Relative incentive rate in a multi-period and multi-task agency

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Abstract: This study explicitly calculates the relative incentive rate in an N-period contract with multiple tasks. The inter-temporal covariance risk, as well as the within-period risk premium, prevents the first best allocation of effort from being endogenously achieved even if the first best allocation is feasible. The inter-temporal covariance risk reduces the effective sensitivity of a performance measure, and thus the performance measure with a bigger inter-temporal covariance risk is assigned a weaker relative incentive rate. From these results, an empirical prediction is derived that a performance measure with larger positive (negative) inter-temporal covariances is assigned a weaker (stronger) relative incentive rate in multi-period contracts.

Keywords: relative incentive rate; performance measures; multi-period; multi-task; inter-temporal covariance.


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1 Introduction

The aggregation of performance measures and the design of a performance evaluation system, more often than not, take place in a multi-period setting with multiple tasks. This study examines what would be the relative incentive rate of performance measures in an institutionally richer setting of multi-period and multi-task.
While inter-temporally correlated performance measures, such as earnings, share prices, and items in balanced-scorecards, are widely used in business and not-for-profit organisations for performance evaluation purposes, no previous research has dealt with the relative incentive rate on performance measures in a multi-period setting. When performance evaluation takes place in a dynamic setting, an evaluatee faces a compensation risk which uniquely arises in the dynamic setting. When performance measures are inter-temporally correlated, an evaluatee is subject to a compensation risk (inter-temporal covariance risk) which results from the inter-temporal covariance among performance measures. The next example illustrates inter-temporal covariance risk from inter-temporally correlated performance measures.

Suppose that a manager is on a compensation contract that is based on the operating income of a branch in a small and remote town. The manager is on a five-year contract and each year the contract is renegotiable upon mutual agreement. The performance measure (the operating income) is affected by two factors: the manager’s own effort and the economic condition of the town that is beyond the manager’s control. Assume that the economic condition of the town (non-controllable factor) has positive inter-temporal correlations to have persistent effects on the yearly operating incomes. In this case, if the current year’s economic condition turns out to be good (bad) and have positive (negative) effect on the current year’s operating income, it is also expected that next year’s (and all remaining years’) economic condition is good (bad) to have positive (negative) effect on the next year’s operating income. As a result, the positive inter-temporal correlations amplify the compensation risk resulting from the non-controllable factor. In equilibrium, the manager expects this high compensation risk and seeks for a high compensation to match the risk. Now assume that the economic condition of the town (non-controllable factor) has negative inter-temporal correlations to have alternating effects on the yearly operating incomes. In this case, if the current year’s economic condition turns out to be good (bad) and have positive (negative) effect on the current year’s operating income, it is expected that next year’s economic condition is bad (good) to have negative (positive) effect on the next year’s operating income. As a result, the negative inter-temporal correlations have off-setting effects and reduce the compensation risk resulting from the non-controllable factor. In equilibrium, the manager expects this low compensation risk and accepts a low compensation to match the risk. Finally, in case that the economic condition of the town (non-controllable factor) is inter-temporally independent, the manager expects a mediocre compensation risk and accepts a mediocre compensation to match the risk, in equilibrium.

This study analyses the relative incentive rate on performance measures in a general N-period setting with two tasks. The endogenous allocation of effort across multiple tasks is examined through the relative incentive rate. This study shows that the inter-temporal covariance risk of performance measures is a factor determining the endogenous allocation of effort.

The modelling setting is endogenous throughout the analysis. In particular, the endogenous allocation of effort is preserved. This study employs a LEN model (linear contract, exponential utility of the agent, normal distribution of performance measure) and the aggregation of performance measures is restricted to linear aggregation.

This study contributes to the literature by first providing the explicit relative incentive rate in a multi-period and multi-task setting. In addition, this study first analyses the endogenous allocation of effort in a multi-period setting. Banker and Datar (1989) and Amershi et al. (1990) discuss the nature and characteristics of optimal aggregation of
performance measures. In particular, Banker and Datar (1989) show that the optimal relative incentive rate in a single-period setting with a single-task is determined only by the signal-to-noise ratio of performance measures.

Using a LEN model, Holmstrom and Milgrom (1991) introduce the allocation of effort across multiple tasks. The allocation of effort in their study depends on whether multiple tasks are complements or substitutes to each other in terms of the agent’s personal action cost. Using a LEN model in a single-period setting with multiple tasks, Datar et al. (2001) analyse the trade-off between the congruity of performance measures and the risk premium by demonstrating that an endogenously determined optimal allocation of effort may not be the first best allocation even if the first best allocation is feasible. Datar et al. (2001) show that as the endogenous allocation of effort comes into the principal’s problem, the optimal relative incentive rate is no longer equivalent to the relative signal-to-noise ratio of performance measures.

The results of Banker and Datar (1989) and Datar et al. (2001) are restricted to a single-period setting. This study not only encompasses the results of Banker and Datar (1989) and Datar et al. (2001), but also shows how the inter-temporal covariance risk of performance measures, in addition to the within-period risk premium, interacts with the congruity of performance measures in determining the optimal endogenous allocation of effort. In equilibrium, the inter-temporal covariance risk reduces the effective sensitivity of a performance measure, and thus the performance measure with a bigger inter-temporal covariance risk is assigned a weaker relative incentive rate. From these results, an empirical prediction is derived that a performance measure with larger positive (negative) inter-temporal covariances is assigned a weaker (stronger) relative incentive rate in multi-period contracts.

The rest of this study is organised as follows: Section 2 explains the modelling features. Section 3 analyses the optimal relative incentive rate. Section 4 concludes the study.

2 Modelling features

This study employs a LEN model in an \( N \)-period setting with two tasks. An \( N \)-period contract is characterised with renegotiation, not only because renegotiation is an important institutional feature but also because, stripping away wealth effects and income smoothing, a long-term contract with full commitment is equivalent to a single-period contract on multiple tasks.

This study mainly analyses the relative incentive rate for inducing the optimal effort level and thus, the analysis focuses on the equilibrium path.

Figure 1  Time line of events: \( I \) for the initial contract and \( R \) for renegotiation offers

![](image)

A risk neutral principal owns a production technology which requires efforts on two tasks \( \bar{a}_t = (a_{t1}, a_{t2}) \), where the first subscript indicates the task and the second subscript indicates the period, in each of \( N \) periods \( t = 1, 2, \ldots, N \), from a risk and effort averse
agent. The economic outcome (e.g., wealth of corporations or effectiveness of not-for-profit organisations) from the agency is not contractible and the principal and the agent write a contract, which is linear on a sequence of two contractible performance measures \((y_t, z_t), t=1, 2, \ldots, N\).

The principal receives economic benefit \(\hat{b}_t = (b_{y_t}, b_{z_t})\) from the agent’s effort \(\hat{a}_t = (a_{y_t}, a_{z_t})\). \(\hat{b}_t\) is equivalent to the first best effort level, which would be induced if the agent’s effort were observable and contractible such that a fixed wage contract (no incentive) would be offered in equilibrium. The principal’s total economic benefit is

\[
B_t(\hat{a}) = \sum_{i=1}^{N} (\hat{b}_t \cdot \hat{a}_t) = \sum_{i=1}^{N} (b_{y_t}a_{y_i} + b_{z_t}a_{z_i})
\]

and his utility is represented by

\[
U_t^p = \sum_{i=1}^{N} (b_t \cdot a_t) - C_t(\bar{y}, \bar{z}),
\]

where \(C_t(\bar{y}, \bar{z})\) is the compensation from the contract. The agent has a multiplicatively separable exponential utility

\[
U_t^a = \exp\left[-r \left\{G_t(\bar{y}, \bar{z}) - K_t(\hat{a})\right\}\right],
\]

where \(r\) is the agent’s absolute risk aversion, and \(K_t(\hat{a})\) is the agent’s personal action cost

\[
K_t(\hat{a}) = \sum_{i=1}^{N} \frac{1}{2} (a_{y_t}^2 + a_{z_t}^2).
\]

The agent has a single consumption date and no time value of money is assumed for simplicity. Thus, only the total amount of compensation matters to the agent. A similar result in a multiple consumption date setting could be obtained if the agent is allowed access to unlimited borrowing and lending opportunities with the same interest rate as the principal’s (Dutta and Reichelstein, 1999). When equal banking opportunities are available, consumption smooth-ing would be done by the agent and the multi-consumption model would be equivalent to one-point-consum ption model1 (only the present value of the total compensation matters). As the contract proceeds, performance measures \((y_t, z_t)\) are realised and the agent’s compensation accrues (savings account) such that future performance does not affect already accrued results.

In each of \(N\) periods \(t = 1, 2, \ldots, N\), the performance measures \(y_t\) and \(z_t\) are joint normally distributed with normally distributed residual terms:

\[
y_t = \bar{m}_t \cdot \hat{a}_t + \epsilon_t \quad (1)
\]

\[
z_t = \bar{k}_t \cdot \hat{a}_t + \delta_t, \quad t = 1, \ldots, N. \quad (2)
\]

Each performance measure \(y_t\) and \(z_t\) has sensitivity \(\bar{m}_t = (m_{y_t}, m_{z_t})\) and \(\bar{k}_t = (k_{y_t}, k_{z_t})\) to the agent’s effort \(\hat{a}_t = (a_{y_t}, a_{z_t})\). The performance measures \(y_t\) and \(z_t\) are also affected by uncontrollable random factors \(\epsilon_t\) and \(\delta_t\) which are zero-mean normally distributed. It is
assumed that the residual terms $\varepsilon_t$ and $\delta_t$ are correlated. Both time-series and cross-sectional correlations among performance measures are allowed. For simplicity, no long-term action is assumed\(^2\). The agent’s effort at affects only the current period performance measures ($y_t, z_t$) and not the future period performance measures.

To make sure that the agent participates in the contract, the principal should guarantee a reservation certainty equivalent, which is alternatively available to the agent in the labour market. Since the amount of a reservation certainty equivalent is independent of the analysis, a zero reservation certainty equivalent is assumed without loss of generality.

Since the performance measures $y_t$ and $z_t$ are normally distributed, the agent’s certainty equivalent is represented by the expected compensation minus the risk premium and action cost:

$$ACE_t = E_t - \frac{1}{2} tVar - k_t (\tilde{a}),$$

where the following notations for conditional expectation and variance are used:

$$E_t[y_t] = E[y_t | y_{t-1}, z_{t-1}]$$

$$Var_t[y_t] = Var[y_t | y_{t-1}, z_{t-1}].$$

A contract offer at $t - 1$ (the start of period $t$, see Figure 1) is denoted by a sequence of incentive rates:

$$C_t = \{a_{t-1}, (\beta^1_t, \beta^2_t), ... , (\beta^1_n, \beta^2_n)\},$$

where the fixed payment $a_{t-1}$ is a function of the history of realised performance measures $a_{t-1} = h(y_{t-1}, z_{t-1})$ ($a_0$ some constant), and the superscript to the incentive rate $\beta$ indicates the performance measure such that 1 is for $y$ and 2 is for $z$ while the subscript to the incentive rate $\beta$ indicates the period. The initial contract $C^I = \{a_0, (\beta^1_1, \beta^2_1), ... , (\beta^1_n, \beta^2_n)\}$ is the contract in effect unless replaced by a subsequent renegotiation offer. In the sequel, the superscript $I$ is used for the initial contract at the renegotiation time and $R$ is used for renegotiation offers.

At the start of the first period, $t = 0$, the principal offers to the agent an initial contract $C^I$. The agent either accepts or rejects it. Once the agent accepts the initial contract offer, the agent provides the period 1 effort $\tilde{a}_t = (a_{11}, a_{21})$. Before the end of period 1, the principal and the agent observe two contractible performance measures $y_1$ and $z_1$. At $t = 1$, the principal makes a take-it-or-leave-it renegotiation offer $C^R_2$. If the renegotiation offer is rejected, $C^2$ (the initial contract\(^1\)) is the contract in effect for period 2. If accepted, $C^R_2$ becomes the contract in effect. The agent provides the period 2 effort $\tilde{a}_2 = (a_{12}, a_{22})$ and the principal and the agent observe two contractible performance measures $y_2$ and $z_2$ before the end of period 2. At the start of each period $t = 3, ..., N$, the same renegotiation procedure occurs with a renegotiation offer $C^R_t$. At the terminal date, $t = N$, the agent receives the compensation based on the realised values of performance measures and the contract is resolved.

Due to the renegotiation-proofness principle for the LEN model (Sabac, 2007), the analysis of linear optimal contract can, without loss of generality, be restricted to a linear renegotiation-proof contract:
where $\alpha_0$ is the initial fixed payment term. At the decision point of the period $t$ incentive $\beta_1$ and $\beta_2$, the incentive rates in the remaining contract periods $\beta_{i+1}$ and $\beta_{i+1}$, $i = 1, \ldots, N - t$ are restricted to be ex-post efficient and rationally expected by the principal and the agent due to the renegotiation-proofness requirement.

The contracting and renegotiation can be summarised by the principal’s problem at the start of period $t$ as follows:

$$\max_{\beta_1, \beta_2} E_t [B_t (\bar{a}) - C^{R_t}], \quad (E_t [B_t (\bar{a}) - C^{H_t}] \text{ for the first period})$$

subject to the renegotiation-proofness constraint

$$\{ (\beta_1, \beta_{1+1}), \ldots, (\beta_N, \beta_{N}) \} \text{ are optimal at the start of periods } t+1, \ldots, N,$$

the incentive compatibility constraint

$$\bar{a}_1, \ldots, \bar{a}_N \in \arg \max_{R_t} ACE_{t-1} (C^{R_t}),$$

and the participation constraint

$$ACE_{t-1} (C^{R_t}) \geq ACE_{t-1} (C^{H_t}). \quad (ACE (C^{H_t}) \geq 0 \text{ for the first period})$$

The LEN approach was developed by Holmstrom and Milgrom (1987) and Hemmer (2004, p.150, 3rd paragraph) states that the study identifies the main assumptions and a structure that can be safely analysed following the LEN approach. Hemmer (2004, p.152) emphasises the two key assumptions in Holmstrom and Milgrom (1987, p.315–316), which are necessary for optimality of the LEN model:

1. The agent acts repeatedly in a dynamic setting, not in ‘one-shot’ game. In particular, the agent should learn the result in each period and should be free to change behaviour in response to past results.

2. The utility of the parties (principal and agent) are exponential, and the technology should be stationary and history-independent such that the principal faces identical problems in each period and actions taken by the agent in each period are indeed identical in equilibrium.

The modelling features in this study well satisfy the first assumption. As renegotiation is offered at the end of each period, the agent can freely change his behaviour after observing the results of his past actions. Renegotiation-proof contract, as opposed to full commitment contract, keeps the LEN model to be optimal in a dynamic setting. On the other hand, for this study to satisfy the second assumption, a specification on time-series feature [e.g., autoregressive of order 1 (AR(1)) process] of performance measures should be made. While the exponential utility of the parties (the principal is risk-neutral as a special case) prevents wealth effects such that the preferences of the principal and agent are constant in each period, the technology regarding performance measures is not stationary unless a specification is made. Appendix shows that when AR(1) process is assumed on the inter-temporally correlated performance measures, the technology is
(second-order or weakly) stationary in that the means and variances of performance measures are constant regardless of periods and the autocovariances depend only on the time-lag among performance measures. Therefore, this study satisfies the two key assumptions in Holmstrom and Milgrom (1987) for optimality of the LEN model.

3 Optimal relative incentive rate

This section provides the optimal relative incentive rate in a multi-period and multi-task setting. The results in this section not only encompass what have been found in Banker and Datar (1989) and Datar et al. (2001), but also explicitly show the role of inter-temporal covariance risk in determining the optimal relative incentive rate. The endogenous allocation of effort is examined through the optimal relative incentive rate on performance measures.

In a single-period setting, Datar et al. (2001) show that there is a trade-off between maximising the congruity of performance measures and minimising the risk premium. The optimal relative incentive rate is affected by the within-period risk premium as well as the congruity of performance measures. In the optimal relative incentive rate, a performance measure with a bigger variance is assigned a less weight because the performance measure causes a bigger within-period risk premium. Thus, as long as a risk premium exists, the first best allocation of effort is not endogenously achieved even if it is feasible.

In a multi-period setting, this section shows that it is not only the within-period risk premium but also the inter-temporal covariance risk of performance measures that trades-off with the congruity of performance measures. The optimal relative incentive rate is affected by the inter-temporal covariance risk of performance measures as well as the within-period risk premium and the congruity of performance measures. In the optimal relative incentive rate, a performance measure with a bigger inter-temporal covariance risk is assigned a less weight because the performance measure causes a bigger risk premium. As a result, the first best allocation of effort is not endogenously achieved, even if it is feasible, in a multi-period setting in which inter-temporal correlations and inter-temporal covariance risk of performance measures exist.

3.1 Optimal relative incentive rate

In maximising the expected utility (8), the principal wants to minimise the expected compensation, which leads to minimising the sum of the risk premium and action cost as the agent certainty equivalent (3) is constant by the binding participation constraint. When a residual term of a performance measure, say $e_{it}$, is correlated with its future residual term $e_{i,t+i}$ or the other performance measure’s future residual term $\delta_{i,t+i}$, the future incentive rates come into the principal’s problem through the inter-temporal covariance risk terms:

$$
\sum_{i=1}^{N} \beta^{t}_{i} \beta^{t-1}_{i} \text{Cov}_{y_{i}, y_{i,t+i}}(y_{i}, \tilde{z}_{i,t+i}) + \sum_{i=1}^{N} \beta^{t}_{i} \beta^{t-1}_{i} \text{Cov}_{y_{i}, y_{i,t+i}}(y_{i}, \tilde{z}_{i,t+i})
$$

and
In a multi-period setting, the principal develops an incentive to minimise the inter-temporal covariance risk regarding the incentive rates in the yet-to-come periods. In analysing this problem, the inter-temporal covariance risk factors (ICR) are defined as follows.

**Definition 1:** N-period: inter-temporal covariance risk factors

$ICR^1$ and $ICR^2$ characterise the inter-temporal covariance risk of the performance measures $y_t$ and $z_t$, respectively:

\[ ICR^1 = rCov_{1:t} \left( y_t, \sum_{i=1}^{N-1} (\beta^1_i y_{t+i} + \beta^2_i z_{t+i}) \right), \]

\[ ICR^2 = rCov_{1:t} \left( z_t, \sum_{i=1}^{N-1} (\beta^1_i z_{t+i} + \beta^2_i y_{t+i}) \right). \]  

Using the inter-temporal covariance risk factors $ICR^1$ [equation (14)] and $ICR^2$ [equation (15)], the following proposition presents the optimal incentive rates on the two performance measures $y_t$ and $z_t$ in an N-period contract with the agent’s effort on two tasks.

**Proposition 1:** Optimal incentive rates

The period $t$ optimal incentive rates $\beta^1_t$ and $\beta^2_t$ are as follows:

\[ \beta^1_t = \frac{\left[ \bar{k}_t \cdot \bar{m}_t - ICR^1 \right] \left[ \bar{k}_t \cdot \bar{m}_t + rVar_{t-1}(z_t) \right] - \left[ \bar{k}_t \cdot \bar{m}_t - ICR^2 \right] \left[ \bar{m_t} \cdot \bar{k}_t + rCov_{t-1}(y_t, z_t) \right]}{D_t}, \]

\[ \beta^2_t = \frac{\left[ \bar{k}_t \cdot \bar{m}_t - ICR^2 \right] \left[ \bar{m}_t \cdot \bar{k}_t + rVar_{t-1}(y_t) \right] - \left[ \bar{k}_t \cdot \bar{m}_t - ICR^1 \right] \left[ \bar{m_t} \cdot \bar{k}_t + rCov_{t-1}(y_t, z_t) \right]}{D_t}, \]

where

\[ D_t = (m_t k_{zt} - m_{zt} k_t)^2 + (\bar{m}_t \cdot \bar{m}_t) rVar_{t-1}(z_t) + (\bar{k}_t \cdot \bar{k}_t) rVar_{t-1}(y_t) \]

\[ - 2(\bar{m}_t \cdot \bar{k}_t) rCov_{t-1}(y_t, z_t) \]

\[ + r^2 \left[ Var_{t-1}(y_t) Var_{t-1}(z_t) - Cov_{t-1}(y_t, z_t)^2 \right] > 0. \]

The optimal incentive rates for the last period are obtained by substituting $ICR^1_t = ICR^2_t = 0$.

From Proposition 1, it follows that the optimal relative incentive rate on the two performance measures $y_t$ and $z_t$ is:

\[ \frac{\beta^1_t}{\beta^2_t} = \frac{\left[ \bar{h}_t \cdot \bar{m}_t - ICR^1 \right] \left[ \bar{h}_t \cdot \bar{m}_t + rVar_{t-1}(y_t) \right] - \left[ \bar{h}_t \cdot \bar{m}_t - ICR^2 \right] \left[ \bar{m}_t \cdot \bar{h}_t + rCov_{t-1}(z_t) \right]}{\left[ \bar{h}_t \cdot \bar{k}_t - ICR^2 \right] \left[ \bar{h}_t \cdot \bar{k}_t + rVar_{t-1}(z_t) \right] - \left[ \bar{h}_t \cdot \bar{k}_t - ICR^1 \right] \left[ \bar{k}_t \cdot \bar{h}_t + rCov_{t-1}(y_t) \right]}, \]
Relative incentive rate in a multi-period and multi-task agency

where

\[ \phi_t = \frac{\langle \tilde{m}_t, \tilde{k}_t + r \text{Cov}(y_t, z_t) \rangle}{\langle \tilde{k}_t + r \text{Var}(z_t) \rangle} \]

and

\[ \phi_t = \frac{\langle \tilde{m}_t, \tilde{k}_t + r \text{Cov}(y_t, z_t) \rangle}{\langle \tilde{m}_t + r \text{Var}(y_t) \rangle}. \]

With a single-task in each period \( \tilde{m}_t = m_t, \tilde{k}_t = k_t, \tilde{h}_t = h_t \), and independent periods \( ICR_1 = ICR_2 = 0 \) (effectively a single-period setting), the optimal relative incentive rate (19) in each period is:

\[
\frac{\beta_1}{\beta_2} = \frac{(m_t - \phi_t k_t)/\text{Var}_t(y_t)}{(k_t - \phi_t m_t)/\text{Var}_t(z_t)}.
\]

where \( \phi_1(t) = \text{Cov}(y_t, z_t)/\text{Var}_t(z_t) \) and \( \phi_2(t) = \text{Cov}(y_t, z_t)/\text{Var}_t(y_t) \). The relative incentive rate (20) is equivalent to the relative signal-to-noise ratio [Banker and Datar, (1989), p.30] with the posterior beliefs. Thus, a multi-period agency problem with independent periods \( ICR_1 = ICR_2 = 0 \) and a single-task is equivalent to a repeated single-period agency problem with a single-task, and the optimal relative incentive rate in such a setting is explained by the relative signal-to-noise ratio (20) in each period.

In addition to the relative signal-to-noise ratio, the optimal relative incentive rate (19) reflects the risk externality from inter-temporally correlated performance measures and the endogenous allocation of effort across multiple tasks. The following sections discuss the risk externality and the endogenous allocation of effort.

3.2 Risk externality

The last period optimal relative incentive rate \( \beta_N \) does not contain the inter-temporal covariance risk factors \( ICR_1 \) and \( ICR_2 \), because there is no further contract period after \( N \) and thus no inter-temporal covariance risk regarding the last period incentive rates \( \beta_N \) and \( \beta_2^2 \):

\[
\frac{\beta_1^2}{\beta_2^2} = \frac{\langle \tilde{m}_N, \tilde{k}_N \rangle - \phi_N \langle \tilde{m}_N, \tilde{k}_N \rangle}{\langle \tilde{k}_N, \tilde{m}_N \rangle - \phi_N \langle \tilde{k}_N, \tilde{m}_N \rangle},
\]

where

\[ \phi_N = \frac{\langle \tilde{m}_N, \tilde{k}_N + r \text{Cov}(y_N, z_N) \rangle}{\langle \tilde{k}_N + r \text{Var}(z_N) \rangle}. \]

and

\[ \phi_N = \frac{\langle \tilde{m}_N, \tilde{k}_N + r \text{Cov}(y_N, z_N) \rangle}{\langle \tilde{m}_N + r \text{Var}(y_N) \rangle}. \]

The principal’s problem at \( t = N - 1 \) is a single-period problem with myopic incentives in the sense that the last period incentive rates \( \beta_N \) and \( \beta_2^2 \) are designed only to induce the optimal effort \( \tilde{a}_N \) in the last period.
On the other hand, all the incentive rates before the start of the last period consist of two components: the myopic incentive and the risk externality adjustment. In the optimal incentive rates (16) and (17), the myopic incentive is the component which is based on the principal’s benefit from the current period effort $\bar{a}_t$ and the sensitivity of the current period measures $\bar{m}_t$ and $\bar{k}_t$, and the posterior variances of the current period measures $\text{Var}_{t-1}(y_t), \text{Var}_{t-1}(z_t), \text{Cov}_{t-1}(y_t, z_t)$. That is, the myopic incentive is the component analogous to the last period optimal incentive rate, respectively:

$$
\left[ (\bar{b}_t \cdot \bar{m}_t) \{ \bar{k}_t \cdot \bar{k}_t + r \text{Var}_{t-1}(z_t) \} - (\bar{b}_t \cdot \bar{k}_t) \{ \bar{m}_t \cdot \bar{k}_t + r \text{Cov}_{t-1}(y_t, z_t) \} \right] / D_t,
$$

$$
\left[ (\bar{b}_t \cdot \bar{k}_t) \{ \bar{m}_t \cdot \bar{m}_t + r \text{Var}_{t-1}(y_t) \} - (\bar{b}_t \cdot \bar{m}_t) \{ \bar{m}_t \cdot \bar{k}_t + r \text{Cov}_{t-1}(y_t, z_t) \} \right] / D_t.
$$

In the optimal incentive rates (16) and (17), the risk externality adjustment is the component which is characterised by the inter-temporal covariance risk factors $ICR^1_t$ and $ICR^2_t$. The risk externality adjustment in the optimal incentive rates (16) and (17) is given as follows, respectively:

$$
\left[ -ICR^1_t \{ \bar{k}_t \cdot \bar{k}_t + r \text{Var}_{t-1}(z_t) \} + ICR^2_t \{ \bar{m}_t \cdot \bar{k}_t + r \text{Cov}_{t-1}(y_t, z_t) \} \right] / D_t,
$$

$$
\left[ -ICR^2_t \{ \bar{m}_t \cdot \bar{m}_t + r \text{Var}_{t-1}(y_t) \} + ICR^1_t \{ \bar{m}_t \cdot \bar{k}_t + r \text{Cov}_{t-1}(y_t, z_t) \} \right] / D_t.
$$

The risk externality (Sabac, 2008) results from the principal’s lack of commitment to future incentive rates. The principal cannot ‘cooperate with himself’ in determining incentive rates at different points in time. The risk externality arises when positive (negative) covariance between current and future performance measures imposes too much (too little) compensation risk to the agent so that the principal lowers (raises) current incentive rates in order to reduce (increase) the current period induced effort.

In studies with a single-period setting such as Banker and Datar (1989) and Datar et al. (2001), the optimal incentive rate consists only of the myopic incentive, because there is no inter-temporal consideration. As the analysis is extended to a multi-period setting, the risk externality adjustment becomes a factor of the optimal incentive rate, because the principal takes into account the impact of incentive rates on the inter-temporal covariance risk as well as effort inducement. Thus, when performance measures are inter-temporally correlated, the optimal relative incentive rate is also affected by the inter-temporal covariance of performance measures.

The risk externality adjustments (24) and (25) can be explained by two special cases: the pure-insurance and the window-dressing (Christensen et al., 2005). The pure-insurance is a special case when the principal induces null current period effort $\bar{a}_t = 0$ because the current period performance measures $y_t$ and $z_t$ (but not the performance measures for future periods $t + 1, \ldots, N$) have no sensitivity to the agent’s effort $\bar{m}_t = \bar{k}_t = 0$. The incentive rates $\beta^1$ and $\beta^2$ are used purely to minimise the risk premium. If $\bar{m}_t = \bar{k}_t = 0$ is substituted in the optimal incentive rates (16) and (17), then the pure-insurance adjustment is obtained:

$$
\left[ -ICR^1 r \text{Var}_{t-1}(z_t) + ICR^2 r \text{Cov}_{t-1}(y_t, z_t) \right] / D_t,
$$

$$
\left[ -ICR^2 r \text{Var}_{t-1}(y_t) + ICR^1 r \text{Cov}_{t-1}(y_t, z_t) \right] / D_t.
$$
Relative incentive rate in a multi-period and multi-task agency

\[
\left[ -ICR^2 \text{rVar}_{i-1}(y_i) + ICR \text{rCov}_{i-1}(y_i, z_i) \right] / D_i, 
\]  
(27)

which are respectively included in the risk externality adjustments (24) and (25).

The window-dressing is a special case when the agent’s effort generates no economic benefit \( h_t = 0 \) (only in the current period and not in the future periods \( t + 1, \ldots, N \)) but the current period performance measures \( y_i \) and \( z_i \) have non-zero sensitivity to effort \( \bar{m}_i \neq 0 \) or \( \bar{k}_i \neq 0 \). In this case, non-zero window-dressing effort is induced \( \bar{a}_i \neq 0 \) and the principal should compensate for it. If \( h_t = 0 \) is substituted in the optimal incentive rates (16) and (17), the risk externality adjustments (24) and (25) are obtained. Therefore, the risk externality adjustment is equivalent to the optimal incentive rate in the case that no productive effort is induced from the agent, but the principal cannot avoid paying for the agent’s window-dressing effort in equilibrium.

The principal respectively includes the risk externality adjustments (24) and (25) in the optimal incentive rates (16) and (17) because the myopic incentives (22) and (23) is too strong or too weak in the presence of covariance among the current and future performance measures. Suppose \( y_i \) has positive covariances with future performance measures: \( \text{Cov}_{i-1}(y_i, y_{i+n}) > 0 \), \( \text{Cov}_{i-1}(y_i, z_{i+n}) > 0 \) and \( ICR^i > 0 \) in equation (14). Then setting the incentive rate \( \beta^i \) as the myopic incentive (22) is too expensive because a stronger incentive rate \( \beta^i \) results in a bigger inter-temporal covariance risk in equation (12). Therefore, the principal reduces \( \beta^i \) by \( -ICR^i |\bar{k}_i \cdot \bar{k}_i + r \text{Var}_{i-1}(z_i)| / D_i \) in equation (24) because \( y_i \) is relatively expensive in inducing effort \( \bar{a}_i \) due to the positive inter-temporal covariance risk. At the same time, the principal raises \( \beta^i \) by \( ICR^i |\bar{m}_i \cdot \bar{k}_i + r \text{Cov}_{i-1}(y_i, z_i)| / D_i \) in equation (25) because \( z_i \) is relatively inexpensive in inducing effort \( \bar{a}_i \).

Now suppose that \( y_i \) has negative covariances with future performance measures: \( \text{Cov}_{i-1}(y_i, y_{i+n}) < 0 \), \( \text{Cov}_{i-1}(y_i, z_{i+n}) < 0 \) and \( ICR^i < 0 \) in equation (14). Then the myopic incentive (22) is too weak for \( \beta^i \) because a stronger incentive rate \( \beta^i \) results in a smaller inter-temporal covariance risk in equation (12). Thus, the principal raises \( \beta^i \) by \( -ICR^i |\bar{k}_i \cdot \bar{k}_i + r \text{Var}_{i-1}(z_i)| / D_i \) in equation (24) because \( y_i \) is relatively inexpensive in inducing effort \( \bar{a}_i \) due to the negative inter-temporal covariance risk. The principal reduces \( \beta^i \) by \( ICR^i |\bar{m}_i \cdot \bar{k}_i + r \text{Cov}_{i-1}(y_i, z_i)| / D_i \) in equation (25) because \( z_i \) is relatively expensive in inducing effort \( \bar{a}_i \). The impact of the risk externality from the inter-temporal covariances \( \text{Cov}_{i-1}(z_i, z_{i+n}) \) and \( \text{Cov}_{i-1}(z_i, y_{i+n}) \) on the optimal incentive rates is symmetric.

3.3 Endogenous effort allocation

In a single-period setting with multiple tasks, Datar et al. (2001, p.82) show that the principal’s utility maximisation problem is equivalent to minimising the sum of the incongruity of performance measures and the risk premium (also see the proof of Proposition 1 of this study). Congruity refers to the degree of congruence between the impacts of the agent’s effort on his performance measure and on the principal’s expected
gross payoff (Feltham and Xie, 1994). For example, residual income is generally more congruent with the firm’s objectives than divisional profit because residual income considers the cost of capital tied up in divisional assets. As a benchmark in discussing the endogenous allocation of effort, below presented is a case with a risk neutral agent, in which both the within-period risk premium and the inter-temporal covariance risk are null.

If the agent is risk neutral \( r = 0 \), then the principal’s problem is reduced to minimising the incongruity of performance measures, which is geometrically explained by the squared distance from the first best effort and the induced effort \( \| \tilde{h} - \tilde{a} \| \). That is, the principal uses the incentive rates in inducing a second best effort \( \tilde{a} = \beta \tilde{m} + \beta \tilde{k} \) as close as possible to the first best effort \( \tilde{b} \) without concerns for the risk premium. When the agent is risk neutral \( r = 0 \), the optimal relative incentive rate (19) is equivalent to the following, which actually achieves perfect congruity:

\[
\frac{(\tilde{b} - \tilde{m})(\tilde{k} - \tilde{k}) - (\tilde{b} - \tilde{m})(\tilde{m} - \tilde{k})}{(\tilde{b} - \tilde{m})(\tilde{m} - \tilde{k})} = \frac{(b_{1}k_{21} - b_{2}k_{12})(m_{1}k_{21} - m_{2}k_{12})}{(b_{1}m_{2} - b_{2}m_{1})(m_{1}k_{21} - m_{2}k_{12})}.
\]

The endogenous allocation of effort among multiple tasks, which is reflected in the optimal relative incentive rate (19) through the numerator and denominator of equation (28), geometrically minimises the angle between the induced effort \( \tilde{a} \) and the first best effort \( \tilde{b} \) in the case of no risk premium. With a fixed first best effort \( \tilde{b} \), the squared distance \( \| \tilde{b} - \tilde{a} \| \) is determined by the length (intensity) of the induced effort \( \| \tilde{a} \| \) and the angle (allocation) between the induced effort \( \tilde{a} \) and the first best effort \( \tilde{b} \). Since the length (intensity) of the induced effort is costless due to the agent’s risk neutrality \( r = 0 \), the principal’s only problem is to decide the angle (allocation) between the induced effort \( \tilde{a} \) and the first best effort \( \tilde{b} \) in minimising the squared distance \( \| \tilde{b} - \tilde{a} \| \). The optimal (minimal) angle is solved by the numerator and denominator of equation (28).

When the two performance measures \( y \) and \( z \) are perfectly aligned \( (m_{1}k_{21} - m_{2}k_{12} = 0) \), effectively a single-performance-measure setting) or in a single-task setting \( \tilde{m} = m \), \( \tilde{k} = k \) and \( \tilde{b} = b \), both the numerator and denominator of equation (28) vanish in the optimal relative incentive rate (19). In a single-performance-measure setting such as Feltham and Xie (1994, p.435, Proposition 1), the induced effort in equilibrium is constrained to a one-dimensional linear subspace and the principal cannot affect the allocation of effort. In a single-task setting such as Banker and Datar (1989), there is no effort allocation problem and thus the allocation of effort is not relevant to the principal. Therefore, either in a single-performance measure setting or in a single-task setting, the allocation of effort is not a relevant problem to the principal. In accordance, the optimal relative incentive rate (19) does not contain the numerator and denominator of equation (28) in a single-performance-measure setting or a single-task setting.

In a general setting with a risk averse agent, the endogenous allocation of effort can be discussed in terms of the geometric analysis in Demski et al. (2008). Their study presents the induced effort \( \tilde{a} \) as a projection of the first best effort \( \tilde{b} \) on the
implementable action space $M(\hat{a}) = \text{Span}(\hat{m}, \hat{k})$, which is the span of the performance sensitivities to effort. The presence of a risk premium in the agency problem makes the projection non-orthogonal, and the non-orthogonal projection $\hat{a} = \text{Proj}_{M(\hat{a})}\hat{b}$ is viewed as first being orthogonally projected on the implementable action space $\text{Proj}_{M(\hat{a})}\hat{b}$ and then adjusted in the implementable action space. The orthogonal projection of the first best effort on the implementable action space $\text{Proj}_{M(\hat{a})}\hat{b}$ represents what would be induced without concerns for the risk premium $r = 0$.

It has been shown by equation (28) that if there is no concern for the risk premium $r = 0$, the two-task and two-performance-measure setting makes it possible to achieve perfect congruity. Thus, the first best effort resides in the implementable action space $\hat{b} \in M(\hat{a})$ in this study. If the number of tasks exceeds the number of performance measures, perfect congruity is not achievable even without concerns for the risk premium $r = 0$ because the first best effort does not reside in the implementable action space $\hat{b} \not\in M(\hat{a})$.

In general, the presence of a risk premium in the agency problem prevents the first best effort from being induced even if the first best effort resides in the implementable action space $\hat{b} \in M(\hat{a})$. In a single-period setting, Datar et al. (2001) show that the principal may not achieve perfect congruity in order to reduce the within-period risk premium even if perfect congruity is feasible. To apply the single-period setting with multiple tasks of Datar et al. (2001), assume the current period measure $z_t$ (but not $y_t$ and not the performance measures for future periods $t + 1, \ldots, N$) has no variance $\text{Var}(z_t) = 0$. Then, it follows that both the cross-sectional covariance and the inter-temporal covariance risk factor of $z_t$ vanish $21$.

In this case, the optimal relative incentive rate (19) is equivalent to the following:

$$
\left( \frac{\hat{b} \cdot \hat{m}_t}{\hat{b} \cdot \hat{k}_t} \cdot \left( \frac{\hat{m}_t \cdot \hat{k}_t}{\hat{m}_t \cdot \hat{m}_t + r\text{Var}_{t-1}(y_t)} \right) - \left( \frac{\hat{m}_t \cdot \hat{k}_t}{\hat{m}_t \cdot \hat{m}_t} \right) \right).
$$

(29)

The relative incentive rate (29) reflects an endogenous allocation of effort which is not the first best. In accordance with the result of Datar et al. (2001), the within-period risk premium $r\text{Var}_{t-1}(y_t)$ in the denominator causes the relative incentive rate (29) to deviate from the case of perfect congruity in equation (28).

Now, while keeping the assumptions in the previous case $\text{Var}(z_t) = \text{Cov}_{t-1}(y_t, z_t) = ICR_t^2 = 0$, assume the existence of an inter-temporal covariance risk from $y_t$, $ICR_t^1 \neq 0$, for analysing the allocation of effort in a multi-period setting. Note that the within-period risk premium from $y_t$, $r\text{Var}_{t-1}(y_t) \neq 0$, is minimal in the sense that without the variance of $y_t$, $\text{Var}_{t-1}(y_t) = 0$, the inter-temporal covariance risk from $y_t$ also vanishes, $ICR_t^1 = 0$, and there is no risk premium in the principal’s problem. With the inter-temporal covariance risk of $y_t$, which is represented by $ICR_t^1$, the optimal relative incentive rate (19) is now equivalent to the following:
The relative incentive rate (30) reflects the endogenous allocation of effort when there exist the inter-temporal covariance risk as well as the within-period risk premium from $y_t$.

The inter-temporal covariance risk factor $ICR_t$ in the numerator and the denominator, as well as the within-period risk premium $rVar_{t-1}(y_t)$ in the denominator, causes the relative incentive rate (30) to deviate from the case of perfect congruity (28).

The inter-temporal covariance risk can be regarded as an ‘expense’ for using a performance measure in inducing effort from the risk averse agent in a multi-period setting. While there is only one ‘expense’ (the within-period risk premium) in a single-period setting, a multi-period setting incurs an additional ‘expense’ (the inter-temporal covariance risk) for using performance measures to maximise the congruity. In a multi-period setting, the existence of the ‘expenses’ (the inter-temporal covariance risk and the within-period risk premium) adjusts the maximisation of congruity and results in the endogenous allocation of effort in equation (30).

As a result, in the presence of the inter-temporal covariance risk, the endogenous allocation of effort is not the first best even if it is feasible. As long as the inter-temporal covariance risk of performance measures exists in a multi-period setting, the endogenously determined optimal allocation of effort is not the first best allocation. In a multi-period setting, the congruity of performance measures trades-off with the inter-temporal covariance risk as well as the within-period risk premium.

### 3.4 Effective signal-to-noise ratio

When the analysis is extended from a single-period and single-task setting to a single-period setting with multiple tasks, the endogenous allocation of effort becomes relevant to the principal. In accordance, the numerator and the denominator of equation (28) come into the optimal incentive rates, and the resulting optimal relative incentive rate is equivalent to equation (21). When the optimal relative incentive rate (21) in a single-period setting with multiple tasks is compared with the relative signal-to-noise ratio (20) in a single-period and single-task setting, it is observed that the terms $(\hat{h}_t \cdot \hat{m}_t)$ and $(\hat{h}_t \cdot \hat{k}_t)$ play the role of ‘effective sensitivity’ and the terms $\{\hat{m}_t \cdot \hat{m}_t + rVar_{t-1}(y_t)\}$ and $\{k_t \cdot k_t + rVar_{t-1}(z_t)\}$ play the role of ‘effective noise’ of the performance measures $y_t$ and $z_t$, respectively.

When the analysis is extended further to a multi-period setting with multiple tasks, the inter-temporal covariance risk of performance measures becomes relevant to the principal. In accordance, the inter-temporal covariance risk factors $ICR_t^1$ [equation (14)] and $ICR_t^2$ [equation (15)] come into the optimal incentive rates, and the resulting relative incentive rate is equation (19). From the optimal relative incentive rate (19) in a multi-period setting with multiple tasks, it is observed that the terms $(\hat{h}_t \cdot \hat{m}_t - ICR_t^1)$ and $(\hat{h}_t \cdot \hat{k}_t - ICR_t^2)$ play the role of ‘effective sensitivity’ and the terms $\{\hat{m}_t \cdot \hat{m}_t + rVar_{t-1}(y_t)\}$ and $\{k_t \cdot k_t + rVar_{t-1}(z_t)\}$ play the role of ‘effective noise’ of the performance measures $y_t$ and $z_t$, respectively.
The effective noise is obviously positive. How about the effective sensitivity? The effective sensitivity is also positive as long as the two performance measures \(y_t\) and \(z_t\) have a non-negative cross-sectional correlation such that \(y_t\) and \(z_t\) are competing alternatives to the principal in inducing the agent’s effort. In particular, from the optimal incentive rates (16) and (17), it can be shown that the following identities hold on the equilibrium path:

\[
\begin{align*}
\left[ \tilde{h}_t \cdot \tilde{m}_t - ICR^1_t \right] &= \beta_1 \left\{ \tilde{m}_t \cdot \tilde{m}_t + r \text{Var}_{y,-1} (y_t) \right\} + \beta_2 \left\{ \tilde{m}_t \cdot \tilde{k}_t + r \text{Cov}_{y,-1} (y_t, z_t) \right\}, \\
\left[ \tilde{h}_t \cdot \tilde{k}_t - ICR^2_t \right] &= \beta_1 \left\{ \tilde{k}_t \cdot \tilde{k}_t + r \text{Var}_{z,-1} (z_t) \right\} + \beta_2 \left\{ \tilde{m}_t \cdot \tilde{k}_t + r \text{Cov}_{y,-1} (y_t, z_t) \right\},
\end{align*}
\]

(31) \hspace{1cm} (32)

where no inter-temporal covariance risk exists in the last period, \(ICR^1_{t} = ICR^2_{t} = 0\).

If the two performance measures \(y_t\) and \(z_t\) have a non-negative cross-sectional correlation, it is clear from equation (19) that the optimal relative incentive rate \(\beta_1 / \beta_2\) on the two performance measures \(y_t\) and \(z_t\) is strictly decreasing in the inter-temporal covariance risk factor of \(y_t\) (\(ICR^1\)) and strictly increasing in the inter-temporal covariance risk factor of \(z_t\) (\(ICR^2\)). Therefore, a performance measure with a bigger inter-temporal covariance risk has a smaller effective sensitivity and thus is assigned a weaker relative incentive rate.

Now, with the positive effective sensitivities \([\tilde{h}_t \cdot \tilde{m}_t - ICR^1]\) and \([\tilde{h}_t \cdot \tilde{k}_t - ICR^2]\), relative incentive rates discussed in the previous section [equations (28), (29) and (30)] decrease monotonically: (28) > (29) > (30). The existence of the within-period risk premium from \(y_t\) causes the relative incentive rate (29) to be less than equation (28). The existence of the inter-temporal covariance risk factor from \(y_t\) causes the relative incentive rate (30) to be further reduced from equation (29).

If the two performance measures \(y_t\) and \(z_t\) have a non-negative cross-sectional correlation, the inter-temporal covariance risk of a performance measure has a monotonic impact on the endogenous allocation of effort through the relative incentive rate. Given the binding incentive compatibility constraints [equations (58) and (59)], the induced allocation of effort \(a_{y1}/a_{z2}\) is monotonically affected by the relative incentive rate \(\beta_1 / \beta_2\) with the direction depending on the sensitivities of performance measures \(y_t\) and \(z_t\). It can be shown that the sign of the derivative:

\[
\frac{d \left( \frac{a_{y1}}{a_{z2}} \right)}{d \left( \frac{\beta_1}{\beta_2} \right)}
\]

is decided by the sign of the term:

\[
(m_{y1}k_{z2} - m_{z2}k_{y1}).
\]

(33) \hspace{1cm} (34)

As the relative incentive rate is monotonically affected by the inter-temporal covariance risk of a performance measure, the resulting impact on the induced allocation of effort is also monotonic, with the direction depending on the sensitivities of performance measures. Therefore, in a multi-period setting, the inter-temporal covariance risk of a performance measure has a monotonic impact on the endogenous allocation of effort through the relative incentive rate.
From the results in this section, a hypothesis for empirical tests is derived in alternative form. With Definition 1, it is straightforward to observe that a performance measure with larger positive (negative) inter-temporal covariances brings about a bigger (smaller) inter-temporal covariance risk and thus will be assigned a weaker (stronger) relative incentive rate:

H1 Other things being equal, a performance measure with larger positive (negative) inter-temporal covariances is assigned a weaker (stronger) relative incentive rate in multi-period contracts.

Because a performance measure with a less inter-temporal covariance risk implies a less ‘expense’ to the principal in inducing productive effort from the risk averse agent, it is rational that the principal prefers a performance measure with a less inter-temporal covariance risk to one with a bigger inter-temporal covariance risk.

4 Conclusions

In a multi-period setting, the inter-temporal covariance risk of performance measures becomes a part of an agency problem and thus relevant to the principal. In particular, the inter-temporal covariance risk weakens the effective sensitivity of a performance measure, and the performance measure with a bigger inter-temporal covariance risk is assigned a weaker relative incentive rate. In accordance with the results, this study proposes a hypothesis for empirical tests that a performance measure with larger positive (negative) inter-temporal covariances is assigned a weaker (stronger) relative incentive rate in multi-period contracts.

This study shows that in determining the optimal endogenous allocation of effort, the congruity of performance measures trades-off not only with the within-period risk premium but also with the inter-temporal covariance risk of performance measures. As a result, in a multi-period setting, the endogenous allocation of effort is not the first best allocation even if it is feasible.

References


Notes

1 What happens if the assumption of equal banking opportunities is not met? First, if there is no banking opportunity available to the agent (very unlikely assumption), then the agent’s effort choice needs ‘memory’ to achieve consumption smoothing by adjusting productive efforts in each period. Second, if access to banking is allowed but the interest rate for the agent is not the same as the interest rate for the principal, then a renegotiation-proof contract cannot be used as the optimal contract. Fudenberg et al. (1990) show that equal banking opportunity is a condition for a renegotiation-proof contract to be optimal in a long-term agency which is free from adverse selection. The result implies that when the assumption of equal banking opportunity is not met, a long-term agency may require full-commitment in the optimal contract to avoid adverse selection such that a renegotiation-proof contract is no more optimal.

2 Christensen and Feltham (2005, p.475) show that performance measures that are inter-temporally correlated (stochastically dependent) without long-term action (technologically independent) can be rewritten as performance measures that are inter-temporally uncorrelated (stochastically independent) with long-term action (technologically dependent).

3 At \( t = 1 \), the initial contract \( C^{12} = \{ \alpha_1, (\beta^1_1, \beta^1_2), \ldots, (\beta^N_1, \beta^N_2) \} \) has the fixed payment term, \( \alpha_1 = \alpha_0 + \beta^1_1 y_1 + \beta^1_2 z_1 \), which includes the accrued compensation from the realised first period measures \( y_1 \) and \( z_1 \).

4 To see this, apply the agent’s risk neutrality \( r = 0 \) in the optimal incentive rates (16) and (17). Then, it follows that \( a_{11} = b_{11} \) and \( a_{22} = b_{22} \) from the binding incentive compatibility constraints [equations (58) and (59)].
Appendix

Stationarity: AR(1) process

A stochastic process is called (second-order or weakly) stationary if its mean is constant and its autocovariance function depends only on the lag [Chatfield, (1989), p.30].

The following shows that when AR(1) process is assumed, the uncontrollable random factor \( \varepsilon_t \) in equation (1) has a constant mean, regardless of \( t \). Given the autocorrelation \( \rho_y \), the uncontrollable random factor \( \varepsilon_t \) can be written as

\[
\varepsilon_t = \rho_y \varepsilon_{t-1} + \eta_t,
\]

where \( \eta_t \) is a white noise, \( \eta_t \sim (0, \sigma_\eta^2) \) and \( \text{Cov}(\eta_t, \eta_s) = 0, t \neq s \). By the recursive process,

\[
\begin{align*}
\varepsilon_t &= \rho_y \varepsilon_{t-1} + \eta_t \\
&= \rho_y \left( \rho_y \varepsilon_{t-2} + \eta_{t-1} \right) + \eta_t \\
&= \rho_y^2 \varepsilon_{t-2} + \rho_y \eta_{t-1} + \eta_t \\
&= \ldots \\
&= \sum_{i=0}^{\infty} \rho_y^i \eta_{t-i}.
\end{align*}
\]

The expected value of \( \varepsilon_t \) is zero for all \( t \),

\[
E[\varepsilon_t] = \sum_{i=0}^{\infty} \rho_y^i E[\eta_{t-i}] = 0,
\]

and it follows from equation (1) that the expected value of \( y_t \) is constant:

\[
E[y_t] = \bar{m} \cdot \bar{a}.
\]

Likewise the uncontrollable random factor \( \delta_t \) in equation (2) can be written as follows with the autocorrelation \( \rho_z \):

\[
\delta_t = \rho_z \delta_{t-1} + \nu_t,
\]

where \( \nu_t \) is a white noise, \( \nu_t \sim (0, \sigma_\nu^2) \) and \( \text{Cov}(\nu_t, \nu_s) = 0, t \neq s \). By symmetry, the expected value of \( \delta_t \) is zero \( (E[\delta_t] = 0) \) for all \( t \) and it follows from equation (2) that the expected value of \( z_t \) is constant:

\[
E[z_t] = \bar{k} \cdot \bar{a}.
\]

The variance of \( y_t \) is constant:
Relative incentive rate in a multi-period and multi-task agency

\[ \text{Var}[y_t] = \text{Var}[\varepsilon_t] \]

\[ = \text{Var} \left[ \sum_{i=0}^{\infty} \rho^i u_{t-i} \right] \quad \text{[from equation (36)]} \]

\[ = \sum_{i=0}^{\infty} (\rho^i)^2 \text{Var}[u_{t-i}] \]

\[ = \sigma_u^2 \frac{1}{1 - \rho^2}, \]

and by symmetry, the variance of \( z_t \) is also constant:

\[ \text{Var}[z_t] = \sigma_z^2 \frac{1}{1 - \rho^2} . \] (42)

Given equations (41) and (42), it is evident that the cross-sectional covariance \( \text{Cov}(y_t, z_t) \) is constant regardless of \( t \).

Now, it is shown below that the autocovariance of \( y_t \) does not depend on \( t \) but depends on the lag \( k \). By the recursive process,

\[ \varepsilon_{t+k} = \rho_j \varepsilon_{t+k-1} + u_{t+k} \]

\[ = \rho_j (\rho_j \varepsilon_{t+k-2} + u_{t+k-1}) + u_{t+k} \]

\[ = \rho^2_j \varepsilon_{t+k-2} + \rho_j u_{t+k-1} + u_{t+k} \]

\[ = \ldots \]

\[ = \sum_{i=0}^{\infty} \rho^i u_{t+k-i}. \] (43)

The autocovariance of \( y_t \) is given as

\[ \text{Cov}[y_{t+k}, y_{t+k}] = \text{Cov}[\varepsilon_t, \varepsilon_{t+k}] \]

\[ = \text{Cov} \left[ \sum_{i=0}^{\infty} \rho^i u_{t-i} \sum_{i=0}^{\infty} \rho^i u_{t+k-i} \right] \]

\[ = \text{Cov}[u_t, \rho^k u_t] + \text{Cov}[\rho_j u_{t-1}, \rho^{k+1} u_{t-1}] \]

\[ + \text{Cov}[\rho^{k+2} u_{t-2}, \rho^{k+2} u_{t-2}] + \ldots \]

\[ = \rho^k \sigma_u^2 \frac{1}{1 - \rho^2} \]

\[ = \rho^k \text{Var}[y_t]. \] (44)

which depends only on the lag \( k \). By symmetry, the autocovariance of \( z_t \) also depends only on the lag \( k \),

\[ \text{Cov}[z_{t+k}, z_{t+k}] = \rho^k \text{Var}[z_t]. \] (45)

Finally, it is shown that \( \text{Cov}[y_t, z_{t+k}] \) and \( \text{Cov}[z_t, y_{t+k}] \) do not depend on \( t \) but depend only on the lag \( k \).
\[ \text{Cov}[y_t, z_{t+k}] = \text{Cov}[\varepsilon_t, \delta_{t+k}] \]
\[ = E[(\varepsilon_t - E[\varepsilon_t])(\delta_{t+k} - E[\delta_{t+k}])] \]
\[ = E[\varepsilon_t \delta_{t+k}] \quad \text{[from equation (37) and by symmetry]} \]
\[ = E[\varepsilon_t \rho \delta_{t+k-1}] \quad (E[\varepsilon_t v_{t+1}] = \text{Cov}[\varepsilon_t, v_{t+1}] = 0 \text{ as } v_t \text{ is a white noise.}) \]
\[ = \rho^k E[\varepsilon_t \delta_{t+k-2}] \]
\[ = \rho^{2k} E[\varepsilon_t \delta_{t+k-2}] \quad \text{(...)} \]
\[ = \rho^{(k+1)} E[\varepsilon_t \delta_{t+1}] \]
\[ = \rho^{2k+1} E[\varepsilon_t \rho \delta_t + v_{t+1}] \]
\[ = \rho^k E[\varepsilon_t \delta_t] \]
\[ = \rho^k \text{Cov}[y_t, z_t]. \]

Because \( \text{Cov}[y_t, z_t] \) does not depend on \( t \), \( \text{Cov}[y_t, z_{t+k}] \) depends only on the lag \( k \), regardless of \( t \). By symmetry, \( \text{Cov}[z_t, y_{t+k}] \) also depends only on the lag \( k \), regardless of \( t \):

\[ \text{Cov}[z_t, y_{t+k}] = \rho^k \text{Cov}[y_t, z_t] \quad (47) \]

**Proof of Proposition 1**

The proof is by backward induction. The optimal incentive rates \( \beta_N^1 \) and \( \beta_N^2 \) in the last period are obtained by solving the principal’s problem at \( t = N - 1 \). The principal maximises his expected utility with the decision variables \( \beta_N^1 \) and \( \beta_N^2 \):

\[ \max_{\beta_N^1, \beta_N^2} U_p^E = (\tilde{b}_N \cdot \tilde{a}_N) - E_{N-1}[C | \tilde{a}_N]. \quad (48) \]

For the last period the agent’s rational action choice is as follows:

\[ a_{1N} = \beta_N^1 m_{1N} + \beta_N^2 k_{1N}, \quad (49) \]
\[ a_{2N} = \beta_N^1 m_{2N} + \beta_N^2 k_{2N}. \quad (50) \]

Given the binding participation and incentive compatibility constraints, the principal’s expected utility maximisation (48) becomes:

\[
\max_{\beta_N^1, \beta_N^2} \left( b_{1N}, b_{2N} \right) \cdot \left( \beta_N^1 m_{1N} + \beta_N^2 k_{1N}, \beta_N^1 m_{2N} + \beta_N^2 k_{2N} \right) \\
- \frac{1}{2} \left[ (\beta_N^1 m_{1N} + \beta_N^2 k_{1N})^2 + (\beta_N^1 m_{2N} + \beta_N^2 k_{2N})^2 \right] - \frac{1}{2} \text{Var}_{N-1}[C | \tilde{a}_N],
\]

which is equivalent to an agency loss minimisation problem:
Relative incentive rate in a multi-period and multi-task agency

\[
\min_{\beta_N, \beta_k} L_N = \frac{1}{2} \left[ (\beta_N m_N + \beta_k^2 k_N - b_N)^2 + (\beta_N m_{2N} + \beta_k^2 k_{2N} - b_{2N})^2 \right] \\
+ \frac{1}{2} r \left[ (\beta_N^2 \text{Var}_{N-1}(y_N)) + (\beta_k^2 \text{Var}_{N-1}(z_N)) \right] \\
+ 2 \beta_N \beta_k \text{Cov}_{N-1}(y_N, z_N). \tag{52}
\]

Minimising \( L_N \) gives the last period optimal incentive rates \( \beta_N^* \) and \( \beta_k^* \):

\[
\beta_N^* = \frac{\left( \bar{m}_N \cdot \bar{k}_N \right) \{ \bar{k}_N \cdot \bar{k}_N + r \text{Var}_{N-1}(z_N) \} - \left( \bar{m}_N \cdot \bar{k}_N \right) \{ \bar{m}_N \cdot \bar{k}_N + r \text{Cov}_{N-1}(y_N, z_N) \}}{D_N}, \tag{53}
\]

\[
\beta_k^* = \frac{\left( \bar{m}_N \cdot \bar{k}_N \right) \{ \bar{m}_N \cdot \bar{m}_N + r \text{Var}_{N-1}(y_N) \} - \left( \bar{m}_N \cdot \bar{k}_N \right) \{ \bar{m}_N \cdot \bar{k}_N + r \text{Cov}_{N-1}(y_N, z_N) \}}{D_N}, \tag{54}
\]

where

\[
D_N = (m_N k_{2N} - m_{2N} k_N)^2 + (\bar{m}_N \cdot \bar{m}_N) r \text{Var}_{N-1}(z_N) + (\bar{k}_N \cdot \bar{k}_N) r \text{Var}_{N-1}(y_N) \\
- 2 (\bar{m}_N \cdot \bar{k}_N) r \text{Cov}_{N-1}(y_N, z_N) \\
+ r^2 \left[ \text{Var}_{N-1}(y_N) \text{Var}_{N-1}(z_N) - \{ \text{Cov}_{N-1}(y_N, z_N) \}^2 \right]. \tag{55}
\]

\( D_N \) is positive since

\[
D_N \geq (m_N k_{2N} - m_{2N} k_N)^2 + r^2 \left( 1 - \rho_{ad}^2 \right) \text{Var}_{N-1}(y_N) \text{Var}_{N-1}(z_N) \\
+ r \left( m_N \sqrt{\text{Var}_{N-1}(z_N)} - k_N \sqrt{\text{Var}_{N-1}(y_N)} \right)^2 \\
+ r \left( m_{2N} \sqrt{\text{Var}_{N-1}(z_N)} - k_{2N} \sqrt{\text{Var}_{N-1}(y_N)} \right)^2, \tag{56}
\]

where \( \rho_{ad} \) is the correlation between \( y_N \) and \( z_N \).

Given the last period optimal incentive rates \( (\beta_N^*, \beta_k^*) \), backward induction allows one to calculate the optimal incentive rates of all periods before the last period \( (\beta_{N-1}, \beta_{k_{N-1}}), \ldots, (\beta_1, \beta_{k_1}), \ldots, (\beta_1^*, \beta_{k_1^*}) \). The optimal period \( t \) \((1 \leq t \leq N - 1)\) incentive rates \( \beta_t \) and \( \beta_t^* \) are calculated by solving the principal’s problem at \( t - 1 \). The principal maximises his expected utility with the decision variables \( \beta_t \) and \( \beta_t^* \):

\[
\max_{\beta_t, \beta_t^*} U_p = \sum_{i=1}^{N} (\beta_t \cdot \bar{a}_t) - E_{t+1}[C | \bar{a}_1, \ldots, \bar{a}_N]. \tag{57}
\]

The agent’s rational action choice for period \( t \) is as follows:

\[
a_{1t} = \beta_t \cdot m_{1t} + \beta_t^* \cdot k_{1t}, \tag{58}
\]

\[
a_{2t} = \beta_t \cdot m_{2t} + \beta_t^* \cdot k_{2t}. \tag{59}
\]
With the binding participation and incentive compatibility constraints, the principal’s expected utility maximisation (57) is:

$$\max_{\beta_1^t, \beta_2^t} \sum_{i=1}^{N} \left[ (b_{i1}, b_{21}) \cdot (\beta_1^t m_{i1} + \beta_2^t k_{i1}, \beta_1^t m_{i2} + \beta_2^t k_{i2}) - \frac{1}{2} \left[ (\beta_1^t m_{i1} + \beta_2^t k_{i1})^2 
+ (\beta_1^t m_{i2} + \beta_2^t k_{i2})^2 \right] \right] = \frac{1}{2} r Var_{t-1}[C | \bar{a}_i, \ldots, \bar{a}_N].$$ (60)

As the optimal incentive rates for the period \(t + 1, \ldots, N\) are taken as effectively fixed at \(t - 1\) due to the renegotiation-proofness requirement, equation (60) is reduced to:

$$\max_{\beta_1^t, \beta_2^t} \sum_{i=1}^{N} \left[ (b_{i1}, b_{21}) \cdot (\beta_1^t m_{i1} + \beta_2^t k_{i1}, \beta_1^t m_{i2} + \beta_2^t k_{i2}) 
- \frac{1}{2} \left[ (\beta_1^t m_{i1} + \beta_2^t k_{i1})^2 
+ (\beta_1^t m_{i2} + \beta_2^t k_{i2})^2 \right] \right] - \frac{1}{2} r Var_{t-1}[C | \bar{a}_i, \ldots, \bar{a}_N].$$ (61)

In addition, the risk premium \(\frac{1}{2} r Var_{t-1}[C | \bar{a}_i, \ldots, \bar{a}_N]\) is reduced to a relevant risk premium \(\frac{1}{2} r Var'_{t-1}[C | \bar{a}_i, \ldots, \bar{a}_N]\) due to the renegotiation-proofness requirement:

$$Var_{t-1}[C | \bar{a}_i, \ldots, \bar{a}_N] = Var_{t-1} \left[ \sum_{i=1}^{N} \beta_1^t y_i + \sum_{i=1}^{N} \beta_2^t z_i \right]$$

$$\sim Var'_{t-1}[C | \bar{a}_i, \ldots, \bar{a}_N]$$

$$= (\beta_1^t)^2 Var_{t-1}(y_i) + (\beta_2^t)^2 Var_{t-1}(z_i) + 2\beta_1^t \beta_2^t Cov_{t-1}(y_i, z_i)$$

$$+ \sum_{i=t+1}^{N} 2\beta_1^t \beta_2^t Cov_{t-1}(y_i, z_i) + \sum_{i=t+1}^{N} 2\beta_1^t \beta_2^t Cov_{t-1}(z_i, z_i)$$

$$+ \sum_{i=t+1}^{N} 2\beta_1^t \beta_2^t Cov_{t-1}(z_i, z_i).$$ (63)

\(Var'_{t-1}[C | \bar{a}_i, \ldots, \bar{a}_N]\) is substituted for \(Var_{t-1}[C | \bar{a}_i, \ldots, \bar{a}_N]\) in equation (61) and the principal’s expected utility maximisation at \(t - 1\) is equivalent to an agency loss minimisation:

$$\min_{\beta_1^t, \beta_2^t} L_t = \frac{1}{2} \left[ (\beta_1^t m_{i1} + \beta_2^t k_{i1} - b_{i1})^2 + (\beta_1^t m_{i2} + \beta_2^t k_{i2} - b_{i2})^2 \right]$$

$$+ \frac{1}{2} r Var'_{t-1}[C | \bar{a}_i, \ldots, \bar{a}_N].$$ (64)

Finally, minimising \(L_t\) gives the period \(t\) optimal incentive rates \(\beta_1^t\) and \(\beta_2^t\):

$$\beta_1^t = \frac{\begin{array}{c} \bar{b}_i \cdot \bar{m}_i - ICR^2 \{ \bar{b}_i \cdot \bar{k}_i + r Var_{t-1}(z_i) \} - \left[ \bar{b}_i \cdot \bar{k}_i - ICR^2 \right] \{ \bar{m}_i \cdot \bar{k}_i + r Cov_{t-1}(y_i, z_i) \} \end{array}} {D},$$

$$\beta_2^t = \frac{\begin{array}{c} \bar{b}_i \cdot \bar{m}_i - ICR^2 \{ \bar{b}_i \cdot \bar{k}_i + r Var_{t-1}(z_i) \} - \left[ \bar{b}_i \cdot \bar{k}_i - ICR^2 \right] \{ \bar{m}_i \cdot \bar{k}_i + r Cov_{t-1}(y_i, z_i) \} \end{array}} {D},$$

where 

$$D = \bar{m}_i \cdot \bar{k}_i + r Var_{t-1}(z_i) - \left[ \bar{m}_i \cdot \bar{k}_i - ICR^2 \right] \{ \bar{m}_i \cdot \bar{k}_i + r Cov_{t-1}(y_i, z_i) \}.$$ (65)
Relative incentive rate in a multi-period and multi-task agency

\[ \beta^2 = \frac{\left[ \tilde{h}_t \cdot \tilde{k}_t - ICR_t^2 \right] \left[ \tilde{m}_t \cdot \tilde{m}_t + r \text{Var}_{t-1}(y_t) \right] - \left[ \tilde{h}_t \cdot \tilde{k}_t - ICR_t^2 \right] \left[ \tilde{m}_t \cdot \tilde{k}_t + r \text{Cov}_{t-1}(y_t, z_t) \right]}{D_t}, \]  

(66)

where

\[ D_t = \left( m_{k_2} - m_{k_1} \right)^2 + \left( \tilde{m}_t \cdot \tilde{m}_t \right) r \text{Var}_{t-1}(z_t) + \left( \tilde{k}_t \cdot \tilde{k}_t \right) r \text{Var}_{t-1}(y_t) \]

\[ - 2 \left( \tilde{m}_t \cdot \tilde{k}_t \right) r \text{Cov}_{t-1}(y_t, z_t) \]

\[ + r^2 \left[ \text{Var}_{t-1}(y_t) \text{Var}_{t-1}(z_t) - \left\{ \text{Cov}_{t-1}(y_t, z_t) \right\}^2 \right]. \]

(67)

It can be shown that \( D_t \) is positive as in equation (56).