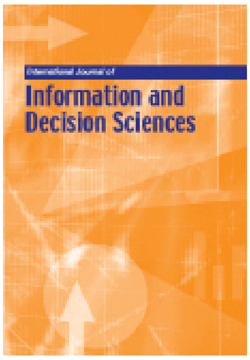

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A multi-view approach to multi-criteria decision making

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Abstract: In this paper, we investigate a new approach to multi-criteria decision making (MCDM) centred upon the application of canonical correlation analysis (CCA) to distinct groups of judgement criteria. By resorting to MV-MCDM (multi-view multi-criteria decision making), one can estimate reliable values for criteria weights via CCA for multi-view multi-criteria problems; reduce the dimensionality of the decision matrix by considering only one of the available views; and easily extend well-known MCDM methods, such as simple additive weighting (SAW) and technique for order of preferences by similarity to ideal solution (TOPSIS). MV-MCDM also allows the adoption of different aggregation methods (such as the Choquet integral and a new heuristic based on radar charts) to generate the overall scores of the alternatives. A numerical example with the multi-view versions of SAW and TOPSIS demonstrates the applicability of the novel approach.

Keywords: multi-criteria decision making; MCDM; multi-view canonical correlation analysis; CCA; TOPSIS; simple additive weighting; SAW; Choquet integral.

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1 Introduction

Multi-view learning (MVL) is a field of machine learning that integrates data from multiple feature sets (a.k.a. views) (Zhao et al., 2017; Sun et al., 2019). Its methods have shown great appeal recently because data are often collected from different sources (or show multiple facets) and because single-view data cannot comprehensively describe the relevant contents of all learning examples. For instance, web pages can be described by both the page contents (one view) and the hyperlink information (another view), whereas each image in a web page can be characterised by its shape, colour, and some metadata (Zhao et al., 2017; Ye et al., 2018).

The different views often contain complementary information, and MVL methods can take advantage of this information to learn representations that are useful for understanding the structure of the data. Due to its strong theoretical underpinnings, MVL has been adopted with great success to improve the generalisation performance of different learning systems. Applications of MVL abound in the literature, covering a wide range of machine learning branches, such as dimensionality reduction, active learning, ensemble learning, clustering, deep learning, and evidence aggregation (Zhao et al., 2017; Sun et al., 2019; Ye et al., 2018; Mukherjee et al., 2019).

On the other hand, the area of multi-criteria decision making (MCDM) deals with decisions involving the choice of a best alternative (i.e., course of action, strategy, solution) from several potential candidates, taking into account various judgment criteria (Triantaphyllou, 2000; Kou et al., 2011; Cuong et al., 2016; Papathanasiou and Ploskas, 2018; Kumar and Kumar, 2018; Loganathan et al., 2020). Typically, the outcome of an MCDM method is a ranking of the alternatives, which allows the decision maker (DM) to compare their relative performance according to his/her preferences with respect to the conflicting criteria.

In MCDM, numerical weights are usually assigned to the decision criteria to quantify their relative importance. The combined effect of the weighted criteria measures the overall performance of the alternatives. In this regard, the available MCDM techniques can be categorised into two groups, namely compensatory and non-compensatory (Banihabib et al., 2017). While in the former poor performance in some criteria can be compensated for by high performance in other criteria, in the latter each individual criterion can independently play a critical role in the aggregated performance of an alternative. Among the several compensatory methods available, simple additive weighting (SAW) (MacCrimmon, 1968) and the technique for order of preferences by similarity to ideal solution (TOPSIS) (Hwang and Yoon, 1981) are two of the most well-known.

Drawing a parallel between MVL and MCDM, it is noticeable that methods pertaining to both areas operate on a series of values arranged in the form of a two-dimensional (data/decision) matrix. While in MVL there is a set of patterns (rows) represented by a set of features (columns), in MCDM one has a set of alternatives/decision options (rows), each assessed according to a set of criteria (columns). By establishing this correspondence, one can realise that in MCDM no distinction is usually made among the elements of the criteria set with respect to the different scenarios/contexts they may be associated with.

Such an issue is evident in the medical field, for example. Disorders, such as acute coronary syndrome, vector-borne diseases, and chronic liver diseases, have been the target of MCDM approaches for the purposes of diagnosis or assessment of mortality/severity levels (Sałabun and Piegat, 2017; Pal et al., 2019; Piegat and Sałabun, 2015). Although relevant, such investigations have focused solely on the unimodal analysis of the clinical conditions of the patients, even though the data related to such criteria may be collected from diverse information sources, such as medical records, pathological exams, blood tests, ultrasounds, etc. The same observation could be made to studies conducted in a very different application domain, namely the analysis of renewable energy (RE) sources via MCDM methods. In Lee and Chang (2018), for instance, the evaluation criteria assessing the RE sources were divided into four main categories, viz. financial, technical, environmental, and social, but all of them were grouped into a common set while performing the decision analysis.

So, even in those cases where the criteria are explicitly modelled as belonging to different groups (Baudry et al., 2018; Munda, 2004; Cambrainha and Fontana, 2018) or a large number of them is somehow reduced by considering the similarities of their score values (Liu et al., 2017), the application of standard MCDM methods takes into consideration neither the peculiarities of each group nor the correlations among the groups while generating the final ranking of alternatives. Besides, it is worth emphasising that criteria belonging to different groups are often dependent to each other, even though their interdependence may be of different levels and due to different reasons. This way, the individual weights associated with different criteria are usually difficult to be properly set (Tervonen et al., 2009). Although these issues are very relevant, as far as we are aware of there is no work published in the MCDM literature that is dedicated to investigating effective approaches to deal directly with them. To help filling this gap, we propose in this paper a new approach, referred to as multi-view multi-criteria decision making (MV-MCDM).

MV-MCDM is centred upon the application of canonical correlation analysis (CCA) (Hotelling, 1936) to the distinct groups of criteria (referred to as 'criteria views'). CCA is a widely used statistical method to measure the linear relationships between two multidimensional variables (Meloun and Militký, 2011; Manly and Navarro Alberto, 2017). It has gotten popularity in machine learning (Hardoon et al., 2004), particularly in MVL, where it is employed to obtain a low-dimensional and closely correlated representation of the original multi-view data (Sun et al., 2019).[1]

By resorting to MV-MCDM, we expect that the DM can deal more naturally with multi-criteria problems having distinct groups of criteria. Moreover, the novel approach enables the estimation of reliable values for criteria weights via CCA. Another interesting advantage is that MV-MCDM entails the reduction of the dimensionality of the decision matrix by considering only a single criteria view for performing the assessment of the alternatives. In addition, we show here that the MV-MCDM methodology allows the easy multi-view extension of well-known MCDM methods, such as SAW and TOPSIS. Our approach is also generic enough to allow the use of different aggregation methods (Marichal, 1999; Grabisch et al., 2009) to yield the overall scores of the alternatives based on distinct criteria weights generated by CCA.

In this regard, we show that the canonical variate correlation coefficients can be used as fuzzy density measurements, thus allowing the application of the Choquet integral (Choquet, 1954). In short, the Choquet integral is an aggregation operator with respect to any fuzzy measure (Grabisch, 1995). Its wide usage is mainly due to the fact that it considers not only the importance of each individual attribute to be aggregated, but also the interactions between them (Cao, 2012). Once a fuzzy measure is identified, a fuzzy integral can be used as an aggregation tool for computing the global scores or ranking the alternatives.

However, the identification of a fuzzy measure is one of the most difficult steps for applying fuzzy integrals in order to solve MCDM problems. Over the years, several studies have emerged aiming at determining the values of fuzzy densities (Leszczyński et al., 1985; Takahagi, 2007; Larbani et al., 2011). Nonetheless, these methods usually require subjective information to be provided by a specialist. To exclude this subjectivity, other methods (using neural networks and genetic algorithms, for example) have been proposed (Lee and Teng, 2000; Liao et al., 2013). Yet, these methods are supervised in nature, requiring prior knowledge of the results and also showing slow convergence problems. Besides the application of the Choquet integral, a new heuristic aggregation method based on radar charts (Wilkinson, 2005) is also considered in this paper.

In short, the main contributions of MV-MCDM are:

1. the estimation of reliable values for criteria weights via CCA
2. the reduction of the dimensionality of the decision matrix by considering only one of the available views
3. the easy multi-view extension of well-known MCDM methods, such as SAW and TOPSIS.

With respect to the third contribution in particular, MV-MCDM is generic enough to allow the adoption of different aggregation methods to generate the overall scores of the alternatives. In this regard, two other contributions of this work involve the use of canonical variate correlation coefficients as fuzzy density measurements for the Choquet integral and the proposition of a novel aggregation method based on radar charts.

The remainder of this paper is structured as follows. In Section 2, we present an overview of the main aspects related to standard MCDM, CCA, and aggregation methods, which compose the main conceptual ingredients of MV-MCDM. In Section 3, we present details about the main steps comprising the new multi-view decision making methodology and also point out some of its relevant properties. In Section 4, a numerical example is given to demonstrate the usefulness of the proposed approach, showing in particular how MVMCDM can be instantiated for SAW and TOPSIS. Section 5 concludes the paper with some remarks on future work. The detailed formulation of the new aggregation method based on radar charts is provided in Appendix.

2 Background

In what follows, we briefly review the basic aspects related to MCDM, with a focus on SAW and TOPSIS. Then, we present the theory behind CCA that is particularly useful for the design of MV-MCDM. Finally, we comment on the role of aggregation methods in the context of MCDM, providing specific details on the formulation of the Choquet integral.

2.1 MCDM

As already mentioned, MCDM refers to the process of making decisions when there are multiple but a finite list of alternative solutions to the decision problem in hand and multiple criteria to assess the pros and cons of such alternatives (Triantaphyllou, 2000; Papathanasiou and Ploskas, 2018). An MCDM problem with m alternatives and n criteria can be formulated in the form of a decision matrix:

$$\mathcal{D} = \left(x_{ij}\right)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ . \\ . \\ . \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & & c_n \end{array} \\ \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix} \end{array} \tag{1}$$

$$\mathrm{w} = \left(w_1, w_2, \ldots, w_n\right)$$

where A_1, A_2, …, A_m denote the feasible alternatives, c_1, c_2, …, c_n refer to the (usually conflicting) evaluation criteria, x_{ij} is the evaluation of alternative A_i under criterion c_j, and w_j stands for the weight of criterion c_j.

The aim is usually to find the best option, taking into account the information available in the decision matrix $\mathcal{D}$ and the weight vector w. Instead of a single solution, a ranking of alternatives may be also output by an MCDM method, allowing one to better compare their relative performance. In classical MCDM (Hwang and Yoon, 1981; Dyer et al., 1992), the ratings and criteria weights should be known precisely. More recently, MCDM approaches aiming at the modelling and handling of uncertain values and weights have been investigated, mostly in the context of fuzzy logic (Mardani et al., 2015). In the sequel, we summarise two of the most well-known MCDM methods, which have been particularly considered in our proposal of MV-MCDM.

SAW is probably the simplest and most well-known compensatory MCDM method (MacCrimmon, 1968). Suppose the DM has assigned importance weights w to the criteria. Then, according to SAW, the most preferred alternative, A^*, is selected such that

$$A^* = \left\{ A_i \mid \max_i \frac{\sum_{j=1}^{n} w_j x_{ij}}{\sum_{j=1}^{n} w_j} \right\}, \tag{2}$$

where x_{ij} is the evaluation of the i^{th} alternative with respect to the j^{th} criterion in accord with a numerically comparable scale. Usually, the weights are normalised so that $\sum_{j=1}^{n} w_j = 1$.

Hwang and Yoon (1981) developed TOPSIS based on the notion that the chosen alternative should have the closest distance to a positive ideal solution (PIS) and the farthest distance to a negative ideal solution (NIS). An assumption of TOPSIS is that the criteria are monotonically increasing or decreasing (Banihabib et al., 2017). The main steps of TOPSIS can be briefly described as follows (Papathanasiou and Ploskas, 2018).

The first step is to compute normalised ratings from the original evaluations. For this purpose, vector normalisation is usually employed:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, i = 1, \ldots, m; \; j = 1, \ldots, n. \tag{3}$$

Alternatively, one can make use of linear normalisation, which should be computed differently for benefit criteria (for which the higher the evaluation, the better is the performance of an alternative) and cost criteria (for which the smaller the evaluation, the better is the performance of a given alternative). For benefit criteria, $r_{ij} = (x_{ij} - x_j^-) / (x_j^+ - x_j^-)$, where $x_j^+ = \max_i \{x_{ij}\}$ and $x_j^- = \min_i \{x_{ij}\}$. Conversely, for cost criteria, $r_{ij} = (x_j^- - x_{ij}) / (x_j^- - x_j^+)$. The values of x_j^+ and x_j^- can be also manually set in order to represent the aspired and worst assessment level, respectively.

The second step is simply to compute the weighted normalised ratings via the linear combination

$$v_{ij} = w_j r_{ij}, i = 1, \ldots, m; \; j = 1, \ldots, n, \tag{4}$$

where we also assume that $\sum_{j=1}^{n} w_j = 1$.

Next, the PIS and the NIS are derived as:

$$\begin{aligned} \text{PIS} = A^+ &= \left\{v_1^+, \ldots, v_j^+, \ldots, v_n^+\right\} \\ &= \left\{\left(\max_i \left\{v_{ij}\right\} \mid j \in J_1\right), \left(\min_i \left\{v_{ij}\right\} \mid j \in J_2\right)\right\}, \end{aligned} \tag{5}$$

$$\begin{aligned} \text{NIS} = A^- &= \left\{v_1^-, \ldots, v_j^-, \ldots, v_n^-\right\} \\ &= \left\{\left(\min_i \left\{v_{ij}\right\} \mid j \in J_1\right), \left(\max_i \left\{v_{ij}\right\} \mid j \in J_2\right)\right\}, \end{aligned} \tag{6}$$

where J_1 and J_2 are the sets of benefit and cost criteria, respectively.

Considering all criteria, one can then compute the Euclidean distances of each alternative to PIS (D_i^+) and NIS (D_i^-), which are given as follows:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2}, i = 1, \ldots, m \tag{7}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}, \; i = 1, \ldots, m. \tag{8}$$

Finally, the relative closeness to the ideal solution (Cl_i) should be computed for each alternative based on equations (7) and (8). This measure is always between 0 and 1, and then the alternatives can be ranked from best (higher values) to worst.

$$Cl_i = \frac{D_i^-}{D_i^- + D_i^+}, \; i = 1, \ldots, m. \tag{9}$$

2.2 Canonical correlation analysis

CCA is a multivariate statistical technique that models the linear relationships between two (or more) sets of variables, usually called dependent and independent sets (Meloun and Militký, 2011). In particular, CCA has been applied with great success to a variety of learning problems related to multi-view data (Zhao et al., 2017; Sun et al., 2019). Like principal component analysis (PCA) and linear discriminant analysis (LDA) (Meloun and Militký, 2011; Manly and Navarro Alberto, 2017), CCA can also reduce the dimensionality of the original variables, since only a few factor pairs are normally needed to represent the relevant information. Another attractive property of CCA is its invariance to affine transformations of the input variables (Donner et al., 2006).

Suppose we have a dataset with two distinct views on the features (columns) describing the data patterns (rows): $X \in \Re^{m \times n_x} = [\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{x}_m]^T$ and $Y \in \Re^{m \times n_y} = [\mathrm{y}_1, \mathrm{y}_2, \ldots, \mathrm{y}_m]^T$, with n_x, $n_y \geq 2$. In a nutshell, CCA computes a series of paired projection vectors (linear transformations), w_x and w_y, so that the resulting pairs of variables, $\mathrm{U} = \mathrm{w}_x^T X$ and $\mathrm{V} = \mathrm{w}_y^T Y$ (known as canonical variates), are maximally correlated. More precisely, $r = \min(n_x, n_y)$ pairs of projection vectors are created so that the correlation between U_1 and V_1 is maximum, the correlation between U_2 and V_2 is maximum, subject to the condition that U_2 and V_2 are uncorrelated with U_1 and V_1, respectively, and so on and so forth, up to the point that the correlation between U_r and V_r is maximum, provided that they correlate with neither $\mathrm{U}_1, \mathrm{U}_2, \ldots, \mathrm{U}_{r-1}$ nor V_1, V2, …, V_{r-1}, respectively.

Each canonical variate can be seen as a latent factor, while the canonical correlations $\rho_{\mathrm{U}_1,\mathrm{V}_1}$ through $\rho_{\mathrm{U}_r,\mathrm{V}_r}$ capture the linear relationships between the corresponding latent variables, so that

$$\rho_{\mathrm{U}_1,\mathrm{V}_1} > \rho_{\mathrm{U}_2,\mathrm{V}_2} > \ldots > \rho_{\mathrm{U}_r,\mathrm{V}_r}. \tag{10}$$

These correlations can be calculated as follows (for a given pair of projection vectors):

$$\rho_{\mathrm{U,V}} = \frac{\operatorname{cov}\left(\mathrm{w}_x^T X, \mathrm{w}_y^T Y\right)}{\sqrt{\operatorname{var}\left(\mathrm{w}_x^T X\right) \operatorname{var}\left(\mathrm{w}_y^T Y\right)}} = \frac{\mathrm{w}_x^T C_{xy} \mathrm{w}_y}{\sqrt{\left(\mathrm{w}_x^T C_{xx} \mathrm{w}_x\right)\left(\mathrm{w}_y^T C_{yy} \mathrm{w}_y\right)}}, \tag{11}$$

where the covariance matrix C_{xy} is defined as

$$C_{xy} = \frac{1}{m}\sum_{j=1}^{m}\left(\mathrm{x}_j - \mathrm{m}_x\right)\left(\mathrm{y}_j - \mathrm{m}_y\right)^T, \tag{12}$$

with m_x and m_y denoting the means from the two views, respectively,

$$\mathrm{m}_x = \frac{1}{m}\sum_{j=1}^{m}\mathrm{x}_j,\ \mathrm{m}_y = \frac{1}{m}\sum_{j=1}^{m}\mathrm{y}_j, \tag{13}$$

and C_{xx} and C_{yy} can be defined analogously.

Since $\rho_{\mathrm{U,V}}$ is invariant to the scaling of w_x and w_y, CCA's optimisation problem can be formulated equivalently as

$$\begin{aligned} &\max_{\mathrm{w}_x,\mathrm{w}_y} \quad \mathrm{w}_x^T C_{xy}\mathrm{w}_y \\ &s.t. \quad \mathrm{w}_x^T C_{xx}\mathrm{w}_x = 1,\ \mathrm{w}_y^T C_{yy}\mathrm{w}_y = 1. \end{aligned} \tag{14}$$

This reduces to the problem of solving the following canonical equations:

$$\left(C_{xx}^{-1}C_{xy}C_{yy}^{-1}C_{yx} - \lambda I\right)\mathrm{w}_x = 0 \tag{15}$$

and

$$\left(C_{yy}^{-1}C_{yx}C_{xx}^{-1}C_{xy} - \lambda I\right)\mathrm{w}_y = 0, \tag{16}$$

where I is the identity matrix and λ is the largest eigenvalue for the characteristic equations

$$\left|C_{xx}^{-1}C_{xy}C_{yy}^{-1}C_{yx} - \lambda I\right| = 0 \tag{17}$$

and

$$\left|C_{yy}^{-1}C_{yx}C_{xx}^{-1}C_{xy} - \lambda I\right| = 0. \tag{18}$$

It turns out that the largest eigenvalue of the matrix products

$$C_{xx}^{-1}C_{xy}C_{yy}^{-1}C_{yx} \tag{19}$$

or

$$C_{yy}^{-1}C_{yx}C_{xx}^{-1}C_{xy} \tag{20}$$

is in fact the squared canonical correlation coefficient, also known as a canonical root.

Furthermore, it can be shown that

$$\mathrm{w}_x = \frac{C_{xx}^{-1}C_{xy}\mathrm{w}_y}{\sqrt{\lambda}} \tag{21}$$

and

$$\mathrm{w}_y = \frac{C_{yy}^{-1}C_{yx}\mathrm{w}_x}{\sqrt{\lambda}}. \tag{22}$$

Therefore, the eigenvectors associated with each canonical root are the vectors of coefficients w_x and w_y, which are usually referred to as canonical weights.

From the formulation above, one can notice that there are many ways to combine the original variables via pairs of canonical variates. However, as mentioned, usually only the first two or three linear combinations (pairs of canonical variates) are reliable and, thus, need to be interpreted.

CCA can address a wide range of objectives, which can be any or all of the following (Meloun and Militký, 2011; Dillon and Goldstein, 1984):

1 Determining whether two sets of variables are independent or determining the magnitude of the relationships that may exist between the two sets.

2 Deriving a set of weights for each set of dependent and independent variables, so that the linear combinations of each set are maximally correlated.

3 Explaining the nature of whatever relationships exist between the two sets of variables, generally by measuring the relative contribution of each original variable to the canonical variates that are extracted.

As a result, some methods to achieve these objectives have been proposed:

1 *Via canonical weights:* This procedure examines the sign and the magnitude of the canonical weight assigned to each original variable in a given canonical variate. Variables with relatively higher (lower) weights contribute more (less) to the variates (Lambert and Durand, 1975).

2 *Via canonical loadings:* This procedure measures the simple linear correlation between an original variable, either in the dependent or independent set, and the corresponding set's canonical variate.

3 *Via canonical cross-loadings:* This procedure, which has been suggested as an alternative to former (Dillon and Goldstein, 1984), involves correlating each of the original variables directly with the canonical variate associated with the other set.

In our proposed approach (Section 3), we have made use of the first procedure above for estimating the weights of the different sets of criteria that may be available to the DM.

2.3 Aggregation methods

Aggregation is an important component in building any evaluation or estimation model that is based upon various pieces of data or information coming from different sources or modalities (Grabisch et al., 2009). In the MCDM context, aggregation usually refers to the process of combining several evaluations related to the judgment criteria, or several decision scores, in order to produce a global quality score for each alternative (Marichal, 1999).

A profusion of aggregation operators have been proposed in the literature, making the problem of choosing the right method for a given application a difficult one (Grabisch et al., 2009). Among the most well-known, we can cite the various types of average, such as the arithmetic, geometric, harmonic, and Bonferroni (1950) average, as well as the power average (Yager, 2001), ordered weighted averaging (OWA) (Yager, 1988), and its several variants (Chiclana et al., 2002).

In MV-MCDM, aggregation is paramount for combining several decision scores associated to one of the views in order to generate the final ranking of the alternatives (see next section). For this purpose, so far, we have experimented with two distinct aggregation operators, a novel one based on radar charts (presented in Appendix) and the well-known Choquet (1954) integral. The latter is a fancy aggregation function defined with respect to a particular fuzzy measure (Murofushi and Sugeno, 1991) and can be considered as extending the weighted arithmetic average or OWA operator by taking into account the interdependence or correlation among the different criteria, offering a great flexibility for aggregation.

Let g be a particular fuzzy measure on X. The Choquet integral of a function $f : X \rightarrow [0, \infty]$ with respect to g is given by

$$\int f dg = \sum_{i=1}^{n} \left(f\left(x_{(i)}\right) - f\left(x_{(i-1)}\right) \right) g\left(A_{(i)}\right) \tag{23}$$

or, equivalently, by

$$\int f dg = \sum_{i=1}^{n} \left[g\left(A_{(i)}\right) - g\left(A_{(i+1)}\right) \right] f\left(x_{(i)}\right), \tag{24}$$

where $x_{(.)}$ indicates a permutation on the elements of X, such that $f(x_{(1)}) \leq \ldots \leq f(x_{(n)})$, $f(x_{(0)}) = 0$, $A_{(i)} = \{x_{(i)}, \ldots, x_{(n)}\}$ and $A_{(n+1)} = \emptyset$.

The great difficulty to use the Choquet integral is precisely to determine what is the fuzzy measure. Because of this, in order to simplify the fuzzy measurement theory, Sugeno proposed the λ-fuzzy measurement (Sugeno, 1974; Murofushi and Sugeno, 1991).

Let X = $\{x_1, x_2, \ldots, x_n\}$ be a finite set and let $\lambda \in (-1, +\infty)$. Sugeno's λ-measure is a function $g : 2^X \rightarrow [0, 1]$ such that

1 $g(X) = 1$.

2 if $A, B \subseteq X, A, B, \in 2^X$ with $A \cap B = \emptyset$ then $g(A \cup B) = g(A) + g(B) + \lambda g(A)g(B)$.

As a convention, the value of g at a singleton set $\{x_i\}$ is called a density and is denoted by $g_i = g(\{x_i\})$. In addition, we have that λ should satisfy the following equation:

$$\lambda + 1 = \prod_{i=1}^{n} (1 + \lambda g_i). \tag{25}$$

Tahani and Keller (1990) showed that once the densities are given, it is possible to use equation (25) to obtain the values of λ uniquely.

The densities g_i can be interpreted as the importance of a certain element within a set. Several approaches have been developed to determine such fuzzy densities, such as via neural networks (Zhenyuan and Jia, 1997) or genetic algorithms (Chen and Wang, 2001). As shown in the next section, this work proposes a different approach, using canonical correlation provided by CCA, to determine the fuzzy densities, which allows the use of the Choquet integral in the context of MVMCDM.

3 A multi-view approach to MCDM

As mentioned, in several decision-making settings (Sałabun and Piegat, 2017; Pal et al., 2019; Piegat and Sałabun, 2015; Lee and Chang, 2018; Baudry et al., 2018; Munda, 2004; Cambrainha and Fontana, 2018; Liu et al., 2017), the assessment of the different alternatives available is usually made considering complementary perspectives on the problem, each perspective related to a group of criteria and a particular context. Hereafter, we refer to each group of related criteria as a view, and emphasise that different views may have different numbers of associated criteria.

In this approach, an MCDM problem with m alternatives, n criteria, and v views (referred to as an MV-MCDM problem) can be formulated in the form of v decision matrices:

$$\mathcal{D}^{v_k} = \left(x_{ij}\right)_{m \times n_k}^{v_k} = \begin{array}{c} \\ A_1 \\ A_2 \\ . \\ . \\ . \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & & c_{n_k} \end{array} \\ \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n_k} \\ x_{21} & x_{22} & \cdots & x_{2n_k} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ x_{m1} & x_{m2} & \cdots & x_{mn_k} \end{pmatrix} \end{array} \tag{26}$$

$$w^{v_k} = \left(w_1, w_2, \ldots, w_{n_k}\right)$$

where $A_1, A_2, \ldots, A_m$ denote the set of feasible alternatives, $c_1, c_2, \ldots, c_{n_k}$ represent the n_k evaluation criteria associated to the k^{th} view ($k = 1, \ldots, v$), x_{ij} is the performance rating of alternative A_i under criterion c_j ($j = 1, \ldots, n_k$), and w_j stands for the weight of this criterion. Notice that $\sum_k n_k = n.$ By this means, one can regard the MV-MCDM formulation [equation (26)] as a generalisation of the standard MCDM problem [equation (1)].

In order to cope with the MV-MCDM problem, we introduce in the sequel a new methodology based on CCA. Such methodology is generic enough to permit the easy extension of well-known MCDM methods, in particular SAW and TOPSIS. Without loss of generality, we consider the existence of only two criteria views (i.e., $v = 2$), since, as shown in Section 2.2, the standard CCA works only on a paired dataset. However, if two or more criteria views are indeed available, a generalised version of CCA could still be used (Kettering, 1971).

The steps of the proposed methodology are given as follows – refer to Figure 1.

Step 1 Identify the alternatives, criteria, and views

First of all, the DM has to clearly define the sets of m alternatives, n criteria, and v views. The alternatives should be the same across the views. The criteria set should be partitioned into the views they are associated with, reflecting the different perspectives, contexts or scenarios related to the decision problem in hand. Usually, two criteria belonging to the same view are more correlated to each other than two criteria belonging to different views, although this rule is not strict. Since we are considering only $v = 2$ views in this modelling, they are hereafter referred to as the independent view (X), with n_x criteria, and the dependent view (Y), with n_y criteria.

Step 2 Identify the correlations between the views and setup the criteria weights via CCA

In this step, CCA is applied to both views, following the steps given in Section 2.2, in order to obtain the $r = \min(n_x, n_y)$ pairs of canonical weights w_x and w_y – refer to equations (21) and (22). As a result, CCA will deliver r pairs of canonical variates, one associated with the independent view and another associated with the dependent view. For each pair of canonical variates, their correlation is computed according to equation (11). Then, the several canonical weights w_x will be used in the next step to weigh the criteria pertaining to the independent view, whereas the canonical weights w_y associated with the dependent criteria are disregarded.

Step 3 Apply the chosen MCDM method on the independent view

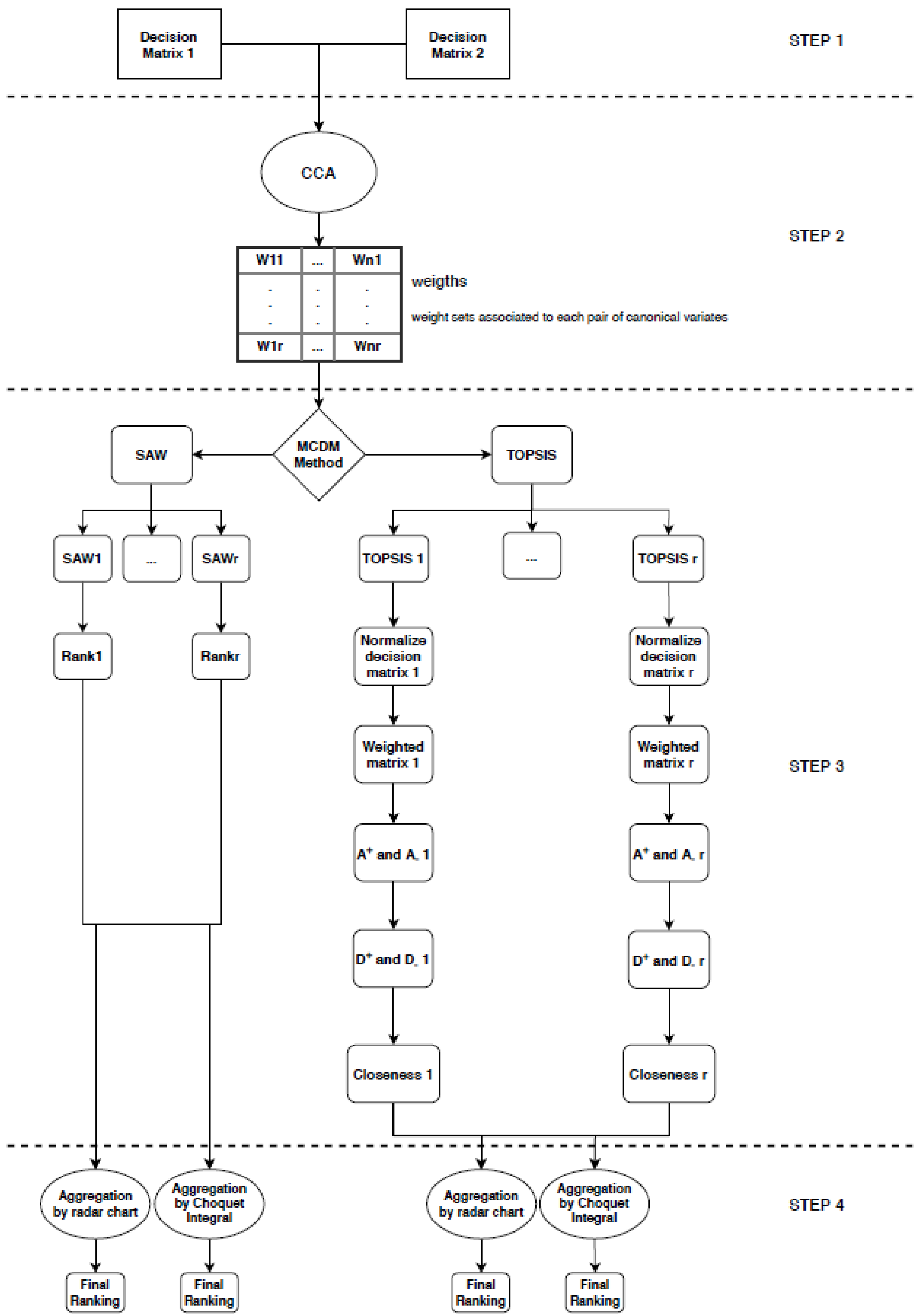
Based on the r canonical weights w_x available for the independent view, the chosen standard MCDM method (either SAW or TOPSIS) is executed r times in this step, one over each set of weights. As a result, each alternative is assessed r times.

Step 4 Aggregate the scores and generate the final ranking

With the r assessments (scores) available for each alternative, one can produce a final, global score via the Choquet integral. For this purpose, the fuzzy densities of the Sugeno measure [equation (25)] are set as the correlations of the canonical variates computed in the second step. An alternative aggregation operator, based on the area of radar charts, may be also employed here. This novel operator is formulated in detail in Appendix. With the aggregated scores in hand, the full ranking of the alternatives can be finally produced (one ranking for each aggregation method used).

Regarding specifically the last step of the methodology, we advocate the adoption of the canonical correlations as fuzzy densities g_i for the Sugeno measure due to the following reason. Making a parallel between the fuzzy density and the canonical correlation, it can be noticed that while the value of fuzzy density g_i is interpreted as the (possibly subjective) importance of a source of information for the decision process, the canonical correlation of a pair of canonical variates determines the degree of importance of this pair in the relationship between the sets of variables. As mentioned, the computation of the canonical variates is performed sequentially, with each pair capturing the residual variance that is still available for the original variables (the variance not explained by the first pair is captured by the following pairs of orthogonal canonical variates). This way, the values of the canonical correlations decrease for each pair in the sequence, as shown in equation (10). Since in the third step of the methodology we generate r scores for each alternative based on the projection weights associated with each pair of canonical variates, we have used their correlations to represent not only the relevance of each score individually but also the relevance of the accumulated aggregation of such scores (Krishnan et al., 2015).

Figure 1 Steps of the proposed MV-MCDM methodology



One relevant property of our multi-view methodology is that it allows the detection of redundant criteria, that is, criteria belonging to the same view showing high correlation of their evaluations (i.e., intra-view correlation). This is made possible by examining the magnitude of the canonical weight assigned to each criterion in a given canonical variate.

Redundant criteria tend to be assigned with almost-null values. In the extreme case that two or more criteria are fully correlated, the application of CCA can not happen, which signals an error indicating the multicollinearity condition (Meloun and Militký, 2011).

Another distinctive feature of our multi-view methodology is that it allows the reduction of the problem size (i.e., the decision matrix) by considering only a single criteria view (i.e., the independent view) for performing the assessment of the alternatives – refer to the second step. This is feasible since CCA estimates the canonical weights so as to maximise the correlation between the different groups of criteria (i.e., inter-view correlation). This way, the values of the canonical weights w_x of the independent-view criteria already reflect (some of) the judgment information available in the dependent-view criteria, which can then be set aside. (Of course, if the DM wishes to consider all criteria in the analysis, this is readily possible by also making use of w_y.) Eventually, if more than two views were available in the decision problem, the same procedure could still be pursued via generalised CCA (Kettering, 1971), leading to a greater reduction in the size of the decision matrix.

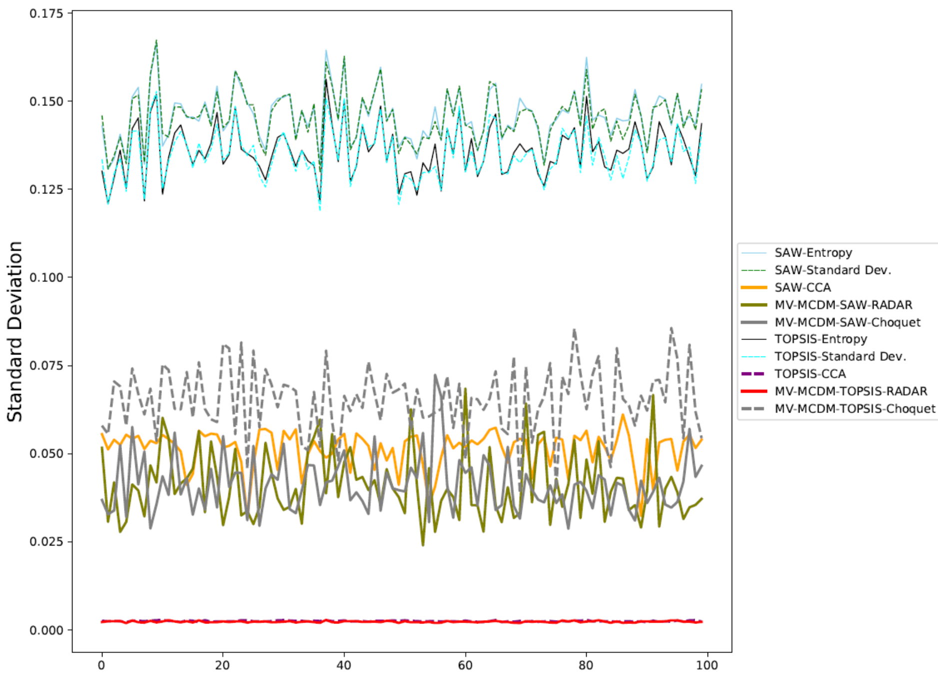
Leveraging on this property, if, for some reason, one of the criteria views were deemed as more significant than the other(s), the DM could regard it as the single independent view. Conversely, if the significance of the different views was quite the same, but their cardinalities were very distinct, gains in computational performance could be obtained by sticking with the view with less criteria. Even when a natural criteria split does not exist, the multi-view methodology could still be adopted using manually crafted splits. This may be particularly useful when the evaluations of some criteria (that will serve as dependent-view criteria) are much uncertain (and thus not trustful) or not easy to be measured/estimated.

Finally, we claim here that using the weights elicited by CCA to weigh the criteria in MCDM is indeed a very reasonable choice, even when considering the standard (single-view) setting. This is because the canonical weights identify the relative importance of the different criteria: The higher the value of a canonical weight, the higher is the contribution of the associated criterion to each induced canonical variate.

In order to empirically support our claim, we conducted experiments on a synthetic, two-view decision problem having $m = 500$ alternatives and $n = 8$ criteria in total. The numbers of criteria associated with the independent and dependent views were set arbitrarily as five and three, respectively. A hundred simulations were run, each time randomly sampling (without replacement) half of the alternatives to conduct the MCDM process. In the simulations, we considered both our multi-view methodology (with the two aggregation methods) and the standard versions of SAW and TOPSIS, the latter operating on all criteria. For setting up the criteria weights for SAW and TOPSIS, three methods were considered, namely via entropy, standard deviation, or CCA. The first two are very well-known and widely used methods in the MCDM literature (Hwang and Yoon, 1981; Wang and Luo, 2010; Dammak et al., 2015).

Figure 2 shows the average spread of the final scores of the randomly chosen 250 alternatives across all runs. The higher the standard deviation of the scores, the more sensitive is the outcome (ranking) of the decision-making process to the way the criteria weights was computed. Regardless of the type of problem (single or multi-view), the scores generated via canonical weights were more stable and robust to the variation in the chosen alternatives. As a result, the entailing decision-making process could be deemed as more consistent.

Figure 2 Effect of the way the criteria weights are computed on the decision scores generated by MV-MCDM, SAW, and TOPSIS (see online version for colours)



4 Results

In order to reveal the convenience of the proposed methodology, we present here a numerical example and results showing the step-by-step application of the MV-MCDM process to a synthetic, two-view decision problem. This type of validation approach has been widely used in the MCDM literature (Zhang et al., 2011; Xian et al., 2018; Kacprzak, 2019).

The first step of the methodology involves the explicit definition of the decision matrices related to the different views. In this example, for the sake of clarity and simplicity, we consider only $m = 10$ alternatives and $v = 2$ views, namely the independent view (with $n_1 = 5$ criteria) and the dependent view (with $n_2 = 3$ criteria). The two decision matrices are shown in Tables 1 and 2.

In order to setup the criteria weights for the independent view, CCA is applied in the second step of MV-MCDM. In this case, there are $r = \min(n_1, n_2) = 3$ sets of canonical weights w_x, which are provided in Table 3. Notice that the sign of the weight associated with a given independent-view criterion determines its type, whether benefit (positive sign) or cost criterion (negative sign) – see Section 2.1.

Table 1 Decision matrix of the first view

	c_1	c_2	c_3	c_4	c_5
1	1.222	1.129	0.905	1.107	0.483
2	0.982	0.565	1.162	0.263	1.417
3	0.239	0.067	0.173	0.600	0.038
4	2.243	0.183	1.605	0.151	2.111
5	0.873	0.602	1.308	0.471	0.740
6	0.540	0.358	0.039	0.045	0.056
7	0.584	0.392	0.682	0.712	0.495
8	0.020	0.017	0.129	0.570	0.426
9	1.497	0.313	1.151	0.123	1.437
10	0.009	0.091	0.262	0.856	0.310

Table 2 Decision matrix of the second view

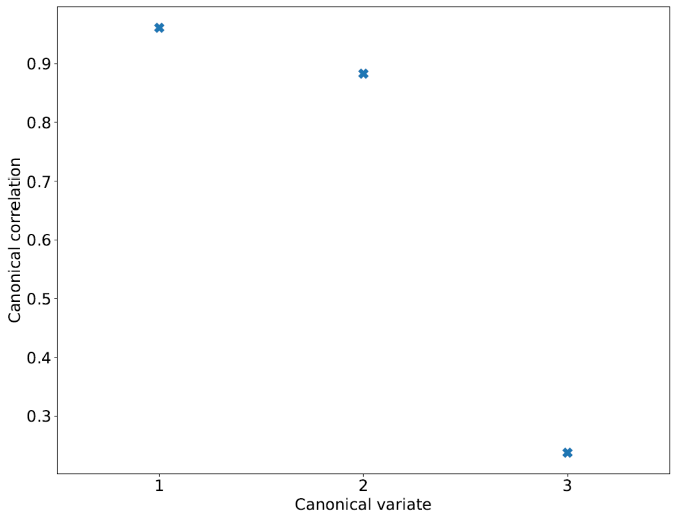
	c_1	c_2	c_3
1	0.797	1.086	0.907
2	1.398	0.875	0.936
3	0.219	0.653	0.685
4	1.576	0.374	2.079
5	1.043	0.586	1.227
6	0.018	0.150	0.228
7	0.507	0.909	0.881
8	0.307	0.571	0.676
9	1.154	0.152	1.120
10	0.028	1.043	0.265

Table 3 Sets of criteria weights achieved with CCA

	$w_x^{(1)}$	$w_x^{(2)}$	$w_x^{(3)}$
1	–0.037	0.033	0.152
2	0.020	–0.061	–0.292
3	–0.045	0.106	–0.237
4	–0.034	–0.113	0.239
5	–0.030	–0.014	0.101

Moreover, the correlations for the r pairs of canonical variates are computed, which are depicted in Figure 3. It is apparent that the correlation of the third pair of canonical variates (0.24) is not as significant as the correlations of the first two pairs (viz. 0.96 and 0.88). Therefore, the decision scores generated with $\mathrm{w}_x^{(3)}$ should be deemed as less significant in the aggregation process than the scores produced with $\mathrm{w}_x^{(1)}$ and $\mathrm{w}_x^{(2)}$.

Figure 3 Correlations for the $r = 3$ pairs of canonical variates (see online version for colours)



In the third step, the standard MCDM methods are invoked r times to calculate the scores for each alternative, each time using a different instance of w_x. Tables 4 and 5 show the scores obtained with SAW and TOPSIS, respectively.

Figure 4 Radar charts for MV-MCDM with SAW (see online version for colours)

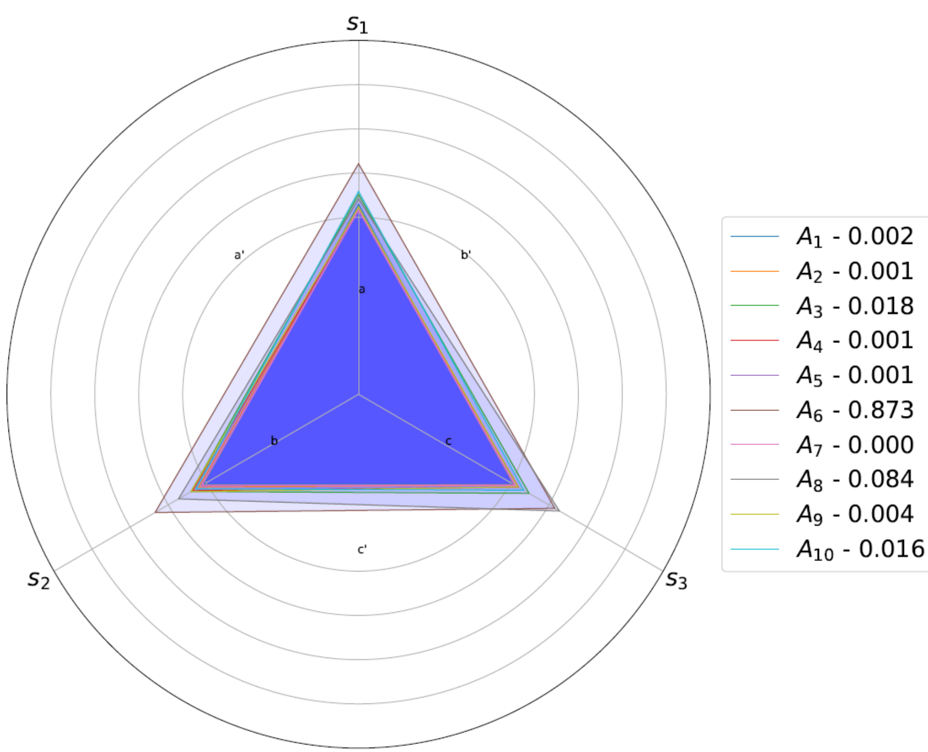


Table 4 Scores produced for each alternative by SAW

	$w_x^{(1)}$	$w_x^{(2)}$	$w_x^{(3)}$
1	0.154	0.024	0.018
2	0.110	0.069	0.030
3	0.271	0.138	0.136
4	0.091	0.124	0.057
5	0.102	0.044	0.023
6	0.639	0.471	0.329
7	0.087	0.039	0.031
8	0.220	0.256	0.363
9	0.123	0.141	0.053
10	0.309	0.095	0.096

Table 5 Scores produced for each alternative by TOPSIS

	$w_x^{(1)}$	$w_x^{(2)}$	$w_x^{(3)}$
1	0.488	0.296	0.405
2	0.482	0.678	0.415
3	0.692	0.449	0.650
4	0.294	0.876	0.546
5	0.496	0.625	0.398
6	0.784	0.539	0.518
7	0.597	0.444	0.571
8	0.689	0.454	0.652
9	0.430	0.777	0.512
10	0.648	0.384	0.667

The last step is to aggregate the r MCDM scores of each alternative into a global score. For this purpose, we have adopted two aggregation approaches, one using the Choquet integral and the other using the operator based on radar charts (hereafter referred to as RADAR).

For SAW as internal MCDM method, the global scores and final rankings yielded by each aggregation operator are shown in Tables 6 and 7. For the first operator, we should first setup the density values of the Sugeno measure as the canonical correlations, which are 0.96, 0.88, and 0.24. Then, we resort to equation (25) to calculate the λ-fuzzy values, which are given in Table 8. Figure 4, in turn, brings the charts and respective area scores for each alternative generated by RADAR.

For TOPSIS as internal decision method, the global scores and final rankings generated by MV-MCDM with the Choquet integral and RADAR are shown in Tables 9 and 10. The canonical correlations for the density values and the λ-fuzzy values of the Sugeno measure are the same given before. Figure 5, in turn, brings the radar charts and respective area scores for each alternative generated by the second aggregation method.

Table 6 Global scores and final ranking achieved by MV-MCDM configured with SAW and Choquet integral

Alternative	*Score*	*Rank*
1	0.014	10
2	0.020	8
3	0.155	4
4	0.050	5
5	0.019	9
6	0.165	3
7	0.031	7
8	0.616	1
9	0.031	6
10	0.519	2

Table 7 Global scores and final ranking achieved by MV-MCDM configured with SAW and RADAR

Alternative	*Score*	*Rank*
1	0.002	6
2	0.001	8
3	0.018	3
4	0.001	7
5	0.001	9
6	0.873	1
7	0.000	10
8	0.084	2
9	0.004	5
10	0.016	4

Table 8 λ-fuzzy values of the Sugeno measure

	λ-fuzzy
1	0.000
2	0.237
3	0.883
4	0.912
5	0.961
6	0.971
7	0.999
8	1.000

Figure 5 Radar charts for MV-MCDM with TOPSIS (see online version for colours)

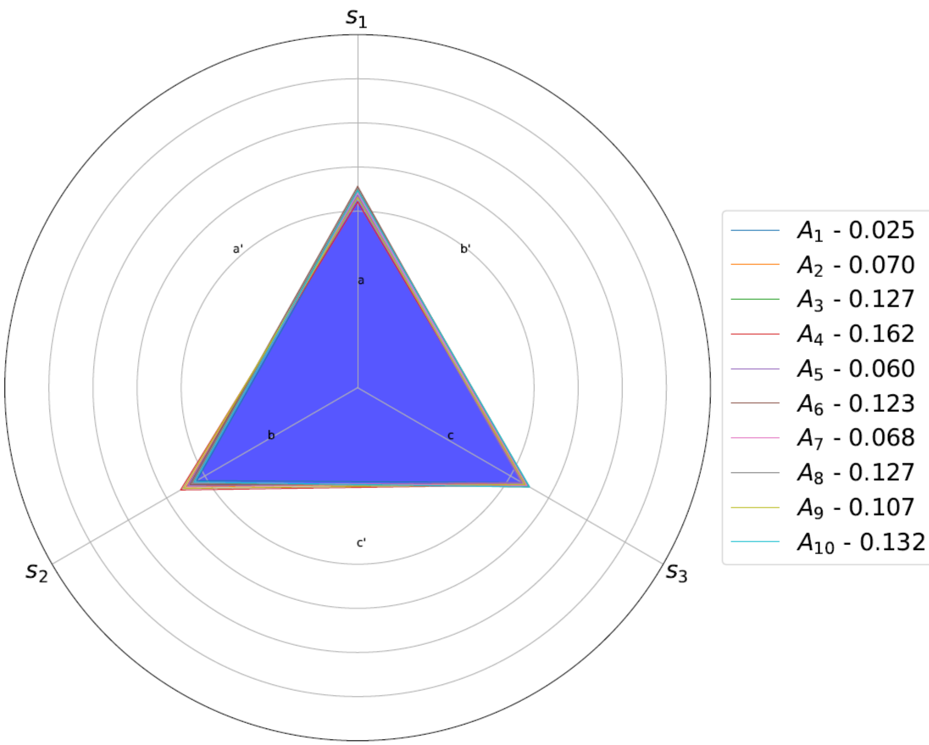


Table 9 Global scores and final ranking achieved by MV-MCDM configured with TOPSIS and Choquet integral

Alternative	*Score*	*Rank*
1	0.421	10
2	0.653	6
3	0.654	5
4	0.837	1
5	0.606	7
6	0.596	8
7	0.574	9
8	0.656	4
9	0.746	2
10	0.658	3

Based on the above results, some comments can be made. Regarding the SAW method, we noticed that the rankings generated using the RADAR and Choquet integral methods agreed only on alternatives #2 and #5 (ranks #8 and #9, respectively). This reveals that the SAW method can be very sensitive to the chosen aggregation method, not keeping stable, for example, the choice of the best and worst alternatives. Regarding TOPSIS, the rankings generated by this method also varied significantly; however, they could at least agree on the choice of the best and worst alternatives. So, this method could be considered relatively more reliable than the previous one.

Table 10 Global scores and final ranking achieved by MV-MCDM configured with TOPSIS and RADAR

Alternative	*Score*	*Rank*
1	0.025	10
2	0.070	7
3	0.127	3
4	0.162	1
5	0.060	9
6	0.123	5
7	0.068	8
8	0.127	4
9	0.107	6
10	0.132	2

5 Conclusions and future work

In this paper, we have introduced a new conceptual framework to MCDM, referred to as MV-MCDM, which is centred upon the notion of criteria views. MV-MCDM is particularly useful for those circumstances in which two or more groups of judgment criteria are available and the DM wishes to capture the linear relationships that may be available between these groups in order to better ascribe the weights to the criteria.

After formulating the MV-MCDM as a generalisation of the standard MCDM problem, we presented a generic, four-step methodology that can easily extend well-known MCDM methods, such as SAW and TOPSIS. The methodology is based on the canonical weights and canonical correlations produced by CCA, when operating on pairs of criteria views. Interestingly, we showed that by resorting to MV-MCDM one could perform the decision analysis by considering only one of the views available, which might lead to a substantial reduction in the problem (decision matrix) size. Besides, we also argued, based on results of simulations, that the usage of canonical weights as criteria weights can be beneficial even to the standard (single-view) setting.

We also established that MV-MCDM can be instantiated with different aggregation methods so as to combine the weighted criteria values generated via different sets of canonical weights. In particular, simulations on a synthetic, two-view problem demonstrated the step-by-step application of MV-MCDM configured with two aggregation operators, namely the Choquet integral and a novel operator based on radar charts. Specifically for the first operator, we provided arguments in favour of adopting the canonical correlations generated by CCA to serve as density values for the Sugeno λ-fuzzy measure.

Although the proposed approach shows distinctive features, its formulation, as derived here, considers solely two criteria views. Another limitation of the present work is that the current version of MV-MCDM assumes only crisp values for the criteria.

Besides, it does not capture nonlinear correlations that may exist among the different criteria groups.

As ongoing work, we are researching how to apply MV-MCDM to group decision making, where two or more DMs make their assessment based on distinct (possibly private and non-overlapping) groups of criteria. The use of a nonlinear version of the CCA is also under consideration. Besides, due to the stability property shown by criteria weights calculated via CCA, we are investigating how this approach could deal with the ranking reversal problem in MCDM. We also plan to deploy the new methodology in a real-case study that involves the comparative assessment of several branches of a regional bank, according to several criteria views. Finally, we shall investigate how the information captured by canonical loadings and cross-loadings could be somehow incorporated into the proposed methodology in order to further improve the decision-making process.

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Notes

1 Although CCA has been widely used as a tool for MVL, we would like to stress that MV-MCDM is not a machine learning approach to solve the MCDM problem. It is solely inspired by the idea of MVL.

Appendix

Aggregation method based on radar charts

In a nutshell, a radar chart (Wilkinson, 2005) is a 2D diagram to display multivariate data that charts multiple quantitative variables (categories) on one graph for easy comparison. Categories become axes on the graph, and the distance of a point in an axis from the centre indicates the value of the score in the associated category. Typically, a radar chart looks like an irregular polygon, and several charts can be stacked on top of each other, all with the same centre.

Based on these features, we propose a new aggregation method for MCDM, whereby the categories in the radar chart represent the results of r MCDM methods operating on a certain alternative. The score of an alternative produced by a given MCDM method is given by the distance from a point to the centre in the associated axis in the chart. The connection between the points corresponding to the different scores of an alternative composes a polygon, the area of which will be interpreted as the final, global score of the alternative. In MV-MCDM, in particular, the interpretation of the chart is a bit different. Each of the r categories in the chart represents the score produced by the internal MCDM method (either SAW or TOPSIS) for a given set of canonical weights to the independent-view criteria.

In a more formal basis, the new operator can be defined as follows.

Theorem 1: A radar-chart operator Ψ of dimension r is a mapping $\Psi : \Re^r \rightarrow \Re$ that has an associated scoring vector s of dimension r, such that:

$$\Psi(s_1, s_2, \ldots, s_r) = \Delta A, \tag{27}$$

where $s_r \in [0, \infty]$ is the r^{th} MCDM score and ΔA denotes the area of the polygon formed by the points corresponding to each score on the radar chart.

As a didactic example, consider an MV-MCDM problem for which there are $r = 3$ pairs of canonical variates produced by CCA. As shown in Figure 6, let a, b and c be the score values produced for the different canonical weights. In this case, the generated polygon is a triangle, whose area should be computed to give the final score. Remember that the area of any polygon boils down to a sum of triangle areas (Vialar, 2016). In this case, it is actually composed of three small triangles, $\Delta ba'a$, $\Delta bb'c$, and $\Delta cc'a$, whose areas can be calculated by the cosine law, as follows:

$$c'^2 = a^2 + c^2 - 2ac\cos(\gamma) \tag{28}$$

where γ denotes the angle formed by the sides of lengths a and c, which is opposite to the side of length c'. The other two relations are analogous:

$$\begin{aligned} a'^2 &= a^2 + b^2 - 2ab\cos(\alpha), \\ b'^2 &= b^2 + c^2 - 2bc\cos(\beta). \end{aligned} \tag{29}$$

Using Heron's formula (Raifaizen, 1971), the semi-perimeter can be calculated as

$$p = \frac{(a' + b' + c')}{2}, \tag{30}$$

whereas the area Δ of a triangle is given by

$$\Delta = \sqrt{p(p-a')(p-b')(p-c')}. \tag{31}$$

Finally, $\Psi(s_1, s_2, \ldots, s_r) = \Delta A = \sum_i \Delta i$ is given as the final score of an alternative by the MV-MCDM approach, where Δ_l is the area of the l^{th} triangle.

Figure 6 Aggregation method based on radar charts – example with $r = 3$ categories (scores) (see online version for colours)