able to capture with a sufficient degree of adequacy the interest rates observed for different maturities. The estimation results show that none of the models analysed seem to be clearly superior to the others in terms of adjustment. With regard to the temporal dimension, DNS and ADNS models have good adjustment in market stability phases, however, in periods of high volatility, the adjustment quality decreases considerably.

5 Results

5.1 Out-of-sample predictive ability

The data set was divided into two subsets: an initial subset – January 1990 to December 2002 – used for the initial estimation, and a second subset – January 2002 to December 2012 – used for forecasting.

To assess the predictive power of the models specified above, the methodology used was based on the expanding window method. Given the forecast period considered, we use expanding windows for the prediction horizons set, h = 1, 3, 6, 9, 12, 15 and 18 months were estimated for each maturity. For each of the 13 maturities estimated (3, 6, 9, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months) the RMSE was determined, as well as the ratio RMSE and RMSE of the random walk model (RW).

The analysis of the model's predictive ability was carried out using the DNS autoregressive models with first-order modelling (DNS-AR (1)) and vector autoregressive modelling of first order (DNS-VAR (1)).

The relative performance of each model was measured by the ratio of the RMSE obtained from the model to the RMSE of the random-walk process. The out-of-sample forecasts results indicate, in general and for all countries, that the model based on a VAR(1) process has superior performance compared to the model based on an AR(1) process, although it assumes similar error values. However, its performance in terms of forecasting is not systematically higher than the RW model for some of the maturities under analysis.

The analysis of out-of-sample and in-sample shows that, in general, all models perform well in terms of forecasting for short-term horizons. However, when the forecast horizon is increased, we see a break in the performance for all models, which is more pronounced in shorter maturities, while still maintaining the good predictive power of the models, and there is tight competition between the RW model and model VAR(1) for forecasting horizons longer than nine months.

Table 2 presents the RMSE ratio for DNS-AR and DNS-VAR models relative to RW of the German case, assuming the use of only latent factors of term structure. When we compare the forecasting performance of AR(1) and VAR(1) models, we can observe some superiority of the VAR(1) model. These results are in line with those obtained by other authors (De Pooter, 2007; Christensen et al., 2011), which indicate that the dependency assumption among the factors implicit in the VAR model improves the predictive ability. The results show the good performance of AR(1) and VAR(1) models against the RW for maturities up to 24 months and for maturities greater than 96 months, especially in the case of the VAR(1) model.

Table 2 RMSE ratio for DNS-AR and DNS-VAR models, Germany

A) RMSE [DNS-AR(1)]/RMSE[RW]													
	3	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
h = 1	1.179	1.022	1.030	1.062	1.013	1.052	1.163	1.204	1.158	1.072	1.005	1.019	1.114
h = 3	1.062	1.003	0.993*	0.993*	0.989*	1.013	1.058	1.081	1.071	1.039	1.001	0.981*	0.985*
h = 6	1.031	0.999*	0.990*	0.987*	0.988*	1.007	1.035	1.051	1.048	1.029	1.002	0.978*	0.961*
h = 9	1.025	0.998*	0.987*	0.982*	0.989*	1.010	1.037	1.051	1.049	1.031	1.002	0.972*	0.945*
h = 12	1.020	0.996*	0.984*	0.979*	0.988*	1.012	1.039	1.053	1.050	1.031	1.002	0.969*	0.938*
h = 15	1.015	0.995*	0.984*	0.978*	0.988*	1.012	1.039	1.052	1.048	1.029	0.999*	0.967*	0.936*
h = 18	1.010	0.994*	0.985*	0.980*	0.988*	1.010	1.034	1.047	1.044	1.026	0.998*	0.967*	0.935*
B) RMSE [DNS-VAR(1)]/RMSE[RW]													
	3	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
h = 1	1.105	0.997	1.024	1.055	1.000	1.055	1.172	1.211	1.160	1.069	0.999*	1.012	1.110
h = 3	1.046	1.002	0.996*	0.996*	0.991*	1.017	1.062	1.083	1.072	1.037	0.997*	0.976*	0.980*
h = 6	1.027	1.001	0.993*	0.990*	0.990*	1.009	1.037	1.051	1.047	1.027	0.998*	0.974*	0.956*
h = 9	1.019	0.995*	0.985*	0.981*	0.988*	1.010	1.037	1.051	1.048	1.028	0.998*	0.967*	0.939*
h = 12	1.014	0.992*	0.982*	0.976*	0.986*	1.012	1.039	1.053	1.048	1.028	0.998*	0.965*	0.933*
h = 15	1.011	0.992*	0.982*	0.976*	0.986*	1.012	1.039	1.052	1.047	1.027	0.997*	0.964*	0.931*
h = 18	1.009	0.994*	0.985*	0.980*	0.988*	1.010	1.034	1.047	1.043	1.025	0.996*	0.964*	0.931*

Notes: Table 2 presents in two panels the RMSE ratio for Germany for out-of-sample forecasts, for the autoregressive of first order process AR(1) in latent factor model (DNS) (panel A) and for the vector autoregressive of first order process VAR(1) in the latent factor model (DNS) (panel B). The values marked with the symbol * correspond to the forecast horizons (h) and maturities for which the models show a superior performance. The forecasts in all cases presented were made for the period between 01:2002 and 12:2012. The forecasts were made for the maturities of 3, 6, 9, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months and for forecast horizons of 1 to 18 months (h = 1, h = 3, h = 6, h = 9, h = 12, h = 15 and h = 18).

Table 3 RMSE ratio for DNS-AR and DNS-VAR models, UK

A) RMSE [DNS-AR(1)]/RMSE[RW]													
	3	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
h = 1	1.471	1.026	0.955*	1.012	1.045	1.060	1.286	1.483	1.599	1.409	1.131	1.153	1.849
h = 3	1.112	1.003	0.978*	0.983*	1.000	1.009	1.063	1.121	1.163	1.123	1.046	0.996*	1.177
h = 6	1.041	0.999*	0.988*	0.990*	0.996*	1.000	1.024	1.053	1.078	1.067	1.031	0.974*	1.005
h = 9	1.025	0.997*	0.989*	0.988*	0.992*	0.999*	1.020	1.046	1.068	1.063	1.031	0.959*	0.942*
h = 12	1.016	0.997*	0.991*	0.990*	0.991*	0.998*	1.018	1.041	1.061	1.061	1.031	0.947*	0.898*
h = 15	1.009	0.997*	0.993*	0.993*	0.992*	0.996*	1.013	1.033	1.050	1.051	1.027	0.946*	0.880*
h = 18	1.004	0.997*	0.995*	0.995*	0.994*	0.996*	1.008	1.025	1.040	1.042	1.023	0.949*	0.874*
B) RMSE [DNS-VAR(1)]/RMSE[RW]													
	3	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
h = 1	1.070	0.835*	0.953*	1.083	1.060	0.989*	1.227	1.458	1.604	1.431	1.158	1.136	1.809
h = 3	0.990*	0.933*	0.943*	0.967*	0.986*	0.987*	1.043	1.108	1.159	1.124	1.048	0.987*	1.156
h = 6	0.990*	0.968*	0.971*	0.979*	0.988*	0.989*	1.012	1.044	1.072	1.062	1.026	0.963*	0.987*
h = 9	0.988*	0.973*	0.972*	0.975*	0.981*	0.986*	1.007	1.034	1.058	1.054	1.022	0.944*	0.921*
h = 12	0.987*	0.976*	0.976*	0.977*	0.979*	0.984*	1.003	1.028	1.050	1.050	1.021	0.933*	0.878*
h = 15	0.987*	0.981*	0.982*	0.983*	0.982*	0.984*	1.000	1.020	1.039	1.041	1.017	0.934*	0.863*
h = 18	0.988*	0.986*	0.987*	0.989*	0.986*	0.986*	0.997*	1.014	1.030	1.033	1.015	0.938*	0.861*

Notes: Table 3 presents in two panels the RMSE ratio for UK for out-of-sample forecasts, for the autoregressive of first order process AR(1) in latent factor model (DNS) (panel A) and for the vector autoregressive of first order process VAR(1) in the latent factor model (DNS) (panel B). The values marked with the symbol * correspond to the forecast horizons (h) and maturities for which the models show a superior performance. The forecasts in all cases presented were made for the period between 01:2002 and 12:2012. The forecasts were made for the maturities of 3, 6, 9, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months and for forecast horizons of 1 to 18 months (h = 1, h = 3, h = 6, h = 9, h = 12, h = 15 and h = 18).

Table 4 RMSE ratio for DNS-AR and DNS-VAR models, Portugal

A) RMSE [DNS-AR(1)]/RMSE[RW]													
	3	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
h = 1	2.691	1.309	1.341	1.301	1.172	1.074	1.056	1.142	1.075	1.145	1.089	1.098	1.857
h = 3	1.343	1.037	1.085	1.081	1.017	0.977*	0.957*	0.990*	0.982*	0.998*	0.965*	1.004	1.356
h = 6	1.070	1.010	1.069	1.073	1.007	0.972*	0.960*	0.992*	0.969*	0.957*	0.952*	1.009	1.248
h = 9	1.002	1.002	1.060	1.065	1.003	0.985*	0.988*	1.024	0.984*	0.957*	0.967*	1.021	1.189
h = 12	0.982*	1.005	1.050	1.053	1.003	1.002	1.012	1.035	0.994*	0.967*	0.987*	1.029	1.126
h = 15	0.981*	0.987*	1.024	1.034	1.011	1.004	1.001	1.004	0.982*	0.973*	0.997*	1.033	1.101
h = 18	0.983*	0.965*	0.985*	0.999*	1.003	0.992*	0.982*	0.977*	0.969*	0.972*	0.994*	1.024	1.075
B) RMSE [DNS-VAR(1)]/RMSE[RW]													
	3	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
h = 1	1.954	1.352	1.551	1.475	1.145	1.048	1.066	1.243	1.057	1.013	0.987*	1.158	2.189
h = 3	1.126	1.052	1.146	1.131	1.005	0.967*	0.955*	1.008	0.961*	0.946*	0.909*	0.980	1.433
h = 6	0.992*	1.032	1.107	1.106	1.016	0.984*	0.976*	1.016	0.977*	0.949*	0.937*	1.001	1.280
h = 9	0.970*	1.038	1.103	1.103	1.024	1.004	1.008	1.050	1.001	0.965*	0.971*	1.029	1.223
h = 12	0.973*	1.030	1.079	1.080	1.020	1.020	1.031	1.056	1.010	0.979*	0.996*	1.040	1.154
h = 15	0.981*	1.013	1.055	1.064	1.032	1.026	1.025	1.031	1.005	0.993*	1.014	1.052	1.129
h = 18	0.988*	0.993*	1.018	1.028	1.022	1.011	1.003	0.999*	0.988*	0.988*	1.009	1.040	1.096

Notes: Table 4 presents in two panels the RMSE ratio for Portugal for out-of-sample forecasts, for the autoregressive of first order process AR(1) in latent factor model (DNS) (panel A) and for the vector autoregressive of first order process VAR(1) in the latent factor model (DNS) (panel B). The values marked with the symbol * correspond to the forecast horizons (h) and maturities for which the models show a superior performance. The forecasts in all cases presented were made for the period between 01:2002 and 12:2012. The forecasts were made for the maturities of 3, 6, 9, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months and for forecast horizons of 1 to 18 months (h = 1, h = 3, h = 6, h = 9, h = 12, h = 15 and h = 18).

Table 5 RMSE ratio for DNS-AR and DNS-VAR models, Spain

A) RMSE [DNS-AR(1)]/RMSE[RW]													
	3	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
h = 1	1.063	0.998*	1.029	1.086	1.063	0.866*	0.779*	0.795*	0.925*	0.995*	0.982*	1.107	1.520
h = 3	0.986*	0.992*	1.030	1.068	1.051	0.910*	0.816*	0.819*	0.897*	0.948*	0.963*	1.042	1.214
h = 6	0.997*	0.996*	1.020	1.048	1.045	0.942*	0.875*	0.878*	0.924*	0.951*	0.961*	1.016	1.127
h = 9	0.983*	0.993*	1.013	1.031	1.022	0.950*	0.902*	0.903*	0.932*	0.952*	0.966*	1.015	1.109
h = 12	0.987*	0.992*	1.005	1.018	1.014	0.956*	0.912*	0.911*	0.938*	0.961*	0.980*	1.032	1.127
h = 15	0.973*	0.991*	1.011	1.026	1.018	0.956*	0.908*	0.906*	0.931*	0.952*	0.972*	1.030	1.134
h = 18	0.980*	0.991*	1.004	1.014	1.008	0.968*	0.938*	0.933*	0.941*	0.949*	0.969*	1.026	1.122
B) RMSE [DNS-VAR(1)]/RMSE[RW]													
	3	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
h = 1	1.040	0.999*	1.052	1.123	1.081	0.847*	0.749*	0.760*	0.886*	0.965*	0.960*	1.096	1.525
h = 3	0.981*	0.999*	1.047	1.091	1.068	0.912*	0.810*	0.811*	0.889*	0.942*	0.959*	1.041	1.216
h = 6	0.997*	1.006	1.040	1.074	1.070	0.955*	0.880*	0.880*	0.925*	0.952*	0.963*	1.020	1.133
h = 9	0.983*	0.998*	1.022	1.043	1.032	0.954*	0.903*	0.902*	0.932*	0.955*	0.971*	1.023	1.119
h = 12	0.989*	0.997*	1.014	1.029	1.025	0.963*	0.916*	0.913*	0.939*	0.963*	0.984*	1.037	1.133
h = 15	0.976*	0.997*	1.019	1.036	1.028	0.962*	0.911*	0.907*	0.933*	0.954*	0.976*	1.036	1.139
h = 18	0.983*	0.996*	1.012	1.023	1.018	0.975*	0.943*	0.938*	0.946*	0.954*	0.976*	1.033	1.128

Notes: Table 5 presents in two panels the RMSE ratio for Spain for out-of-sample forecasts, for the autoregressive of first order process AR(1) in latent factor model (DNS) (panel A) and for the vector autoregressive of first order process VAR(1) in latent factor model (DNS) (panel B). The values marked with the symbol * correspond to the forecast horizons (h) and maturities for which the models show a superior performance. The forecasts in all cases presented were made for the period between 01:2002 and 12:2012. The forecasts were made for the maturities of 3, 6, 9, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months and for forecast horizons of 1 to 18 months (h = 1, h = 3, h = 6, h = 9, h = 12, h = 15 and h = 18).

Table 6(a) RMSE ratio of the DNS-VAR model with macroeconomic factors

A) RMSE [DNS-VAR(1)]/RMSE[RW] GERMANY													
	3	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
h = 1	1.116	0.995*	1.021	1.055	1.021	1.056	1.150	1.176	1.127	1.049	1.004	1.047	1.168
h = 3	1.039	0.987*	0.978*	0.978*	0.983*	1.010	1.052	1.070	1.057	1.025	0.991*	0.978*	0.992*
h = 6	1.022	0.990*	0.980*	0.976*	0.980*	1.001	1.028	1.042	1.037	1.017	0.991*	0.969*	0.955*
h = 9	1.018	0.990*	0.978*	0.973*	0.979*	1.001	1.027	1.040	1.037	1.017	0.988*	0.959*	0.933*
h = 12	1.015	0.992*	0.979*	0.973*	0.980*	1.004	1.030	1.043	1.038	1.018	0.988*	0.956*	0.925*
h = 15	1.014	0.994*	0.983*	0.976*	0.983*	1.007	1.032	1.044	1.038	1.018	0.988*	0.955*	0.923*
h = 18	1.014	0.998*	0.989*	0.984*	0.989*	1.009	1.031	1.042	1.036	1.017	0.987*	0.955*	0.922*
B) RMSE [DNS-VAR(1)]/RMSE[RW] UK													
	3	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
h = 1	1.058	0.835*	0.979*	1.123	1.080	0.954*	1.182	1.427	1.587	1.443	1.202	1.198	1.839
h = 3	0.997*	0.934*	0.944*	0.967*	0.975*	0.964*	1.018	1.087	1.144	1.118	1.052	1.001	1.168
h = 6	1.000	0.974*	0.974*	0.980*	0.983*	0.979*	1.002	1.036	1.067	1.064	1.035	0.981*	1.009
h = 9	0.997*	0.978*	0.976*	0.978*	0.981*	0.982*	1.002	1.029	1.054	1.053	1.025	0.954*	0.936*
h = 12	0.994*	0.982*	0.981*	0.982*	0.982*	0.986*	1.003	1.028	1.050	1.052	1.025	0.942*	0.894*
h = 15	0.993*	0.986*	0.987*	0.988*	0.987*	0.988*	1.003	1.023	1.042	1.046	1.024	0.946*	0.881*
h = 18	0.994*	0.991*	0.992*	0.994*	0.991*	0.991*	1.002	1.019	1.036	1.041	1.025	0.954*	0.883*

Notes: Table 6(a) shows the RMSE ratio for Germany for the out-of-sample predictions of the vector autoregressive first order VAR(1) for latent factors model (DNS) and macroeconomic variables. The values marked with the symbol * correspond to the forecast horizons (h) and maturities for which the models perform better. The predictions in all cases were carried out for the period from 1:2002 to 12:2012. The forecasts were made for the maturities of 3, 6, 9, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months, and for forecast horizons of 1 to 18 months (h = 1, h = 3, h = 6, h = 9, h = 12, h = 15 and h = 18).

Table 6(b) RMSE ratio of the DNS-VAR model with macroeconomic factors

C) RMSE [DNS-VAR(1)]/RMSE[RW] PORTUGAL													
	3	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
h = 1	2.074	1.367	1.542	1.466	1.140	1.042	1.058	1.235	1.051	1.011	0.988*	1.168	2.209
h = 3	1.135	1.037	1.134	1.122	1.002	0.966*	0.953*	1.006	0.961*	0.948*	0.912*	0.985*	1.442
h = 6	0.986*	1.013	1.092	1.095	1.012	0.983*	0.976*	1.017	0.978*	0.951*	0.940*	1.005	1.285
h = 9	0.963*	1.020	1.089	1.093	1.020	1.002	1.007	1.049	1.002	0.966*	0.972*	1.031	1.227
h = 12	0.967*	1.014	1.066	1.069	1.014	1.015	1.027	1.053	1.008	0.978*	0.995*	1.040	1.154
h = 15	0.975*	1.002	1.046	1.056	1.029	1.023	1.023	1.029	1.003	0.992*	1.013	1.052	1.129
h = 18	0.983*	0.991*	1.018	1.030	1.024	1.012	1.004	0.999	0.989*	0.989*	1.010	1.040	1.096
D) RMSE [DNS-VAR(1)]/RMSE[RW] SPAIN													
	3	*6*	*9*	*12*	*24*	*36*	*48*	*60*	*72*	*84*	*96*	*108*	*120*
h = 1	1.023	0.977*	1.029	1.101	1.061	0.832*	0.740*	0.740*	0.851*	0.943*	0.951*	1.098	1.532
h = 3	0.951*	0.959*	1.002	1.043	1.023	0.876*	0.785*	0.791*	0.875*	0.935*	0.958*	1.043	1.220
h = 6	0.973*	0.975*	1.004	1.035	1.031	0.924*	0.859*	0.864*	0.914*	0.947*	0.962*	1.022	1.136
h = 9	0.964*	0.974*	0.996*	1.015	1.002	0.928*	0.882*	0.886*	0.920*	0.947*	0.968*	1.023	1.120
h = 12	0.979*	0.985*	1.000	1.014	1.009	0.950*	0.907*	0.907*	0.935*	0.961*	0.984*	1.038	1.134
h = 15	0.974*	0.995*	1.018	1.035	1.026	0.960*	0.911*	0.907*	0.934*	0.956*	0.979*	1.038	1.141
h = 18	0.987*	1.002	1.018	1.030	1.025	0.981*	0.949*	0.944*	0.952*	0.959*	0.980*	1.036	1.129

Notes: Table 6(b) shows the RMSE ratio for Germany for the out-of-sample predictions of the vector autoregressive first order VAR(1) for latent factor model (DNS) and macroeconomic variables. The values marked with the symbol * correspond to the forecast horizons (h) and maturities for which the models perform better. The predictions in all cases were carried out for the period from 1:2002 to 12:2012. The forecasts were made for the maturities of 3, 6, 9, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months, and for forecast horizons of 1 to 18 months (h = 1, h = 3, h = 6, h = 9, h = 12, h = 15 and h = 18).

However, the values for maturities between 36 and 84 months show that the AR(1) and VAR(1) models cannot outperform the RW model. For short-period forecasting (1-month forecast period) the results show superior performance of the random walk model as documented by several empirical papers [among others, Diebold and Li (2006), De Pooter et al. (2010), Nyholm and Vidova-Koleva (2012) and Caldeira et al. (2016)], once the information incorporated into the structure of yield curve tends to be dominated by random noises.

Tables 3 to 5 present a comparison of out-of-sample results of DNS-AR and DNS-VAR models relative to RW using the RMSE ratio for UK, Portugal and Spain, respectively.

A similar pattern to the German case can be observed for the other countries, although with worse relative performance in the case of Portugal and Spain, whose yields are subject to more significant trajectory changes. For medium- and long-term forecasting periods and for medium and long time to maturity, with some exceptions, the results show that DNS-AR(1) and DNS-VAR(1) generate better performance when compared with the random walk model.

5.2 Contribution of macroeconomic variables

In order to assess the impact of the introduction of macroeconomic variables in the dynamic models, we added to the state vector of the first order VAR(1) model two macroeconomic variables representing the inflation rate and the annual change in the industrial production index.

Similarly to the previous analysis, for the model with macroeconomic variables, we also present the ratio of the RMSE of each of the dynamic processes and the RMSE of the RW model.

The results indicate the existence of a positive contribution to the performance of dynamic models resulting from the incorporation of macroeconomic data for all the countries under analysis.

Tables 6(a) and 6(b) present the results for all the countries considering the inclusion of macroeconomic variables in the forecasting model. We found that including macroeconomic data in the Nelson and Siegel dynamic model improved its performance in terms of prediction, particularly for medium- and long-term maturities for longer forecasting horizons. Comparing the performance of the VAR(1) and RW model, Tables 6(a) and 6(b) show a clear improvement on the performance of the VAR(1), which exceeds the RW in 3–46 month and 84–120 month maturities.

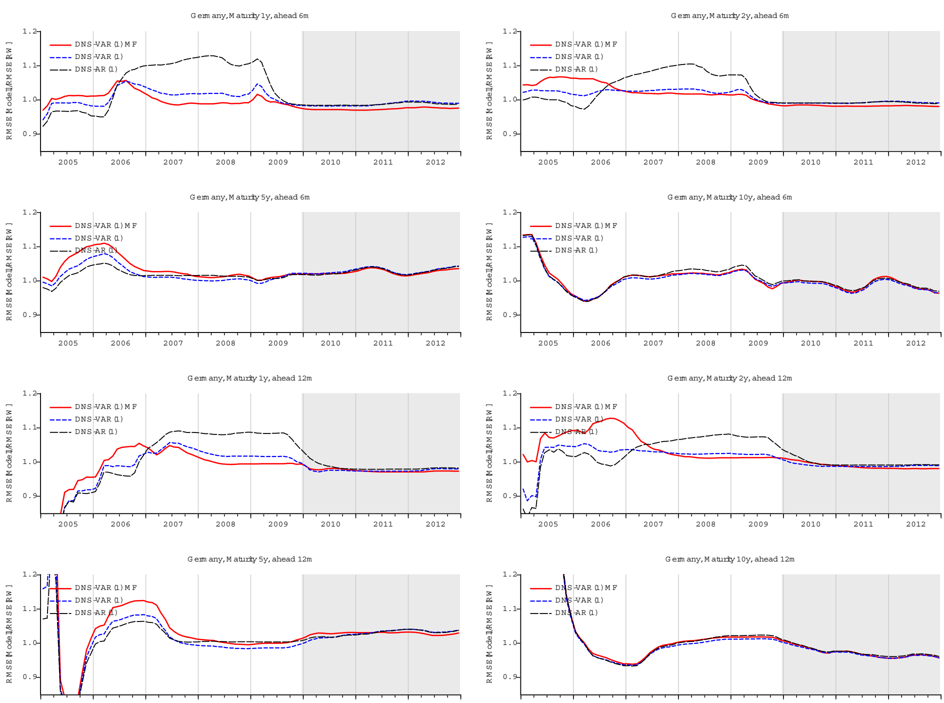
The results obtained after macroeconomic variables are incorporated point to an improvement in the forecast ability of the dynamic Nelson and Siegel model. The RMSE for most of the maturities and forecast horizons macroeconomic information contributes positively to the predictive ability of the model. The positive contribution of the inclusion of macroeconomic data is reflected in an improvement of the RMSE of the models as compared to the RMSE of the random walk model. Even if, in many cases, the RW model continues to provide a superior forecast, the model VAR(1) considering the two mentioned macroeconomic factors performs better for short maturities in UK, and for short maturities as well as long maturities in the case of Germany, Spain and Portugal.

5.3 *Temporal evolution of error analysis*

The results based on the introduction of two macroeconomic variables in the model indicate a performance improvement in terms of the prediction ability of the Nelson and Siegel dynamic model.

In order to analyse the evolution of the model's performance compared to models that do not include macroeconomic data, as presented in Figures 1 to 4, we study the evolution of the RMSE for the latent factor models (DNS) compared to the RMSE of the random walk model for the autoregressive of first order process (DNS-AR(1)), the vector autoregressive of first order process (DNS-VAR(1)) and the autoregressive of first order process, incorporating macroeconomic factors (DNS-VAR(1)MF). The data are for the period from January 2002 to December 2012, and for maturities of one, two, five and ten years and forecasting horizons of six and 12 months.

Figure 1 Germany – RMSE ratio evolution (see online version for colours)



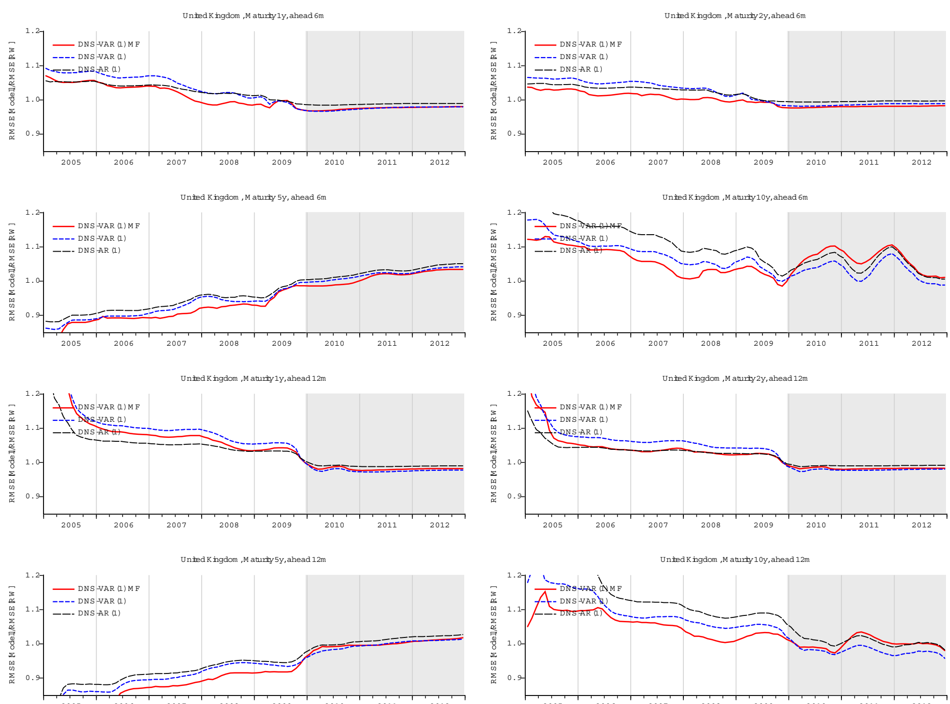
Notes: RMSE evolution for latent factor models (DNS) with autoregressive of first order process [DNS-AR (l)], with vector autoregressive of first order processes [DNS-VAR (l)] and vector autoregressive of first order processes incorporating macroeconomic factors [DNS-VAR (l) MF] related with the RMSE of the RW specification. The shaded area stands for the sovereign debt crises.

Figure 1 presents the evolution of the RMSE of dynamic models (DNS) for Germany. We can observe a relative stability over the period under analysis for all models, except the AR(1) model, especially for maturities of one year in which there is poor performance against the VAR(1). By combining macroeconomic variables with latent factors, one can improve the accuracy forecasts for yields at all maturities, but the accuracy gain is not constant over the sample.

In Figure 2, we show the results for UK. According to these results, the positive contribution trend of the inclusion of macroeconomic data from the first part of the sample was reversed from the beginning of the sovereign debt crisis. Although the forecast accuracy for shorter maturities of the different models remains relatively stable, we can observe an increase in the RMSE for all the models in the final period of the sample, especially for longer maturities.

Figure 3 shows the results for Portugal. From the beginning of the sovereign debt crisis, when the bond markets faced high uncertainty, there is deterioration in the forecasting performance compared to the previous period.

Figure 2 UK – RMSE ratio evolution (see online version for colours)

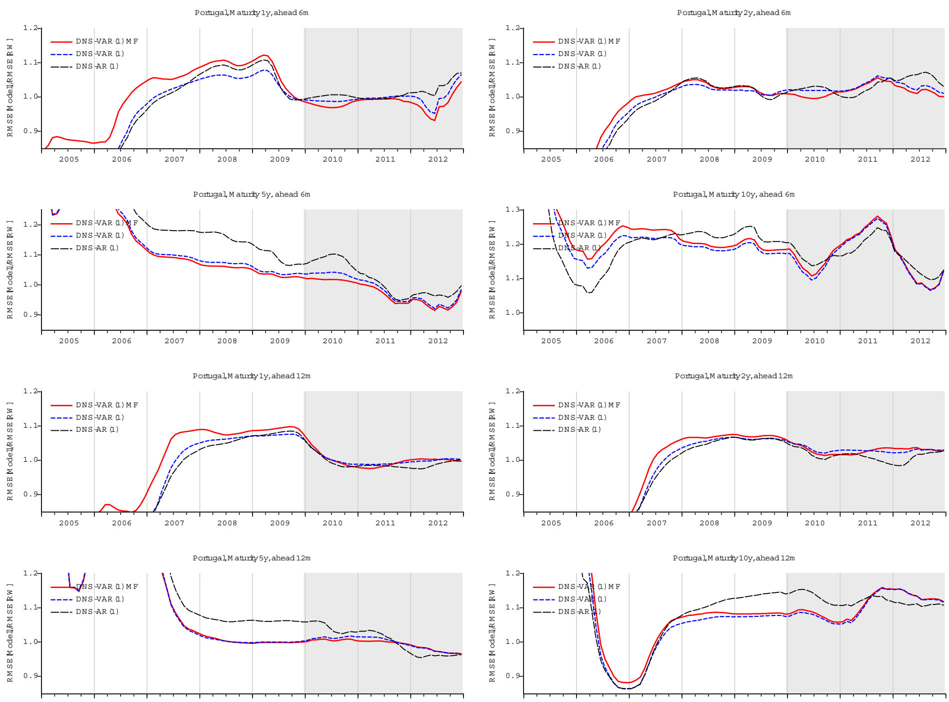


Notes: RMSE evolution for latent factor models (DNS) with autoregressive of first order process [DNS-AR (l)], with vector autoregressive of first order processes [DNS-VAR (l)] and vector autoregressive of first order processes incorporating macroeconomic factors [DNS-VAR (l) MF] related with the RMSE of the RW specification. The shaded area stands for the sovereign debt crises.

However, it is precisely in this period that we see the clear contribution of including macroeconomic data in the VAR(1) model. The performance of VAR(1)-MF clearly exceeds the AR(1) and VAR(1) that generally present higher values to RMSE. This effect is particularly evident in maturities of one, five and ten years.

Figure 4 displays the results for Spain. We can observe great instability in the performance of the models in the first part of the sample, especially in 1-year maturity. For maturities of five and ten years, the VAR model incorporating macroeconomic variables VAR(1)MF has a better performance in relation to AR(l) and VAR(l) models that include only latent factors of term structure.

Figure 3 Portugal – RMSE ratio evolution (see online version for colours)



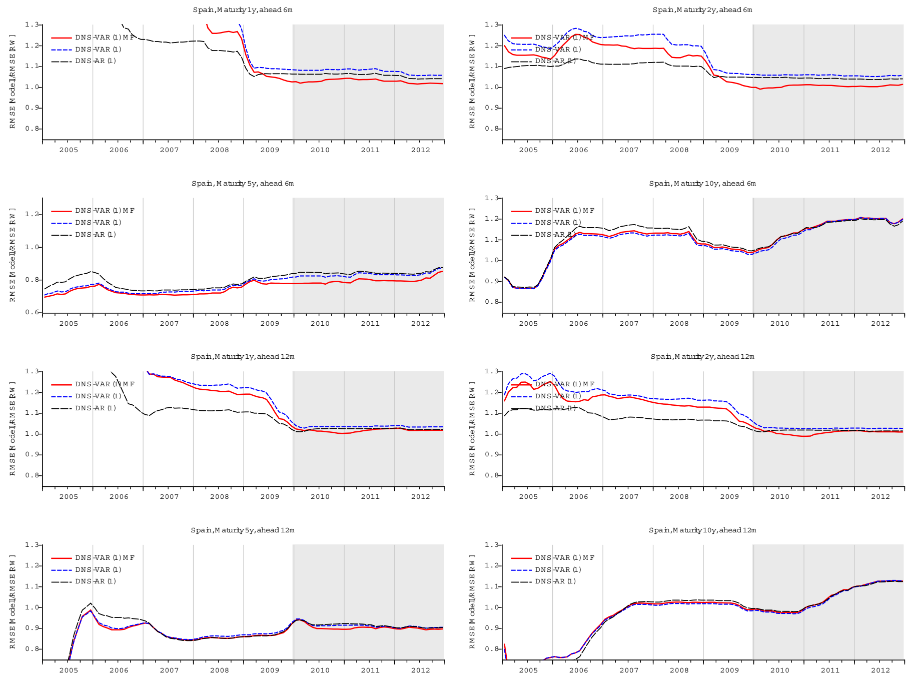
Notes: RMSE evolution for latent factor models (DNS) with autoregressive of first order process [DNS-AR (l)], with vector autoregressive of first order processes [DNS-VAR (l)] and vector autoregressive of first order processes incorporating macro-economic factors [DNS-VAR (l) MF] related with the RMSE of the RW specification. The shaded area stands for the sovereign debt crises.

In order to study the behaviour of the forecast accuracy over time, using the information presented in Figures 1 through 4, we can observe that the performance of AR(1), VAR(1) and VAR(1)-MF models varies over time.

Although the interaction between latent factors of yield curve with macroeconomic variables improves the forecast accuracy on most of maturities and forecasting horizons, the magnitude of its contribution depends on the country and sub-period.

Overall, our results show the sample dependence of forecast accuracy, which depends at least partly on yields trajectories with sudden changes as observed in the sovereign debt crisis.

Figure 4 Spain – RMSE ratio evolution (see online version for colours)



Notes: RMSE evolution for latent factor models (DNS) with autoregressive of first order process [DNS-AR (l)], with vector autoregressive of first order processes [DNS-VAR (l)] and vector autoregressive of first order processes incorporating macro-economic factors [DNS-VAR (l) MF] related with the RMSE of the RW specification. The shaded area stands for the sovereign debt crises.

6 Conclusions

In this paper we analyse the predictive ability of the term structure of interest rates for one of the most popular models in the literature: the Nelson and Siegel dynamic model proposed by Diebold and Li. For this purpose, we started by analysing the model's adjustment capacity by considering data on public debt interest rates from four European countries, Germany, UK, Spain and Portugal.

The main empirical results can be summarised as follows.

First, when we evaluated the adjustment capacity of the model to the observed data based on specifications AR(1) and VAR(1), all the models presented robust performance. In conditions with low volatility, all models show a high degree of adjustment to the data, however, their performance decreases considerably during periods of high market volatility. None of the models presented proved to be clearly superior to all the others for all countries.

Second, based on the analysis of the predictive power of dynamic models for the latent factors of the term structure of interest rates (DNS), we can verify that, while for short-period forecasting the results show a superior performance of the random walk model, for middle and long forecasting periods on the other hand, and for medium and long time to maturity yields, with some exceptions, the results show that dynamic models for the latent factors generate better performance compared with the random walk model.

Third, the inclusion of macroeconomic variables representative of inflation and the annual growth of industrial production index shows a positive contribution to the forecast improvement for the four countries analysed, for all maturities and for all forecast horizons, even if, in many cases, the random walk model continues to provide superior foresight to the model VAR(l) considering these two macroeconomic factors. However, the forecasting gain is very small when we incorporate macroeconomic information.

Fourth, the paper presents clear evidence of time-varying forecast accuracy, not only across yield maturities and forecast horizon, but also over sub-periods, which seem to be formed by sudden yield movements.

As the results of time-varying forecast accuracy present a strong sample dependence, in future research, we plan to explore the incorporation of macroeconomic information into a term structure model using observable and latent factors in order to improve forecasts and delimit the different phases or sub-periods formed by their positive, neutral and negative contributions, as well as their main drivers.

References

Ang, A. and Piazzesi, M. (2003) 'A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables', *Journal of Monetary Economics*, Vol. 50, No. 4, pp.745–787.

Ang, A., Piazzesi, M. and Wei, M. (2006) 'What does the yield curve tell us about GDP growth?', *Journal of Econometrics*, Vol. 131, Nos. 1–2, pp.359–403.

Bikbov, R. and Chernov, M. (2013) 'Monetary policy regimes and the term structure of interest rates', *Journal of Econometrics*, Vol. 174, No. 1, pp.27–43.

Caldeira, J., Moura, G., Santos, A. and Tourrucôo, F. (2016) 'Forecasting the yield curve with the arbitrage-free dynamic Nelson-Siegel model: Brazilian evidence', *EconomiA*, Vol. 17, No. 2, pp.221–237.

Carleton, W. and Cooper, I. (1976) 'Estimation and uses of the term structure of interest rates', *Journal of Finance*, Vol. 31, No. 49, pp.1067–1083.

Carriero, A. (2011) 'Forecasting the yield curve using priors from no-arbitrage affine term structure models', *International Economic Review*, Vol. 52, No. 2, pp.425–459.

Christensen, J., Diebold, F. and Rudebusch, G. (2011) 'The affine arbitrage-free class of Nelson-Siegel term structure models', *Journal of Econometrics*, Vol. 164, No. 1, pp.4–20.

Coroneo, L., Giannone, D. and Modugno, M. (2016) 'Unspanned macroeconomic factors in the yield curve', *Journal of Business and Economic Statistics*, Vol. 34, No. 3, pp.472–485.

Cox, J., Ingersoll, J. and Ross, S. (1985) 'A theory of the term structure of interest rates', *Econometrica*, Vol. 53, No. 2, pp.385–407.

Dai, Q. and Singleton, K. (2000) 'Specification analysis of affine term structure models', *Journal of Finance*, Vol. 55, No. 5, pp.1943–1978.

De Pooter, M. (2007) *Modeling and Forecasting Stock Return Volatility and the Term Structure of Interest Rates*, Unpublished PHD dissertation, Erasmus School of Economics (ESE), Amsterdam.

De Pooter, M., Ravazzolo, F. and van Dijk, D. (2010) *Term Structure Forecasting Using Macro Factors and Forecast Combination*, International Finance Discussion Papers 933, Norges Bank.

Diebold, F. and Li, C. (2006) 'Forecasting the term structure of government bond yields', *Journal of Econometrics*, Vol. 130, No. 2, pp.337–364.

Diebold, F. and Rudebusch, G. (2013) *Yield Curve Modeling and Forecasting*, Princeton University Press, Princeton, NJ.

Diebold, F., Rudebusch, G. and Aruoba, S. (2006) 'The macroeconomy and the yield curve, a dynamic latent factor approach', *Journal of Econometrics*, Vol. 131, Nos. 1–2, pp.309–338.

Duffee, D. and Kan, R. (1996) 'A yield factor model of interest rates', *Mathematical Finance*, Vol. 6, No. 4, pp.379–406.

Duffee, G. (2002) 'Term premia and interest rate forecasts in affine models', *Journal of Finance*, Vol. 57, No. 1, pp.405–443.

Duffee, G. (2007) *Are Variations in Term Premia Related to the Macroeconomy?*, Working Papers Series, Johns Hopkins University.

Favero, C., Niu, L. and Sala, L. (2012) 'Term structure forecasting, no-arbitrage restrictions versus large information set', *Journal of Forecasting*, Vol. 31, No. 2, pp.124–156.

Hördahl, P., Tristani, O. and Vestin, D. (2006) 'A joint econometric model of macroeconomic and term-structure dynamics', *Journal of Econometrics*, Vol. 131, Nos. 1–2, pp.405–444.

Kaya, H. (2013) 'Forecasting the yield curve and the role of macroeconomic information in Turkey', *Economic Modelling*, May, Vol. 33, pp.100–107.

Kim, D. and Wright, J. (2005) *An Arbitrage-free Three-factor Term Structure Model and the Recent Behavior of Long-term Yields and Distant-horizon Forward Rates*, Finance and Economics Discussion Series 33, Board of Governors of the Federal Reserve System US.

Matsumura, M., Moreira, A. and Vicente, J. (2011) 'Forecasting the yield curve with linear factor models', *International Review of Financial Analysis*, Vol. 20, No. 5, pp.237–243.

McCulloch, J. (1971) 'Measuring the term structure of interest rates', *Journal of Business*, Vol. 44, No. 1, pp.19–31.

McCulloch, J. (1975) 'The tax-adjusted yield curve', *Journal of Finance*, Vol. 30, No. 3, pp.811–830.

Merton, R. (1973) 'An intertemporal capital asset pricing model', *Econometrica*, Vol. 41, No. 5, pp.867–887.

Moench, E. (2008) 'Forecasting the yield curve in a data-rich environment: a no-arbitrage factor-augmented VAR approach', *Journal of Econometrics*, Vol. 146, No. 1, pp.26–43.

Nelson, C. and Siegel, A. (1987) 'Parsimonious modeling of yield curves', *Journal of Business*, Vol. 60, No. 4, pp.473–489.

Nyholm, K. and Vidova-Koleva, R. (2012) 'Nelson-Siegel, affine and quadratic yield curve specifications: which one is better at forecasting?', *Journal of Forecasting*, Vol. 31, No. 6, pp.540–564.

Orphanides, A. and Wei, M. (2012) 'Evolving macroeconomic perceptions and the term structure of interest rates', *Journal of Economic Dynamics and Control*, Vol. 36, No. 2, pp.239–254.

Paccagnini, A. (2016) 'The macroeconomic determinants of the US term structure during the great moderation', *Economic Modelling*, Vol. 52, Part A, pp.216–225.

Rubaszek, M. (2016) 'Forecasting the yield curve with macroeconomic variables', *Econometric Research in Finance*, Vol. 1, No. 1, pp.1–21.

Rudebusch, G. and Wu, T. (2008) 'A macro-finance model of the term structure, monetary policy and the economy', *The Economic Journal*, Vol. 118, No. 530, pp.906–926.

Svensson, L. (1994) *Estimating and Interpreting Forward Interest Rates, Sweden 1992–1994*, Technical Report 4871, National Bureau of Economic Research.

Van Dijk, D., Koopman, S.J., Van der Wel, M. and Wright, J.H. (2014) 'Forecasting interest rates with shifting endpoints', *J. Appl. Econometrics*, Vol. 29, No. 5, pp.693–712.

Vasicek, O. (1977) 'An equilibrium characterization of the term structure', *Journal of Financial Economics*, Vol. 5, No. 2, pp.177–188.

Vasicek, O. and Fong, H. (1982) 'Term structure modeling using exponential splines', *Journal of Finance*, Vol. 37, No. 2, pp.39–348.

Vieira, F., Chague, F. and Fernandes, M. (2017) 'A dynamic Nelson-Siegel model with forward-looking indicators for the yield curve in the us', *CEQEF 32*, Escola de Economia de São Paulo da Fundação Getúlio Vargas.