The equilibrium model of dual channel closed-loop supply chain network based on carbon trading and carbon tax

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Abstract: In this paper, on the basis that carbon trading and carbon tax are brought into the category of the carbon management, an equilibrium model of a dual channel closed-loop supply chain network is developed. Many decision-makers and their independent behaviours are fully processed based on the variational inequality theory and Lagrange duality theory. Based on the above analysis, a finite-dimensional variational inequality formulation is established. Besides, qualitative properties of the equilibrium model and a modified projection algorithm are given.

Keywords: dual channel; carbon trading; carbon tax; closed-loop supply chain network.

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1 Introduction

Supply chain is in a functional network of complex and dynamics and this network is composed of upstream suppliers, manufacturers, downstream retailers, end users and recycling centres. In today’s society, the competition among enterprises is mainly the competition among the supply chains based on the supply chain network. Therefore, supply chain network has become an important research subject, which has attracted much attention from both academic and business community. One of the goals of each member of the supply chain network is to seek to maximise their own interests. The problem scholars concerned is to obtain the equilibrium condition of supply chain network through depicting the behaviour of each member and the relationship between the members of supply chain network.

With environmental pollution and global warming, more and more attention has been paid to the problem of carbon emissions. Enterprises should pay attention to the low carbon operation of the supply chain while pursuing the maximum profits. Matthews et al. (2008) believes that only through the effective operation of the enterprise internal is very difficult to realise the target of reduction of carbon emission in the whole supply chain, we should put the emphasis of carbon reduction in the cooperation of key business. Keskin and Plenbeck (2011) also found that carbon emissions from the world’s largest 2500 companies accounted for 20% of the total emissions of greenhouse gases and the carbon emissions produced by supply chain is far more than enterprise’s own carbon emissions. Nowadays, companies began to actively look for measures to reduce carbon emissions. Yang and Ji (2013) found that the main measure taken by companies is to increase investment in technology of reducing emissions. But the technology is mainly to reduce carbon emissions in the production plant or a single manufacturing process, ignoring the interaction between members in supply chain. In the low carbon supply chain, it is necessary to consider that not only the compatible degree of behaviour subject and its environment, but also the carbon emission index of the entire supply chain. De Benedetto and Klemes (2009) proposed life cycle assessment method and made a quantitative analysis of relevant data which achieved through tracking carbon emissions of the whole process of supply chain. Abdallah et al. (2012) studied the influence of carbon emissions on supply chain network on the basis of low-carbon economy. Payman and Cory (2013) took environment factors into account in the design of supply chain network. Yan and Jingling (2014) studied the multi criteria supply chain network equilibrium under the government energy saving and emission reduction and achieve equilibrium between maximising profits and minimising emissions. Based on low-carbon supply chain researches at home and abroad currently, Lin et al. (2015) researched on the following four hot spots: supply chain carbon footprints, carbon emissions, supplier’s selection, supply chain network optimisation and supply chain performance evaluation.

Along with the proposed of the idea of circular economy, people began to pay attention to treatment and recycling of worn-out products and a closed-loop cycle have formed. Nagurney and Toyasaki (2005) assumed that the electronic waste can be recycled and then processed into other products, thus established a equilibrium model of closed-loop supply chain network, in which the relation of the members in same layer is horizontal competition, the relation of members in different layer is vertical cooperation. Hammond and Beullens (2006) established a supply chain network equilibrium model to study the closed-loop supply chain network which assumes that different manufacturers produce same products, the products formed by raw material and worn-out products are
homogeneous and the society mainly concern environment performance as an additional participant of supply chain. El-Sayed et al. (2010) studied the equilibrium conditions of closed-loop supply chain network through establishing a random integer programming model. Turan et al. (2011) optimised the design of closed-loop supply chain network and studied how to reduce the carbon emissions by choosing different mode of transportation. Fahimnia et al. (2013) established an optimisation model of closed-loop supply chain network and studied the impact of carbon pricing on the performance of supply chain network. On the basis of comprehensive consideration of environmental constraints, government subsidies and other factors, Sun et al. (2015) established a multi-loop supply chain network equilibrium model which the manufacturers are responsible for recycling. This model provided support for enterprises to make dynamic decision and government departments to make subsidy policy. Effective operation of closed-loop supply chain can realise the strategy of sustainable development.

With the advent of the internet era, e-commerce has great impact on the behaviour of enterprises, the order mode of consumers and the transport of goods. Nowadays, consumers are increasingly tended to online shopping. In response to this phenomenon, Hu and Li (2012) assumed that the products trading between retailers and consumers could be realised through the way of express delivery services and customer’s self proposed approach and established an equilibrium model of online shopping supply chain network which consist of manufacturers, retailers, express service providers and consumers. This article also indicated that consumers’ decision behaviour is different from common shopping, consumers choose different providers based on the perceived generalised cost consisting of commodity price, express delivery price and delivery time. E-commerce has brought new opportunities for the management and research of supply chain network with the development of modern information technology and thus formed dual channel supply chain. With regard to the network design problem of the low-carbon supply chain based on online shopping, the concave function is used to minimising the supply chain operation cost including the carbon emission cost, Wu and Bai (2015) analysis results show that resources allocation of the low carbon supply chain can be optimised if the carbon emission cost is considered during supply chain designing. Cai (2010) proposed increase Pareto zone model and contract coordination Pareto zone model, which studied the influence of dual channel to supply chain coordination and analysed the option method of transaction channels. Huang et al. (2012) proposed a two-stage decision model and discussed the impact of uncertain demand on production and trading price of dual channel supply chain. Man et al. (2013) studied the dual channel supply chain network structure which consist of manufactures, retailers and demand markets under the environment of carbon emissions, made a performance evaluation of the network and analysed the effect of carbon emissions factors on trading patterns and trading volume. This paper studies in the form of B2C.

In supply chain management, the supply chain structure model includes two kinds: chain model and network model. The chain model refers to the model which the direction
of logistics is generally from nature to suppliers, manufacturers, distributors and finally to users and the direction of supply chain is defined according to the direction of logistics. However, network model can cover all the enterprises in the supply chain. It takes each enterprise as a node and considers that there is a link between these nodes. In comparison, the supply chain network model can reflect the complex products trading relationship better in real world and simulate the competition between enterprise members better in the supply chain, market competition, as well as the complexity of demand. Therefore, in recent years, the research on the equilibrium of supply chain network has become one of the hot spots. The equilibrium model of supply chain network is a new way to study the supply chain and this idea comes from the equilibrium theory of micro economics. Nagurney et al. (2002) first proposed the concept of supply chain network equilibrium, which assumed that the manufacturer pursues the maximisation of profit and the retailer pursues the maximisation based on the price which customers are willing to pay. The network was also analysed using the variational inequality theory. Nagurney et al. (2005) take both traditional transaction channel and e-commerce transaction channel into account and assumed that retailer should consider both increase profits and reduce pollution and studied the optimisation conditions of the members in supply chain. Dong and Ma (2011) made a review of equilibrium model of supply chain network which is composed by decentralised decision-making multi-layer and multi-decision and illustrated the main application range of equilibrium model, including: power supply, financial engineering, global supply chain management, social network, e-commerce, environmental protection, products recovery and reproduction. The pricing strategies and coordination mechanism are studied in the closed-loop supply chain by Cao et al. (2015) based on channel competition. The equilibrium pricing strategies of the manufacturer leading Stackeberg game and the Bertrand competition game are obtained. Tang et al. (2016) proposed a non-deterministic polynomial model considering the factors affecting bulk cargo ports scheduling, then, it solves the model based on MPPSO algorithm and MATLAB. Zhang et al. (2016) expands previous work to the closed-loop supply chain network with dual transaction channels in which the two transaction channels compete with each other and the consumers in the demand markets have certain preference for e-commerce transaction channel. Gong et al. (2017) considered production coordination is a common phenomenon in supply chains and he studied how a manufacturer coordinates the relationships with its subsidiary firms from the perspective of asymmetric information, the research shows manufacturers could obtain production coordination through applying incentive mechanism.

This paper studied the closed-loop supply chain network with dual channel consisting of multiple suppliers, multiple manufactures, multiple retailers, multiple demand markets and multiple recycling centres based on the minimise carbon emissions. Besides, transactions between manufactures and consumers can be conducted not only through physical chain, but also through e-commerce channel. In today’s competitive market environment, it will be more significant.

2 The reduction methods of carbon emission in low carbon supply chain

The structure of the low carbon supply chain is shown in Figure 1.
Dong (2015) found that there are two kinds of environmental policies of carbon emissions, that is, the mandatory policy which mainly insist of carbon tax and subsidy and the regulation policy of carbon emissions trading which based on market. The research of carbon emissions trading mechanism is originally derived from the emissions trading and the forest carbon trading which is put forward in ‘United Nations Environmental and Development Conference of the United Nations Framework Convention on Climate Change’ in 1992. The ultimate goal of carbon trading is to create a market mechanism in order to reduce the cost of enterprise in the process of reducing carbon emissions. At present, the carbon trading mechanism which is most widely used is the European Union Emission Trading Scheme (EUETS), that is, certain amount of carbon emissions are assigned to each enterprise freely by the carbon emissions regulatory agencies in accordance with the historical emissions levels of the enterprises and the enterprise members can not only reduce carbon emissions in order to achieve redundant emissions permit, but also reap the benefits through carbon credits trading in the carbon exchange. Avi Yonah and Uhlmann (2009) considered that the mainly goal of carbon emissions trading mechanism was to convey a message to the public: as long as the company is willing to pay for the cost of pollution, it can pollute the environment. Caro et al. (2011) reached a conclusion that we could mandatory set emissions caps for each member in the supply chain and thus make the members of supply chain try their best to reduce carbon emissions. Li and Zhao (2012) summarised the carbon emissions trading mechanism which include free distribution, threshold allocation, fixed price and open auction and then compare them through the simulation and drew a conclusion: compared with the free allocation mechanism, the paid allocation mechanism of carbon emissions can more effectively promote the enterprise to choose environmental priority strategy to reduce the carbon emissions; compared with fixed price mechanism, the price achieved through the open auction mechanism can be more reasonable to reflect the real situation of the market and more conductive to effective decision-making.

From the perspective of government implementation, carbon tax is a kind of tool which cost the least in the process of reaching the goal of reducing the carbon emissions. Chung et al. (2013), in order to get the needs of consumers timely and accurately,
considered that manufacturers and retailers in the supply chain would form a relationship of non-cooperative game and each decision of enterprise will affect the decision-making of other enterprise. He constructed a space model of supply chain on the base of the idea and studied the impact of environmental tax on the decision-making members in supply chain. Li and Su (2015) argued that interaction between upstream enterprise and downstream enterprise had an important impact on carbon emissions of supply chain; enterprises must make relevant effective measures under the supervision of government carbon regulatory policy. Besides, the enterprises also need to make strategic choices to achieve their own profit maximisation.

In practice, the carbon emissions trading and carbon tax are not exclusive. Nowadays, in logistics industry, it is difficult for government to use a single carbon regulatory policy to supervise carbon emissions because of the diversification of service products. Therefore, it is inevitable for government to use the policies of both carbon trading and carbon tax in the process of reducing the carbon emissions in supply chain. Pigou (1952) believed that the essence of carbon emissions trading and carbon tax is internalisation of the external costs of carbon emissions with the power of market on the base of price and quantity and then affected the decision-making behaviour of enterprises through the change of profit function of enterprises. On the base of LCA principle, Chaabane et al. (2012) conducted optimal design for each node in the supply chain, which consist of suppliers, manufacturers, distributors, demand markets and recycling centres and indicated that the environmental protection policy is more conducive to the sustainable development of the supply chain. Yang et al. (2012) proposed that we can reduce the carbon emissions of supply chain by setting up reasonable emissions caps and reasonable carbon tax rate. Benjaafar (2013) introduced carbon emissions into supply chain and built some models which insist of carbon tax and carbon trading model, carbon emissions cap model, carbon neutral model and calculated the carbon footprint and cost of supply chain respectively.

In the carbon emissions trading mechanism of this paper, we assume that the carbon emissions will be distributed to enterprise members freely in accordance with the industry level by government. When the carbon emissions of the enterprise has a surplus, it can be transferred, that is, enterprises can get the remaining carbon emissions to the carbon trading market and sale it to the enterprises which are short of carbon emissions, the sales can be added to the enterprises’ own profits. At the same time, the government levy carbon tax on the carbon emissions of products.

3 Supply chain network equilibrium model

In this paper, we develop the supply chain network model with suppliers, manufacturers, retailers, demand markets and recycling centres. Specifically, we consider suppliers provide raw materials to manufacturers for the production process; n recycling centres recycle worn-out products from the end consumers and sell these products to manufacturers for the re-manufacturing process. The quality of the products which produced in the two processes has no difference. I manufacturers sell the products to retailers to meet the demands of consumers through traditional channel and to demand markets through e-commerce channel. In the figures, the solid lines represent the entity transactions made through traditional channels; the dotted lines represent the online transactions made through e-commerce channels.
Figure 2  Forward structure and reverse structure of the supply chain

Given the complexity of the recovery process, we assume: consider the process of recycling and re-manufacturing of products and there is no difference between remanufacturing products and new products. All cost functions are continuous differentiable convex functions. Manufactures have no capacity constraints. The carbon emissions of each supplier is always not less than the allocate quotas of government and each manufacturer’s carbon emissions is always not higher than the emissions limits.

3.1 The behaviour of the suppliers and their optimality conditions

\( \alpha_i \) indicates the coefficient of carbon emissions when products are supplied to manufactures by suppliers. Carbon emissions \( \alpha_i, q_i \) are produced in the process of transaction between supplier \( s \) and manufacturer \( i \). We assume that there always are \( \text{cap}_s \leq \sum'_i \alpha_i q_i \), supplier need pay \( P_s \Delta e_s \) for buying carbon emissions \( \Delta e_s = \sum'_i \alpha_i q_i - \text{cap}_s \). The transaction risk of carbon market is \( r_s \), the carbon emissions given by government to supplier \( s \) are \( \text{cap}_s \). The model of profit maximisation of supplier \( s \) is as following:

\[
\max \sum_{i=1}^{I} \left[ \rho_{si} q_{si} - c_{si} \left( q_{si} \right) - \alpha_i q_{si} u \right] - f_s \left( q_s \right) - P_s \Delta e_s - r_s \left( \Delta e_s \right) \\
\text{s.t} \ \text{cap}_s \leq \sum_{i=1}^{I} \alpha_i q_{si} \\
\sum_{i=1}^{I} q_{si} \leq q_s
\]
Table 1  The decision variables and parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>The meaning of variable and parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td>The supplier, $s \in {1, 2, \ldots, S}$</td>
</tr>
<tr>
<td>$i$</td>
<td>the manufacture, $i \in {1, 2, \ldots, I}$</td>
</tr>
<tr>
<td>$j$</td>
<td>the retailer, $j \in {1, 2, \ldots, J}$</td>
</tr>
<tr>
<td>$d$</td>
<td>The consumer, $d \in {1, 2, \ldots, D}$</td>
</tr>
<tr>
<td>$N$</td>
<td>the recycling centre, $n \in {1, 2, \ldots, N}$</td>
</tr>
<tr>
<td>$q_s$, $q_i$</td>
<td>The number of products supplied by the supplier $s$; the number of products produced by the manufacturer $i$</td>
</tr>
<tr>
<td>$q_{si}$, $q_{ij}$, $q_{id}$, $q_{jd}$</td>
<td>The trading volume of products between supplier $s$ and manufacture $i$, manufacture $i$ and retailer $j$, manufacture $i$ and consumer market $d$, retailer $j$ and consumer market $d$</td>
</tr>
<tr>
<td>$q_{nd}$, $q_{ni}$</td>
<td>The quantity of products recovered from consumer market $d$ by the recycling centre $n$; the quantity of worn-out products recovered from the recycling centre $n$ by manufacture $i$</td>
</tr>
<tr>
<td>$f_i(q_s)$, $f_i(q_i)$</td>
<td>The function of purchasing cost of supplier $s$; the function of production cost of manufacture $i$</td>
</tr>
<tr>
<td>$c_{oi}(q_{oi})$, $c_{ij}(q_{ij})$, $c_{jd}(q_{jd})$</td>
<td>The function of transaction cost between supplier $s$ and manufacture $i$, manufacture $i$ and retailer $j$, manufacture $i$ and consumer market $d$, retailer $j$ and consumer market $d$</td>
</tr>
<tr>
<td>$c_{nd}(q_{nd})$, $c_{ni}(q_{ni})$</td>
<td>The function of transaction cost between recycling centre $n$ and consumer market $d$, recycling centre $n$ and manufacture $i$</td>
</tr>
<tr>
<td>$c_j(v_j)$, $c_j(q_{jd})$</td>
<td>the activity cost of retailer $j$ for promotional products; the function of product storage cost of retailer $j$</td>
</tr>
<tr>
<td>$c_n(q_{nd})$, $c_n(w_n)$</td>
<td>The function of product storage cost of recycling centre $n$; the cost function for dealing with worn-out products of recycling centre $n$</td>
</tr>
<tr>
<td>$\rho_{1s}$, $\rho_{1d}$</td>
<td>The wholesale price of raw materials of supplier $s$; the price consumers are willing to pay for the products</td>
</tr>
<tr>
<td>$\rho_{oi}$, $\rho_{ij}$, $\rho_{jd}$</td>
<td>The transaction price between supplier $s$ and manufacture $i$, manufacture $i$ and retailer $j$, manufacture $i$ and consumer market $d$, retailer $j$ and consumer market $d$, recycling centre $n$ and manufacture $i$, recycling centre $n$ and consumer market $d$</td>
</tr>
<tr>
<td>$\phi_1$, $\phi_2$, $\phi_3$, $\phi_4$</td>
<td>The conversion rate of raw materials; the re-manufacturing rate stipulated by the government; the conversion rate of worn-out product; the minimum recovery rate of worn-out products provided by the government to the manufacturer</td>
</tr>
<tr>
<td>$\rho_1$, $\rho_2$, $\rho_3$</td>
<td>The disposal cost of the unit discarded raw material caused by the manufacture, the disposal cost of the unit worn-out products which produced in the process of re-manufacturing by the manufacturer; the penalty cost charged by government for unit not recycling products and unit not reuse of material</td>
</tr>
<tr>
<td>$U$</td>
<td>Carbon tax and $u &gt; t$</td>
</tr>
<tr>
<td>$G$</td>
<td>The carbon emissions cap of the whole supply chain</td>
</tr>
<tr>
<td>$P_e = a - bG$</td>
<td>The price of carbon transaction</td>
</tr>
<tr>
<td>$\theta$</td>
<td>The trading fees of unit carbon emissions</td>
</tr>
</tbody>
</table>
Because of the non-cooperative competition game among suppliers, the optimality condition of all suppliers can be equivalent to the following variational inequality: determine \((Q^1, Q^2, \lambda_1, \lambda_2) \in R^S\) satisfying
\[
\sum_{s=1}^S \left\{ \frac{\partial f_s(q^*_s)}{\partial q_s} \right\} \times (q_s - q^*_s) + \sum_{s=1}^S \sum_{i=1}^I \left( \frac{\partial c_{si}(q^*_u)}{\partial q_{si}} + \frac{\partial r_i(\Delta e_{is})}{\partial q_{si}} + \alpha_i u + \alpha_i P_s + \lambda_2^s - \lambda_2^s - \alpha_i \lambda_{is} \right) \times (q_{si} - q_{isi}) + \sum_{s=1}^S \left( \sum_{i=1}^I \alpha_i u - \text{cap}_i \right) \times (\lambda_{is} - \lambda_{isi}) + \sum_{s=1}^S \left( q_{si} - \sum_{i=1}^I q_{isi}^* \right) \times (\lambda_{is} - \lambda_{isi}) \geq 0
\]
\[
\forall (Q^1, Q^2, \lambda_1, \lambda_2) \in R^S
\]
\[
\lambda_1 = (\lambda_{11}, \lambda_{12}, ..., \lambda_{1s}), \lambda_2 = (\lambda_{21}, \lambda_{22}, ..., \lambda_{2s}) \) is Lagrange coefficient to ensure the constraint (1) established.

3.2 The behaviour of the manufacturers and their optimality conditions

\(\beta_{ij}\) indicates the coefficient of carbon emissions when products are supplied to retailers by manufacturers, \(\gamma_{id}\) indicates the coefficient of carbon emissions when products supplied to consumer markets by manufacturers. Carbon emissions \(\beta_{ij}q_{ij}\) are produced in the process of transaction between manufacturer \(i\) and retailer \(j\), carbon emissions \(\gamma_{id}q_{id}\) are produced in the process of transaction between manufacturer \(i\) and consumer market \(d\). We assume that there always have \(\sum_{j=1}^J \beta_{ij}q_{ij} + \sum_{d=1}^D \gamma_{id}q_{id} \leq \text{cap}_i\), manufacturer \(i\) can gain \(P_i \Delta e_{2i}\) through selling carbon emissions \(\Delta e_{2i} = \text{cap}_i - \sum_{j=1}^J \beta_{ij}q_{ij} + \sum_{d=1}^D \gamma_{id}q_{id}\) to suppliers.

The transaction risk of carbon market is \(r_i\), the carbon emissions given by government to manufacturer \(i\) are \(\text{cap}_i\). The manufactures is mainly to create product and sell the products to retailers or consumers directly. We can express the criterion of profit maximisation for manufacturer \(i\) as following
\[
\max \sum_{j=1}^J \left[ \beta_{ij}q_{ij} - c_{ij} (q_{ij}) - \beta_{ij}q_{ij} q_{ij} \right] + \sum_{d=1}^D \left[ \rho_{id}q_{id} - c_{id} (q_{id}) - \gamma_{id}q_{id} \right] + f_i(q_{ij}) - \sum_{j=1}^J \rho_{ij}q_{ij} + c_{ij} (q_{ij}) - \sum_{n=1}^N \rho_{in}q_{in} + c_{in} (q_{in}) \right] - \rho_i (1 - \phi_i) \sum_{s=1}^S q_{si} - \rho_i \sum_{j=1}^J q_{ij} + \sum_{d=1}^D q_{id} - \phi_i \sum_{n=1}^N q_{in} + \Delta e_{2i} \right) \]
\[
\sum_{j=1}^J q_{ij} + \sum_{d=1}^D q_{id} \leq q_i \quad (4)
\]
\[
q_i \leq \phi_i \sum_{s=1}^S q_{si} + \phi_i \sum_{n=1}^N q_{in} \quad (5)
\]
\[ \phi_2 q_i \leq \phi_3 \sum_{n=1}^{N} q_{il} \]  
(6)

\[ \phi_4 \left( \sum_{j=1}^{J} q_{ij} + \sum_{d=1}^{D} q_{id} \right) \leq \sum_{n=1}^{N} q_{il} \]  
(7)

\[ \sum_{j=1}^{J} \beta_j q_{ij} u + \sum_{d=1}^{D} \gamma_d q_{id} \leq \text{cap}_i \]  
(8)

Because of the non-cooperative competition game among manufactures, the optimality condition of all manufactures can be equivalent to the following variational inequality: determine \((Q^2, Q^3, Q^4, Q^5, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7) \in \mathbb{R}^{S+J+D+NI}\) satisfying

\[
\sum_{i=1}^{J} \left( \frac{\partial f_i(q^*_i)}{\partial q_i} + \phi_2 \lambda_3 - \lambda_5 \right) \times (q_i - q^*_i) \\
+ \sum_{s=1}^{S} \sum_{i=1}^{J} \left( \frac{\partial c_{ui}(q^*_u)}{\partial q_u} + \phi_4 \lambda_4 + \phi_6 \lambda_6 + \phi_7 \lambda_7 \right) \times (q_u - q^*_u) \\
+ \sum_{i=1}^{J} \sum_{j=1}^{J} \left( \frac{\partial c_{iy}(q^*_y)}{\partial q_y} + \phi_4 \lambda_4 + \phi_6 \lambda_6 + \phi_7 \lambda_7 \right) \times (q_y - q^*_y) \\
+ \sum_{d=1}^{D} \left( \frac{\partial c_{id}(q^*_d)}{\partial q_d} + \phi_4 \lambda_4 + \phi_6 \lambda_6 + \phi_7 \lambda_7 \right) \times (q_d - q^*_d) \\
+ \sum_{n=1}^{N} \sum_{i=1}^{J} \left( \frac{\partial c_{ni}(q^*_n)}{\partial q_n} + \phi_4 \lambda_4 + \phi_6 \lambda_6 + \phi_7 \lambda_7 \right) \times (q_n - q^*_n) \\
+ \sum_{j=1}^{J} \left( q^*_j - \sum_{j=1}^{J} q^*_j - \sum_{d=1}^{D} q^*_d \right) \times (\lambda_3 - \lambda_3) \\
+ \sum_{i=1}^{J} \left( \phi_4 \sum_{s=1}^{S} q^*_u + \phi_6 \sum_{n=1}^{N} q^*_n - q^*_i \right) \times (\lambda_4 - \lambda_4) \\
+ \sum_{i=1}^{J} \left( \phi_6 \sum_{n=1}^{N} q^*_n + \phi_7 q^*_i \right) \times (\lambda_5 - \lambda_5) \\
+ \sum_{d=1}^{D} \left( q^*_d + \phi_4 \sum_{j=1}^{J} q^*_j - \phi_6 \sum_{d=1}^{D} q^*_d \right) \times (\lambda_6 - \lambda_6) \\
+ \sum_{j=1}^{J} \left( \phi_7 q^*_j - \sum_{d=1}^{D} \gamma_d q^*_d \right) \times (\lambda_7 - \lambda_7) \geq 0 \\
\forall (Q^2, Q^3, Q^4, Q^5, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7) \in \mathbb{R}^{S+J+D+NI}\]

\(\lambda_3 = (\lambda_{31}, \lambda_{32}, \ldots, \lambda_{33}), \lambda_4 = (\lambda_{41}, \lambda_{42}, \ldots, \lambda_{44}), \lambda_5 = (\lambda_{51}, \lambda_{52}, \ldots, \lambda_{55}), \lambda_6 = (\lambda_{61}, \lambda_{62}, \ldots, \lambda_{66}), \lambda_7 = (\lambda_{71}, \lambda_{72}, \ldots, \lambda_{77})\) is Lagrange coefficient to ensure the constraint (4), (5), (6), (7) and (8) established.
3.3 The behaviour of the retailers and their optimality conditions

The retailers are involved in transactions both with the manufacturers since they wish to obtain the product for their retail outlets, as well as with the consumers, who are the ultimate purchasers of the product. Therefore, a retailer conducts transactions with not only manufacturers but also need markets. We assume $\delta_{j,d}$ indicates the coefficient of carbon emissions when products supplied to demand markets by retailers. The model of profit maximisation of retailer $j$ is as following.

$$\max \sum_{d=1}^{D} \left( \rho_{j,d} q_{j,d} - c_{j,d}(q_{j,d}) \right) - \sum_{i=1}^{t} \left( \rho_{j,i} q_{j,i} - c_{j,i}(q_{j,i}) \right)$$

$$- c_{j}(v_{j}) - \sum_{d=1}^{D} c_{j}(q_{j,d}) - \sum_{d=1}^{D} \delta_{j,d} q_{j,d}\theta$$

$$\text{s.t.} \sum_{d=1}^{D} q_{j,d} \leq \sum_{i=1}^{t} q_{j,i} \quad (10)$$

Because of the non-cooperative competition game among retailers and the handling cost for each retailers is continuous and convex, the optimality condition of all retailers can be equivalent to the following variational inequality: determine $(Q^{1}, Q^{1}, \lambda^{1}) \in R_{+}^{t \times JD}$ satisfying

$$\sum_{j=1}^{t} \sum_{i=1}^{t} \left( \frac{\partial c_{j,i}(q_{j,i})}{\partial q_{j,i}} + \rho_{j}^{\ast} - \lambda_{j,i} \right) \times \left( q_{j,i} - q_{j,i}^{\ast} \right)$$

$$+ \sum_{j=1}^{t} \sum_{d=1}^{D} \left( \frac{\partial c_{j,d}(q_{j,d})}{\partial q_{j,d}} + \frac{\partial c_{j}(v_{j})}{\partial q_{j,d}} + \frac{\partial c_{j,l}(q_{j,l})}{\partial q_{j,d}} + \delta_{j,d} \theta + \lambda_{j,d} - \rho_{j,d} \right)$$

$$\times \left( q_{j,d} - q_{j,d}^{\ast} \right) + \sum_{j=1}^{t} \left( \sum_{i=1}^{t} q_{j,i}^{\ast} - \sum_{d=1}^{D} q_{j,d}^{\ast} \right) \times \left( \lambda_{j,d} - \lambda_{j,d}^{\ast} \right) \geq 0$$

$$\forall (Q^{1}, Q^{1}, \lambda^{1}) \in R_{+}^{t \times JD}$$

where $v_{j} = \sum_{d=1}^{D} q_{j,d}$, besides, $\lambda_{j} = (\lambda_{j,1}, \lambda_{j,2}, \ldots, \lambda_{j,q})$ is Lagrange coefficient to ensure the constraint (10) established.

3.4 The consumers at demand markets and the equilibrium conditions

The consumers at demand markets need to make the decisions as following: the quantity of products purchased from the manufacturer and retailer; the price they willing to pay for the products; the quantity of worn-out products returned to recycling centres.

In the electronic commerce channels of forward supply chain, consumers need to consider both products price and transaction cost. It needs to meet the condition:

$$\rho_{j,d} + c_{j,d}(q_{j,d}^{\ast}) \begin{cases} \geq \rho_{j,d}^{\ast}, & q_{j,d} > 0 \\ \geq 0, & q_{j,d} = 0 \end{cases}$$
The second complementary condition shows that the market demand is equal to purchases when the equilibrium transaction price between consumers and manufactures is positive.

In the traditional physical channels of forward supply chain, consumers also need to consider both products price and transaction cost. It needs to meet the condition:

\[
\rho_{jd} + c_d (q_{jd}^*) \begin{cases} 
= \rho_{d}, & q_{jd} > 0 \\
\geq \rho_{d}, & q_{jd} = 0 
\end{cases}
\]

\[
h_{jd} (\rho_{d}) \begin{cases} 
= \sum_{i=1}^{I} q_{id}^*, & q_{jd} > 0 \\
\geq \sum_{i=1}^{I} q_{id}^*, & q_{jd} = 0 
\end{cases}
\]

The second complementary condition shows that the market demand is equal to purchases when the equilibrium transaction price between consumers and retailers is positive.

In reverse logistics, the higher the price of recycling worn-out products, the more willing of consumers to sell the worn-out products to recycling centres. Some consumers will sell the worn-out products to recycling centres when the recycling price is fixed. Assuming \( c_m (Q^k) \) represents the recycling inclination coefficient of consumers. The model of profit maximisation of consumer m is as following.

\[
c_{nd} (q_{nd}) + c_d (Q^k) \begin{cases} 
= \rho_{nd}, & q_{nd} > 0 \\
\geq \rho_{nd}, & q_{nd} = 0 
\end{cases}
\]

s.t \[\sum_{n=1}^{N} q_{nd} \leq \sum_{i=1}^{I} q_{id} + \sum_{j=1}^{J} q_{jd} \quad (12)\]

The variational inequality in the condition of equilibrium can given by: determine \((Q^k, \lambda_0) \in R_{ND}^\mathbb{R}\) such that

\[
\sum_{d=1}^{D} \sum_{n=1}^{N} (c_d (Q^k) + c_{nd} (q_{nd}) - \rho_{nd} - \lambda_{nd}) \times (q_{nd} - q_{nd}^*) \\
+ \sum_{j=1}^{J} \sum_{d=1}^{D} (\rho_{jd} + c_d (q_{jd}^*) - \rho_{jd} - \lambda_{jd}) \times (q_{jd} - q_{jd}^*) \\
+ \sum_{j=1}^{J} \sum_{d=1}^{D} q_{jd} (\rho_{jd} + c_d (q_{jd}^*) - \rho_{jd} - \lambda_{jd}) \times (q_{jd} - q_{jd}^*) \\
+ \sum_{d=1}^{D} \left( \sum_{i=1}^{I} q_{id}^* - h_{id} (\rho_{d}) \right) \times (\rho_{d} - \rho_{d}^*) + \sum_{d=1}^{D} \left( \sum_{j=1}^{J} q_{jd}^* - h_{jd} (\rho_{d}) \right) \\
\left( \lambda_{nd} - \lambda_{nd}^* \right) \geq 0 \quad \forall (Q^k, Q^*, \lambda_0) \quad (13)\]

\(\lambda_0 = (\lambda_{n1}, \lambda_{n2}, \ldots, \lambda_{nD})\) is Lagrange coefficient to ensure the constraint (12) established.
3.5 The behaviour of the recycling centres and their optimality conditions

The recycling centres need to make the decisions as following: the price and quantity of worn-out products which recovered from consumers and the price of these products when they are sold to manufacturers. We assume that $\rho_4$ indicates the subsidies for recycling of unit worn-out products. The model of profit maximisation of recycling centre $n$ is as following:

$$
\max \sum_{i_1=1}^{I} \left( \rho_{ad} q_{ad} - c_{ad} \left( q_{ad} \right) \right) + \rho_4 \sum_{d=1}^{D} q_{ad} - \sum_{d=1}^{D} \left( \rho_{ad} q_{ad} - c_{ad} \left( q_{ad} \right) \right) -
$$

$$
\sum_{d=1}^{D} c_{ad} \left( q_{ad} \right) - c_a \left( w_a \right)
$$

s.t

$$
\sum_{i_1=1}^{I} q_{ad} \leq \sum_{d=1}^{D} q_{ad}
$$

(14)

Because of the non-cooperative competition game among recycling centres and the handling cost for each recycling centres is continuous and convex. The optimality condition of all recycling centres can be equivalent to the following variational inequality: determine $(Q', Q^*, \lambda_{10}) \in R^{N_I + N_D}$ satisfying

$$
\sum_{n=1}^{N} \sum_{i_1=1}^{I} \left( \frac{\partial c_{ad}}{\partial q_{ad}} \left( q_{ad}^{*} \right) + \lambda_{10n}^{*} - \rho_n \right) \times \left( q_{ad} - q_{ad}^{*} \right) + 
$$

$$
\sum_{n=1}^{N} \sum_{d=1}^{D} \left( \frac{\partial c_{ad}}{\partial q_{ad}} \left( q_{ad}^{*} \right) + \frac{\partial c_{ad}}{\partial q_{ad}} \left( q_{ad}^{*} \right) + \frac{\partial c_{ad}}{\partial q_{ad}} \left( w_a^{*} \right) + \rho_n^{*} - \rho_4 - \lambda_{10n} \right) \times \left( q_{ad} - q_{ad}^{*} \right) + 
$$

$$
\sum_{n=1}^{N} \left( \sum_{d=1}^{D} q_{ad}^{*} - \sum_{i_1=1}^{I} q_{ad}^{*} \right) \times (\lambda_{10a} - \lambda_{10a}^{*}) \geq 0
$$

(15)

$$
\forall (Q', Q^*, \lambda_{10}) \in R^{N_I + N_D}
$$

where $w_a \sum_{d=1}^{D} q_{ad}$, besides, $\lambda_{10} = (\lambda_{101}, \lambda_{102}, \ldots, \lambda_{10a})$ is Lagrange coefficient to ensure the constraint (14) established

3.6 The optimum conditions of the trading centre of carbon

In this paper, we assume that there is only one carbon trading centre. In the trading centre, the supply of carbon emissions is from the manufacturers and the demand of carbon emissions is to the suppliers. The trading centre will charge a certain percentage of transaction cost in the middle.

The demand of carbon emissions of the suppliers is as following:

$$
\sum_{i_1=1}^{S} T_{i_1} = \sum_{s=1}^{S} \sum_{i_1=1}^{I} \left( \alpha_{s} q_{ad} - cap_s \right)
$$

The supply of carbon emissions of the manufacturers is as following.

$$
\sum_{j=1}^{J} T_{j_2} = \sum_{i_1=1}^{I} \left( cap_j - \sum_{j=1}^{J} \beta_{j_2} q_{j_2} - \sum_{d=1}^{D} \lambda_{ad} q_{ad} \right)
$$
The profits of the trading centre are as following:
\[ \sum_{i=1}^{S} \theta T_{is} = \sum_{i=1}^{I} \theta T_{2i} \]

The function of transaction risk and transaction cost undertaken by the trading centre when it sell carbon emissions to suppliers are as following:
\[ t_i = t_s(T_{is}) \]
\[ c'_i = c'_s(T_{is}) \]

The function of transaction risk and transaction cost caused by the carbon products which come from manufacturers are as following:
\[ t_i = t_s(T_{2i}) \]
\[ c'_i = c'_s(T_{2i}) \]

The optimal objective function of the carbon trading centre is:
\[
\max \sum_{s=1}^{S} \left( \sum_{i=1}^{I} \theta \alpha_{si} q_{si} - \theta \text{cap}_s - t_s(T_{is}) - c'_s(T_{is}) \right) \\
+ \sum_{i=1}^{I} \left( \theta \text{cap}_i - \sum_{j=1}^{J} \theta \beta_{ij} q_{ij} - \sum_{d=1}^{D} \theta \gamma_{id} q_{id} - t_s(T_{2i}) - c'_s(T_{2i}) \right) \\
\text{s.t} \quad \sum_{s=1}^{S} \left( \sum_{i=1}^{I} \alpha_{si} q_{si} - \text{cap}_s \right) = \sum_{i=1}^{I} \left( \text{cap}_i = \sum_{j=1}^{J} \beta_{ij} q_{ij} - \sum_{d=1}^{D} \gamma_{id} q_{id} \right) \\
(16)
\]

The constraints above balance the carbon product flow of both parties. The optimality condition of this planning can be equivalent to the following variational inequality:
\[
\text{determine } (Q^2, Q^3, Q^4, \lambda_{11}) \in R^{S+I+JD} \text{ satisfying}
\]
\[
\sum_{s=1}^{S} \sum_{i=1}^{I} \left( \frac{\partial c'_s(T_{is})}{\partial q_{si}} + \frac{\partial t_s(T_{is})}{\partial q_{si}} + \alpha_{si} \lambda_{11c} - \alpha_{si} \theta \right) \times (q_{si} - q^*_{si}) \\
+ \sum_{i=1}^{I} \sum_{j=1}^{J} \left( \frac{\partial c'_s(T_{ij})}{\partial q_{ij}} + \frac{\partial t_s(T_{ij})}{\partial q_{ij}} + \beta_{ij} \lambda_{11c} + \beta_{ij} \theta \right) \times (q_{ij} - q^*_{ij}) \\
+ \sum_{i=1}^{I} \sum_{d=1}^{D} \left( \frac{\partial c'_s(T_{id})}{\partial q_{id}} + \frac{\partial t_s(T_{id})}{\partial q_{id}} + \gamma_{id} \lambda_{11c} + \gamma_{id} \theta \right) \times (q_{id} - q^*_{id}) \\
+ \left( \sum_{i=1}^{I} \left( \text{cap}_i - \sum_{j=1}^{J} \beta_{ij} q^*_{ij} - \sum_{d=1}^{D} \gamma_{id} q^*_{id} \right) - \sum_{s=1}^{S} \left( \sum_{i=1}^{I} \alpha_{si} q_{si} - \text{cap}_s \right) \right) \\
\times (\lambda_{11c} - \lambda_{11c}^*) \geq 0 \\
\forall(Q^2, Q^3, Q^4, \lambda_{11}) \in R^{S+I+JD}
\]
\[
\lambda_{11} = (\lambda_{111}, \lambda_{112}, \ldots, \lambda_{11c}) \text{ Lagrange coefficient to ensure the constraint (16) established.}
\]
4 Equilibrium analysis

4.1 The equilibrium condition of the closed-loop supply chain

In equilibrium, the shipments of the product that the suppliers ship to the manufacturers must be equal to the manufacturers accept from the suppliers and for retailers, the output of the product is same as the input, besides, the amounts of the product purchased by the consumers at the demand markets must be equal to the amounts sold by the manufacturers and the retailers to the demand markets.

Definition 1: the equilibrium of dual channel closed-loop supply chain network is the production of products, the forward and reverse flows and the prices in the supply chain satisfy the sum of the optimality conditions (3), (9), (11), (15) and (17).

The following can now be established:

Theorem 1: the equilibrium conditions governing the closed-loop supply chain model with dual channel are equivalent to the solution of the following variational inequalities problem: that is to seek \( \mathbf{Q}, \mathbf{Q'}, \mathbf{Q''}, \mathbf{Q''}, \mathbf{P'}, \mathbf{P''}, \mathbf{p}_1, \mathbf{p}_2, \mathbf{\lambda}_1, \mathbf{\lambda}_2, \mathbf{\lambda}_3, \mathbf{\lambda}_4, \mathbf{\lambda}_5, \mathbf{\lambda}_6, \mathbf{\lambda}_7, \mathbf{\lambda}_8, \mathbf{\lambda}_9, \mathbf{\lambda}_{10}, \mathbf{\lambda}_{11} \) satisfying

\[
\sum_{i=1}^{S} \left( \frac{\partial f_i(q_i^*)}{\partial q_i} - \mathbf{z}_i \right) \times (q_i - q_i^*) + \sum_{j=1}^{l} \left( \frac{\partial f_j(q_j^*)}{\partial q_j} + \mathbf{z}_j + p_j \mathbf{z}_j - \mathbf{z}_j \right) \times (q_j - q_j^*) + \sum_{m=1}^{N} \sum_{i=1}^{l} \left( \frac{\partial c_{mi}(q_{mi}^*)}{\partial q_{mi}} + \mathbf{\delta}_m + \mathbf{\lambda}_m \right) \times (q_{mi} - q_{mi}^*)
\]

\[
\sum_{i=1}^{S} \left( \frac{\partial f_i(q_i^*)}{\partial q_i} - \mathbf{z}_i \right) \times (q_i - q_i^*) + \sum_{j=1}^{l} \left( \frac{\partial f_j(q_j^*)}{\partial q_j} + \mathbf{z}_j + p_j \mathbf{z}_j - \mathbf{z}_j \right) \times (q_j - q_j^*) + \sum_{m=1}^{N} \sum_{i=1}^{l} \left( \frac{\partial c_{mi}(q_{mi}^*)}{\partial q_{mi}} + \mathbf{\delta}_m + \mathbf{\lambda}_m \right) \times (q_{mi} - q_{mi}^*)
\]

\[
\sum_{i=1}^{S} \left( \frac{\partial f_i(q_i^*)}{\partial q_i} - \mathbf{z}_i \right) \times (q_i - q_i^*) + \sum_{j=1}^{l} \left( \frac{\partial f_j(q_j^*)}{\partial q_j} + \mathbf{z}_j + p_j \mathbf{z}_j - \mathbf{z}_j \right) \times (q_j - q_j^*) + \sum_{m=1}^{N} \sum_{i=1}^{l} \left( \frac{\partial c_{mi}(q_{mi}^*)}{\partial q_{mi}} + \mathbf{\delta}_m + \mathbf{\lambda}_m \right) \times (q_{mi} - q_{mi}^*)
\]

\[
\sum_{i=1}^{S} \left( \frac{\partial f_i(q_i^*)}{\partial q_i} - \mathbf{z}_i \right) \times (q_i - q_i^*) + \sum_{j=1}^{l} \left( \frac{\partial f_j(q_j^*)}{\partial q_j} + \mathbf{z}_j + p_j \mathbf{z}_j - \mathbf{z}_j \right) \times (q_j - q_j^*) + \sum_{m=1}^{N} \sum_{i=1}^{l} \left( \frac{\partial c_{mi}(q_{mi}^*)}{\partial q_{mi}} + \mathbf{\delta}_m + \mathbf{\lambda}_m \right) \times (q_{mi} - q_{mi}^*)
\]

\[
\sum_{i=1}^{S} \left( \frac{\partial f_i(q_i^*)}{\partial q_i} - \mathbf{z}_i \right) \times (q_i - q_i^*) + \sum_{j=1}^{l} \left( \frac{\partial f_j(q_j^*)}{\partial q_j} + \mathbf{z}_j + p_j \mathbf{z}_j - \mathbf{z}_j \right) \times (q_j - q_j^*) + \sum_{m=1}^{N} \sum_{i=1}^{l} \left( \frac{\partial c_{mi}(q_{mi}^*)}{\partial q_{mi}} + \mathbf{\delta}_m + \mathbf{\lambda}_m \right) \times (q_{mi} - q_{mi}^*)
\]

\[
\sum_{i=1}^{S} \left( \frac{\partial f_i(q_i^*)}{\partial q_i} - \mathbf{z}_i \right) \times (q_i - q_i^*) + \sum_{j=1}^{l} \left( \frac{\partial f_j(q_j^*)}{\partial q_j} + \mathbf{z}_j + p_j \mathbf{z}_j - \mathbf{z}_j \right) \times (q_j - q_j^*) + \sum_{m=1}^{N} \sum_{i=1}^{l} \left( \frac{\partial c_{mi}(q_{mi}^*)}{\partial q_{mi}} + \mathbf{\delta}_m + \mathbf{\lambda}_m \right) \times (q_{mi} - q_{mi}^*)
\]

\[
\sum_{i=1}^{S} \left( \frac{\partial f_i(q_i^*)}{\partial q_i} - \mathbf{z}_i \right) \times (q_i - q_i^*) + \sum_{j=1}^{l} \left( \frac{\partial f_j(q_j^*)}{\partial q_j} + \mathbf{z}_j + p_j \mathbf{z}_j - \mathbf{z}_j \right) \times (q_j - q_j^*) + \sum_{m=1}^{N} \sum_{i=1}^{l} \left( \frac{\partial c_{mi}(q_{mi}^*)}{\partial q_{mi}} + \mathbf{\delta}_m + \mathbf{\lambda}_m \right) \times (q_{mi} - q_{mi}^*)
\]
Proof: We establish that the equilibrium conditions (18). Indeed, the summation of the optimal conditions (3), (9), (11), (15) and (17) yields, after algebraic, simplification, condition (18).

We now establish the converse, to variational inequality (18), add *() to the term in the brackets preceding the forth multiplication sign, add the term *() to the term in the brackets preceding the fifth multiplication sign, add the term *() to the term in the brackets preceding the sixth multiplication sign, add the term *() to the term in the brackets preceding the seventh multiplication sign, add the term *() to the term in the brackets preceding the eighth multiplication sign. Such term do not change the value of the inequality and variational inequality (18) is exactly the sum of (3), (9), (11), (15) and (17). The proof is completed.

Variational inequality (18) can be rewritten as following: determine \( X^* \) satisfying

\[
\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in R^{S+1+SD+IN+ID+ND+NI}
\]

where \( F(X) \) is given by the respective functional terms preceding the multiplication signs in variational inequality (18) and \( F(X) = \left( F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{11}, F_{12} \right) \) the symbol \( \langle, \rangle \) represents inner product of Euclid space of finite dimension.
It is needed to note that some price variables are offset in the process of solving the operation of summing of the equilibrium problem in the supply chain. The solution of these variables can be obtained through quantity variables and Lagrange multipliers which have been solved. When $\rho_{ij}^*, \rho_{id}^*, \rho_{jd}^*, \rho_{nd}^*$ can be retrieved from an equilibrium solution by setting the third term and forth term in inequality (9), the second term in inequality (11), the first, second and third term in inequality (13), the first and second term in inequality (15) to 0.

$$\rho_{ij}^* = \frac{\partial c_{ij}(q_{ij})}{\partial q_{ij}} + \beta_{ij}u + \rho_3 + \beta_{ij}p_e + \lambda_{ij1} + \rho_{ij}* \lambda_{ij1}$$

$$\rho_{id}^* = \frac{\partial c_{id}(q_{id})}{\partial q_{id}} + \gamma_{id}u + \rho_5 + \gamma_{id}p_e + \lambda_{id1} + \rho_{id}* \lambda_{id1}$$

or $\rho_{id}^* = \rho_{id}^* + \lambda_{id} - c_{id}(q_{id})$

$$\rho_{jd}^* = \frac{\partial c_{jd}(q_{jd})}{\partial q_{jd}} + \frac{\partial c_{jd}(q_{jd})}{\partial q_{jd}} + \frac{\partial c_{jd}(q_{jd})}{\partial q_{jd}} + \gamma_{jd}u + \lambda_{jd}$$

or $\rho_{jd}^* = \rho_{jd}^* + \lambda_{jd} - c_{jd}(q_{jd})$

$$\rho_{nd}^* = c_{d}(Q^d) + c_{nd}(q_{nd}) \lambda_{nd}$$

or $\rho_{nd}^* = \rho_{nd} + \lambda_{nd} - c_{nd}(q_{nd})$

$$\rho_{ui}^* = \frac{\partial c_{ui}(q_{ui})}{\partial q_{ui}} + \lambda_{ui}$$

4.2 Qualitative properties

It is important for us to determine whether there is a solution exists before solving any variational inequality. Nagurney (1993) indicated that if the function $F$ is continuous and the feasible region is compact, then the variational inequality has at least one solution.

Therefore, let $K_b = \left\{Q^1, Q^2, Q^3, Q^4, Q^5, Q^6, Q^7, Q^8, p_1^*, p_2^*\right\}$, where $b = (b_1, b_2, \ldots, b_{21})$ $\geq 0$ and $Q^i \leq b_i$, $Q^i \leq b_i$, $p_1^* \leq b_0$, $p_2^* \leq b_0$, $b_0 \leq b_0$, $b_1 \leq b_1$, $b_1 \leq b_1$. Then $K_b$ is a bounded closed convex subset of $R^{21}$. Then the following variational inequality:

$$\left\{F(X^b), X - X^b\right\} \geq 0, \forall X^b \in K_b$$

admit at least one solution. Therefore, we have:
Lemma 1: Variational inequality (19) has a solution if and only if there is \( b = (b_1, b_2, \ldots, b_{17}) > 0 \) such that the variational inequality (20) has a solution in \( K_b \) with \( Q^1 \leq b_1, \ldots, Q^{17} \leq b_8, \rho_1^1 \leq b_9, \rho_2^1 \leq b_{10}, \lambda_1 \leq b_{11}, \ldots, \lambda_{11} \leq b_{21} \).

Theorem 2: (existence) Supposed the existence of positive constants \( M, N, R \) and \( M > R \).

Then variational inequality (18) admits at least one solution.

Lemma 2: (monotonicity) In inequality (18), If the functions \( f_s, f_i, c_{si}, c_{ni}, c_{ij}, (v_j), c_j(q_{jd}), c_n(w_n), c_{nd}(q_{nd}) \) are differentiable convex function, \( h_{i\beta}, h_{j\beta}, c_{i\beta}, c_{i\beta}, c_{j\beta}, c_{nd} \) are continuous function of monotone increasing, then the function \( F \) in variational inequality (19) is monotone function, that is
The equilibrium model of dual channel closed-loop supply chain network are continuous function of monotone increasing, then the function \( F \) is strictly monotone function, that is for any different \( X^t, X^n \in k \), there is
\[
\left( \left( F(X^t) - F(X^n) \right)^T, X^t - X^n \right) \geq 0, \forall X^t, X^n \in k
\]

**Theorem 3:** (uniqueness) variational inequality (18) has unique solution under the condition of Lemma 3. That is existing unique equilibrium flow and unique equilibrium price meet the network equilibrium of closed-loop supply chain with dual channel based on carbon emissions.

5 Model algorithm

The feasible region of variational inequality (18) on non-negative quadrant, so network equilibrium solution can be obtained by modified projection method (2004). Besides, the optimal Lagrange multipliers can also be obtained through the algorithm. Suppose \( P \) \( \Omega X \) represent the projection of \( X \) on non-negative quadrant \( \Omega = R_+^{s+3+5+7+11+14+16} \).

Specific steps of the algorithm are as following:

1. **Initialisation.** Suppose initial point \( X^0 \), iterations \( T = 1, k \) meet \( 0 < k < \frac{1}{L} \), \( L \) is Lipschitz continuity constant, set \( \epsilon > 0 \).
2. **Computation.** Computer \( \Xi^T \) by solving the variational inequality sub problem:
   \[
   \left( \left( \Xi^T + kF(\Xi^{t-1}) - X^{t-1} \right)^T, X - \Xi^T \right) \geq 0, X \in \Omega
   \]
3. **Adaptation.** Computer \( \Xi^T \) by solving the variational inequality sub problem:
   \[
   \left( \left( \Xi^T + kF(\Xi^{t-1}) - X^{t-1} \right)^T, X - \Xi^T \right) \geq 0, X \in \Omega
   \]
4. **Convergence verification.** If \( \max |X^T - X^{t-1}| \leq \epsilon \), a prespecified tolerance, then stop; else, set \( T = T + 1 \) and go to step (2) to continue iterating.

6 Conclusions and future research

In this paper, considering the conversion rate of raw materials, the remanufacturing rate stipulated by government, the conversion rate and the minimum recovery rate of worn-out products, an equilibrium model of dual-channel closed-loop supply chain network with carbon trading and carbon tax has been developed based on variational inequalities theory. Prices associated with manufacturers, retailers and consumers are endogenous. Besides, the article discussed the products output and pricing decisions in the addition which suppliers as the carbon buyer and manufacturers as the carbon sellers.

Qualitative properties of the equilibrium pattern were established, especially, the paper validates the existence and uniqueness of a solution under some reasonable assumptions on the underlying functions. Finally, the modified projection method was proposed for the computation of the product shipments and equilibrium prices.
For future research, we can consider the situation of multiple products and random demand and study the equilibrium of supply chain under some other measures to reduce carbon emissions.

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The equilibrium model of dual channel closed-loop supply chain network


