A new algorithm for synthesising locally most consistent priorities in analytic hierarchy process for group decision making

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Abstract: The analytic hierarchy process (AHP) is an efficient tool in supporting group decision making processes because it is offering various possibilities to aggregate individual opinions, judgments and/or priorities into group equivalents, allowing control over the consistency of participating individuals. This paper proposes a straightforward procedure to locally identify: 1) the most consistent decision maker; 2) an associated prioritisation method; 3) the corresponding best local priority vector (set of weights in given node of hierarchy), to be used for the final AHP synthesis. Minimum total Euclidean distance as a universal consistency measure for the hierarchy is guaranteed on the group of decision makers and on a set of most used matrix and optimisation prioritisation methods. The algorithm is named as AHP-DPE and is applicable to any complete hierarchy (which is a philosophical pillar of AHP), for any size of a group and for any number of prioritisation methods. Selected example from agriculture illustrates how proposed methodology can be efficiently applied to obtain trustful solution in the group decision making context.

Keywords: group decision making; GDM; new algorithm for AHP synthesis; complete hierarchy; Euclidean distance.

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1 Introduction

Group decision making (GDM) when supported by AHP – analytic hierarchy process (Saaty, 1980), recently called AHP-GDM – offers different possibilities for aggregating individual opinions, judgments and/or priorities into so-called group equivalents. The final stage of any aggregation is to determine the weights of alternatives with respect to the stated goal while respecting a given set of criteria (possibly also sub-criteria, sub-sub-criteria, etc.). Reported results of research on different approaches indicate that there are many options for synthesising individual decisions within a group in real-life situations and decision-making scenarios. However, there exist many open questions that are shared among these studies, such as: how to treat differences in decision makers' (DMs) knowledge, education, and willingness to participate? Is it justifiable to allocate different weights to DMs when aggregating their opinions based on their demonstrated consistency? How to measure (in)consistencies of a group of DMs both on the level of the individual and as participants in a group and how to relate that information to the prioritisation methods used? How to measure quality of group aggregations of judgments/priorities at either individual comparison (AHP pairwise) matrix, or, even more important, complete hierarchy of the decision-making problem?

Regarding the final outcome of the AHP-GDM, some aggregations are straightforward, some are not. Scientific discussion about aggregation schemes is underway and many researchers offer various solutions (e.g., Forman and Peniwati, 1998; Chiclana et al., 2001; Beynon, 2005; Tsyganok, 2010; Aguaron et al., 2016; Escobar and Moreno-Jimenez, 2007; Dong et al., 2010; Dong and Zhang, 2014; Srdjevic, 2005; Xu and Cai, 2011; Myronidis et al., 2016). Detail literature review showed that it is not an easy task to draw possible deficiencies in current AHP-GDM aggregations and to get a stronghold on the shortcomings of previous approaches. Neither completely and consistently responds to some of above put questions can be found in cited sources, which is probably also valid in case of an approach we developed here. In fact, we refer to our earlier published approach called multi-criteria group prioritisation synthesis within AHP (MGPS algorithm) (Srdjevic and Srdjevic, 2013) and offer its generalised version, called hereafter as AHP-DPE synthesis algorithm (acronym of AHP – DMs – prioritisation methods – Euclidean distance). We found that AHP-DPE has an advantage over MGPS in that the global (universal) consistency measure, namely the total Euclidean L^2 distance (TD), is minimised over the complete hierarchy of the decision problem, across all most commonly used prioritisation methods, and across all DMs participating in the group. This norm is used as a unique criterion which helps to distinguish opinions of DMs and priorities produced by prioritisation methods and to point out where is the minimum difference between semantic/numerical judgments of various decision elements elicited from DMs and computed final weights of alternatives (by different prioritisation methods) with respect to the stated goal.

The main feature of AHP-DPE synthesis algorithm is that it performs across the complete hierarchy of the problem, not at just one local matrix. This is important because it complies with the fundamental logic of AHP: solve the decision-making problem by working at its complete hierarchy until the very end when the final priorities (weights) of alternatives at the tip of the hierarchy are obtained by synthesising the local priorities of other decision elements (such as criteria or sub criteria) in upper levels of the hierarchy. In short, the leading idea is that at each node of the hierarchy all DMs make pair-wise comparisons to set their semantic judgments and in turn produce individualised numerical comparison matrices. If all matrices are subjected to a set of selected (most commonly used) prioritisation methods, a set of corresponding priority vectors can be generated and TD can be immediately computed. Based on the minimum value of TD at a given node obtained by any prioritisation method and by any DM, the corresponding priority vector can be selected for the final AHP synthesis. The process is repeated at all nodes of the hierarchy and through standard AHP synthesis produces the group priority vector with absolute minimum TD. If inconsistency is measured by TD only, AHP-DPE synthesis guarantees that the group priority vector remains the most consistent throughout the contexts of the hierarchy, prioritisation methods and the DMs.

The AHP-DPE synthesis works quite well and it is directly related to earlier research by Srdjevic (2005), and by Srdjevic et al. (2013). Although some contexts are similar, at least GDM supported by AHP, other approaches we could find in pertinent literature (such as iterative improvements of priorities for one matrix at a time, group aggregations, consensus models and else) are not directly comparable with AHP-DPE.

The structure of the paper is as follows. The outline of the decision-making problem and brief state-of-art review is given in Section 2. The proposed synthesis algorithm is described in Section 3. Section 4 evaluates this method by presenting the results obtained by AHP-DPE in example related to agriculture. Conclusions given in Section 5 are followed by selected references.

2 Decision problem and group decision

2.1 Context

The research activities in GDM are easily visible from mid-90s of the last century, but, as correctly pointed in Kabak and Ervural (2017), have dramatically increased over the last decade. Although the related literature is rapidly growing, still there is no systematic classification scheme for these researches and authors of this paper presented a generic conceptual framework and a classification scheme for doing that based on a set of top cited papers. A classification scheme included GDM methodologies widely used in many diverse and/or interconnected fields such as preference analysis, utility theory, social choice theory, game theory, expert evaluation analysis, aggregation of qualitative factors, economic equilibrium theory, etc. Authors also give some useful suggestions for future research.

There are many different decision making scenarios treated in different contexts, varying from simplest individual single criterion decision situations to group multi-criteria decision making processes. If multiple actors are involved in making a decision, then various methodologies can be applied to derive a single common decision, usually called the group decision. Multi-criteria (multi-objective) based approaches and social choice theory (voting) based approaches are two typical classes of approaches for deriving a group decision, with many extensions in directions such as: consensus and no-consensus; fairness and dictatorship; consistency and inconsistency (individual agents and group); positive, negative and neutral manipulation of the decision process; a priori and a posterior aggregations of individual judgments and/or priorities (e.g., aggregation of individual nodal judgments – AIJ, and aggregation of the final individual priorities – AIP, both in AHP); and weighting or equalising importance of agents in the group. Research in this field is continuously reported in scientific papers and discussion about best approaches and solutions for different decision-making scenarios and broader contexts is interesting and expectedly controversial simply because of humans behaviour, attitudes and other components of their background, either they are involved as decision-makers, modellers or process moderators.

From contextual point of view, worth to mention is the work reported in Kou et al. (2016) as a review of the literature with focus on the main developments in treating pairwise comparison matrices created in each node of the AHP hierarchy. Measurement scales, consistency index, inconsistency issues, missing judgment estimation and priority derivation methods are analysed; all these aspects have been extensively studied using data from 37 peer reviewed international journals from 2010 to 2015 (searched via ISI Web of Science). Although comprehensive, it seems to be incomplete because there are many researches underway which are not (or will not ever be) published in top ranked international journals.

What follows is a brief state-of-the art review of researches in diverse directions of the AHP application in individual and GDM frameworks, primarily those in agricultural engineering and management. Both aspects relevant for this paper are treated:

1 prioritisation methods in AHP

2 aggregation of nodal weights (priorities) obtained from participating DMs in the group usage of complete AHP.

2.2 Prioritisation methods

For performing prioritisation in each node of AHP hierarchy there are many well-known methods ranging from simple matrix calculus to linear and nonlinear optimisation. One of first comparison analysis presented by Mikhailov and Singh (1999) included five methods (EV-eigenvector, WLS – weighted least squares; LLS – logarithmic least squares, FPP – fuzzy preference programming and LGP – logarithmic goal programming) applied to large number of generated pairwise comparison matrices of sizes up to 7. They concluded that analysed prioritisation methods perform differently regarding the selected evaluation criteria. Similar conclusion is derived after comprehensive analysis performed by Srdjevic (2005) who searched for optimal combination of different prioritisation methods to perform AHP synthesis in individual application. In later research, LGP method is excluded and AN – additive normalisation included for combining five methods in synthesis. Analysing results of multiple examples, we concluded that in applications of either matrix methods (AN, EV), or optimisation methods (LLS, WLS and FPP) different results could be produced and that there are no proofs that any of these models is better from the others. For instance, it was shown that even the simplest one, AN, may outperform more sophisticated models if TD, or order reversal indicator known as minimum violations (MV) criterion (Golany and Kress, 1993; Chiclana et al., 2001), is used as consistency measure. Main conclusion of Golany and Kress (1993) is that there is no prioritisation method that is superior to the others in all cases, and that the choice of the prioritisation method should be dictated by the objective of the analysis. As pointed in Srdjevic (2005), this conclusion relates to only evaluating criteria and measures of a quality of estimations made, and that once judgments are elicited from the DM (or generated), the remaining is simply a mathematical manipulation of the numerical equivalents of judgments contained in the matrix which has nothing in common with objectives of the analysis. Similar conclusion is derived in Yuen (2010) who analysed a problem of selecting prioritisation method. A list of seven measurement criteria is adopted to enable selection of the most appropriate prioritisation method among nine such methods. The results obtained at several examples from literature showed once more again that the most appropriate prioritisation method is dependent of the content of the reciprocal matrix and clear indication of which prioritisation model and consistency measuring mechanism is left as an open issue for future discussion.

New developments in this subject area led to recently proposed prioritisation method named the cosine maximisation (Kou and Lin, 2014) (from now on COS), with associated measure of consistency and defined tolerance level in range 0.9–1. The COS method is included as a module in our MCDM/AHP software named DECIDE and tested over millions of generated matrices of sizes 5, 6, and 7 satisfying eigenvector consistency check-up criterion CR (up to 0.10 and 0.15). Compared to methods AN, EV, LLS, WLS and FPP, this method produced lower values of consistency indicators in significant number of matrices so we decided to include it in our set of prioritisation methods for AHP-DPE synthesis. An example in the Section 4 justifies the decision we made.

For the sake of completeness, worth to mention is that Khatwani and Kar (2016) listed 14 more methods besides six methods we selected to demonstrate how our algorithm performs. More complete list includes: gradient eigen-weight and least-distance method, geometric least squares method, goal (and logarithmic goal) programming method, linear programming method, data envelopment analysis method, correlation coefficient maximisation, Bayesian prioritisation procedure, and heuristics and re-evaluation-based method. Obviously, it is hard to theoretically prove that inclusion or exclusion of any of listed prioritisation methods into our algorithm could not have an impact on the final solution. Our tests showed that changes are not significant because our algorithm in a way performs fine tuning of the AHP synthesis across similar vectors generated by different methods. In fact, different methods produce similar vectors in most cases, without order reversal. Small differences in weights of decision elements (criteria and/or alternatives) produced by different prioritisation methods in our multiple tests led to a conclusion that if certain method is excluded from the selection process, a group decision structure (based on values of TD measure) may only slightly change which will not significantly influence the final outcome – priorities of alternatives versus goal.

2.3 Aggregation methods

There are several well-known base aggregation schemes in AHP applications aimed to support GDM processes and produce the group decision as a final outcome. Aggregations of individual nodal judgments (AIJ) and the final individual priorities (AIP) are discussed in Forman and Peniwati (1998). Consensus-based models of aggregation are described in Tavana et al. (1996), Regan et al. (2006), Moreno-Jiménez et al. (2008) and Dong et al. (2010) with many extensions in different directions, e.g., considering the mutual respect of agents in the group (Srdjevic et al., 2013; Dragincic et al., 2015), iterative modification of individual judgments while reaching certain consistency criterion (Herrera-Viedma et al., 2002; Chiclana et al., 2008) or handling ordinal information about priorities of alternatives in various ways in a group contexts. An interesting aggregation procedure for obtaining a priority vector within AHP supported GDM processes is presented in Blagojevic et al. (2016). A heuristic aggregation procedure is proposed to obtain a group priority vector at any node of an AHP hierarchy. Simulated annealing algorithm is used to perform aggregation process by minimising the group Euclidean distance as a consensus measure across both group weights and judgments. The group vector obtained in this way is invariant to any prioritisation method, i.e., there is no need to have individual priority vectors as is required by some other aggregation procedures. Comparison analysis on five examples (taken from literature) is performed with respect to several reported combinations of different prioritisation methods and group aggregation procedures. It is shown that proposed heuristic procedure performs better or at least equally to several other well-known combinations of prioritisation and aggregation in AHP GDM frameworks.

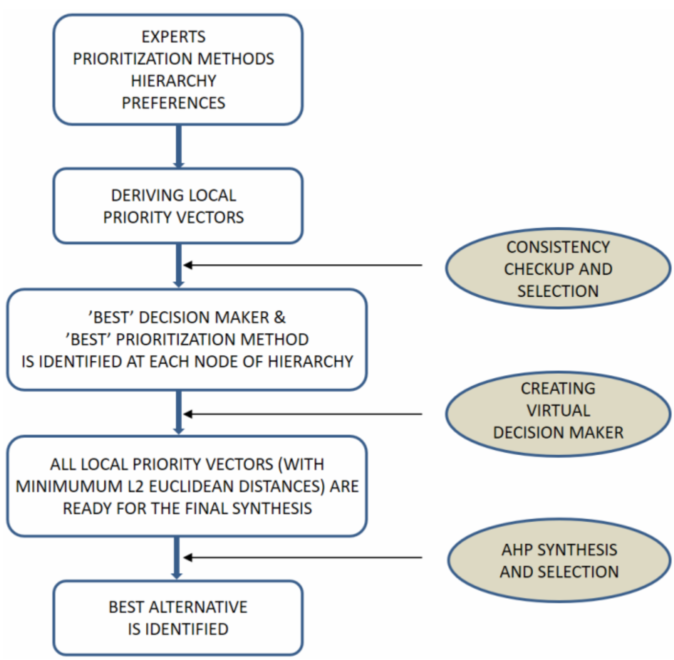
In short, a real 'forest of approaches and comparisons' exists and it is rather hard to find a proper position for any new proposal within a broad subject framework. From the aforementioned cases and situations, here we deal with a decision-making problem which is, by assumption, structured as a multi-levelled hierarchy. Without losing in generality, a three-level hierarchy is treated, composed of a goal at the top of hierarchy, a set of criteria in a level below, and a set of alternatives at the lowest level. An AHP GDM context means that a certain number of individuals will participate in creating the group, that the same hierarchy will be adopted by all participants and that the same scale (e.g., Saaty's nine-points ratio scale) will be used by all while making pair-wise comparisons, known as elicitation of DMs' judgments. The final outcome of the process should be a unique group priority vector representing cardinal information. Entries of this vector are the weights of alternatives against the stated goal.

Besides different direct aggregation schemes applicable within the AHP framework, there are 'indirect' approaches to this important part of a GDM processes. For instance, a consensus decision making field occupies a vast area in the literature. In a study (Dong and Zhang, 2014) a direct consensus framework is proposed for multi-person decision making with different preference representation structures such as preference orderings, utility functions, multiplicative preference relations and fuzzy preference relations. The individual selection methods are used to obtain individual preference vectors of alternatives and the standardised preference vectors are directly aggregated. Based on the group vector of preferences, the feedback adjustment rules enable the DMs to reach consensus. The authors of this study argue that proposed framework satisfies two properties: avoiding internal inconsistencies when the transformation functions among different preference representation structures are used, and that this framework satisfies the Pareto principle of social choice theory (without rigorous proof, however). More recently, Dong et al. (2016a) proposed a consensus reaching model in the complex and dynamic multi-attribute group decision making (MAGDM) which enables a consensus among DMs by generating adjustment suggestions for individual sets of attributes, alternatives, and individual preferences. This study is also focused on providing a flexible framework for deriving group decisions in real-world MAGDM problems. A consensus framework is also presented in Dong et al. (2016b) aimed to manage the consensus and weights of the experts and attributes in iterative multiple-attribute GDM process. An optimisation-based model enables modifications of preferences assigned by DMs by minimising Manhattan distances between the original and adjusted preferences.

3 New algorithm for synthesis priority vectors in group AHP applications

Being aware of what exists within the theory and applications of AHP for GDM, we propose a new algorithm named AHP-DPE as an improvement of the MGPS algorithm published in Srdjevic and Srdjevic (2013). The AHP-DPE is aimed to derive as much as possible consistent group decision. The main feature of AHP-DPE is that it is applicable to the *complete hierarchy which is a spiritual and philosophical pillar of AHP*. The algorithm has not restriction on number of levels, number of decision elements in a hierarchy and size of a group of DMs. Differently from MGPS which uses only one selected prioritisation method, in AHP-DPE the number of prioritisation methods to be used in each node of a hierarchy to identify most consistent DM is not limited; what is recommended is that for local prioritisations a combination of established and commonly used matrix and optimisation methods should be employed.

Figure 1 Flowchart of the AHP-DPE synthesis algorithm (see online version for colours)



It is important to note that any well-established consistency criterion can be used for local prioritisation process, but it must be uniquely applied at all nodes of a hierarchy and be common for all involved prioritisation methods. For example, natural consistency index μ associated with FPP method (Mikhailov, 2000), is not applicable in AHP-DPE because it is method specific, differently from TD and MV as general consistency measures applicable to all prioritisation methods in either individual or group applications. Note also that if only one prioritisation method is used for all local nodes and all DMs, AHP-DPE becomes MGPS.

In this section, a step-by-step use case of AHP-DPE synthesis shown in Figure 1 is described with only necessary mathematics, in order to not lose sight of the base goal, which is generating the final AHP group priority vector of alternatives against a goal with an absolute minimum of TD as a measure of DMs' consistency. Details on prioritisation methods and other consistency measures commonly used in AHP – in general and in AHP-GDM in particular – can be found in rich pertinent literature partially given in the list of references of this paper.

3.1 AHP-DPE synthesis step-by-step

Given

1. Standard three level hierarchy with a goal on the top, criteria set (*nc*) at the level below and alternatives set (*na*) at the bottom level.
2. A group of DMs (*nd*) composed of individuals willing to participate in the decision-making process on fair and competent basis, assuming that their decisions will receive equal importance in any further aggregation to come up to the final group decision.
3. All local pair-wise comparison matrices produced by DMs. In total, there are $nd \cdot (1 + nc)$ matrices, that is:
   a. at criteria level there are *nd* matrices of size $nc \cdot nc$
   b. at the alternatives level there are $nd \cdot nc$ matrices of size $na \cdot na$.

   All matrices are complete, positive and reciprocal with respect to the main diagonal; entries in matrices are from fundamental nine-point Saaty's scale and correspond to semantic judgments elicited from the DMs.
4. The set of scientifically sound matrix and optimisation methods for extracting so-called priority vectors from any comparison matrix. The number of methods can be denoted as *nm*.

Assumptions

1. Any aggregation is considered to be performed within the whole-hierarchy context, not just at a single node of the hierarchy.
2. Participating DMs will be treated as having the same importance within the group and will accordingly receive equal weights summing to 1.
3. DMs' judgments and the resulting priorities of decision elements (criteria and alternatives) will be evaluated with consistency measure defined as the total

Euclidean L^2 norm over elicited judgments a_{ij} and corresponding computed weights w_i and w_j, where i and j are row and column indices.

4 Algorithm does not assume any consensus between participating members in the group.

Algorithm

Step #1 The hierarchy with *nc* criteria and *na* alternatives is adopted by all DMs. Number of DMs in the group is *nd*, and the number of selected prioritisation methods is *nm*.

Step #2 Each DM DM_{id} (id = 1, 2, ..., nd) individually makes judgments on the mutual importance of criteria versus the goal, and do the same for the mutual importance of alternatives versus criteria. Each DM produces a structure of 1 + *nc* comparison matrices (one matrix for criteria versus goal, and *nc* matrices for alternatives versus criteria). In total, a set of $nd \cdot (1 + nc)$ matrices is created for the group.

Each DM inserts numerical judgments into the upper triangles of all comparison matrices in the 1 + *nc* structure. Reciprocals are automatically generated in the lower triangles, while spaces on the main diagonals are set to 1.

For given DM DM_{id}, corresponding matrices are $P_{0,id}$, $P_{1,id}$, ..., $P_{nc,id}$. Matrix $P_{0,id}$ contains comparisons of criteria versus goal, while matrices $P_{1,id}$, ..., $P_{nc,id}$ contain comparisons of alternatives versus criteria.

Step #3 Prioritisation is performed with each prioritisation method *im* (*im* = 1, 2, ..., *nm*) for all matrices produced by all DMs resulting in $nm \cdot nd \cdot (1 + nc)$ local priority vectors. Priority vectors contain local weights of related decision elements that were compared in the previous step of the procedure. The sum of all weights in each priority vector is always 1.

Once prioritisation is completed, TD is computed over elicited judgments and corresponding weights obtained by prioritisation. Computation is performed for each DM, each matrix and each prioritisation method [equation (1)].

$$TD_k^{id,im} = \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \left(a_{ij,k}^{id} - w_{i,k}^{id,im} \big/ w_{j,k}^{id,im} \right)^2 \right]^{1/2}, \tag{1}$$

$$k = 0, 1, ..., nc;\ id = 1, ..., nd;\ im = 1, ..., nm$$

In equation (1), indexed variables in double summation are: judgment a obtained for node k in the hierarchy by DM *id* while comparing decision elements i and j in a matrix of size n; weights w_i and w_j computed for elements i and j are entries of priority vector $w_k^{id,im}$ for the same node and the same DM derived by prioritisation method *im*. Size of the matrix is n and $TD_k^{id,im}$ is the total Euclidean distance for node k, DM *id* and prioritisation method *im*, and *nc*, *nd* and *nm* are as defined above.

To summarise, for given DM there are 1 + *nc* comparison matrices P_k (k = 0, 1, ..., *nc*) and associated nm·(1 + *nc*) priority vectors. Total distances (TDs) are computed for each node in the 1 + *nc* structure for all DMs and all prioritisation methods, i.e., $TD_k^{id,im}\,(k = 0, 1, ..., nc;\, id = 1, ..., nd;\, im = 1, ..., nm)$.

Step #4 Based on the values of $TD_k^{id,im}$ computed in step #3, identify in each node of the hierarchy the DM with the lowest TD across all used prioritisation methods. In other words, identify 'best ordered pair (*id*, *im*)' (DM, prioritisation method) so that the TD in any given node of the hierarchy structure 1 + *nc* is minimum:

$$TD_k^{id,im*} = \min_{id,im} TD_k^{id,im},\ k = 0, 1, ..., nc \tag{2}$$

Adopt corresponding nodal vector $w_k^{id,im}$ as the best local vector $w_k^{id,im*}$ for the final synthesis.

Step #5 Perform standard AHP synthesis with 1 + *nc* locally best priority vectors $w_k^{id,im*}$ identified in step #4. Declare the final priority vector as the group priority vector.

3.2 Remarks

Several remarks should be given related to above statements and algorithm.

Remark #1: The total difference TD obtained by the AHP-DPE synthesis is by definition absolutely minimal compared to any other AHP synthesis based on priority vectors computed by selected set of prioritisation methods.

Remark #2: Minimum violation criterion MV is excluded from AHP-DPE because our earlier research show that it has significantly lower importance compared to the Euclidean distance, regardless the size of a pairwise comparison matrix.

Remark #3: In the MGPS (multi-criteria group prioritisation) synthesis most consistent DMs are identified in the hierarchy structure 1 + *nc* and their locally best priority vectors are used for the final synthesis. One prioritisation method is used for all matrices and all DMs, namely eigenvector (EV) method. The most locally consistent DMs, meaning their priority vectors, are identified based on different weightings of two consistency measures: TD and MV (change of orders). In the AHP-DPE, only the dominant measure of consistency is used (TD), but several prioritisation methods are employed to help identify locally best priorities for the final AHP synthesis. In this way, the AHP-DPE synthesis displays an improvement of the MGPS synthesis (Srdjevic and Srdjevic, 2013) and also is the group extension of the MULS methodology based on combining different prioritisation methods for an individual DM (Srdjevic, 2005).

Remark #4: In a group decision-making, participating individuals usually have different educational background, different attitudes and specific subjective and objective interests while exposing their judgments about decision elements. Sometimes, it is opportune to organise the decision-making process in two stages. In the first one, managers should decide about criteria importance, and in the second stage engineers and designers have to validate alternatives against criteria. In such cases, AHP-DPE can be transformed and applied locally at only goal-criteria level in the hierarchy. This is the one-matrix case and AHP-DPE enables computing criteria weights based on a judgment of just one DM. The weights are output of that prioritisation method which (for judgments of identified DM) generates minimum total Euclidean distance L^2 between judgments and weights of criteria obtained by that method. The remaining evaluations of alternatives versus criteria can be individually performed by the DMs with the same weights of criteria obtained in described way by AHP-DPE algorithm, or in any other well-known aggregation techniques. Of course, a procedure can be organised in such a way to proceed with AHP-DPE or MGPS or combination of these two algorithms.

Remark #5: Respecting state-of-art in the field of prioritisation methods, AHP-DPE algorithm should use six concurrent methods: AN – additive normalisation; EV – eigenvector; LLS – logarithmic lest squares: WLS – weighted least squares; FPP and COS – cosine maximisation. All these methods are well-established and described in pertinent literature as referenced above.

Remark #6: The proposed method is general and applicable in only group decision-making contexts. It does not assume any attempt toward reaching consensus among DMs. That is, aggregation of selected priority vectors at nodes of the hierarchy is straightforward governed by the minimum total Euclidean L^2 distance at all nodes and therefore for the complete hierarchy. Selection of the 'best' decision-maker and 'best' prioritisation method is exclusively based on the minimum consistency measured by universal error criterion total Euclidean L^2 distance. One must be aware of the fact that not only consistencies in judgments provided by the DMs represent quality decisions. Consistency is merely about precision, while on the other hand accuracy can also be considered as an important descriptive parameter of the quality of decision. Accuracy issue must be treated seriously, but before proposed algorithm is applied. There are many different cases that may occur. For instance, DM can be precise but not accurate. Is he accurate depends on DM's overall background which includes education, expertise in the subject, attitudes, understanding of judgment process, way of using ratio-scale, etc. In relation to this, in Cooke's (1991) method, DMs are previously interviewed about the subject to get information how much one can trust them when they start making decisions, and a lot of statistics may be employed to check their accuracy.

Remark #7: It can happen that one or more DMs at one or more nodes of the hierarchy judge elements with 'all 1' (all elements are equally important; no preferences actually) and ED = 0. Such cases may rarely happen in practice. However, if such a case happen – certain DM can hardly be considered as relevant for decision-making process at all (or at least for that node), but even then derived final group decision is valid. That is, minimum distance at given node(s) is zero, any prioritisation method can be selected, and all elements have same weights. The final result (group decision), after the synthesis, is valid and, of course, its quality will be as was the quality of judgements provided by one or more DMs. Mediating the d-m process can avoid such case(s) and algorithm can be performed with appropriately reduced number of DMs at nodes where it happened. To check implications of such case(s), we performed tests with reasonable 'all 1' cases and algorithm generated satisfactory results.

Remark #8: Size of the group is not limited, set of prioritisation methods (matrix, optimisation, heuristic etc.) is not closed, and other possible conditions for putting AHP thru the decision making process is not restricted. To our best knowledge, there are quite interesting applications of AHP via internet where DMs do not know each other, or nobody can check individual competences, accuracy and conditions while they are individually judging decision elements. AHP group applications are sometimes conducted with distant and very different decision-makers (participants), sometimes AHP applications are supported by questionnaires and with only brief explanations about AHP, how to fill in matrices or answer to pair wise comparison questions and so on. Even in such cases, proposed algorithm can perform well and will provides quality group decision which may serve as threshold (ideal point) solution. Derived group decision may eventually be used in attempts to harmonise group in any sense, reach consensus etc.

Remark #9: Possible changes in proposed algorithm could be to allocate the weights to DMs based on their consistency. If extended to consensus scenarios, possibility could be to perform aggregations of DMs' priorities (at nodes) based on mutual recognition of importance of participating members in the group, but this must be done in advance, before aggregation start. Intentionally, we do not propose it in our approach because it can be very tedious process with uncertain outcome. Moreover, so far we could not find reported real-life applications where different weights are given to DMs in a group based on theoretical proofs. On the contrary, we found only suggestions when certain DMs could be directly or indirectly excluded from the decision-making process (e.g., based on Sammon's mapping). Here again comes argumentation given in Cooke (1991) and Cooke and Goossens (2004) about competence of DMs and activities to be performed in advance, before AHP really start.

4 Numerical example

The example application of proposed algorithm is provided from a real-life decision making process performed across a representative group of 13 out of 42 PhD students from many countries in Europe. They participated in a human-computer interaction (HCI) related workshop with one session dedicated to the AHP. The students validated importance of five sources of agriculture related information on the Internet against five typical criteria.

Saaty's nine-point scale is used in this example and AHP-DPE synthesis is performed with the six commonly used prioritisation methods (AN, EV, LLS, WLS, FPP and COS) being applied in all nodes of the hierarchy. The first two are matrix, while the remaining four are optimisation methods. All these methods are described in detail in pertinent literature (e.g., Saaty, 1980; Srdjevic, 2005; Mikhailov and Singh, 1999; Mikhailov, 2000, 2004; Dong et al., 2008; Siraj et al., 2015; Kou and Lin, 2014).

After a brief half-hour introduction to the AHP, over the next half-hour students assessed and evaluated by importance sources of information related to agriculture on the internet against typical criteria. Their (in) consistency has been controlled through eigenvector prioritisation and computation of nodal and total CR coefficients. Only relatively consistent students (with CR less than 0.18, keeping in mind that the suggested tolerant limit is 0.10) are included in the group for performing AHP-DPE synthesis. Selected 13 students evaluated the same hierarchy with the following decision elements:

- Goal: rank by importance for agricultural research, planning and management the set of five typical internet sources of information across given set of criteria.
- Alternatives:

  A1 conferences, exhibitions, fairs

  A2 citation databases

  A3 discussion forums/social networks

  A4 employment/career

  A5 maps.
- Criteria:

  C1 *Understandability* measures how well a source presents information, so that the user is able to comprehend its semantic value. For instance, this quality measure can be given as a score between 1 and 10.

  C2 *Extent* of a given set of information is the average length of the individual pieces of information (for instance, the number of columns in a table). As a general measure a scale 1 to 5 can be used (associated with semantic expression such as: 1 – poor, 5 – excellent).

  C3 *Availability* of an information source as the probability that a feasible query is correctly or at least satisfyingly answered in a given time range. If understood as a statistical value it can be represented with a percentage.

  C4 *Context coverage* relates to the degree to which the internet source can be used with effectiveness, efficiency, freedom from risk and satisfaction in both specified context of use and in contexts beyond those initially explicitly identified. A score can be given, e.g., between 1 and 10.

  C5 *Keeping up to date* is the measure of continuous coverage of incoming information by the information source on the internet. A general measure can be again a scale 1 to 5 (1 – poor, 5 – excellent).

The selected criteria are mostly qualitative. Presented measuring scales were suggested to all students to keep in mind while judging the importance of each criterion, but to use the nine-points scale recommended by Saaty (1980) when finally reporting the judgments. This two-fold thinking worked surprisingly well and most of students demonstrated a satisfactory consistency.

Individual AHP judgments and corresponding comparison matrices are taken from 13 selected students (DMs DM1, ..., DM13) and used to perform AHP-DPE synthesis. For each student as DM, six matrices are created during elicitation of individual judgments:

a one matrix (P_0) of size 5 × 5 for criteria vs. goal

b five matrices (P_1–P_5) of size 6 × 6 for alternatives vs. given criterion(s).

The eigenvector method is used for prioritisation over each matrix. In a way, the eigenvector method served as a control mechanism aimed to identify (in) consistencies of DMs at local nodes of the hierarchy, and for the hierarchy as a whole.

Best local priority vectors at each node of the hierarchy (priority matrices P_0–P_5) are taken for the final AHP synthesis. A global minimum TD is identified from both aspects, i.e., DM is selected if his TD is the absolute minimum across all DMs and all prioritisation methods. For each node in the hierarchy (priority matrices P_0–P_5), the 'best DM' and his 'best prioritisation method' are selected, i.e., corresponding 'best local priority vector' is selected for the final AHP synthesis. Table 1 summarises minimum total TDs identified for all DMs.

Table 1 Minimum total differences TD across DMs and prioritisation methods

DM	*P_0*		*P_1*		*P_2*	
	TD	*Method*	*TD*	*Method*	*TD*	*Method*
DM1	5.457	WLS	4.052	WLS	6.488	WLS
DM2	6.515	AN	6.626	COS	4.437	WLS
DM3	5.052	FPP	4.336	WLS	4.264	COS
DM4	5.591	WLS	4.447	FPP	8.840	FPP
DM5	3.265	COS	2.359	COS	2.689	AN/COS
DM6	5.591	WLS	5.591	WLS	4.606	FPP
DM7	4.188	COS	7.906	COS	4.505	COS
DM8	*1.438*	*WLS*	10.337	COS	8.332	FPP
DM9	3.008	WLS	*2.049*	*COS*	6.783	WLS
DM10	4.189	WLS	3.808	WLS	5.195	WLS
DM11	6.450	WLS	9.618	COS	5.073	COS
DM12	3.842	WLS	4.135	COS	*2.037*	COS
DM13	7.578	COS	4.177	WLS	6.800	FPP
	Selected: DM8 (WLS)		Selected: DM9 (COS)		Selected: DM12 (COS)	

DM	*P_3*		*P_4*		*P_5*	
	TD	*Method*	*TD*	*Method*	*TD*	*Method*
DM1	11.536	WLS	7.562	WLS	5.148	WLS
DM2	8.705	FPP	5.512	WLS	6.848	WLS
DM3	11.018	COS	5.792	EV	6.215	WLS
DM4	5.020	WLS	11.697	FPP	5.578	WLS
DM5	*1.535*	*FPP*	2.577	FPP	4.280	COS
DM6	5.591	WLS	5.327	FPP	5.942	FPP
DM7	8.250	COS	6.388	AN	7.608	COS
DM8	9.800	COS	7.047	AN	7.130	WLS
DM9	7.217	COS	7.766	COS	3.824	WLS
DM10	5.146	COS	3.565	WLS	3.808	WLS
DM11	3.731	COS	5.366	WLS	4.889	WLS
DM12	2.399	WLS	2.212	COS	*2.049*	*WLS*
DM13	11.794	COS	*2.105*	*WLS*	8.150	WLS
	Selected: DM5 (FPP)		Selected: DM13 (WLS)		Selected: DM12 (WLS)	

Interesting to note is that out of 78 cases (13 DMs, each with six matrices) in 38 cases (49%) WLS produced minimum nodal TD. The COS method produced the second-best results with achieving a minimum nodal TD in 25 cases (32%), the next was FPP method with 11 cases (14%), and the last two were AN and EV only three and one nodal minimum (both 5%), respectively. LLS method did not produce any nodal minimum TD.

In this example, the AHP-DPE synthesis used local priority vectors for five DMs (DM5, DM8, DM9, DM12 and DM13). This means that priorities derived from judgments made by other eight DMs were more inconsistent and therefore were excluded from the synthesis. As far as prioritisation methods are considered, in this example it happened like in the first example that WLS was selected in three of six cases (local matrices), COS in two cases and FPP is selected in one case (matrix); AN, EV and LLS are not selected because they performed worse than aforementioned methods, i.e., TDs between local judgments and weights of compared decision elements were higher when compared with values obtained for the other methods.

Figure 2 presents how AHP-DPE selection is performed, and Table 2 contains the final priorities of Internet sources of information for the group of 13 PhD students. Notice that students' satisfaction with information provided by citation databases is at the second place, although the weight of this source is very close to the first and third positioned sources (conferences ... and maps). At the time of this experiment, social networks were not that popular and expectedly they have not received high recognition by the PhD students; even discussion forums weren't able to compensate for the low recognition of social networks.

Figure 2 AHP-DPE selection of DMs and prioritisation methods for the final group AHP synthesis (see online version for colours)

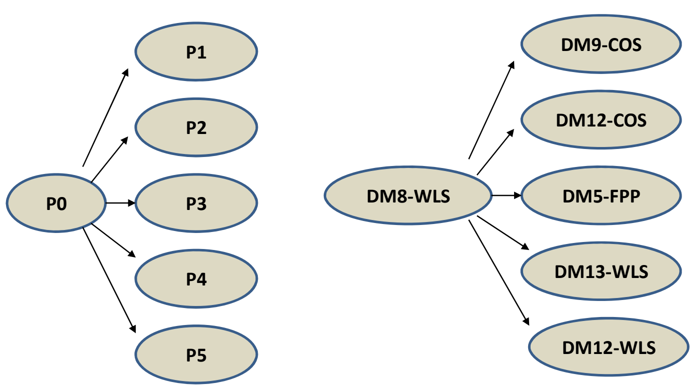


Table 2 The final group priority vector of internet information sources

Alternatives	*AHP-DPE*	*Rank*
A1 – Conferences, exhibitions, fairs	0.265	1
A2 – Citation databases	0.264	2
A5 – Maps	0.261	3
A4 – Employment/career	0.130	4
A3 – Discussion forums/social networks	0.083	5

5 Conclusions

GDM supported by AHP (known also as AHP-GDM) is attracting significant attention from both the scientific community and practitioners. This paper offers an improvement of earlier published approach named MGPS – multi-criteria group prioritisation synthesis (in AHP). Following a philosophy of AHP and real life circumstances which commonly happen if more DMs are involved in deriving a group decision, we found that it is possible to perform a 'fine tuning' of the weights of decision elements (criteria/ sub criteria/alternatives) while prioritising decision elements with more than one prioritisation techniques in all local pair-wise comparison matrices within a hierarchy. The outcome is expectedly a set of best final weights of alternatives at the bottom level with respect to a stated goal at the top of hierarchy.

The logic behind and premise in our work is that none of the well-known prioritisation techniques performs better than the others in all cases, and that assessment of individual consistencies of DMs participating in the group is globally justified (equal for all) if Euclidean distance is used to measure differences between original DMs' judgments and weights of decision elements computed by selected prioritisation techniques. Proposed algorithm, called AHP-DPE synthesis, uses Euclidean L^2 norm as the global (universal) consistency measure which is minimised over the complete hierarchy of the decision problem, across all most commonly used prioritisation methods, and across all DMs participating in the group. This norm is used as a unique criterion which helps to distinguish opinions of DMs and priorities produced by any prioritisation methods and to point out the minimum difference between semantic/numerical judgments of various decision elements elicited from DMs and the computed final weights of alternatives (by different prioritisation methods) with respect to the stated goal.

In a way, AHP-DPE synthesis performs so that in each node of the hierarchy it adopts for the final synthesis a set of weights of decision elements relevant for that node from a DM whose judgments 'produced' in this node a minimum total Euclidean distance across all prioritisation methods he could use. The AHP-DPE synthesises local priority vectors taken node-by-node from more than one DM and from more than one prioritisation method. Such a scheme leads to the final AHP synthesis of a set of absolutely best local priority vectors with a minimum total Euclidean distance. The decision to stick to only using the Euclidean distance as a consistency measure is based on previous research and results which indicated that other measures are less representative and significantly less used. For example, MV (order reversal) measure is used in various researches but we did not find any clear indication of how much it is relevant in comparison with a universal measure such as Euclidean distance.

Worth to mention is that proposed algorithm does not perform any normalisation of Euclidean distances regarding sizes of matrices. It handles only total Euclidean L^2 values at nodes, because at each node in the hierarchy its computation is unique for all involved prioritisation methods and all DMs, and therefore values are directly comparable. This way, and especially by inspecting the final priorities of alternatives versus goal, possible violation of Pareto principle at some node(s) could be identified and additional actions undertaken to fix the problem(s), e.g., by repeating part of the decision-making process. This issue is not considered in presented approach and requires further research.

To conclude, contribution of this work is the AHP-DPE synthesis performed by a virtual DM, created of locally most consistent DMs identified after mutual comparisons of their individually demonstrated consistencies. The final group priority vector of alternative solutions derived by AHP-DPE is in a way a consensual group solution which may serve as a benchmark for other group AHP applications and aggregation schemes. We see AHP-DPE as a general framework which may be applicable for any size of hierarchy, number of individuals in the group and any set of prioritisation methods. Possible limitations of this work we see in relation to:

1 impossibility to precisely justify that 'good consistency' really means 'good DM' (very difficult issue to treat)

2 treatment of Pareto optimality principle in GDM contexts.

In this regard we see our future research directions for improving decision making processes with more than one participating individuals.

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