

An EOQ model for deteriorating items when demand is cloud fuzzy

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Abstract: In this study, an EOQ model with constant rate of deterioration is developed and solved in crisp, fuzzy and cloud fuzzy environment. Demand is considered as a normal triangular fuzzy number and cloud normal triangular fuzzy number in uncertain environments to get the total minimum inventory cost. Yager's ranking index is used for defuzzification. The objective is to minimise the total inventory cost and compare the results in crisp, fuzzy and cloud fuzzy environment. With the help of numerical example, it is observed that the cloud fuzzy environment produce better results. Sensitivity analysis has also been performed to understand the change in optimal solution as various parameters changes.

Keywords: economic order quantity; EOQ; deterioration; fuzzy demand; cloud fuzzy demand.

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1 Introduction

One of the most important tasks in business is inventory management as it helps manager for accurate order fulfilment, increased productivity, organise warehouses which ultimately results in customer's satisfaction.

In order to solve inventory related problems using mathematical ideas, Harris (1913) has introduced classical economic order quantity (EOQ) model. The stringent assumptions of the classical model fail to represent the real scenario. To make this model more realistic, many researchers have worked out different models by relaxing certain assumptions. Urban and Baker (1997) developed a model to find quantity of order when demand depends upon price, time and inventory level. An inventory model in which the demand is a convex function of price was investigated by Shinn and Hwang (2003). You (2005) has developed an EOQ model under price and time dependent demand.

Deterioration has been considered as one of the most vital factor in inventory system. Deteriorating items are those items which losses its utility as time progresses. Whiting (1957) was the pioneer to study the effect of deterioration while analysing the inventory system. He considered fashion items to study the effect of deterioration. Ghare (1963) has derived an EOQ model for exponentially decaying inventory. Goswami and Chaudhuri (1991) formulated an EOQ model by taking into account the deteriorating item with time varying demand. Kim (1995) considered deteriorating items with linear demand and constructs an inventory model. For more literature on deteriorating inventory models one can refer to Nahmias (1982), Raafat (1991), Shah and Shah (2000), Goyal and Giri (2001), Janssen et al. (2016), and Pérez Mantilla and Torres (2014).

Generally, parameters associated with inventory modelling like demand rate, deterioration rate, etc. are considered to take a crisp value. But to survive in global competition and for better decision making, we must take into account uncertainty while formulating inventory model. For example, when we say that the demand rate is '100', in practice it is not exactly 100, but it is about 100. The concept of 'about' can be well handles by fuzziness. Here, the concept of fuzzy set theory plays a vital role in formulating such kind of uncertainty. Fuzzy set theory is a very close approach to reality. Due to practical implementation, fuzzy inventory modelling is an emerging research area in the field of decision making.

The invention of fuzzy set theory by Zadeh (1965) laid a foundation stone of a new research area and it gives a new dimension to the field of decision making. Park (1987) has re-examined the EOQ model with fuzzy set theory approach. Yao and Lee (1996) calculated optimal stock quantity with backorder by considering order quantity as normal triangular fuzzy number (NTFN). Kao and Hsu (2002) invented the single period inventory model with fuzzy demand. Dutta and Kumar (2013) developed fuzzy inventory model without shortage to determine total cost and optimal order quantity by introducing trapezoidal fuzzy number. They have used signed distance method for defuzzification. Soni et al. (2017) constructed a continuous review inventory model with backorder by considering fuzzy demand and learning effect. Maity et al. (2019) has developed an EOQ model by considering learning experience of decision maker under lock fuzzy environment. Under intuitionistic dense fuzzy environment, Maity et al. (2020) has studied a backorder inventory problem.

When some variables involved in inventory system are fuzzy in nature, the solution will also be a fuzzy number. To apply that solution to our problem, we need to convert

the fuzzy number to a crisp value. For that defuzzification is required. Defuzzification is the process which transforms fuzzy solution to a crisp value. For ordering a fuzzy subset of unit interval, Yager (1981) introduced a new function for defuzzification. Filev and Yager (1991) developed a generalised defuzzification method with the help of basic defuzzification distribution transformations.

In practice, it is observed that in any management system uncertainty reduces from the system as time progresses. Therefore, we need such fuzzy number in which fuzzy number converge to a singleton crisp set as time tends to infinite. Recently, De and Mahata (2017) has introduced a new concept of cloudy fuzzy number and applied it on classical backorder EOQ model. Karmakar et al. (2018) extended that research and apply cloudy fuzzy approach to solve EOQ model under cloudy fuzzy demand rate. De and Mahata (2019) has developed an EOQ model for imperfect quality items with allowable proportionate discounts using the concept of cloudy fuzzy number. In this paper, we have solved an EOQ model with deterioration in crisp environment, fuzzy environment and cloud fuzzy environment. The Yager's ranking index is used for defuzzification. A numerical example is given to understand the difference between the results in crisp, fuzzy and cloud fuzzy environment. Graphical representation followed by sensitivity analysis is given for better understanding and visualisation of results.

2 Notations and assumptions

2.1 Notations

T is total cycle time; in year

θ is constant deterioration rate; $0 \leq \theta < 1$

h is inventory holding cost per unit per cycle; in \$

R is demand rate per year

c is purchase cost per unit; in \$

A is ordering cost per order; in \$

Q is order quantity per cycle; in units (decision variable)

K is total inventory cost; in \$

Q_f is fuzzy order quantity per cycle; in units (decision variable)

R_f is fuzzy demand rate per year; in units

Q_c is cloud fuzzy order quantity per cycle; in units (decision variable)

R_c is cloud fuzzy demand rate per year; in units.

2.2 Assumptions

- 1 Replenishment rate is infinite and lead time is zero.
- 2 No shortages are permitted.
- 3 Planning period for inventory system is of infinite length.

- 4 Purchase price of an item, holding cost and deterioration rate are constant during the cycle time.
- 5 Demand rate for crisp model is constant.
- 6 During given cycle, no replenishment or repair of deteriorating items occur.

3 Definitions and preliminary concepts

3.1 Normal fuzzy number

A fuzzy number is a fuzzy set of the real line having normal, convex and continuous membership function of bounded support.

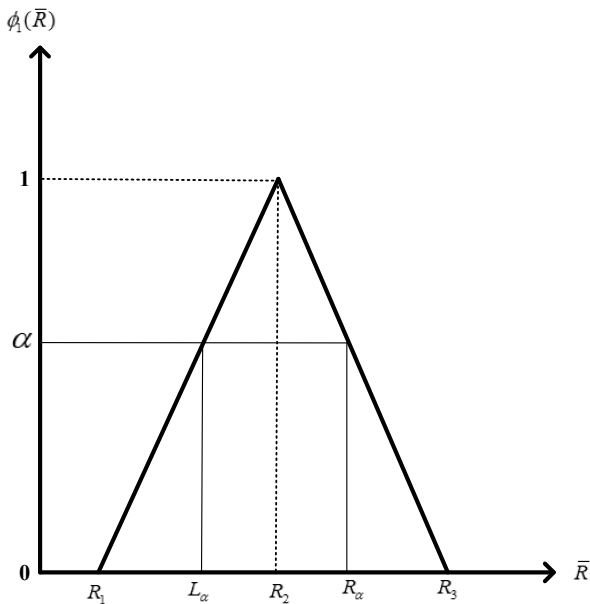
3.2 Normal triangular fuzzy number

NTFN is a fuzzy number of the form $\bar{R} = (R_1, R_2, R_3)$ having membership function

$$\phi(\bar{R}) = \begin{cases} 0, & x < R_1 \\ \frac{x - R_1}{R_2 - R_1}, & R_1 \leq x \leq R_2 \\ \frac{x - R_3}{R_2 - R_3}, & R_2 \leq x \leq R_3 \\ 0, & x > R_3 \end{cases} \tag{1}$$

as shown in Figure 1.

Figure 1 Membership function and α -cuts of NTFN



3.3 Left and right α -cut of NTFN

Left α -cut (L_α) and right α -cut (R_α) of NTFN are given by,

$$L_\alpha = R_1 - \alpha(R_1 - R_2) \quad \& \quad R_\alpha = R_3 - \alpha(R_3 - R_2) \tag{2}$$

respectively as shown in Figure 1.

3.4 Cloud normal triangular fuzzy number

A NTFN which converge to a crisp number after a long time is known as cloud normal triangular fuzzy number (CNTFN).

For example,

$$\tilde{R} = \left(R_2 \left(1 - \frac{\beta}{1+t} \right), R_2, R_2 \left(1 + \frac{\gamma}{1+t} \right) \right)$$

where $\beta, \gamma \in (0, 1)$.

Note that as $t \rightarrow \infty, \tilde{R} \rightarrow \{R_2\}$.

Membership function of CNTFN is given by,

$$\phi_2(\tilde{R}) = \begin{cases} 0, & x < R_2 \left(1 - \frac{\beta}{1+t} \right) \\ \frac{x - R_2 \left(1 - \frac{\beta}{1+t} \right)}{\frac{\beta R_2}{1+t}}, & R_2 \left(1 - \frac{\beta}{1+t} \right) \leq x \leq R_2 \\ \frac{R_2 \left(1 + \frac{\gamma}{1+t} \right) - x}{\frac{\gamma R_2}{1+t}}, & R_2 \leq x \leq R_2 \left(1 + \frac{\gamma}{1+t} \right) \\ 0, & x > R_2 \left(1 + \frac{\gamma}{1+t} \right) \end{cases} \tag{3}$$

3.5 Left and right α -cut of CNTFN

Left α -cut ($L_{\alpha,t}$) and right α -cut ($R_{\alpha,t}$) of CNTFN are given by,

$$L_{\alpha,t} = R_2 \left(1 - \frac{\beta}{1+t} \right) + \frac{\alpha\beta}{1+t} R_2 \quad \& \quad R_{\alpha,t} = R_2 \left(1 - \frac{\gamma}{1+t} \right) + \frac{\alpha\gamma}{1+t} R_2 \tag{4}$$

respectively.

3.6 Defuzzification method for NTFN (Yager's ranking index)

We follow Yager's (1981).

Given a NTFN \bar{R} with left α -cut (L_α) and right α -cut (R_α) then defuzzification formula by Yager's ranking index is,

$$I(\bar{R}) = \frac{1}{2} \int_0^1 (L_\alpha + R_\alpha) d\alpha$$

Using equation (2),

$$I(\bar{R}) = \frac{1}{4} (R_1 + 2R_2 + R_3) \tag{5}$$

3.7 Defuzzification method for CNTFN (extension of Yager's ranking index)

We follow De and Mahata (2017).

Given a CNTFN \tilde{R} with left α -cut ($L_{\alpha,t}$) and right α -cut ($R_{\alpha,t}$) then defuzzification formula by extension of Yager's ranking index is,

$$I(\tilde{R}) = \frac{1}{2} \int_{\alpha=0}^{\alpha=1} \int_{t=0}^{t=T} (L_{\alpha,t} + R_{\alpha,t}) d\alpha dt$$

Using equation (4)

$$I(\tilde{R}) = R_2 \left(1 - \frac{(\beta - \gamma) \log(1+T)}{4T} \right) \tag{6}$$

4 Crisp mathematical modelling

Let us review one cycle. Assume that initially, we have ordered Q quantity which decreases as time progresses with demand satisfaction and constant deterioration rate θ . The rate of change of inventory level can be explained by,

$$\frac{dI}{dt} = -R - \theta I(t) \tag{7}$$

with initial conditions $I(0) = Q$ and $I(T) = 0$.

The solution of linear differential equation (7) is

$$I(t) = \frac{R}{\theta} (e^{\theta(T-t)} - 1) \tag{8}$$

The crisp total cost of an inventory system comprises of

1 Inventory holding cost

$$IHC = \frac{hR}{\theta^2} (e^{\theta T} - \theta T - 1) \tag{9}$$

2 Cost due to deterioration

$$CD = \frac{cR}{\theta}(e^{\theta T} - \theta T - 1) \tag{10}$$

3 Ordering cost

$$OC = A \tag{11}$$

Using equations (9), (10) and (11), total inventory cost per unit time (K) is given by,

$$K = \frac{1}{T}(IHC + CD + OC)$$

$$K = \left(\frac{h + c\theta}{\theta^2 T}\right)(e^{\theta T} - \theta T - 1)R + \frac{A}{T} \tag{12}$$

The objective is to

$$\text{minimise } K = \left(\frac{h + c\theta}{\theta^2 T}\right)(e^{\theta T} - \theta T - 1)R + \frac{A}{T} \tag{13}$$

with $Q = RT$.

Using classical optimisation technique, optimal time

$$T^* = \sqrt{\frac{2A}{(h + c\theta)R}} \tag{14}$$

5 Fuzzy mathematical modelling

Now let us discuss fuzzy behaviour of demand rate during inventory process.

Therefore, objective function reduces to,

$$\text{minimise } K_f = \left(\frac{h + c\theta}{\theta^2 T}\right)(e^{\theta T} - \theta T - 1)R_f + \frac{A}{T} \tag{15}$$

with $Q_f = R_f T$.

Now consider NTFN as per Section 3.2 and by using equation (1) we have,

1 Membership function for total fuzzy inventory cost,

$$\phi(K) = \begin{cases} 0, & K < K_1 \\ \frac{K - K_1}{K_2 - K_1}, & K_1 \leq K \leq K_2 \\ \frac{K - K_3}{K_2 - K_3}, & K_2 \leq K \leq K_3 \\ 0, & K > K_3 \end{cases} \tag{16}$$

where

$$K_i = \left(\frac{h+c\theta}{T\theta^2} \right) (e^{\theta T} - \theta T - 1) R_i + \frac{A}{T}, \text{ for } i=1, 2, 3.$$

2 Membership function for order quantity

$$\phi(Q) = \begin{cases} 0, & Q < Q_1 \\ \frac{Q-Q_1}{Q_2-Q_1}, & Q_1 \leq Q \leq Q_2 \\ \frac{Q-Q_3}{Q_2-Q_3}, & Q_2 \leq Q \leq Q_3 \\ 0, & Q > Q_3 \end{cases} \tag{17}$$

where $Q_i = R_i T$, for $i = 1, 2, 3$.

Using equation (5), defuzzified value of objective function and order quantity can be written as,

$$\left. \begin{aligned} I(K_f) &= \frac{1}{4} \left(\frac{h+c\theta}{T\theta^2} \right) (e^{\theta T} - \theta T - 1) (R_1 + 2R_2 + R_3) + \frac{A}{T} \\ I(Q_f) &= \frac{1}{4} (R_1 + 2R_2 + R_3) T \end{aligned} \right\} \tag{18}$$

6 Cloud fuzzy mathematical modelling

Now consider demand rate as cloud fuzzy number as in 3.4 and by using equation (3) we have

1 Membership function for total cloud fuzzy inventory cost,

$$\psi_1(K, T) = \begin{cases} 0, & K < K_1 \\ \frac{K-K_1}{K_2-K_1}, & K_1 \leq K \leq K_2 \\ \frac{K-K_3}{K_2-K_3}, & K_2 \leq K \leq K_3 \\ 0, & K > K_3 \end{cases} \tag{19}$$

where

$$K_1 = \left(\frac{h+c\theta}{T\theta^2} \right) (e^{\theta T} - \theta T - 1) R_c \left(1 - \frac{\beta}{1+T} \right) + \frac{A}{T}$$

$$K_2 = \left(\frac{h+c\theta}{T\theta^2} \right) (e^{\theta T} - \theta T - 1) R_c + \frac{A}{T}$$

$$K_3 = \left(\frac{h+c\theta}{T\theta^2} \right) (e^{\theta T} - \theta T - 1) R_c \left(1 + \frac{\gamma}{1+T} \right) + \frac{A}{T}$$

2 Membership function for total cloud fuzzy order quantity,

$$\psi_2(Q, T) = \begin{cases} 0, & Q < Q_1 \\ \frac{Q - Q_1}{Q_2 - Q_1}, & Q_1 \leq Q \leq Q_2 \\ \frac{Q - Q_3}{Q_2 - Q_3}, & Q_2 \leq Q \leq Q_3 \\ 0, & Q > Q_3 \end{cases} \tag{20}$$

where

$$Q_1 = R \left(1 - \frac{\beta}{1+T} \right) T, \quad Q_2 = RT, \quad Q_3 = R \left(1 + \frac{\gamma}{1+T} \right) T$$

Using the equation in Section 3.7, defuzzified value of objective function and order quantity can be obtained as,

$$\left. \begin{aligned} I(K_c) &= \frac{R_c(h+c\theta)T}{4} + \frac{R_c(h+c\theta)(\gamma-\beta)}{8} \left[1 - \frac{\log(1+T)}{T} \right] + \frac{A}{T} \log \left| \frac{T}{\varepsilon} \right| \\ I(Q_c) &= \frac{R_c T}{2} + \frac{R_c(\gamma-\beta)}{4} \left[1 - \frac{\log(1+T)}{T} \right] \end{aligned} \right\} \tag{21}$$

where

$$R_c = R_2 \left[1 + \frac{(\gamma-\beta) \log(1+T)}{4T} \right].$$

7 Numerical experiment and sensitivity analysis

For numerical experiment, consider $c = \$10$, $\theta = 0.05$, $h = \$2.5$, $R = 100$ and $A = \$120$ for crisp model. For fuzzy model consider fuzzy demand rate as a NTFN $\bar{R} = (R_1, R_2, R_3) = (60, 100, 120)$ and keep other parameters same as in the crisp model. For cloud fuzzy model assume $\beta = 0.12$, $\gamma = 0.15$, $\varepsilon = 0.05$. From equations (12), (18) and (21) the following results are obtained.

7.1 Analysis of Table 1 and Table 2

By analysing Table 1 we see that there is not much difference between the optimal solutions in crisp environment and fuzzy environment. But when we consider the cloud fuzzy environment, significant difference is observed among the solutions in crisp, fuzzy and cloud fuzzy environments. The same behaviour is observed for order quantity also. In Table 2 we have considered different cycle times from 1.21 year to 1.3 year and observed the values of ordered quantity and total inventory cost. It is observed that as cycle time increases, the value of order quantity and total inventory cost also increases in both crisp and fuzzy environment. But when we work in cloud fuzzy environment, the total inventory cost starts decreasing until the cycle time reached at 1.24 years. After

1.24 years it starts increasing. So we get the optimal solution (\$228.95) with order quantity (62.394 units) at 1.24 years.

Figure 2 Total inventory cost variation under different environments (see online version for colours)

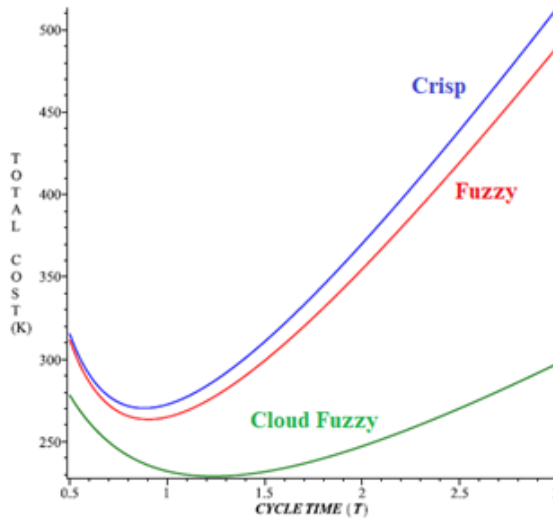


Table 1 Optimal solution of EOQ model with deterioration under different environments

Environment	Cycle time (T) in years	Order quantity (Q)	Minimum total inventory cost (K) in \$
Crisp	0.89	89.442	270.350
Fuzzy	0.91	87.177	263.557
Cloud fuzzy	1.24	62.394	228.952

Table 2 Values of order quantity and total inventory cost under different environments over different cycle time

Cycle time (T)	Crisp environment		Fuzzy environment		Cloud fuzzy environment	
	Q	K	Q	K	Q	K
1.21	121	284.390	114.950	275.129	60.888	228.988
1.22	122	285.139	115.900	275.800	61.390	228.966
1.23	123	285.902	116.850	276.485	61.892	228.955
1.24	124	286.678	117.800	277.183	62.394	228.952
1.25	125	287.468	118.750	277.894	62.896	228.960
1.26	126	288.270	119.700	278.618	63.398	228.976
1.27	127	289.085	120.650	279.355	63.899	229.001
1.28	128	289.912	121.600	280.104	64.401	229.035
1.29	129	290.751	122.550	280.865	64.903	229.078
1.30	130	291.602	123.500	281.637	65.405	229.130

Note: N.B: *Italic* represents optimal solution in cloud fuzzy environment.

Next, we study variations in optimum by varying one parameter at a time by -20% , -10% , 10% and 20% in cloud fuzzy environment.

Table 3 Sensitivity analysis

<i>Parameter</i>	<i>Change in parameter (in %)</i>	<i>Total cloud fuzzy cost (K)</i>	<i>Difference in optimal solution (in %)</i>	<i>% change in total cost</i>	
				<i>Crisp v/s cloud fuzzy environment</i>	<i>Fuzzy v/s cloud fuzzy environment</i>
<i>c</i>	20	232.05	1.35	18.41	15.44
	10	230.50	0.68	18.25	15.28
	-10	227.38	-0.69	17.91	14.94
	-20	225.80	-1.38	17.73	14.77
<i>h</i>	20	243.81	6.49	19.70	16.69
	10	236.57	3.33	18.91	15.92
	-10	220.92	-3.51	17.20	14.26
	-20	212.40	-7.23	16.28	13.36
<i>A</i>	20	254.88	11.33	16.27	13.36
	10	242.18	5.78	17.12	14.18
	-10	215.12	-6.04	19.18	16.18
	-20	200.58	-12.39	20.46	17.42
ε	20	221.05	-3.45	22.30	19.23
	10	224.88	-1.78	20.22	17.20
	-10	233.32	1.91	15.87	12.96
	-20	238.06	3.98	13.56	10.71
θ	20	232.04	1.35	18.60	15.62
	10	230.50	0.68	18.34	15.37
	-10	227.38	-0.69	17.82	14.86
	-20	225.79	-1.38	17.56	14.60
<i>R</i>	20	246.62	7.72	20.00	16.99
	10	238.04	3.97	19.07	16.08
	-10	219.25	-4.24	17.03	14.08
	-20	208.84	-8.78	15.89	12.98
γ	20	229.96	0.44	17.56	14.61
	10	229.45	0.22	17.83	14.86
	-10	228.45	-0.22	18.34	15.36
	-20	227.95	-0.44	18.60	15.62
β	20	228.14	-0.35	18.50	15.52
	10	228.55	-0.17	18.29	15.31
	-10	229.35	0.17	17.88	14.91
	-20	229.76	0.35	17.67	14.71

7.2 Analysis of Table 3

By conducting the sensitivity analysis of various parameters, it is observed that inventory holding cost, ordering cost and demand rate are the most sensitive parameters. Some parameters like purchase cost, parameter ε and deterioration rate shows a very small difference in optimal solution when we vary these parameters. Whereas, parameters like β and γ have negligible sensitivity. Further, the percentage of change in total cost in two different environments is also shown in Table 3. It is observed that difference between the optimal solution in crisp and cloud fuzzy environment is greater than the crisp and fuzzy environment.

8 Conclusions

The concept of cloud fuzzy which we have used in this paper is a new concept in inventory problems. In this study, we observed that there is a significant difference among the solutions in crisp, fuzzy and cloud fuzzy environment. In cloud fuzzy environment, if we consider the higher cycle time then the total inventory cost will be much lesser as compare to other environments. In practice also the same results are observed. As time increases, the decision maker gains more experience and the decision will be much accurate. Hence, the concept of cloud fuzzy is a very close approach to reality. Further research can be done by applying the concept of cloud fuzzy in different variants of inventory models.

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