Inventory management in the presence of inventory inaccuracies: an economic analysis by discrete-event simulation

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Abstract: Today, inventory management is almost entirely based on information systems. However, inventory records can always be prone to errors, leading to underestimated or overestimated inventory. In this study, the effect generated by inventory record inaccuracies on the economic performance of a warehouse of fast moving consumer goods is analysed. A simulation model is developed under Microsoft Excel™ to reproduce the inbound and outbound flows of product at the warehouse, together with the generation of errors in these flows. Different types of errors, correction mechanisms and reorder policies are taken into account in the analysis. As output, the simulation model returns the total cost of the inventory management as a function of the error level, the reorder policy and the correction mechanism adopted. The results of the study provide useful guidelines for warehouse managers. More specifically, outcomes can support strategic decisions, such as the choice among different correction mechanisms, the opportunity to adopt them (or not) or the opportunity to modify (or not) the operating leverages of the reorder policy to adapt them to the presence of errors in inventory records.

Keywords: inventory inaccuracy; simulation; reorder policies; cost analysis; warehouse.


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Martina Mantovani graduated in Management Engineering at the University of Parma in 2011. During her studies and Master’s degree thesis, she explored the theme of inventory management. Upon graduation, she was employed in the reorganisation of logistic processes of engineering companies and implementation of ERP systems. She has worked as a Project Manager in a multinational leader in the construction of important food plants for liquid food.

Roberto Montanari is a Full Professor of Mechanical Plants at the University of Parma. He graduated (with distinction) in 1999 in Mechanical Engineering at the University of Parma. His research activities concern inventory management, logistics, supply chain management, equipment maintenance, power plants, food plants, process modelling and simulation. He has published his research in approx. 60 papers, which appear in qualified international journals and conferences. He acts as a referee for several scientific journals, is editorial board member of two international scientific journals and editor of a scientific journal.

Giuseppe Vignali graduated in 2004 in Mechanical Engineering at the University of Parma. In 2009, he received his PhD in Industrial Engineering at the same university. Since March 2015, he worked as an Associate Professor at the Department of Industrial Engineering of the University of Parma, and since 2007, he taught several courses for the management and food industry engineering classes. His research activities concern food processing, packaging and safety/security of industrial plants. Results of his studies have been published in more than 80 scientific papers, some of which appear both in national and international journals, as well in national and international conferences.

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1 Introduction

Inventory management is crucial to the competitiveness of any organisation, which should be able to ensure the availability of the product required by the customer. The importance of managing the inventory efficiently led to study several methods for the optimisation of stocks, and to implement new technologies to support decision making and streamline operational processes of the warehouse. In recent years, innovations in information technologies have allowed the performance of inventory management systems to be improved significantly, by leading to a more integrated supply chain, where data relating to customer’s demand and inventory levels can be shared among supply chain players and used to support process analysis and strategic decisions. Today, companies depend on the accuracy of the computerised information system for critical decision making.

Information concerning the quantity and location of products must be provided accurately to coordinate the movement of goods effectively in the supply chain.
Conversely, if the data recorded in the information system is incorrect, the ability to provide the product to the final customer at the minimum cost is compromised (Kang and Gershwin, 2004). The term ‘inventory inaccuracy’ was introduced in the 1960’s to describe the mismatches between the inventory record (i.e., the ‘theoretical’ stock data recorded in the company’s information system) and the ‘physical’ stock available in the warehouse (Rinehart, 1960; Iglehart and Morey, 1972; Morey, 1985). That is to say, a misalignment is observed between the information flow and the physical one. Having a perfect alignment between the inventory record and the physical stock could seem obvious, especially if the company exploits, for instance, automatic identification technologies for product identification, such as radio frequency identification (RFID) or barcodes (Sahin, 2004; Bertolini et al., 2015). Unfortunately, despite the use of advanced technological solutions and the significant improvement of warehouse management systems, the stock levels recorded in the company’s information system are often found to be inaccurate (Rekik, 2011). Raman et al. (2001) has estimated that more than 65% of the data relating to the stock keeping units (SKUs) in a retail store are incorrect. Similarly, DeHoratius and Raman (2008) carried out a research involving 37 retail stores and analysed about 370,000 inventory data, highlighting that more than 60% of these data were incorrect. Delaunay et al. (2007) have estimated that correcting inventory record inaccuracies can lead the US retailers to save up to 10 billion $ per year. In addition, based on a survey among supply chain managers, McMullan (1996) has indicated inventory accuracy as one of the most popular metrics for warehouse performance.

Inventory record inaccuracy has attracted the attention of several researchers, as well as of business companies (Wild, 2004). Morey (1972) presented one of the first studies related to the problem of inventory inaccuracy, for which the author has identified the main causes and the corresponding impact on the cost of a company. Fleisch and Tellkamp (2005) have analysed the relationship between inventory inaccuracies (with a particular attention to misalignments in the demand data) and the performance of the inventory management in a supply chain with one retailer. Similarly, Rekik et al. (2007) have studied the issue of inventory inaccuracies within a supply chain. They used a newsvendor model to reproduce the reorder policy of the supply chain. The aim of their analysis is to identify the optimal quantity to be ordered, i.e., the lot size that minimises the total cost of the system. The same authors deepened the study with further works, whose aim was to identify the cost function of inventory management for the wholesaler (Rekik et al., 2009; Sahin et al., 2008). Hovelaque and Thiel (2010) evaluated the effect of inventory inaccuracies on the customer service level for a retail store operating under an economic order quantity (EOQ) policy. Gel et al. (2010) carried out a further study in the EOQ scenario. The authors considered the errors due to inaccuracies occurred during cash transactions. Rekik et al. (2008) studied the impact of thefts on the inventory inaccuracy and provided an estimate of the benefits achievable thanks to the use of automatic identification technologies, such as RFID. The authors considered three scenarios where:

1. the thefts are not known
2. the distribution of thefts is known
3. the RFID technology is used to realign the inventory.
Relating to this latter point, a previous study by Sahin (2004) correlated the use of RFID technology with the errors made in replenishing the shelf of a retail store. De Kok et al. (2008) highlighted and quantified the problem of shrinkage (stock loss) in inventory management, with a particular attention to the potential of RFID technology to reduce shrinkage. As an alternative to analytic models, simulation approaches have been frequently exploited by researchers to analyse logistics issues including inventory inaccuracy, and to investigate different aspects of the problem. A recent review of simulation approaches in logistics and operations management can be found in Jeon and Kim (2016). Among these studies, Curcio and Longo (2009) have developed a simulation model focusing on the inventory and internal logistics management problem for a distribution centre. The model is used to investigate the performance of different inventory control policies under distinct scenarios, with the aim to minimise the logistic cost of the system as a function of some key parameters. Again in the context of logistics, Al-Hawari et al. (2013) have proposed a simulation model to analyse the performance of a four-level/three-product supply chain, composed of retailer, distributor, manufacturer and supplier. They studied the effects of assignment policies, inventory policies and demand patterns on some key performance indicators, including the amount of on-hand inventory and the percentage of customers satisfied. Focusing expressively on the analysis of inventory inaccuracy, related studies have been proposed by Petuhova and Merkuryev (2007), Fleisch and Tellkamp (2005), Brown et al. (2001), Kang and Gershwin (2004), Ernst et al. (1993), Sari (2008), Waller et al. (2006) and Kamaludin (2010). These authors carried out simulation experiments with a number of different configurations of the inventory management system, demonstrating the effectiveness of simulation to analyse inventory management issues.

In line with the above works, this study makes use of simulation to assess the impact of inventory record inaccuracies on the (economic) performance of a warehouse of fast moving consumer goods. An ad hoc simulation model was used to reproduce different operating conditions of the warehouse and to generate inventory inaccuracies in the system, as well as to evaluate, for each configuration, the corresponding total cost of inventory management.

The remainder of the paper is organised as follows. The next section describes the different kinds of error that can affect the inventory data. Section 3 describes the simulation model, the related assumptions, the input data and the simulation strategy. Section 4 details the implementation of the simulation model under Microsoft Excel™ and the simulation steps. Section 5 discusses the main simulation outcomes and the related findings. Section 6 concludes and indicates future research steps.

2 Inventory inaccuracy: causes and related errors

Kang and Gershwin (2004) have categorised the causes of inventory inaccuracy into stock loss (or shrinkage), transaction errors, inaccessible inventory and incorrect product identification. Stock loss includes all forms of loss of the product available for sale. Thefts, unauthorised consumption and expired shelf life are typical examples of this kind of loss. Some product losses can be known: for instance, expired products can be detected by the employees and removed from both the shelf and the inventory records.
Conversely, the unknown shrinkage generates inaccuracy of the inventory records. Transaction errors occur typically at the inbound and outbound sides of the facility. At the inbound side, they can be due, for instance, to discrepancies between what is registered in the information system when a shipment is received and the actual content of a shipment, resulting in an inventory record that does not reflect the physical stock. Inaccessible inventory describes the situation where products are somewhere in a facility but are not available for sale, because they cannot be found. Chappel et al. (2003) identified three situations that can cause inaccessible inventory, i.e.,

1. the final customer has modified the location of an item by picking it and placing it elsewhere
2. during replenishment, the products have been misplaced by the employees
3. products were expected to be in a given shelf of the retail store but their location has changed.

Finally, incorrect product identification can occur in several different ways. For instance, wrong labels can be placed on the products by both the suppliers and the retailers, resulting in the registration of a different product in the company’s information system when the barcode is scanned.

The causes described above can lead to two main types of inventory inaccuracy. According to the literature, most of the times the physical inventory of the product is overestimated compared to the physical amount available, generating a ‘phantom’ inventory or an understock situation. This is due, primarily, to the undetected stock losses, which are not reflected by the inventory records (Kang and Gershwin, 2004). Alternatively, the physical inventory can be underestimated, generating a ‘hidden’ inventory or an overstock situation (Gruen and Corsten, 2007). This latter circumstance is due, for instance, to human errors in inventory operations (e.g., incorrect checking or inventory count), transaction errors, inaccessible inventory and incorrect product identification (Kang and Gershwin, 2004).

Both underestimated and overestimated inventory involve important consequences to a supply chain player, although the effects of the misalignments can vary depending on the type of business and consumers. Whenever the inventory is underestimated, the typical consequence is that the supply chain player will experience a relevant cost of holding stocks, and, depending on the kind of product, may incur in obsolescence risk, shrinkage or markdown cost. However, the overestimated inventory is generally the most critical circumstance; indeed, its direct effect is the occurrence of an out-of-stock situation, as the supply chain player could fail to issue an order for the product, on the basis of the overestimated inventory (Hardgrave, 2009). Further consequences of the overestimated inventory may include the payment of penalties because of the delay in shipping the products to the customer or the need for urgent or supplementary orders. Moreover, the presence of misalignments forces a company to implement some compensation methods (Kang and Gershwin, 2004), thus incurring in additional costs. Examples of these methods include, among others, the increase in the safety stock level (to counteract the uncertainty of the inventory data) or the manual count of inventory (to correct the inventory record and realign the physical and information flows).
3 The problem modelled

3.1 System modelled and types of error

The system investigated consists in a supplier, a company and a customer, according to the scheme in Figure 1. The focus of the analysis is the inventory management problem at the company’s site. The system includes both physical (solid line) and information (dashed line) flows. From the procurement side, the supplier ships its product to the company; the product is received at the warehouse (inbound flow). At the same time, the shipment data are recorded in the company’s information system. According to the fact that, at present, many companies have implemented appropriate software (e.g., enterprise resources planning systems) to support the warehouse management and reorder process, we assume that the inventory data stored in the company’s information system are used to make decisions about the order. From the distribution side, the company faces the demand from the final customer (delivery data); this information is recorded in the company’s information system and compared to the inventory data stored in the same system, to assess whether the demand can be fulfilled. In the case, the company will send the required product to the customer (outbound flow).

Figure 1 Scheme of the system investigated (see online version for colours)

With respect to the inbound flow, it is assumed that the supplier always delivers the right quantity of product to the warehouse, i.e., the product received reflects what requested by the company in its order to the supplier. This means that there are no errors in the physical flow from the supplier to the warehouse. As far as the information flow is concerned, instead, inbound activities at the warehouse can be prone to errors, meaning that the quantity of product recorded in the information system could not reflect the shipment received. Specifically, two kinds of error are taken into account:

- **Error type 1 – positive**: the amount of product recorded in the information system is higher than that actually received. Consequently, the inventory is overestimated.
Inventory management in the presence of inventory inaccuracies

- Error type 2 – negative: the amount of product recorded in the information system is lower than that actually received. The available stock is, consequently, underestimated.

Looking at the outbound flow, i.e., the delivery to the customers, it is assumed that the demand is received and recorded without errors in the company’s information system. Nonetheless, errors in the physical flow can always occur. More precisely, the following kinds of errors are taken into account:

- Error type 1 – negative: the customer receives a lower amount of product than that ordered (and therefore recorded in the warehouse information system). Consequently, the physical inventory at the warehouse is higher than that recorded in the information system.

- Error type 2 – positive: the customer receives a higher amount of product than that ordered. The physical inventory at the warehouse is, therefore, lower than that recorded in the information system.

3.2 Error correction process

According to Kang and Gershwin (2004), inventory managers can exploit different techniques to fix the inaccuracy of the inventory records. In this study, we consider three situations where the errors in the inventory records can be detected and amended, thus realigning the physical stock and the inventory records.

- Error correction type 1: inventory count. The warehouse manager can carry out periodical inventory counts (at least once a year), which allows to fully realign the physical inventory and the theoretical one, thus amending possible errors. In the simulation model, one inventory count per year (i.e., every 250 working days) is introduced to this extent;

- Error correction type 2: realignment due to out-of-stock occurrence. Out-of-stock situations are a direct consequence of the overestimated inventory. Out-of-stock occurs when the physical inventory of the warehouse is null (or lower than the final customer’s demand), but, because of the presence of inaccuracies, the inventory records indicate a stock higher than zero (or higher than the customer’s demand). Under than circumstance, the warehouse will not be able to fulfil the customer’s request. This forces the warehouse manager to check the inventory and to realign the theoretical and physical data. In the simulation model, this correction mechanism is introduced anytime an out-of-stock situation is observed;

- Error correction type 3: alerts. A third set of situations where errors can be identified and corrected refers to possible alerts, generated either by the final customer or by the warehouse management system. Alerts do not allow realigning the physical and information flows completely; rather, the realignment is limited to the process (e.g., receiving or shipping) for which the alert is observed. In the simulation model, four scenarios are set for the alerts generated in the inbound/outbound flows, according to the description below.
a  *Inbound flow.* For the inbound flow, the information system will generate an alert whenever the amount of product received is lower than that expected. Such an alert will force the employees to check the receiving process. We have previously mentioned that the supplier is assumed to provide an amount of product that always corresponds to the quantity ordered. Therefore, the alert generated by the information system indicates a type 2 error of inbound flows (cf., Section 3.1) which will be identified and corrected. Otherwise, the warehouse management system could generate an alert in the case the amount of product recorded is higher than that received. Such a warning allows correcting both type 1 and type 2 errors on the inbound flow. Overall, the two configurations considered for the correction of the inbound flow are:

1. alert generated when the amount of product received is lower than that expected
2. alert generated when the amount of product received is lower or higher than that expected.

b  *Outbound flow.* With respect to the outbound flow, it is assumed that the error is detected by the final customer. More precisely, the customer will never notify an error to the company if the amount of product it receives is higher than that ordered, for convenience. Conversely, whenever it receives a lower amount of product than that ordered, the customer will notify the warehouse, allowing type 1 error of the outbound flow to be corrected. Specifically, the two configurations considered for the error correction in the outbound flow are:

1. no alerts generated
2. alert generated when the amount of product shipped to the customer is lower than that ordered.

**Figure 2** Error correction type 3 – scenarios (see online version for colours)

<table>
<thead>
<tr>
<th>Corrections on inbound flows</th>
<th>Corrections on outbound flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>If quantity received &lt; quantity ordered</td>
<td>If quantity received &lt; quantity ordered and quantity received &gt; quantity ordered</td>
</tr>
<tr>
<td>Scenario 1</td>
<td>Scenario 2</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>Scenario 4</td>
</tr>
</tbody>
</table>

Overall, by combining the possible configurations relating to the error correction type 3, four scenarios can be identified, as shown in Figure 2. As mentioned earlier, the analysis carried out in this paper always considers the error correction mechanisms type 1 and 2;
conversely, with respect to the correction mechanism type 3, the four scenarios listed in Figure 2 will be investigated separately.

### 3.3 Inventory policies

The analysis covers two inventory management policies, i.e., EOQ and economic order interval (EOI), which can regulate the reorder process of the company.

EOQ (Harris, 1913) is a well-known inventory management policy, which aims at identifying the order quantity that minimises the total inventory holding cost and order cost of a system. The decision process is grounded on two operating leverages, called the order point (OP) and the order lot size (EOQ). According to this policy, the warehouse manager will check the inventory position daily and will place an order if the inventory level is found to be lower than the OP; the quantity ordered will reflect the EOQ and will be available after a given lead time.

EOI is the period between two consecutive orders that minimises the total cost of inventory management, in terms of order cost and holding cost. Again, the decision process is controlled by two operating leverages, namely the target inventory level (also called order-up-to level, OUTL) and the reorder interval (EOI). At fixed time intervals, i.e., every EOI days, the warehouse manager will check the inventory and will place an order whose amount is obtained as the difference between the OUTL and the current inventory level. Again, the quantity ordered will be available after a given lead time.

Under both the EOI and the EOQ policies, the decision about the issuing of an order or of the quantity to be ordered is typically made taking into account the ‘theoretical’ inventory position rather than the ‘physical’ one (Bottani and Montanari, 2010; Bottani et al., 2012). Roughly speaking, the physical stock is the amount of product really available at the warehouse, while the theoretical inventory position includes also the product that has been ordered but not yet received by the company. More in general, in this study, by ‘theoretical’ we mean the amount of inventory recorded in the company’s information systems (which also includes the product ordered but not yet received at the warehouse). According to that logic, in this study we assume that the warehouse manager checks the inventory records to make decisions about the reorder process.

### 3.4 Decision process

A schematic representation of the decision process for the system considered is proposed in Figure 3. Again, dashed lines denote the information flow and solid lines denote the physical flow of product.

The decision process of the company is triggered by the demand for finished product, received from a final customer. When a request is received, the warehouse manager will first check the inventory record, to evaluate whether the theoretical stock is sufficient to fulfil the demand seen. In the case the theoretical stock is found to be sufficient, a picking list will be issued to the warehouse employees, who will prepare the order and deliver the product to the customer. As mentioned, the delivery process can be prone to errors. More precisely, the product shipped to the final customer can be different from the order received, or alternatively, errors can be made in updating the inventory position, recording a higher/lower quantity of product compared to that shipped. Whenever the order from the final customer cannot be fulfilled, an out-of-stock situation will occur. Because of that circumstance, a realignment between the physical and information flow
will be carried out by the warehouse employees (see Section 3.2). After realignment, the next step will be to check whether an order should be issued. The reorder process and the related equations are fully detailed in Appendix 1, as a function of the reorder policy and of the error correction mechanism. Whenever an order is issued to the supplier, this latter will prepare the product required and ship it to the warehouse. We recall that no errors can occur in the physical process of shipping, because the supplier is assumed to always deliver the right quantity of product. Conversely, errors can occur at the company when updating the inventory position with the data related to the product received. Inventory update ends the process. In the case the order is not issued, there is no need for updating the inventory position, thus the process is automatically ended.

Figure 3 General scheme of the decision process implemented in the simulation model (see online version for colours)

4 Software implementation

4.1 Simulation model and input data

A simulation model has been developed under Microsoft Excel™ to reproduce the system described in the previous section. The final customer’s demand is modelled as a stochastic variable, with uniform distribution ranging from 900 to 3,200 pallets/day ($\mu = 2,050$ and $\sigma = 665$ pallets/day), which are typical values for the shipment of a single product from a manufacturer’s distribution centre to its customers (see Bottani and Rizzi, 2008). Some examples related to the simulation of a reorder process under Microsoft Excel™ can be found in Bottani and Montanari (2010, 2011) or Bottani et al. (2007, 2012, 2013, 2014).

With respect to the error affecting the different processes, this is modelled as a stochastic variable with uniform distribution and null mean. Ten different error levels are
considered; in particular, the error made in each process can range from $-\epsilon$ to $+\epsilon$, with $\epsilon \in \{0; 0.1\}$, step 0.01. For instance, an error level of 0.01 means that for each transaction (i.e., receiving or shipping), the inventory record will differ from the physical flow by up to $\pm 1\%$. To take into account the presence of errors, the model computes separately the theoretical inventory of the warehouse and the physical stock level.

Table 1  
Input data set in the simulation model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nomenclature</th>
<th>Numerical value</th>
<th>Measurement unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Procurement lead time</td>
<td>$LT$</td>
<td>5</td>
<td>(days)</td>
</tr>
<tr>
<td>Frequency of the inventory count operations</td>
<td>-</td>
<td>250</td>
<td>(days$^{-1}$)</td>
</tr>
<tr>
<td>Unitary cost of holding stock</td>
<td>$c_{stocks}$</td>
<td>0.42</td>
<td>(€/pallet/day)</td>
</tr>
<tr>
<td>Unitary handling cost</td>
<td>$c_{handling}$</td>
<td>0.99</td>
<td>(€/pallet)</td>
</tr>
<tr>
<td>Unitary cost of stock-out</td>
<td>$c_{stock-out}$</td>
<td>47.5</td>
<td>(€/pallet/day)</td>
</tr>
<tr>
<td>Unitary cost of inventory check</td>
<td>$c_{check}$</td>
<td>7.5</td>
<td>(€/check)</td>
</tr>
<tr>
<td>Unitary order cost</td>
<td>$c_{order}$</td>
<td>10</td>
<td>(€/order)</td>
</tr>
<tr>
<td>Unitary cost of inventory count</td>
<td>$c_{inv _count}$</td>
<td>1,200</td>
<td>(€/inventory count)</td>
</tr>
<tr>
<td>Order lot size</td>
<td>EOQ</td>
<td>From 13,950 to 15,000 (step 5)</td>
<td>(pallets)</td>
</tr>
<tr>
<td>Reorder point</td>
<td>OP</td>
<td>From 11,700 to 13,200 (step 5)</td>
<td>(pallets)</td>
</tr>
<tr>
<td>Reorder interval</td>
<td>EOI</td>
<td>From 5 to 8 (step 1)</td>
<td>(days)</td>
</tr>
<tr>
<td>Targeted inventory level</td>
<td>OUTL</td>
<td>From 21,700 to 22,900 (step 5)</td>
<td>(pallets)</td>
</tr>
</tbody>
</table>

The input data listed in Table 1, which describe a typical warehouse of fast moving consumer goods (Bottani and Rizzi, 2008), were set in the simulation. Both the unitary cost of inventory count and the unitary cost of inventory check are expressed as a fixed share, computed starting from the hourly cost of the work force. Implicitly, we assumed that each check (either on the product received or shipped) requires on average 30 minutes to be completed, while the inventory check requires the work of eight employees for one day (i.e., eight working hours).

4.2 Simulation setting and output

As can be seen from Table 1, some input data of the model are fixed and kept unchanged during the simulation. This is the case, for instance, of the economic parameters, the lead time and the frequency of realignments because of the general inventory counts. Conversely, the operating leverages of the reorder policies (i.e., the lot size and OP for the EOQ policy and the reorder interval and OUTL for the EOI policy) vary in a range of possible values. For instance, in the case of the EOQ, the range of variation of the lot size is from 13,950 to 15,000 pallets (step 5), while the range of variation of OP is from 11,700 and 13,200 pallets (step 5), resulting in more than 63,000 combinations of EOQ and OP. These combinations are all tested in the simulation, with the purpose of identifying the couple of values that returns the minimum total cost of the system. The
same approach is followed for the combinations of reorder interval and OUTL in the case of the EOI policy.

Overall, the simulation strategy used in the analysis consists of three steps, according to the description provided in the sub-sections below.

4.2.1 Simulation step 1 – minimum cost configuration without inaccuracy of inventory records

At first, the model is used to identify the minimum cost settings of each reorder policy, under absence of errors \((x = 0)\). For instance, let us suppose that we are looking for the optimal setting of the EOQ policy, i.e., the combination of lot size and OP that generates the minimum total cost of the inventory management. The optimal setting will be identified by varying the operating leverages of the reorder policy (i.e., lot size and OP) within the respective ranges (according to Table 1) and by computing, for each couple of values, the total cost of the inventory policy. In line with the published literature (Kang and Gershwin, 2004; Hardgrave, 2009; Bottani and Montanari, 2010; Bottani et al., 2014), the total cost of inventory management in presence of inaccuracies consists of several cost components, namely:

1. The inventory holding cost. It is computed starting from the unitary cost of holding stocks and the average amount of pallets in stock. The physical inventory is used in the computation of this cost component.

2. The cost of handling. Material handling operations refer to the situations where pallets are received, shipped or simply moved inside the warehouse (to change their location). This cost is computed starting from the unitary cost of handling and the number of pallets handled.

3. The order cost. This cost component results from the fixed order cost and the number of orders placed to the supplier, this latter being a direct outcome of each simulation run.

4. The stock-out cost. It is computed starting from the unitary cost of the out-of-stock and the out-of-stock quantity (i.e., the number of pallets that cannot be shipped to the final customer). The information stored in the warehouse management system is used to quantify the occurrence of out-of-stock situations.

5. The cost of check. In the case the products received should be checked (because of an alert), the warehouse will incur the corresponding cost, which is computed starting from the unitary cost of check and the number of checks to be made. A similar evaluation is made for the inventory counts.

The detailed equations for the computation for the cost components listed above are proposed in Appendix 2. By computing the cost components, the simulation model returns, as output, the total cost of the inventory management as a function of the operating leverages of the reorder policy, allowing to identify the minimum cost setting of each policy in absence of errors.
4.2.2 Simulation step 2 – minimum cost configuration as a function of the inaccuracy level

This step is the same as the previous one with respect to the range of variation of the operating leverages of the reorder policies. The only difference is that, as a further variable, a given level of error in the inventory records is introduced in the simulator ($x \neq 0$). Again, the aim is to identify the setting of the reorder policy parameters that generates the minimum cost of the inventory management, with that level of error. For instance, let us suppose that we are looking for the minimum cost setting of the EOQ policy, with $x = \pm 0.01$. This means that we are looking for the optimal values of lot size and OP in the case the inbound and outbound flows are affected by a $\pm 1\%$ error in the inventory data. As per the previous scenario, the minimum cost setting is identified by computing the total cost of the system, for each level of error set in the simulation.

4.2.3 Simulation step 3 – performance evaluation and analysis

Using the outcomes obtained in steps 1 and 2, some performance parameters are finally computed to quantify the economic impact of inaccuracies on the inventory management system. To this extent, three different types of daily average total costs (€/day) of the inventory management system are evaluated and compared. These costs are:

1. $C_{totI}$ (€/day) = the minimum daily average total cost experienced by a warehouse that operates in presence of inaccuracy of inventory records, but without being aware of the level of error. This cost gives an estimate of the total cost of the inventory management whenever the inventory records are actually affected by a given level of inaccuracy, but the warehouse manager is not aware of such inaccuracy. Consequently, the inventory decisions (e.g., the definition of the quantity to be ordered or of the reorder interval) are made regardless of the presence of the error. $C_{totI}$ is, therefore, obtained from the simulation when the warehouse operates according to the optimal setting of its policy, as they were obtained in simulation step 1 (i.e., with $x = 0$). However, the simulator includes a given error in inventory records ($x \neq 0$).

2. $C_{totII}$ (€/day) = the minimum daily average total cost observed when the warehouse manager is aware of the inventory inaccuracies. $C_{totII}$ reflects the cost of inventory management when the inventory records are affected by a given level of error, whose statistical distribution, mean and standard deviation are known to the warehouse manager. Therefore, the inventory decisions are made taking into account the information about the error, and the parameters of the reorder policy are adapted accordingly. $C_{totII}$ can be obtained by setting the reorder policy parameters at their optimal value depending on the error level, as derived from simulation step 2, and the simulator includes the corresponding error level ($x \neq 0$).

3. $C_{totIII}$ (€/day) = the minimum average total cost resulting when inventory errors are removed. This is the direct outcome of simulation step 1, i.e., the total cost resulting in the case the warehouse is free from errors ($x = 0$).

By comparing the above costs, the following additional performance parameters can be derived:
\[ \Delta C_1 = C_{\text{totI}} - C_{\text{totII}} \text{ (€/day)}, \] which expresses the saving generated in the warehouse by the knowledge of errors in inventory records

\[ \Delta C_2 = C_{\text{totI}} - C_{\text{totIII}} \text{ (€/day)}, \] which reflects the saving associated to the elimination of errors in inventory records.

4.3 Simulation strategy

The simulation duration was set at 25,000 days. Such a long period is required because we always included a quite low level of errors in the inventory records; therefore, the simulation duration should be long enough to allow observing a significant number of inaccuracies in the system. Results are collected starting at day 251, to avoid warm-up effects of the simulation.

By combining the reorder policies (EOQ vs. EOI) and the four scenarios for error correction type 3 (see Figure 2), we obtain, overall, eight system configurations. For each configuration, the simulation model investigates all the possible settings of the reorder policy, as previously described, combined with the ten error levels. Each scenario was replicated ten times, to ensure significance of the outcomes obtained.

5 Results and discussion

In this section, we present the main results from the simulation runs. Outcomes are organised as a function of the reorder policy (EOQ vs. EOI) and of the scenario considered for the error correction type 3, according to the numbering in Figure 2.

5.1 Result for the EOI policy

5.1.1 Scenario 1

Scenario 1 of error correction considers a situation where alerts are generated for errors made in the inbound flow whenever the quantity received is lower than that expected. These errors are, therefore, detected and fixed. Conversely, errors in the outbound flows are never detected. The corresponding results are shown in Table 2; their graph representation is proposed in Figure 4.

A first consideration from Table 2, which provides the optimal combination of EOI and OUTL values as a function of the error level, is that the presence of errors does not affect the optimal reorder interval of the warehouse to an appreciable extent. Indeed, regardless of the error level, the optimal value of EOI is always five days. Nonetheless, the cost of warehouse management (C_{\text{totI}}) is significantly affected by the error level and, in particular, it tends to increase with the error level, although the results become particularly appreciable when the error level exceeds 7%. Being aware of the error, as mentioned, could allow the warehouse manager to adapt the parameters of the reorder policy to the presence of inventory inaccuracies. In this scenario, however, it can be seen from Table 2 that C_{\text{totII}}, although lower than C_{\text{totI}}, does not differ from the former value to an appreciable extent. This suggests that simply knowing the error level (without correcting it) would not improve the performance of the system significantly. On the
Inventory management in the presence of inventory inaccuracies

contrary, removing the error can considerably improve the economic performance of the system, as can be seen from the lower $C_{\text{totIII}}$.

Table 2  Optimal setting, average daily total costs (€/day) and performance parameters for EOI policy with Scenario 1 of error correction

<table>
<thead>
<tr>
<th>$x$ [%]</th>
<th>EOI</th>
<th>OUTL</th>
<th>$C_{\text{totI}}$</th>
<th>$C_{\text{totII}}$</th>
<th>$C_{\text{totIII}}$</th>
<th>$\Delta C_1$</th>
<th>$\Delta C_2$</th>
<th>$\Delta C_2/C_{\text{totI}}$</th>
<th>$\Delta C_1/C_{\text{totI}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>21,820</td>
<td>5,503.3</td>
<td>5,502.8</td>
<td>5,452.4</td>
<td>0.5</td>
<td>50.9</td>
<td>0.92%</td>
<td>0.01%</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>21,900</td>
<td>5,555.0</td>
<td>5,553.2</td>
<td>5,452.4</td>
<td>1.8</td>
<td>102.6</td>
<td>1.85%</td>
<td>0.03%</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>21,935</td>
<td>5,608.2</td>
<td>5,604.4</td>
<td>5,452.4</td>
<td>3.8</td>
<td>155.8</td>
<td>2.78%</td>
<td>0.07%</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>22,000</td>
<td>5,660.9</td>
<td>5,651.5</td>
<td>5,452.4</td>
<td>9.4</td>
<td>208.5</td>
<td>3.68%</td>
<td>0.17%</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>22,075</td>
<td>5,721.3</td>
<td>5,708.0</td>
<td>5,452.4</td>
<td>13.4</td>
<td>268.9</td>
<td>4.70%</td>
<td>0.23%</td>
</tr>
<tr>
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<td>5</td>
<td>22,165</td>
<td>5,788.2</td>
<td>5,769.9</td>
<td>5,452.4</td>
<td>18.3</td>
<td>335.8</td>
<td>5.80%</td>
<td>0.32%</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>22,255</td>
<td>5,881.9</td>
<td>5,842.7</td>
<td>5,452.4</td>
<td>39.1</td>
<td>429.5</td>
<td>7.30%</td>
<td>0.67%</td>
</tr>
<tr>
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<td>5</td>
<td>22,205</td>
<td>5,937.1</td>
<td>5,900.2</td>
<td>5,452.4</td>
<td>36.9</td>
<td>484.8</td>
<td>8.16%</td>
<td>0.62%</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>22,365</td>
<td>6,044.2</td>
<td>5,983.0</td>
<td>5,452.4</td>
<td>61.2</td>
<td>591.9</td>
<td>9.79%</td>
<td>1.01%</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>22,415</td>
<td>6,115.3</td>
<td>6,019.9</td>
<td>5,452.4</td>
<td>95.4</td>
<td>662.9</td>
<td>10.84%</td>
<td>1.56%</td>
</tr>
</tbody>
</table>

Figure 4  Trend of the total cost (€/day) (y-axis) as a function of the error level (x-axis) for EOI policy with Scenario 1 of error correction (see online version for colours)

5.1.2 Scenario 2

Scenario 2 of error correction is the same as scenario 1 with respect to the correction of the errors in the outbound flow (i.e., no correction); moreover, with respect to the inbound flow, an alert is generated anytime the quantity received differs from that ordered. The optimal setting of this scenario, together with the resulting economic performance, is proposed in Table 3 and the graph representation is shown in Figure 5.

As per the previous scenario, the optimal value of EOI is constant (five days) and does not depend on the level of error of inventory records. Similarly, the optimal OUTL, although not exactly constant, varies (and, in particular, increases) to a very limited
extent (less than 65 pallets) with the error level, showing a limited impact of this latter parameter on the target inventory level of the system. Looking at the economic parameters, this scenario experiences a very low cost of inventory inaccuracies. Indeed, from Table 3 it can be seen that CtotI and CtotII are very similar and that the presence of errors, even at the highest level, increases the total cost of the system only by 1% compared to a warehouse where the inventory records are free from errors. In turn, this result is probably due to the fact that, in the scenario under examination, the error in the inbound flow is always identified and corrected; hence, the only error component that affects the system is the inaccuracy in the outbound flow, which is never corrected. However, because the error is modelled as a stochastic variable with null mean, it is likely that the (positive and negative) inaccuracies of the outbound flow somehow compensate during the simulation. On the other hand, this outcome also suggests that just detecting and correcting the errors in the inbound flow, thus keeping the related inventory record accurate, can generate interesting savings for the warehouse.

Table 3  Optimal setting, average daily total costs (€/day) and performance parameters for EOI policy with Scenario 2 of error correction

<table>
<thead>
<tr>
<th>x [%]</th>
<th>EOI</th>
<th>OUTL</th>
<th>CtotI</th>
<th>CtotII</th>
<th>CtotIII</th>
<th>ΔC1</th>
<th>ΔC2</th>
<th>ΔC2/CtotI</th>
<th>ΔC1/CtotI</th>
</tr>
</thead>
<tbody>
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<td>5</td>
<td>21,795</td>
<td>5,471.3</td>
<td>5,471.4</td>
<td>5,452.4</td>
<td>-0.2</td>
<td>18.9</td>
<td>0.35%</td>
<td>0.00%</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>21,795</td>
<td>5,477.4</td>
<td>5,478.5</td>
<td>5,452.4</td>
<td>-1.1</td>
<td>25.0</td>
<td>0.46%</td>
<td>-0.02%</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>21,815</td>
<td>5,483.1</td>
<td>5,482.8</td>
<td>5,452.4</td>
<td>0.4</td>
<td>30.8</td>
<td>0.56%</td>
<td>0.01%</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>21,825</td>
<td>5,491.7</td>
<td>5,493.2</td>
<td>5,452.4</td>
<td>-1.5</td>
<td>39.3</td>
<td>0.72%</td>
<td>-0.03%</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>21,820</td>
<td>5,492.9</td>
<td>5,493.6</td>
<td>5,452.4</td>
<td>-0.7</td>
<td>40.5</td>
<td>0.74%</td>
<td>-0.01%</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>21,820</td>
<td>5,504.5</td>
<td>5,506.7</td>
<td>5,452.4</td>
<td>-2.2</td>
<td>52.2</td>
<td>0.95%</td>
<td>-0.04%</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>21,830</td>
<td>5,508.7</td>
<td>5,513.1</td>
<td>5,452.4</td>
<td>-4.4</td>
<td>56.3</td>
<td>1.02%</td>
<td>-0.08%</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>21,835</td>
<td>5,516.6</td>
<td>5,514.6</td>
<td>5,452.4</td>
<td>1.9</td>
<td>64.2</td>
<td>1.16%</td>
<td>0.04%</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>21,845</td>
<td>5,526.6</td>
<td>5,522.4</td>
<td>5,452.4</td>
<td>4.2</td>
<td>74.2</td>
<td>1.34%</td>
<td>0.08%</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>21,860</td>
<td>5,533.2</td>
<td>5,531.3</td>
<td>5,452.4</td>
<td>1.9</td>
<td>80.9</td>
<td>1.46%</td>
<td>0.03%</td>
</tr>
</tbody>
</table>

Figure 5  Trend of the total cost (€/day) (y-axis) as a function of the error level (x-axis) for EOI policy with Scenario 2 of error correction (see online version for colours)
5.1.3 Scenario 3

Scenario 3 of error correction considers a situation where both the error made in the inbound flow and that made in the outbound flow are detected and corrected if the quantity (received or shipped) is lower than that expected. The relevant outcomes of this scenario are presented in Table 4; their graph representation is proposed in Figure 6.

### Table 4
Optimal setting, average daily total costs (€/day) and performance parameters for EOI policy with Scenario 3 of error correction

<table>
<thead>
<tr>
<th>x [%]</th>
<th>EOI</th>
<th>OUTL</th>
<th>Ctot I</th>
<th>Ctot II</th>
<th>Ctot III</th>
<th>ΔC1</th>
<th>ΔC2</th>
<th>ΔC2/Ctot I</th>
<th>ΔC1/Ctot I</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>5</td>
<td>21,860</td>
<td>8,189.00</td>
<td>8,188.61</td>
<td>5,452.37</td>
<td>0.38</td>
<td>2,736.63</td>
<td>33.42%</td>
<td>0.00%</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>21,955</td>
<td>10,965.88</td>
<td>10,959.47</td>
<td>5,452.37</td>
<td>6.41</td>
<td>5,513.51</td>
<td>50.28%</td>
<td>0.06%</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>21,995</td>
<td>13,583.70</td>
<td>13,568.58</td>
<td>5,452.37</td>
<td>15.12</td>
<td>8,131.33</td>
<td>59.86%</td>
<td>0.11%</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>22,115</td>
<td>16,489.17</td>
<td>16,471.79</td>
<td>5,452.37</td>
<td>17.39</td>
<td>11,036.80</td>
<td>66.93%</td>
<td>0.11%</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>22,175</td>
<td>19,304.60</td>
<td>19,258.34</td>
<td>5,452.37</td>
<td>46.26</td>
<td>13,852.23</td>
<td>71.76%</td>
<td>0.24%</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>22,265</td>
<td>21,696.37</td>
<td>21,654.04</td>
<td>5,452.37</td>
<td>42.33</td>
<td>16,243.99</td>
<td>74.87%</td>
<td>0.20%</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>22,445</td>
<td>24,822.90</td>
<td>24,724.83</td>
<td>5,452.37</td>
<td>98.08</td>
<td>19,370.53</td>
<td>78.03%</td>
<td>0.40%</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>22,460</td>
<td>27,292.12</td>
<td>27,207.08</td>
<td>5,452.37</td>
<td>85.04</td>
<td>21,839.75</td>
<td>80.02%</td>
<td>0.31%</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>22,605</td>
<td>29,683.85</td>
<td>29,550.58</td>
<td>5,452.37</td>
<td>133.26</td>
<td>24,231.48</td>
<td>81.63%</td>
<td>0.45%</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>22,620</td>
<td>33,015.55</td>
<td>32,858.14</td>
<td>5,452.37</td>
<td>157.41</td>
<td>27,563.18</td>
<td>83.49%</td>
<td>0.48%</td>
</tr>
</tbody>
</table>

Figure 6 Trend of the total cost (€/day) (y-axis) as a function of the error level (x-axis) for EOI policy with Scenario 3 of error correction (see online version for colours)

Once again, in this scenario the optimal value of the reorder interval does not vary as a function of the error level, confirming the findings of the previous configurations. Conversely, the OUTL varies (and in particular, it increases) with the increase in the error, following a roughly linear trend. The economic analysis of this scenario show that misalignments in the inventory records generate significant costs, and that, in the presence of errors, managing the warehouse can cost up to 80% more compared to a
situation free from errors. From Figure 6 it can also be appreciated that there is a very
limited difference between $C_{tot}$ and $C_{tot II}$, which indicates that acting only on the
operating leverages of the reorder policy, by adapting them to the presence of errors, does
not allow the cost of the warehouse to be reduced significantly. In turn, this outcome
suggests that most of the cost of this scenario is actually due to the error made in shipping
a higher quantity of product to the customer, which generates relevant sale losses.

5.1.4 Scenario 4

Scenario 4 of error correction describes a situation where the errors in the inbound flow
are always detected and fixed, while the errors in the outbound flows are detected and
fixed only if the quantity shipped is lower than that ordered by the final customer. The
optimal setting of this scenario, together with the resulting economic performance, is
proposed in Table 5. The trend of the total cost as a function of the error level is shown in
Figure 7.

With respect to the operating leverages of the reorder policy, the last scenario
confirms the findings on the previous ones. Specifically, the optimal reorder interval does
not vary as a function of the error level, while the OUTL increases with the increase in
the error level. Looking at the economic performance, it can be seen from Figure 7 that,
similarly to Scenario 3, $C_{tot}$ and $C_{tot I}$ almost overlap, indicating, once again, that
simply adapting the parameters of the reorder policy to the presence of errors does not
allow the total cost of the inventory management to be reduced to an appreciable extent.
Conversely, $C_{tot III}$ is significantly higher, which suggests that most of the cost is actually
due to the presence of errors. In particular, the key error component, which is not
detected and amended in the present scenario, is that due to the shipment of product in
quantity higher than that requested by the final customer.

Table 5 Optimal setting, average daily total costs (€/day) and performance parameters for EOI
policy with Scenario 4 of error correction

<table>
<thead>
<tr>
<th>$x$ [%]</th>
<th>$EOI$</th>
<th>OUTL</th>
<th>$C_{tot I}$</th>
<th>$C_{tot II}$</th>
<th>$C_{tot III}$</th>
<th>$\Delta C_1$</th>
<th>$\Delta C_2$</th>
<th>$\Delta C_2/C_{tot I}$</th>
<th>$\Delta C_1/C_{tot I}$</th>
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<td>21,855</td>
<td>8,235.48</td>
<td>8,234.10</td>
<td>5,452.37</td>
<td>1.38</td>
<td>2,783.11</td>
<td>33.79%</td>
<td>0.02%</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>21,930</td>
<td>10,980.41</td>
<td>10,974.46</td>
<td>5,452.37</td>
<td>5.96</td>
<td>5,528.04</td>
<td>50.34%</td>
<td>0.05%</td>
</tr>
<tr>
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<td>5</td>
<td>22,045</td>
<td>13,558.66</td>
<td>13,543.34</td>
<td>5,452.37</td>
<td>15.31</td>
<td>8,106.28</td>
<td>59.79%</td>
<td>0.11%</td>
</tr>
<tr>
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<td>22,120</td>
<td>16,421.52</td>
<td>16,403.29</td>
<td>5,452.37</td>
<td>18.23</td>
<td>10,969.15</td>
<td>66.80%</td>
<td>0.11%</td>
</tr>
<tr>
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<td>5</td>
<td>22,205</td>
<td>19,153.07</td>
<td>19,128.64</td>
<td>5,452.37</td>
<td>24.44</td>
<td>13,700.70</td>
<td>71.53%</td>
<td>0.13%</td>
</tr>
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<td>22,160</td>
<td>22,037.63</td>
<td>21,989.98</td>
<td>5,452.37</td>
<td>47.66</td>
<td>16,585.26</td>
<td>75.26%</td>
<td>0.22%</td>
</tr>
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<td>22,310</td>
<td>24,507.00</td>
<td>24,447.13</td>
<td>5,452.37</td>
<td>59.87</td>
<td>19,054.63</td>
<td>77.75%</td>
<td>0.24%</td>
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<td>22,315</td>
<td>27,644.57</td>
<td>27,543.11</td>
<td>5,452.37</td>
<td>101.46</td>
<td>22,192.20</td>
<td>80.28%</td>
<td>0.37%</td>
</tr>
<tr>
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<td>22,485</td>
<td>30,104.08</td>
<td>29,982.64</td>
<td>5,452.37</td>
<td>121.43</td>
<td>24,651.71</td>
<td>81.89%</td>
<td>0.40%</td>
</tr>
<tr>
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<td>5</td>
<td>22,490</td>
<td>32,229.14</td>
<td>32,057.96</td>
<td>5,452.37</td>
<td>171.18</td>
<td>26,776.77</td>
<td>83.08%</td>
<td>0.53%</td>
</tr>
</tbody>
</table>
Figure 7 Trend of the total cost (€/day) (y-axis) as a function of the error level (x-axis) for EOI policy with Scenario 4 of error correction (see online version for colours)

5.2 Results for EOQ policy

5.2.1 Scenario 1

Numerical results for the EOQ policy with Scenario 1 of error correction are reported in Table 6. The trend of the corresponding total cost is provided in Figure 8.

Table 6 optimal setting, average daily total costs (€/day) and performance parameters for EOQ policy with Scenario 1 of error correction

<table>
<thead>
<tr>
<th>$x$ [%]</th>
<th>EO1</th>
<th>OUTL</th>
<th>$C_{totI}$</th>
<th>$C_{totII}$</th>
<th>$C_{totIII}$</th>
<th>$\Delta C_1$</th>
<th>$\Delta C_2$</th>
<th>$\Delta C_2/C_{totI}$</th>
<th>$\Delta C_1/C_{totI}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11,990</td>
<td>14,255</td>
<td>6,041.07</td>
<td>6,048.74</td>
<td>5,966.53</td>
<td>−7.67</td>
<td>74.54</td>
<td>1.23%</td>
<td>−0.13%</td>
</tr>
<tr>
<td>2</td>
<td>11,985</td>
<td>14,240</td>
<td>6,085.69</td>
<td>6,087.09</td>
<td>5,966.53</td>
<td>−1.40</td>
<td>119.16</td>
<td>1.96%</td>
<td>−0.02%</td>
</tr>
<tr>
<td>3</td>
<td>12,020</td>
<td>14,335</td>
<td>6,133.40</td>
<td>6,144.99</td>
<td>5,966.53</td>
<td>−11.59</td>
<td>166.87</td>
<td>2.72%</td>
<td>−0.19%</td>
</tr>
<tr>
<td>4</td>
<td>12,045</td>
<td>14,315</td>
<td>6,174.14</td>
<td>6,177.85</td>
<td>5,966.53</td>
<td>−3.71</td>
<td>207.61</td>
<td>3.36%</td>
<td>−0.06%</td>
</tr>
<tr>
<td>5</td>
<td>12,125</td>
<td>14,330</td>
<td>6,240.93</td>
<td>6,206.01</td>
<td>5,966.53</td>
<td>34.92</td>
<td>274.39</td>
<td>4.40%</td>
<td>0.56%</td>
</tr>
<tr>
<td>6</td>
<td>12,195</td>
<td>14,325</td>
<td>6,285.11</td>
<td>6,285.10</td>
<td>5,966.53</td>
<td>16.00</td>
<td>334.57</td>
<td>5.31%</td>
<td>0.25%</td>
</tr>
<tr>
<td>7</td>
<td>12,250</td>
<td>14,320</td>
<td>6,338.63</td>
<td>6,301.56</td>
<td>5,966.53</td>
<td>37.07</td>
<td>372.10</td>
<td>5.87%</td>
<td>0.58%</td>
</tr>
<tr>
<td>8</td>
<td>12,305</td>
<td>14,375</td>
<td>6,406.46</td>
<td>6,388.93</td>
<td>5,966.53</td>
<td>17.54</td>
<td>439.93</td>
<td>6.87%</td>
<td>0.27%</td>
</tr>
<tr>
<td>9</td>
<td>12,305</td>
<td>14,335</td>
<td>6,490.45</td>
<td>6,420.74</td>
<td>5,966.53</td>
<td>69.71</td>
<td>523.92</td>
<td>8.07%</td>
<td>1.07%</td>
</tr>
<tr>
<td>10</td>
<td>12,400</td>
<td>14,430</td>
<td>6,587.88</td>
<td>6,529.04</td>
<td>5,966.53</td>
<td>58.83</td>
<td>621.35</td>
<td>9.43%</td>
<td>0.89%</td>
</tr>
</tbody>
</table>
As can be seen from Table 6, the optimal settings of the reorder policy vary, although slightly, as a function of the error level. More precisely, both the OP and the lot size increase (from 11,990 to 12,400 and from 14,255 to 14,430, respectively) with the increase in the error level. In addition, the increase in the OP follows a roughly linear trend. Looking at the economic performance, $C_{\text{tot}I}$ increases with the increase in the error level, although the impact of the inventory inaccuracy on this performance parameter does not seem to be particularly relevant. Hence, knowing the error level, and therefore adapting the reorder policy parameters to the presence of inventory inaccuracy, does not decrease the total cost of the system to an appreciable extent: even in the case the error is set at 10%, the difference between $C_{\text{tot}I}$ and $C_{\text{tot}II}$ is less than 60 €/day. Moreover, in the case the error is lower than 4%, $C_{\text{tot}I}$ and $C_{\text{tot}II}$ show almost the same values (less than 1% difference).

### 5.2.2 Scenario 2

Numerical results for the EOQ policy with Scenario 2 of error correction are reported in Table 7; the trend of the total cost as a function of the error level is shown in Figure 9.

From Table 7 it can be seen that, overall, the optimal settings of the EOQ policy vary very little with the presence of errors. Moreover, it can be seen from the same table that the optimal values oscillate, and, in particular, they seem to follow the same trend shown in the case of perfect accuracy of the inventory records. Indeed, under this scenario of error correction, the inbound flow is always checked and therefore it is not affected by errors; the outbound flow is not checked, but the (positive and negative) errors somehow compensate because of the null mean of the error set in the simulation. As per the previous scenario, the difference between $C_{\text{tot}I}$ and $C_{\text{tot}II}$ is very low, indicating, once again, that correcting the reorder policy parameters to adapt them to the presence of inaccuracies does not allow the total cost of inventory management to be significantly reduced. It can also be seen that the cost of inventory inaccuracy is lower than in the previous scenario: for instance, with error = 10%, we have $C_{\text{tot}I} = 6,077$ €/day under
Scenario 2 of error correction vs. $C_{\text{tot}1} = 6,587$ €/day under Scenario 1 of error correction. This outcome suggests that perfect alignment of physical stock and inventory records in the inbound flow could allow relevant savings in the inventory management.

### Table 7
Optimal setting, average daily total costs (€/day) and performance parameters for EOQ policy with Scenario 2 of error correction

<table>
<thead>
<tr>
<th>$x$ [%]</th>
<th>$EOI$</th>
<th>$OUTL$</th>
<th>$C_{\text{tot}1}$</th>
<th>$C_{\text{tot}II}$</th>
<th>$C_{\text{tot}III}$</th>
<th>$\Delta C_1$</th>
<th>$\Delta C_2$</th>
<th>$\Delta C_2/C_{\text{tot}I}$</th>
<th>$\Delta C_1/C_{\text{tot}I}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12,045</td>
<td>14,295</td>
<td>6,027.91</td>
<td>6,014.62</td>
<td>5,966.53</td>
<td>13.29</td>
<td>61.38</td>
<td>1.02%</td>
<td>0.22%</td>
</tr>
<tr>
<td>2</td>
<td>11,975</td>
<td>14,280</td>
<td>6,033.19</td>
<td>6,027.02</td>
<td>5,966.53</td>
<td>6.16</td>
<td>66.65</td>
<td>1.10%</td>
<td>0.10%</td>
</tr>
<tr>
<td>3</td>
<td>11,915</td>
<td>14,360</td>
<td>6,034.60</td>
<td>6,032.33</td>
<td>5,966.53</td>
<td>2.27</td>
<td>68.07</td>
<td>1.13%</td>
<td>0.04%</td>
</tr>
<tr>
<td>4</td>
<td>11,985</td>
<td>14,275</td>
<td>6,043.65</td>
<td>6,057.83</td>
<td>5,966.53</td>
<td>–14.18</td>
<td>77.12</td>
<td>1.28%</td>
<td>–0.23%</td>
</tr>
<tr>
<td>5</td>
<td>12,040</td>
<td>14,280</td>
<td>6,058.66</td>
<td>6,064.52</td>
<td>5,966.53</td>
<td>–5.86</td>
<td>92.13</td>
<td>1.52%</td>
<td>–0.10%</td>
</tr>
<tr>
<td>6</td>
<td>11,975</td>
<td>14,335</td>
<td>6,044.54</td>
<td>6,040.57</td>
<td>5,966.53</td>
<td>3.97</td>
<td>78.01</td>
<td>1.29%</td>
<td>0.07%</td>
</tr>
<tr>
<td>7</td>
<td>12,050</td>
<td>14,285</td>
<td>6,060.40</td>
<td>6,075.80</td>
<td>5,966.53</td>
<td>2.60</td>
<td>93.87</td>
<td>1.55%</td>
<td>0.04%</td>
</tr>
<tr>
<td>8</td>
<td>12,015</td>
<td>14,290</td>
<td>6,079.33</td>
<td>6,073.89</td>
<td>5,966.53</td>
<td>5.45</td>
<td>112.80</td>
<td>1.86%</td>
<td>0.09%</td>
</tr>
<tr>
<td>9</td>
<td>11,980</td>
<td>14,370</td>
<td>6,080.40</td>
<td>6,103.64</td>
<td>5,966.53</td>
<td>–23.25</td>
<td>113.86</td>
<td>1.87%</td>
<td>–0.38%</td>
</tr>
<tr>
<td>10</td>
<td>12,025</td>
<td>14,290</td>
<td>6,077.08</td>
<td>6,085.95</td>
<td>5,966.53</td>
<td>–8.88</td>
<td>110.54</td>
<td>1.82%</td>
<td>–0.15%</td>
</tr>
</tbody>
</table>

### Figure 9
Trend of the total cost (€/day) (y-axis) as a function of the error level (x-axis) for EOQ policy with Scenario 2 of error correction (see online version for colours)

5.2.3 Scenario 3

Table 8 details the numerical outcomes for the optimal settings of the EOQ policy with Scenario 3 of error correction; the graph representation of the resulting total cost is shown in Figure 10.
Table 8  Optimal setting, average daily total costs (€/day) and performance parameters for EOQ policy with Scenario 3 of error correction

<table>
<thead>
<tr>
<th>x [%]</th>
<th>EOI</th>
<th>OUTL</th>
<th>CtotI</th>
<th>CtotII</th>
<th>CtotIII</th>
<th>ΔC1</th>
<th>ΔC2</th>
<th>ΔC2/CtotI</th>
<th>ΔC1/CtotI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12,055</td>
<td>14,350</td>
<td>8,739.46</td>
<td>8,719.57</td>
<td>5,968.4</td>
<td>19.88</td>
<td>2,771.06</td>
<td>31.71%</td>
<td>0.23%</td>
</tr>
<tr>
<td>2</td>
<td>12,180</td>
<td>14,270</td>
<td>11,659.79</td>
<td>11,610.90</td>
<td>5,968.4</td>
<td>48.88</td>
<td>5,691.39</td>
<td>48.81%</td>
<td>0.42%</td>
</tr>
<tr>
<td>3</td>
<td>12,210</td>
<td>14,355</td>
<td>14,356.94</td>
<td>14,342.35</td>
<td>5,968.4</td>
<td>14.59</td>
<td>8,388.54</td>
<td>58.43%</td>
<td>0.10%</td>
</tr>
<tr>
<td>4</td>
<td>12,260</td>
<td>14,500</td>
<td>17,168.15</td>
<td>17,141.49</td>
<td>5,968.4</td>
<td>26.66</td>
<td>11,199.75</td>
<td>65.24%</td>
<td>0.16%</td>
</tr>
<tr>
<td>5</td>
<td>12,360</td>
<td>14,505</td>
<td>19,899.12</td>
<td>19,757.99</td>
<td>5,968.4</td>
<td>141.12</td>
<td>13,930.72</td>
<td>70.01%</td>
<td>0.71%</td>
</tr>
<tr>
<td>6</td>
<td>12,505</td>
<td>14,495</td>
<td>22,790.79</td>
<td>22,644.625</td>
<td>5,968.4</td>
<td>146.16</td>
<td>16,822.39</td>
<td>73.81%</td>
<td>0.64%</td>
</tr>
<tr>
<td>7</td>
<td>12,710</td>
<td>14,550</td>
<td>25,447.75</td>
<td>25,247.17</td>
<td>5,968.4</td>
<td>200.57</td>
<td>19,479.35</td>
<td>76.55%</td>
<td>0.79%</td>
</tr>
<tr>
<td>8</td>
<td>12,680</td>
<td>14,705</td>
<td>28,342.99</td>
<td>28,017.03</td>
<td>5,968.4</td>
<td>325.96</td>
<td>22,374.59</td>
<td>78.94%</td>
<td>1.15%</td>
</tr>
<tr>
<td>9</td>
<td>12,755</td>
<td>14,710</td>
<td>31,548.05</td>
<td>31,201.93</td>
<td>5,968.4</td>
<td>346.12</td>
<td>25,579.65</td>
<td>81.08%</td>
<td>1.10%</td>
</tr>
<tr>
<td>10</td>
<td>13,015</td>
<td>14,635</td>
<td>34,590.59</td>
<td>34,209.44</td>
<td>5,968.4</td>
<td>381.14</td>
<td>28,622.19</td>
<td>82.75%</td>
<td>1.10%</td>
</tr>
</tbody>
</table>

Figure 10  Trend of the total cost (€/day) (y-axis) as a function of the error level (x-axis) for EOQ policy with Scenario 3 of error correction (see online version for colours)

With this error correction mechanism, the optimal settings of the EOQ policy vary as a function of the error level; in particular, both the OP and the lot size increase with the increase in the error set in the simulation. The increase in the OP, however, does not follow a rigorous linear trend. The optimal lot size is higher than that of the previous scenarios, which indicates that, to avoid stock-out occurrence and fulfil the customer’s request, the quantity to be ordered should increase. It is worth remarking, in this respect, that in this scenario the shipments to the final customer can be affected only by positive errors, since negative ones (i.e., the situation when the quantity shipped is lower than the customer’s request) will be detected and corrected.
From an economic perspective, from Table 8 it is easy to see that the impact of inventory inaccuracies is particularly relevant. Indeed, even when setting the error at its lowest level (i.e., ±1%), the cost of the warehouse management system in presence of errors (C_{totI}) exceeds the corresponding cost under absence of error (C_{totIII}) by more than 30%. With the error set at ±2%, the difference between C_{totI} and C_{totIII} reaches 50%. Obviously, such a high C_{totI} is due to the fact that the warehouse ships more product than that requested to the customers, incurring in relevant sale losses. Knowing the inaccuracy of inventory records allows to decrease the cost of the system only if the error level is higher than 5%; nonetheless, the potential cost reduction is quite limited, reaching 1% at most.

5.2.4 Scenario 4

The optimal settings of the EOQ policy with Scenario 4 of error correction, together with the resulting economic performance, are proposed in Table 9; their graph representation is shown in Figure 11. We recall that, in this scenario, the errors in the inbound flow are always detected and fixed, while the errors in the outbound flow are detected and fixed only when the quantity shipped is lower than that ordered by the final customer.

It can be seen from Table 9 that the optimal settings of the reorder policy in presence of error vary compared to the situation of perfect alignment of inventory data. Although it is not immediate to identify a relationship between OP, lot size and error level, the results in Table 9 show that the EOQ parameters tend to increase with the increase in the error level. From an economic perspective, the cost of inventory management increases with the increase in the inaccuracy of inventory data: with the error level set at ±1%, the inventory inaccuracy generates an increase of more than 30% in the total cost of the system, compared to a situation free from errors. When the error level is higher (e.g., 8%), the increase in the total cost can reach 80%. Knowing the error level allows the cost of the system to be partially reduced, although the decrease is appreciable only with error higher than ±5%.

<table>
<thead>
<tr>
<th>$x$ [%]</th>
<th>$EOI$</th>
<th>$OUTL$</th>
<th>$C_{totI}$</th>
<th>$C_{totII}$</th>
<th>$C_{totIII}$</th>
<th>$\Delta C_1$</th>
<th>$\Delta C_2$</th>
<th>$\Delta C_2/C_{totI}$</th>
<th>$\Delta C_1/C_{totI}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12,025</td>
<td>14,285</td>
<td>8,756.61</td>
<td>8,754.37</td>
<td>5,966.53</td>
<td>2.23</td>
<td>2,790.07</td>
<td>31.86%</td>
<td>0.03%</td>
</tr>
<tr>
<td>2</td>
<td>12,040</td>
<td>14,355</td>
<td>11,554.11</td>
<td>11,526.03</td>
<td>5,966.53</td>
<td>28.08</td>
<td>5,587.57</td>
<td>48.36%</td>
<td>0.24%</td>
</tr>
<tr>
<td>3</td>
<td>12,125</td>
<td>14,345</td>
<td>14,297.35</td>
<td>14,304.09</td>
<td>5,966.53</td>
<td>–6.74</td>
<td>8,330.82</td>
<td>58.27%</td>
<td>–0.05%</td>
</tr>
<tr>
<td>4</td>
<td>12,145</td>
<td>14,420</td>
<td>16,957.02</td>
<td>16,919.56</td>
<td>5,966.53</td>
<td>37.46</td>
<td>10,990.49</td>
<td>64.81%</td>
<td>0.22%</td>
</tr>
<tr>
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<td>12,115</td>
<td>14,525</td>
<td>19,795.87</td>
<td>19,779.11</td>
<td>5,966.53</td>
<td>16.76</td>
<td>13,829.34</td>
<td>69.86%</td>
<td>0.08%</td>
</tr>
<tr>
<td>6</td>
<td>12,190</td>
<td>14,600</td>
<td>22,674.23</td>
<td>22,624.79</td>
<td>5,966.53</td>
<td>49.44</td>
<td>16,707.69</td>
<td>73.69%</td>
<td>0.22%</td>
</tr>
<tr>
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<td>12,445</td>
<td>14,515</td>
<td>25,072.72</td>
<td>24,985.87</td>
<td>5,966.53</td>
<td>86.85</td>
<td>19,106.19</td>
<td>76.20%</td>
<td>0.35%</td>
</tr>
<tr>
<td>8</td>
<td>12,405</td>
<td>14,625</td>
<td>28,089.21</td>
<td>27,955.09</td>
<td>5,966.53</td>
<td>134.12</td>
<td>22,122.68</td>
<td>78.76%</td>
<td>0.48%</td>
</tr>
<tr>
<td>9</td>
<td>12,460</td>
<td>14,635</td>
<td>30,601.31</td>
<td>30,482.07</td>
<td>5,966.53</td>
<td>119.25</td>
<td>24,634.78</td>
<td>80.50%</td>
<td>0.39%</td>
</tr>
<tr>
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<td>12,410</td>
<td>14,605</td>
<td>32,938.47</td>
<td>32,731.99</td>
<td>5,966.53</td>
<td>206.49</td>
<td>26,971.94</td>
<td>81.89%</td>
<td>0.63%</td>
</tr>
</tbody>
</table>
Discussion and conclusions

Data accuracy is of fundamental importance for inventory management, because nowadays inventory management is almost entirely based on information stored in warehouse management systems. As a consequence, ensuring correspondence between the inventory records and the stock level is vital to make sure that the reorder process is managed correctly.

In line with this consideration, this study has proposed an evaluation of the impact of inventory inaccuracies on the total cost of inventory management for a warehouse. The analysis carried out has targeted the errors in inbound and outbound flows of the warehouse, i.e., the inaccuracies made when recording inventory data relating to product received at the warehouse and product shipped to the final customer. A simulation model was developed under Microsoft Excel™ to reproduce the reorder process of the warehouse and the generation of inventory inaccuracies, as well as to compute the total cost of inventory management. The outcomes of the simulation allowed to estimate:

- the optimal (minimum cost) settings of each reorder policy in absence of error and as a function of the error level
- the total cost of the inventory management in absence of error, in presence of error, and in the case the error is known to the warehouse manager, yet it is not removed.

The results obtained show interesting practical implications for warehouse managers. First, we found that not always the optimal settings of the reorder policy vary in presence of errors in inventory records. To be more precise, under almost all the EOI scenarios the optimal reorder interval is not affected by the error level. Therefore, when adopting such policy, the warehouse manager does not need to reset the reorder interval because of errors in the inventory records. Similar considerations hold true for the OUTL of
Inventory management in the presence of inventory inaccuracies

inventory, whose variation, as a function of the error level, is very limited. The EOI policy is very widely used in practical contexts; indeed, the common practice in retailing to replenish inventories frequently (e.g., daily or weekly) suggests the use of EOI as the replenishment strategy in grocery retailing (Disney and Lambrecht, 2008; Chen and Disney, 2007). Therefore, these outcomes have interesting consequences for the practical management of inventories in retailing.

A further practical consideration from the analyses made is that, in most of the scenarios examined, simply knowing the inventory inaccuracy does not generate appreciable savings in the cost of warehouse management. Indeed, knowing the error level would allow the warehouse manager to adapt the parameters of the reorder policy to the presence of inaccuracies of inventory records, thus operating with the optimal settings, but without correcting the error itself. Adapting the parameters of the reorder policy to the error level, however, does not decrease the cost of the warehouse to an appreciable extent. Therefore, just knowing the error level is not sufficient for the cost of inventory management to be reduced. Conversely, counteracting the inventory inaccuracy, by realigning the physical stock to the information flow, appears as the most effective strategy to reduce the cost of inventory management. For instance, moving from Scenario 1 (partial realignment of the inbound flows) to Scenario 2 (full realignment of the inbound flows) of error correction involves a relevant decrease in the total cost of inventory management, which becomes very similar to the situation free from errors. Therefore, a correction mechanism focusing on the full realignment of the inbound flow could be considered for practical implementation with the purpose of decreasing the total cost of inventory.

More in general, actions that can be undertaken to improve the accuracy of inventory records can be grouped into different sets, according to DeHoratius and Raman (2008) (2008). ‘Prevention’ strategies aim at removing the causes of inventory inaccuracy, by enhancing the workforce training and improving its skills in identifying products, labelling them correctly, stocking them in the right place and so on. These strategies are expected to decrease the error, but not to remove it completely; indeed, job rotation, human actions or dysfunctions of the equipment used for labelling or identification of product can always introduce errors in the system. To complement ‘prevention’ strategies, ‘control’ strategies are often suggested. These strategies aim at identifying misalignments between the physical and information flows and to correct them. For instance, periodic inventory counts (besides the annual one) can be made by a company to support the reorder process on some key products. Obviously, implementing control strategies, as well as any other intervention aimed at decreasing the inaccuracy of inventory records, can involve relevant cost to a company. In this regard, the results of this study can be useful to estimate the potential savings generated by removing inventory inaccuracies and justify the implementation of tailored interventions.

From a theoretical perspective, the simulation model developed in this paper complements the published literature, in that it is quite articulated and enables a detailed analysis of the inventory inaccuracy issues. Indeed, the model considers: ten different error levels, either positive (i.e., the quantity of product recorded is higher than that stored or shipped) or negative (i.e., the quantity of product recorded is lower than that stored or shipped); three order correction mechanisms; and two reorder policies, i.e., the EOQ and EOI. Overall, by combining the reorder policy and the error correction mechanisms, the model allowed to analyse eight scenarios and to derive the optimal combination of the reorder policy parameters for each scenario, as a function of the error
level. The use of simulation appears as particularly effective in reproducing the decision process of the warehouse investigated, as well as to reproduce the occurrence of errors in the physical or information flows. The strong points of simulation refer, in particular, to the possibility of reproducing numerous scenarios (e.g., by changing the operating leverages of the inventory management policy or the level of error) in a relatively short time, thus allowing to derive conclusions from the analysis of different operating conditions of the warehouse. Moreover, although there are some studies in literature that investigate the issue of inventory inaccuracy, very few quantitative analyses are available.

Future research activities can be directed to the analysis of a more structured supply chains, or of alternative reorder policies, or of different scenarios for the correction of inventory inaccuracies, to gain a more complete understanding of this issue.

References


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**Appendix 1**

1.1 Detailed equations for the EOI and EOQ reorder policies

1.1.1 Nomenclature

In addition to the variables defined in Table 1, the nomenclature in Table A1 is used to detail the equations embodied in the simulation model.
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Table A1  Nomenclature used to detail the decision process

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit of measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subscripts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>i</td>
<td>Simulation day ((i = 1, \ldots N_{\text{days}}))</td>
<td>-</td>
</tr>
<tr>
<td>(i, \text{II, III})</td>
<td>Steps of the inventory update</td>
<td>-</td>
</tr>
<tr>
<td>Superscripts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P)</td>
<td>Physical</td>
<td>-</td>
</tr>
<tr>
<td>(T)</td>
<td>Theoretical</td>
<td>-</td>
</tr>
<tr>
<td>Simulation parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(N_{\text{days}})</td>
<td>Simulation duration ((\text{days}))</td>
<td></td>
</tr>
<tr>
<td>(N_{\text{orders}})</td>
<td>Number of orders generated during the simulation ((\text{orders}))</td>
<td></td>
</tr>
<tr>
<td>(N_{\text{alerts}})</td>
<td>Number of alerts generated during the simulation</td>
<td>-</td>
</tr>
<tr>
<td>(N_{\text{inv_counts}})</td>
<td>Number of general inventory counts during the simulation ((\text{inventory counts}))</td>
<td></td>
</tr>
<tr>
<td>Warehouse parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(G_i^T, G_i^P)</td>
<td>Theoretical (i.e., recorded in the company’s information system) and physical stock of product at day (i) ((\text{pallets}))</td>
<td></td>
</tr>
<tr>
<td>(D_i^T, D_i^P)</td>
<td>Theoretical and physical demand of product at day (i) ((\text{pallets}))</td>
<td></td>
</tr>
<tr>
<td>(O_i^T, O_i^P)</td>
<td>Theoretical and physical order issued to the supplier at day (i) ((\text{pallets}))</td>
<td></td>
</tr>
<tr>
<td>(S_i^T, S_i^P)</td>
<td>Theoretical and physical shipment to the customer at day (i) ((\text{pallets}))</td>
<td></td>
</tr>
<tr>
<td>(x_i)</td>
<td>Error in a generic transaction (demand, order or shipment) at day (i) ((%)</td>
<td></td>
</tr>
<tr>
<td>(Q_{\text{stock-out}, i})</td>
<td>Amount of out-of-stock product at day (i) ((\text{pallets}))</td>
<td></td>
</tr>
</tbody>
</table>

1.1.2 EOQ reorder process

Because of the similarities of the decision process of the EOQ policy across the different scenarios of error correction, we describe in detail the decision process under Scenario 1 and limit the description of the remaining scenarios to the differences observed compared to this scenario.

1.1.2.1 Scenario 1

- **Start of the process.** According to the scheme in Figure 3, the process is triggered by the demand received from the final customer, \(D_i^P\). As we assumed that the demand data is correctly received and recorded in the company’s information system, we always have \(D_i^P = D_i^T\). Once the warehouse has received the request, the first check made is whether the demand can be fulfilled with the available inventory. As mentioned, the inventory record stored in the company information system is used to
make such evaluation. Therefore, the first check made reduces to assess whether \( D_i^p \leq G_{i-1}^T \).

- **Order fulfilment.** Whenever \( D_i^p \leq G_{i-1}^T \), the demand will be fulfilled. Because of possible errors in the delivery process, however, the shipment can differ from the request of the customer; in particular, the following set of equations applies:

\[
S_i^T = D_i^p \\
S_i^p = D_i^p \times (1 + x_i)
\]

(1)

We recall that, in this scenario, there is no correction on the outbound flow; hence, \( S_i^p \neq S_i^T \). The inventory position in the company’s information system will be updated as follows (first update):

\[
G_{i-1}^T = G_{i-1} - S_i^T + O_i^T
\]

(2)

which takes into account also the amount of product ordered but not yet received at the warehouse. Conversely, the physical stock available will account for:

\[
G_i^p = G_i^p - S_i^p - O_{i-LT}
\]

(3)

- **Out-of-stock.** If \( D_i^p > G_{i-1}^T \), the demand of the customer cannot be fulfilled, resulting in an out-of-stock situation. The corresponding out-of-stock quantity accounts for \( Q_{stock-out} = D_i^p - G_{i-1}^T \) and will be backlogged. Under that circumstance, the inventory will be checked and realigned; therefore, we will have (second update):

\[
G_{i-1}^T = G_{i-1}^T
\]

(4)

- **Order issuing.** Regardless of the possibility to fulfill the customer’s request, the next step is to check whether an order is required. According to the EOQ policy, orders are made if the inventory level is lower than OP; therefore, the check reduces to assess whether \( G_i^T \leq OP \). Whenever \( G_i^T > OP \), no orders will be placed and the process ends. Conversely, if \( G_i^T \leq OP \) an order will be issued to the supplier, for a defined quantity (lot size). It is assumed that the supplier always provides the right amount of product ordered, i.e., \( O_i^p = EOQ \); nonetheless, errors can be made in recording the information about the inbound flow.

- **Inventory update.** In this scenario, an alert is generated if the quantity received at the warehouse is lower than that expected, i.e., if the quantity recorded in the company’s information system is lower than EOQ. Under that circumstance, the product received is checked and the error is corrected. Overall, the transition equations for the theoretical and physical inventory position (third update) are as follows:

\[
G_{i-1,iii}^T = G_{i-1}^T - S_i^T + O_i^T = G_{i-1}^T - S_i^T + EOQ \times x_i
\]

\[
G_{i-1,iii}^p = G_{i-1}^p - S_i^p + O_{i-LT} = G_{i-1}^p - S_i^p + EOQ \quad \text{if } O_i^T \geq EOQ
\]

\[
G_i^T = G_i^T - S_i^T + O_i^T = G_i^T - S_i^T + EOQ
\]

(5)
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\[ G_{i,ii}^{P} = G_{i-1}^{P} - S_{i}^{P} + O_{i-LT}^{P} = G_{i-1}^{P} - S_{i}^{P} + EOQ \quad \text{if } O_{i}^{T} < EOQ \]  

(6)

Obviously, the update in the inventory record because of an alert, as described in equations (5) to (6), will be made once the product is received at the warehouse. Inventory update ends the process.

1.1.2.2 Scenario 2

Scenario 2 differs from Scenario 1 in that the correction of the inbound flow is made anytime there is a difference (either positive or negative) between the quantity ordered by the warehouse and that recorded in the company’s information system. Therefore:

- **Start of the process.** See Scenario 1.
- **Order fulfilment.** See Scenario 1.
- **Out-of-stock.** See Scenario 1.
- **Order issuing.** See Scenario 1.
- **Inventory update.** The final (third) update of the inventory will be made as follows:

\[ G_{i,iii}^{T} = G_{i-1}^{T} - S_{i}^{T} + O_{i-LT}^{T} = G_{i-1}^{T} - S_{i}^{T} + EOQ \]

\[ G_{i,iii}^{P} = G_{i-1}^{P} - S_{i}^{P} + O_{i-LT}^{P} = G_{i-1}^{P} - S_{i}^{P} + EOQ \]  

(7)

Again, the update in the inventory record because of an alert will be made once the product is received at the warehouse.

1.1.2.3 Scenario 3

Scenario 3 differs from Scenario 1 in that the correction of the outbound flow is introduced. Specifically, if the amount of product shipped by the warehouse is lower than that expected, the customer will alert the company and ask for the remaining quantity of product. Therefore:

- **Start of the process.** See Scenario 1.
- **Order fulfilment.** If the order is fulfilled, whenever \( S_{i}^{P} < S_{i}^{T} \) the quantity shipped to the final customer will be checked and the error amended. Therefore, equation (1) should be rewritten as follows:

\[ S_{i}^{T} = D_{i}^{p} \]

\[ S_{i}^{P} = D_{i}^{p} \ast(1 + x_{i}) = \begin{cases} D_{i}^{p} \ast(1 + x_{i}) & \text{if } x_{i} \geq 0 \\ D_{i}^{p} & \text{if } x_{i} < 0 \end{cases} \]  

(8)

Equation (3), describing the (first) inventory update, can still be used, with the above definition of \( S_{i}^{P} \) and \( S_{i}^{T} \).
• Out-of-stock. See Scenario 1.
• Order issuing. See Scenario 1.
• Inventory update. See Scenario 1

1.1.2.4 Scenario 4
Under Scenario 4, alerts are generated for both the outbound flows (as per Scenario 3) and the inbound flow (as per Scenario 2). Therefore, overall, the decision process of this scenario can be described as follows:

• Start of the process. See Scenario 1.
• Order fulfilment. See Scenario 3.
• Out-of-stock. See Scenario 1.
• Order issuing. See Scenario 1.
• Inventory update. See Scenario 2.

1.1.3 EOI reorder process
As per the EOQ reorder process, in the following subsections we detail the decision process of the EOI policy under Scenario 1 and limit the description of the remaining scenarios to the differences observed compared to the first scenario.

1.1.3.1 Scenario 1
• Start of the process. The start of the process does not differ from that described for the EOQ policy.
• Order fulfilment. Equations (1) to (3 used for the EOQ policy apply also to the EOI policy.
• Out-of-stock. Equation (4) holds also for the EOI policy.
• Order issuing. Under the EOI policy, the decision whether to place an order is made as follows. The inventory level $G_i^{t, j}$ is checked at regular time intervals, which are integer multiples of the reorder interval (i.e., at day $i = a_{EOI}, a \in N$). If $i \neq a_{EOI}$, inventory is not checked and no orders will be placed. Conversely, if the inventory is checked, the warehouse manager will place an order to the supplier, with the purpose of restoring the target inventory level. The quantity ordered thus accounts for $PT_i^{t, i} = O_i^{t} - G_{i, j}^{t}$, As mentioned, it is assumed that the supplier delivers the right quantity of product; nonetheless, errors can be made in updating the inventory records after the product is received.
• Inventory update. In Scenario 1, an alert is generated if the quantity received at the warehouse is lower than that expected, i.e., if the quantity recorded in the company’s information system $O_i^{t} = O_i^{t} \ast x_i$ is lower than $O_i^{t}$. Under that circumstance, the product received is checked and the error is corrected. Overall, the transition
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Equations for the theoretical and physical inventory position (third update) are as follows:

\[ G_{i,III}^T = G_{i-1}^T - S_i^T + O_i^T = G_{i-1}^T - S_i^T + O_i^T \times x_i \]

\[ G_{i,III}^P = G_{i-1}^P - S_i^P + O_{i-LT}^P = G_{i-1}^P - S_i^P + EOQ \quad \text{if } O_i^T \geq O_i^P \]

\[ G_{i,III}^T = G_{i-1}^T - S_i^T + O_i^T = G_{i-1}^T - S_i^T + O_i^P \]

\[ G_{i,III}^P = G_{i-1}^P - S_i^P + O_{i-LT}^P = G_{i-1}^P - S_i^P + O_i^P \quad \text{if } O_i^T < O_i^P \]

As per the EOQ policy, the update in the inventory record because of an alert equations (9) to (10), will be made once the product is received at the warehouse. Inventory update ends the process.

1.1.3.2 Scenario 2

We recall that the main difference between Scenarios 1 and 2 is that, in this latter one, the correction of the inbound flow is made anytime there is a difference (either positive or negative) between the quantity ordered by the warehouse and that recorded in the company’s information system. Therefore, the decision process is as follows:

- **Start of the process.** See Scenario 1.
- **Order fulfillment.** See Scenario 1.
- **Out-of-stock.** See Scenario 1.
- **Order issuing.** See Scenario 1.
- **Inventory update.** The final (third) update of the inventory is made as follows:

\[ G_{i,III}^T = G_{i-1}^T - S_i^T + O_i^T = G_{i-1}^T - S_i^T + O_i^P \]

\[ G_{i,III}^P = G_{i-1}^P - S_i^P + O_{i-LT}^P = G_{i-1}^P - S_i^P + O_i^P \]

1.1.3.3 Scenario 3

Under Scenario 3, the correction of the inventory records is made also to the outbound flow. Specifically, if the amount of product shipped by the warehouse is lower than that expected, the customer will alert the company and ask for the quantity lacking. Therefore:

- **Start of the process.** See Scenario 1.
- **Order fulfillment.** If the order is fulfilled, whenever \( S_i^P < S_i^T \) the quantity shipped to the final customer will be checked and the error corrected. Therefore, equation (1) should be rewritten as follows:
Equation (3), describing the (first) inventory update, can still be used, with the above definition of $S_{i}^{P}$ and $S_{i}^{T}$.

- **Out-of-stock.** See Scenario 1.
- **Order issuing.** See Scenario 1.
- **Inventory update.** See Scenario 1

### 1.1.4 Scenario 4

Under Scenario 4, alerts are generated for both the outbound flows (as per Scenario 3) and the inbound flow (as per Scenario 2). Therefore, overall, the decision process of this scenario can be described as follows:

- **Start of the process.** See scenario 1.
- **Order fulfilment.** See scenario 3.
- **Out-of-stock.** See scenario 1.
- **Order issuing.** See scenario 1.
- **Inventory update.** See scenario 2.

### Appendix 2

#### 2.1 Cost computation

The detailed equations for the computation of the cost components listed in Section 4.2.1 and of the total cost of inventory management are proposed below.

\[
C_{\text{stocks}} = \text{inventory holding cost} = \frac{\sum_{i=1}^{N_{\text{days}}} G_{i}^{P} \cdot c_{\text{stocks}}}{N_{\text{days}}} \text{ (€ / day)} \quad (13)
\]

\[
C_{\text{handling}} = \text{cost of material handling} \quad (€ / day)
\]

\[
= \frac{\sum_{i=1}^{N_{\text{days}}} \left( G_{i}^{P} + S_{i}^{P} + O_{i \rightarrow \text{LT}}^{P} \right) \cdot c_{\text{handling}}}{N_{\text{days}}} \quad (14)
\]

\[
C_{\text{order}} = \text{order cost} = \frac{\rho_{\text{orders}} \cdot c_{\text{order}}}{N_{\text{days}}} \text{ (€ / day)} \quad (15)
\]
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\[ C_{stock-out} = \text{cost of stock-out} = \frac{\sum_{i=1}^{N_{days}} Q_{stock-out,i} \cdot c_{stock-out}}{N_{days}} \text{ (€ / day)} \]  

(16)

\[ C_{check} = \text{cost of inventory check} \\
= \frac{n_{alerts} \cdot c_{check} + n_{inv\_counts} \cdot c_{inv\_count}}{N_{days}} \text{ (€ / day)} \]  

(17)