Genetic algorithm hybridised by a guided local search to solve the emergency coverage problem

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Abstract: The management of emergency logistics is addressed by several researchers. This paper addresses the ambulance allocation in order to cover sectors in the Rabat region of Morocco. Our model takes into account the dynamic and stochastic nature of emergency calls arrival. This work proposes a mathematical model of the coverage problem, resolved using a genetic algorithm (GA) initialised by a heuristic and hybridised by a guided local search (GLS). We consider 12 emergency locations; seven hospitals of the region and five fire stations. These algorithms are approved comparing to the optimal solutions done by Cplex software. As a result, the GA hybridised by a GLS provides a distribution of ambulances in each potential waiting site (hospital or fire station), and minimises the total lateness of emergency intervention.

Keywords: coverage model; emergency; genetic algorithm; hospital logistics; local search; simulation.

Reference to this paper should be made as follows: Benabdouallah, M., El Yaakoubi, O. and Bojji, C. (2017) ‘Genetic algorithm hybridised by a guided local search to solve the emergency coverage problem’, *Int. J. Mathematical Modelling and Numerical Optimisation*, Vol. 8, No. 1, pp.23–41.
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This paper is a revised and expanded version of a paper entitled ‘Deployment and redeployment of ambulances using a heuristic method and an Ant Colony Optimization—Case study’, presented at Sysco 2016 conference, November, 2016, Casablanca Morocco, DOI: 10.1109/SYSCO.2016.7831330.

1 Introduction

Coverage problems are an optimisation issue aimed at covering demands expressed from an intervention sector taking into account the ambulances location and their fleet size. To respond to an incoming emergency call, researchers applied a planning of ambulance route; deployment of ambulance from a waiting site to the patient location, then to the hospital, and its redeployment from this hospital to another waiting site. Li et al. (2011) classified coverage models in static, probabilistic and dynamic classes. The static class contains the earlier models. They have to minimise the ambulance number (Toregas et al., 1971) or to maximise the coverage of demand points (Church and Revelle, 1974). These models have not satisfied all emergency locations. However, the probabilistic models have been more realistic than the static ones; ambulances are independent and have the possibility to not satisfy an incoming call due to their availability rate (Daskin, 1983; Revelle and Hogan, 1989). Dynamic model introduced by Gendreau et al. (2001) stated the dynamic double standard model (DDSMt) having the objective of maximising the coverage and minimising the relocation cost at time \( t \). In literature, coverage problems are solved using heuristic algorithms, simulation techniques and some exact methods. The remainder of paper is divided into five sections. Section 2 deals with a review of literature about GA and local search (LS) as heuristics comparing to the particle swarm optimisation (PSO). Section 3 defines the mathematical formulation of our model. Section 4 presents, respectively, the heuristic and the GA hybridised by a
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guided local search (GLS) as resolution methods. Section 5 is devoted to the computational results. The last section concludes and outlines our future work.

2 Review of literature

GA is one of the most widely used heuristic approaches. Known as an intelligent probabilistic search algorithm, it has been applied to a wide range of optimisation problems (Li et al., 2011). GAs are metaheuristics that represent natural evolution. They are based on random generation of an initial population within a given size. The population contains several chromosomes; each chromosome is composed by genes describing the problem addressed. The evolution of population is guaranteed by selection of two parent chromosomes which cross according to a crossover method. The following step of the genetic algorithm is improvement of the new child chromosome. The concept of the algorithm consists to reorder the population and return the best solution. Beasley and Chu (1996) used the genetic algorithm in the earlier time. They contributed on set covering problem by introducing some parameters in order to validate the efficiency about their genetic algorithm approach. They implemented a crossover-fusion operator, a variable mutation rate and a heuristic feasibility operator to generate optimal or near-optimal solutions depending on the instance size. Aickelin (2002) used the genetic algorithm to solve the set covering problem. Their approach differs from the ordinary one by the fact that he divides the solution into three steps. At first, the genetic algorithm locates best permutations. Secondly, it improves the solution. At the end, solution is completely optimised using another heuristic. Aytug and Saydam (2002) compared the solution obtained by the genetic algorithm and the Daskin solution done in his probabilistic coverage model named Maximal Extended Coverage Location Problem (MEXCLP). Genetic algorithm offers best results in reasonable computational time.

In contrast to this evolutionary algorithm, Eberhart and Kennedy (1995) developed an algorithm through social behaviour named PSO. The PSO does not have genetic operators such as crossover and mutation. Particles update themselves with the internal velocity. The best particle gives out the information to others. The PSO algorithm is more recent than the GA, and it was proposed only on 1995 contrary to the GA that it has more than 40 years. On the other hand, PSO is successfully used to solve various optimisation problems like functions optimisation (Kennedy et al., 2001). It is very popular due to its simplicity.

Local search is an optimisation method applied to improve the fitness of a solution. Among heuristic approaches that use local search techniques, we find the tabu search. It is introduced by Glover and Laguna (1997) and they proved its efficiency to solve NP-hard problems. In fact, from an initial solution \( s \) belongs to a solution set \( S \), solution subsets \( N(s) \) are generated. Glover and Laguna (1997) retained the best solution from the subset \( N(s) \) belonging from the neighbourhood of \( S \) set. This algorithm consists to implement a tabu list that contains the last visited solutions. Thus, it avoids solutions already found to be accepted and stocked in the tabu list. Furthermore, the solution is chosen from the neighbourhood \( N(s) \) except the tabu list items. It is used in several domains especially the coverage and allocation problems. Rajagopalan et al. (2007) used the tabu search as metaheuristic to optimise a probabilistic coverage model, especially to maximise the expected number of calls that can be covered. Tabu search gives good
results in a few computational times. Gendreau et al. (1997) developed a tabu search approach to optimise the static coverage model named double standard model (DSM). The DSM consists to cover all demands within $r_2$ time and a proportion of demands is covered within $r_1$ minutes having that $r_1 < r_2$. According to emergency medical services in the United States in 1973, $r_1 = 10$ min. Tabu search avoids optimal solutions either on real instances or randomly generated instances. Gendreau et al. (2001) implemented a tabu search in their DDSMt, in order to maximise dynamically the coverage and minimise the relocation cost of ambulances. To prove the effectiveness of their heuristic, the authors compared the tabu search with results obtained by CPLEX software. They obtained near-optimal solution. Doerner et al. (2005) implemented a tabu search heuristic to optimise an extended double coverage ambulance location from a real data of Austria city. They compared the results of tabu search with those of an ant colony optimisation. The heuristic of local search gives optimal solutions consuming less time than the ant colony method. Earlier than these studies, Teitz and Bart (1968) used the standard local search procedure that is based on swapping facilities on a median problem. They determined which facility improves the solution.

In this study, we use the GA as metaheuristic issue to minimise the total lateness of emergency intervention. This is due to the typical use of such algorithms to provide good approximate solution to NP-hard problems that cannot be solved easily using other techniques. The GA is hybridised by the GLS with swap movement to improve the quality of solution. Results will be compared to optimal solutions using the mathematical formulation in the following section and the CPLEX software.

3 Mathematical formulation

In this section, an integer programming formulation is proposed; our model aims to minimise the total lateness of the emergency intervention. To do this, it proposes to ensure an optimal allocation of vehicles in potential waiting sites that are either hospitals or fire stations.

Our problem satisfies the following constraints:

- The ambulance speed is fixed to 60 kph.
- Each ambulance is deployed to the nearest hospital from the intervention sector.
- The sum of ambulances in waiting sites has to not exceed the given number of ambulances.
- Each intervention demand is covered by one ambulance.
- The ambulance deployed to meet the intervention $k$ should be redeployed directly to the intervention $l$ if $l$ comes before the end of $k$ intervention.

Emergency calls are received from intervention sectors. We consider the following inputs:

\( N \) : the number of vehicles

\( P \) : the set of fire stations
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$H$: the set of hospitals

$K$: the set of intervention demands

$T$: the simulation horizon

$t_k$: the date of intervention demand arrival ($k \in K$, $t_0 = 0$ and $t_k > t_{k+1}$)

Such as:

$i$: the potential waiting site ($i \in P \cup H$)

$k$: the intervention demand

$\pi$: ambulances number in waiting site $i$

$\pi^k$: ambulances number in site $i$ at $t_k$

$d_k$: the end of intervention $k$ (the patient arrival to the hospital by ambulance).

To model this problem, we need to define the following decision variables:

$$x^i_k := \begin{cases} 1, & \text{if site } i \text{ responds to intervention } k \\ 0, & \text{otherwise} \end{cases}$$

$$z_{ij}^i := \begin{cases} 1, & \text{if hospital } i \text{ covers sector } j \\ 0, & \text{otherwise} \end{cases}$$

The objective function and the constraints are then expressed as follows:

$$\text{minimize } \sum_{i \in K} \sum_{k \in P \cup H} (d_k - t_k)$$

$$\sum_{i \in P \cup H} \pi_i = N \quad (1)$$

$$\pi_i \geq 0 \quad \forall i \in (P \cup H) \quad (2)$$

$$\pi^0_i = \pi_i \quad \forall i \in (P \cup H) \quad (3)$$

$$\sum_{i \in P \cup H} x^i_k = 1 \quad \forall k \in K \quad (4)$$

$$\pi^k_i \leq \pi_i \quad \forall k \in K, \forall i \in (P \cup H) \quad (5)$$

$$\sum_{i \in P \cup H} r^i_k \leq 1 \quad \forall k \in K \quad (6)$$

$$\sum_{i \in P \cup H} h_{ij} = \sum_{i \in P \cup H} r^i_k \quad \forall k \in K \quad (7)$$

$$d_k + \sum_{j \in S} \sum_{i \in P \cup H} r^i_k y_{ij} z_{ij} \frac{d_j}{V} \leq t_i + (1 - h_{ij})^* M \quad \forall k \in K, \forall l \geq k \quad (8)$$
The objective function represents the sum of lateness associated with each intervention demand $k$. At the beginning, we assign to each waiting site a positive number of vehicles [Eq. (2)]. Constraint (1) states that the sum of ambulances in all waiting sites equals to the available number of ambulances $N$. At each intervention demand, a single waiting site fulfils this demand by giving one vehicle [Eq. (4)]. The number of available ambulances in each waiting site decreases after a deployment and increases after a redeployment [Eqs. (10) and (11)] without exceeding the initial distribution of vehicles in hospitals and fire stations [Eq. (5)]. The ambulance goes to take the patient, in each deployment movement, from the appropriate intervention sector to the nearest hospital [Eq. (9)]. The deployment for the demand $l$ ends in the hospital, and then we distinguish two scenarios; either the available ambulance is redeployed to a waiting site [Eqs. (6) and (7)] then it increases the vehicles number of this site once it arrives [Eq. (8)], or the redeployment is not made [Eq. (16)]. Instead of that, the same ambulance is used to fulfil one intervention demand $k$ [Eq. (13)] providing that $l$ ends after $t_k$ [Eq. (12)] or it is served to meet other interventions [Eqs. (14) and (15)].

The total lateness (fitness) is calculated using a discrete event simulation model during a horizon $T$ and repeated for a number of iterations. The model regroups two levels. The first one is tactical. It consists to have the best distribution of ambulances at hospitals and fire stations. The second one interests the operational level which is deployment and redeployment of ambulances when an emergency call arrives. Emergency demands are addressed from intervention sectors. The period between two calls is expressed according to the Poisson distribution. The periodicity of calls equals to two; day and night.

Once a call $k$ comes at instant $t_k$, we try to deploy an ambulance from a waiting site $i$ to an intervention sector $j$. The ambulance reaches the sector $j$ at the instant
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\[ r_i + t_i + \frac{d_{ij}}{V} \]. \( r_i \) is the lateness related to a lack of vehicles. \( d_{ij} \) is the distance between the site \( i \) and the sector \( j \). \( V \) is the ambulance speed. So the number of ambulances in this site will be decremented. A waiting site can be either a hospital or a fire station. Emergency demands are expressed from sectors that are districts of the city. We consider the closest available ambulance. The number of ambulances increases after redeployment in waiting site.

When the patient comes to the hospital \( i' \) at instant \( d_i = r_i + t_i + \frac{d_{ij}}{V} + \frac{dt'j}{V} \) (\( d_i - t_i \) is then the latency of the emergency intervention \( k \) minimised in the objective function), the ambulance becomes free; the following simulation scenario is deployment of the available ambulance. If the next intervention demand \( l \) cannot be satisfied due to the lack of ambulances in the waiting sites, we try to affect the available ambulance located at the hospital directly to this new intervention demand. Otherwise, the ambulance will be redeployed to the nearest waiting site \( i \) and the number of vehicles \( \pi \) in this site will be incremented.

The number of ambulances in each waiting site is called a gene \( \pi \) in the genetic language. The number of ambulances in each waiting site forms the chromosome \( \pi \). This chromosome has different changes during the simulation; deployment (Dep) and redeployment (Redep). Simulation starts with a clone of the initial chromosome. The clone chromosome is called a transitory chromosome and it contains genes \( \pi, (t) \) changing during \( t \). We must ensure that the number of ambulances in the transitory gene \( \pi, (t) \) is less than or equals to the number of ambulances in the initial chromosome \( \pi \), during all iterations of simulation. Table 1 indicates the deployment and redeployment movements of ambulances in different waiting sites. The first site \( i \) contains 2 ambulances in the simulation begin, in the second iteration, one of these ambulances is deployed to a sector \( j \).

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Different steps of simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ambulance distribution</td>
</tr>
<tr>
<td>Initial chromosome ( \pi )</td>
<td>2</td>
</tr>
<tr>
<td>Transitory chromosome ( \pi(t) )</td>
<td>2</td>
</tr>
<tr>
<td>Transitory chromosome ( \pi(t) )</td>
<td>1</td>
</tr>
<tr>
<td>Transitory chromosome ( \pi(t) )</td>
<td>1</td>
</tr>
<tr>
<td>Transitory chromosome ( \pi(t) )</td>
<td>2</td>
</tr>
<tr>
<td>Final chromosome ( \pi )</td>
<td>2</td>
</tr>
</tbody>
</table>

The following section describes the heuristic method applied to our model, and the GA hybridised by a GLS for the ambulance fleet distribution.
4 Problem resolution

This section contains the methods used for the resolution of the problem: a heuristic method and a GA hybridised by a GLS. We consider firstly that demand is expressed from an intervention sector. Then, the time between two phone calls is distributed according to the Poisson law of a periodicity equals to two (day and night). The most used allocation rule is to allocate the nearest available vehicle to the intervention sector. Another assignment rule is to deploy the nearest vehicle with pre-emption of the most urgent cases (Savas, 1969; Lubicz and Mielezarek, 1987). Our model allocates the nearest available ambulance in order to minimise the total lateness of the intervention. The ambulance being in hospital after patient transfer leaves to meet a call that did not yet receive a response or will be allocated to the nearest waiting site (Figure 1). Allocate an ambulance from waiting site to an intervention sector, then from the sector to the closest hospital is the deployment event. The redeployment concerns ambulance back from hospital to waiting site (Figure 2). In our model, the nearest hospital is considered as a destination.

**Figure 1** Event simulation

**Figure 2** Deployment and redeployment movements (see online version for colours)
4.1 Heuristic to initialise the genetic population

We develop a heuristic in order to initialise the genetic population. The application [Eq. (17)] defines the chromosome $\pi$ which contains the number of ambulances in fire stations $P$ and hospitals $H$

$$\pi : P \cup H \rightarrow \mathbb{N} \text{ such as } \sum_{i \in (P \cup H)} \pi_i = N$$  \hspace{1cm} (17)

$\mu_{ij}$: Average inter-arrival time of calls such as $i \in S$ and $j \in \{1, \ldots, p\}$ with $p$ is the number of periods.

Whether the set $S_i = \{s \in S, \text{ such as } d_{s,i} = \min_{j \in P \cup H} d_{s,j}\} \subset S$, $S_i$ contains the sectors closest to the waiting site $i \in (P \cup H)$. We can characterise $S_i$ by $F_i$ the frequency sum of interventions related to $s \in S_i$ [Eq. (18)]:

$$F_i = \sum_{j \in S_i} f_j \text{ and } f_j = \frac{1}{\mu_j} = \frac{p}{\sum_{i \in \mathcal{S}} \mu_{ij}}$$  \hspace{1cm} (18)

$S_i$ is also characterised by $D_i$, the sum of the distances between the site $i$ and the sectors $s \in S_i$, plus the distances between these sectors and the nearest hospitals [Eq. (19)]:

$$D_i = \sum_{s \in S_i} d_{s,i} + \sum_{s \in S_i, h \in H} \min d_{s,h}$$  \hspace{1cm} (19)

Thus, we determine the number of vehicles $\pi_i$ based on a given weight to the site $i$ according to the calculated distances and frequencies:

$$\pi_i = \frac{F_i D_i}{\sum_{j \in (P \cup H)} F_j D_j} \ast N$$  \hspace{1cm} (20)

4.2 Genetic algorithm hybridised by a GLS

4.2.1 The chromosome encoding

The solution contains a vehicle distribution in hospitals and fire stations. The size of the chromosome equals to the sum of hospitals and fire stations. The first part of the genes concerns the set of ambulances located in hospitals, the second part of chromosome concerns ambulances located in fire stations (Table 2).
Our goal is to ensure an allocation of ambulances in hospitals and fire stations in order to minimise the total lateness of interventions. The lateness equals to the duration between the demand call reception and the patient arrival to the hospital.

### 4.2.2 Genetic crossover

During this process, two chromosomes exchange parts of their genes, to give birth to children chromosomes. These crossings can be single or multiple. In our model, the child chromosome contains, gene by gene, the average integer part of the parent genes. We consider a constraint on the total number of ambulances to deploy; therefore, a chromosome correction algorithm has developed if the chromosome is not feasible (Table 3).

#### Table 3 Genetic crossover and genes correction

<table>
<thead>
<tr>
<th>Parent 1</th>
<th>Parent 2</th>
<th>Child chromosome</th>
<th>Corrected child</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 10</td>
<td>n = 10</td>
<td>s = 9</td>
<td>n = 10</td>
</tr>
<tr>
<td>2 1 3 0 4</td>
<td>4 2 1 0 3</td>
<td>3 1 2 0</td>
<td>3 1 2 1 3</td>
</tr>
</tbody>
</table>

### 4.2.3 The random correction algorithm

This algorithm is used to adjust the total number of vehicles $n$ if it is violated after genetic crossover. After calculating the sum $s = \sum_{i=1}^{n} \pi_i$ of ambulances in the $\pi$ chromosome, a gene is randomly selected, if $s$ is smaller than the number of available ambulances $n$ we add a vehicle to the corresponding gene. The algorithm below summarises the correction of genes.

#### Algorithm 1. Correction of genes

**Inputs:** sum of ambulances $s$, ambulances number $n$, chromosome $\pi$

**Start**

While $(s < n)$ do:

Randomly select a gene $i$:

\[
\pi_i \leftarrow \pi_i + 1
\]

$s \leftarrow s + 1$

**End while**

**End**
4.2.4 Local search on child chromosome

Local search is usually used to search around the current solution all possible improvements of the fitness. The fitness is the total lateness resulting from the distribution of ambulances to hospitals and fire stations. We define two movements called Swap and Move for our problem. The first movement consists to choose two genes randomly and swap them in the chromosome after calculating fitness improvement. This algorithm shows the Swap movement.

**Algorithm 2. Swap movement**

**Inputs:** Auxiliary variable aux, chromosome \( \pi \)

**Start**

for \( i \) and \( j \) two genes selected randomly do:

If \( i \neq j \) do:

\[ \text{aux} \leftarrow \pi_i \]
\[ \pi_i \leftarrow \pi_j \]
\[ \pi_j \leftarrow \text{aux} \]

If not improved fitness do:

\[ \text{aux} \leftarrow \pi_i \]
\[ \pi_i \leftarrow \pi_j \]
\[ \pi_j \leftarrow \text{aux} \]

End If

End If

End for

End

**Algorithm 3. Move movement**

**Inputs:** chromosome \( \pi \)

**Start**

for \( i \) and \( j \) two genes selected randomly do:

If \( (i \neq j) \) and \( (\pi_j > 0) \) do:

\[ \pi_i \leftarrow \pi_i + 1 \]
\[ \pi_j \leftarrow \pi_j - 1 \]

If not improved fitness do:

\[ \pi_i \leftarrow \pi_i - 1 \]
\[ \pi_j \leftarrow \pi_j + 1 \]

End if

End if

End for
The second movement Move consists to take a vehicle and move it from one to another gene in the same chromosome. Applying this movement does not change the sum of vehicles in the chromosome, but we must check that the set of available vehicles in the source gene is not null. This is repeated until there is no more improvement of the fitness. The algorithm (3) presents the Move movement.

4.2.5 The GLS

During the simulation, \( m_i \) is calculated by the set ambulance average available in the site \( i \) [Eq. (21)]:

\[
m_i = \frac{1}{T} \int_{0}^{T} \pi_i(t) \, dt
\]

(21)

\( T \) : the simulation horizon

\( \pi_i(t) \): the function that describes the set of vehicles available in the site \( i \) at time \( t \) during the simulation (Figure 3).

![Figure 3](image)

The GLS serves to guide the movement of the local search. In our problem, the movement of local search targets genes of chromosome by Move after calculating the ratio \( m_i / \pi_i \).

The gene \( \pi_i \) having the minimum value of \( m_i / \pi_i \) ratio is the site where a vehicle is added. However, the gene having the maximal value of this ratio is the gene which gives the vehicle.

In this algorithm, we propose an auxiliary table \( aux \) which contains the genes receiving a vehicle during simulation; furthermore, the gene does not get more than one local search movement; the size of this table does not exceed the chromosome size. The gene with zero vehicles is not designed by the GLS, so it is targeted only by Swap movement. We present below the detailed algorithm of the GLS (algorithm 4).
Algorithm 4. Guided local search

\textbf{Inputs:} \( i = 0, \ j = 0 \) \ two genes of chromosome \( \pi \); auxiliary table \( \text{aux} \), chromosome size \( n \)

\textbf{Start}

Do

\begin{itemize}
  \item launch the simulation
  \item \( \min = \) positive infinity
  \item \( \max = \) negative infinity
\end{itemize}

\textbf{for} each gene \( k \) \textbf{do}:

\textbf{if} the gene \( (\pi_k > 0) \) \textbf{do}:

\begin{align*}
  x & \leftarrow \frac{m_k}{\pi_k} \\
  \text{if} \ (x < \min) \text{and} \ (k \notin \text{aux}) \text{ do:} & \\
  \min & \leftarrow x; \\
  i & \leftarrow k; \\
\end{align*}

\textbf{end if}

\textbf{if} \ (x > \max) \text{and} \ (k \notin \text{aux}) \text{ do:} \ \max \leftarrow x; \ \ j \leftarrow k; \\

\textbf{end if}

\textbf{end if}

\textbf{end for}

\( \pi_i \leftarrow \pi_i + 1; \)

\( \pi_j \leftarrow \pi_j - 1; \)

\textbf{Launch the simulation}

\textbf{If the fitness is not improved do:}

\begin{align*}
  \pi_i & \leftarrow \pi_i - 1; \\
  \pi_j & \leftarrow \pi_j + 1; \\
\end{align*}

\textbf{Add} \( i \) \textbf{to} \( \text{aux} \)

\textbf{End if}

\textbf{while size} \ (\text{aux}) < n - 1

\textbf{End}
4.2.6 The GA for distribution of ambulances

We define the genetic algorithm of ambulance distribution.

Algorithm 5. Genetic algorithm

Inputs: population size \( n \),

Start
   Randomly generate an initial population of \( n \) given size.
   Improve the initial population by a local search.
   Sort the initial population in ascending order according to their fitness.
   for each iteration do:
      Select two parent chromosomes of the population.
      Apply a crossing by the average of genes
      Improve the best chromosome resulting from the crossing by the local search and randomly added to the second half of population if it improves its bad self.
      Reorder the population.
   end for
   Return the best solution of the population.

End

5 Computational results

The approach to solve our problem is implemented using the Java language on an Intel® Xeron® CPU E31240 @ 3.30 GHz. Firstly, we test the exact method and the GA hybridised by the GLS to an instance randomly generated. The size of the population is 20, the number of iterations equals to 20 and the crossover operator is applied for a probability 80%. The mutation operator is replaced by the correction algorithm. The CPU of Cplex increases when the number of sectors and the horizon increase. Thus, the complexity of our model is shown in Figure 4. The Cplex CPU is long comparing to the GA computational time. Results obtained by Cplex and the GA are summarised in Table 4.

Figure 4 Complexity dependency of the problem to the sectors number and the horizon (see online version for colours)
Table 4  Total fitness done by exact and approached methods

<table>
<thead>
<tr>
<th>Simulation horizon</th>
<th>1/2 day</th>
<th>1 day</th>
<th>3/2 days</th>
<th>2 days</th>
<th>5/2 days</th>
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</thead>
<tbody>
<tr>
<td>2 sectors GA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPU (s)</td>
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<td>0.005</td>
<td>0.005</td>
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<td>0.007</td>
</tr>
<tr>
<td>exact method</td>
<td>321.90233</td>
<td>333333333</td>
<td>643.804666</td>
<td>1103.6128</td>
<td>1448.5605</td>
</tr>
<tr>
<td>CPU (s)</td>
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<td>0.005</td>
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<td>0.003</td>
<td>0.008</td>
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<td>cplex</td>
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<td>31.62</td>
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<td>*</td>
</tr>
<tr>
<td>3 sectors GA</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>CPU (s)</td>
<td>0.002</td>
<td>0.004</td>
<td>0.002</td>
<td>0.007</td>
<td>0.006</td>
</tr>
<tr>
<td>exact method</td>
<td>335.90233</td>
<td>333333333</td>
<td>685.804666</td>
<td>1173.6128</td>
<td>1532.5605</td>
</tr>
<tr>
<td>CPU (s)</td>
<td>0.16</td>
<td>0.27</td>
<td>34.01</td>
<td>55.66</td>
<td>*</td>
</tr>
<tr>
<td>cplex</td>
<td>0.64</td>
<td>6.49</td>
<td>630.17</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>4 sectors GA</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>CPU (s)</td>
<td>0.002</td>
<td>0.012</td>
<td>0.006</td>
<td>0.006</td>
<td>0.005</td>
</tr>
<tr>
<td>exact method</td>
<td>400.78136</td>
<td>666666666</td>
<td>830.562733</td>
<td>1396.7494</td>
<td>1840.8512</td>
</tr>
<tr>
<td>CPU (s)</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>*</td>
</tr>
<tr>
<td>cplex</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>*</td>
</tr>
</tbody>
</table>

*Out of memory.

The second part of experimentation concerns the case study about Rabat region of Morocco. Due to the complexity of our model, Cplex goes out of memory once we launch it about the real instance that contains 7 hospitals, 5 fire stations and 16 intervention sectors. Thus, we run the GA hybridised by the GLS according to the parameters in Table 5. Emergency location and sectors of Rabat are represented in Figure 5.
Table 5  Computational parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crossover ratio</td>
<td>0.8</td>
</tr>
<tr>
<td>Mutation</td>
<td>Replaced by the correction algorithm</td>
</tr>
<tr>
<td>Simulation iterations</td>
<td>100</td>
</tr>
<tr>
<td>Horizon computation</td>
<td>7 days</td>
</tr>
<tr>
<td>Vehicle speed</td>
<td>60 kph</td>
</tr>
<tr>
<td>Genetic population size</td>
<td>30</td>
</tr>
<tr>
<td>Genetic algorithm iterations</td>
<td>100</td>
</tr>
</tbody>
</table>

Figure 5  Emergency location and demand sectors of Rabat region (see online version for colours)

The intervention sectors are defined using the administrative classification of the city. We consider demands expressed from 16 sectors of Rabat region. The Poisson parameter describes two periods; the first parameter Poisson\_day concerns the duration between two emergency calls in the day, i.e. from 6 a.m to 6 p.m, the second one Poisson\_night is about the same duration in the night, i.e. from 6 p.m to 6 a.m. We refer to the historic of the emergencies calls received at the call centre of the region from March 2015 to March 2016 (Table 6).

Table 6  Poisson parameter of intervention sectors

<table>
<thead>
<tr>
<th>Sector</th>
<th>Coordinate x (m)</th>
<th>Coordinate y (m)</th>
<th>Poisson_day (min)</th>
<th>Poisson_night (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>YAAKOUB AL MANSOUR</td>
<td>34005.343</td>
<td>−6862.973</td>
<td>319</td>
<td>345</td>
</tr>
<tr>
<td>TOUARGA</td>
<td>34005.752</td>
<td>−6832.899</td>
<td>355</td>
<td>361</td>
</tr>
<tr>
<td>AGDAL</td>
<td>34000.157</td>
<td>−6845.532</td>
<td>189</td>
<td>364</td>
</tr>
<tr>
<td>SWISSI</td>
<td>33978.391</td>
<td>−6839.075</td>
<td>379</td>
<td>418</td>
</tr>
<tr>
<td>YO USSOUFIA</td>
<td>33997.489</td>
<td>−6814.764</td>
<td>199</td>
<td>416</td>
</tr>
<tr>
<td>HASSAN</td>
<td>34022.894</td>
<td>−6786.101</td>
<td>322</td>
<td>367</td>
</tr>
<tr>
<td>BAB LAMRISSA</td>
<td>34033.07</td>
<td>−6820.047</td>
<td>298</td>
<td>384</td>
</tr>
</tbody>
</table>
Table 6  Poisson parameter of intervention sectors (continued)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Coordinate x (m)</th>
<th>Coordinate y (m)</th>
<th>Poisson_day (min)</th>
<th>Poisson_night (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TABRIQUET</td>
<td>34032.012</td>
<td>-6818.47</td>
<td>216</td>
<td>350</td>
</tr>
<tr>
<td>BETTANA</td>
<td>34029.521</td>
<td>-6808.643</td>
<td>278</td>
<td>404</td>
</tr>
<tr>
<td>LAYAYDA</td>
<td>34055.919</td>
<td>-6748.61</td>
<td>353</td>
<td>411</td>
</tr>
<tr>
<td>HSSAINE</td>
<td>33988.375</td>
<td>-6736.5</td>
<td>353</td>
<td>384</td>
</tr>
<tr>
<td>SIDI BOUKNADEL</td>
<td>34120.224</td>
<td>-6739.941</td>
<td>239</td>
<td>361</td>
</tr>
<tr>
<td>SHOUL</td>
<td>34026.467</td>
<td>-6755.201</td>
<td>316</td>
<td>369</td>
</tr>
<tr>
<td>SKHIRATE</td>
<td>33855.634</td>
<td>-7033.201</td>
<td>253</td>
<td>402</td>
</tr>
<tr>
<td>HARHOURA</td>
<td>33944.55</td>
<td>-6934.779</td>
<td>326</td>
<td>389</td>
</tr>
<tr>
<td>TEMARA</td>
<td>33915.542</td>
<td>-6933.838</td>
<td>232</td>
<td>369</td>
</tr>
</tbody>
</table>

The total fitness is 25,198 min during a horizon of 7 days relative to 130 min as an average fitness. The appropriate number of ambulances is 15. Beyond 15 vehicles, the fitness cannot be improved anymore. In this case, we say that the system is stabilised (Figure 6).

Figure 6  Relationship between the fitness and the fleet size of ambulances (see online version for colours)

The resulting chromosome defines the distribution of 15 ambulances in potential waiting sites. Table 7 recapitulates results.

Table 7  Distribution of ambulances in potential waiting sites done by GA

<table>
<thead>
<tr>
<th>Waiting site type</th>
<th>Waiting site name</th>
<th>Number of vehicles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hospitals</td>
<td>Hospital ‘Ibn sina’</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Hospital ‘Militaire’</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Hospital ‘d’enfants’</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Hospital ‘les spécialités’</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Hospital ‘Maternité des orangers’</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Hospital ‘prince Moulay abdellah’</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Hospital ‘ar-razi’</td>
<td>3</td>
</tr>
</tbody>
</table>

Figure 6  Relationship between the fitness and the fleet size of ambulances (see online version for colours)
Table 7  Distribution of ambulances in potential waiting sites done by GA (continued)

<table>
<thead>
<tr>
<th>Waiting site type</th>
<th>Waiting site name</th>
<th>Number of vehicles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fire stations</td>
<td>Center ‘Ibn Rochd Rabat’</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Center ‘Rabat Agdal’</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Center ‘Rabat Akkari’</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Center ‘Sala al jadida’</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Center ‘Temara’</td>
<td>3</td>
</tr>
</tbody>
</table>

6 Conclusion

We have considered a dynamic stochastic coverage model to cover the emergency location about Rabat region. We targeted seven hospitals and five fire stations to respond calls incoming from sixteen intervention sectors. Our model aims to minimise the total lateness of emergency intervention. This model is based on the deployment and redeployment of ambulances. It computes the total lateness of intervention at each movement during the simulation horizon. The available ambulance is affected to the intervention sector, then to the nearest hospital. After achieving the service, the ambulance comes back to the nearest waiting site. The computational experiments prove that the algorithm can give good results and the most important strength is that the model considers the randomness of incoming emergency calls expressed according to the Poisson distribution. We propose an exact method that we have tested on an instance randomly generated. Then, we have compared the results obtained to the GA hybridised by a GLS as an approached method. The exact method gives optimal solutions but goes out of memory once the number of sectors and horizon are increased. GA gives good results. The solution of the genetic algorithm hybridised by a GLS provides a distribution of ambulances that minimises the total lateness of emergency intervention. One future issue is to consider a criticality operator relative to each intervention sector, taking into account the population density, social and industrial facts to categorise regions.

References


Genetic algorithm to solve the emergency coverage problem


