A multi-objective solid transportation problem with reliability for damageable items in random fuzzy environment

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Abstract In this paper, a multi-objective solid transportation problem (MOSTP) for damageable item is formulated and solved. First, we minimised the total cost of transportation and transportation time and maximise the reliability of transportation system. Here, transportation costs, resources, demands and capacities of conveyances are random fuzzy in natures. The transported item is likely to be damaged during transportation and damageability are different for different conveyances along different roots. The solid transportation problem (STP) is formulated as a decision making model optimising possibilistic value at risk (pVaR) by incorporating the concept of value at risk (VaR) into possibility and necessity measure theory. The reduced deterministic constrained problem is solved using generalised reduced gradient (GRG) method (LINGO-14.0). Some particular models has been presented. The model is illustrated with numerical examples and some sensitivity analysis is made on damageability.

Keywords: solid transportation; random fuzzy variables; possibility; damageability; reliability.
1 Introduction

The classical transportation problem (Hitchcock transportation problem) is one of the sub-classes of linear programming problem in which all the constraints are of equality type. In many industrial problems, a homogeneous product is delivered from an origin to a destination by means of different modes of transport called conveyances, such as trucks, cargo flights, goods trains, ships, etc. A solid transportation problem (STP) can be converted to a classical transportation problem by considering a single type of conveyance. In general, the real life problems are modelled with multi-objective functions which are measured in different respects and they are non-commensurable and conflicting in nature. Furthermore, it is frequently difficult for the decision maker to combine the objective functions in one overall utility function. In a STP more than one objective is normally considered. In many practical applications, it is realistic to assume that the amount which can be sent on any particular route is restricted by the capacity of that route. Further, when a route is altogether excluded, this can be expressed by limiting its capacity to zero. This is an alternative to attach a very high cost to that route.
Omar and Samir (2003) and Ebrahimnejad (2014) discussed the solution algorithm for solving the transportation problem in fuzzy environment. Recently, a few research papers have published on multi-objective solid transportation problems (MOSTPs) in uncertain environment. Kundu et al. (2013), Ojha et al. (2011), Narayanamoorthy and Anukokila (2015) and Dewess (2014) developed a MOSTP in fuzzy environment. Pramanic et al. (2014) proposed a MOSTP in fuzzy and bi-fuzzy environments. Giri et al. (2014) developed fuzzy fixed charge multi-item solid transportation problem. In this year, they also published a fuzzy stochastic STP using fuzzy goal programming approach (cf., Giri et al., 2014; Chakraborty et al., 2015).

In the transportation model, the entropy function acts as a measure of dispersal of trips between origins and destinations. It will be more practical to minimise the transportation penalties as well as to maximise entropy amount. In this situation, a capacitated, multi-objective, solid transportation system with fuzzy parameters may be developed considering entropy function as an additional objective function.

In this paper, for the first time, a MOSTP is formulated in random fuzzy environment. Here, the transportation costs, resources and demands at various centres, capacity of different modes of transport between origins and destinations are imprecise, i.e., uncertain in non-stochastic sense. Here, we maximise the reliability in transportation system. For the first time, the random fuzzy interactive satisfied method (FISM) has been introduced in the transportation to transformed multi-objective problems into single objective problem. Two special models have been derived in fuzzy and stochastic environments respectively from random fuzzy transportation problem. The reduced single-objective transportation problem is solved using generalised reduced gradient (GRG) techniques. These models are illustrated with an example. The effect of damageability on the proposed model has numerically pointed out.

2 Literature review

et al. (2013) and Chakraborty et al. (2014) have developed a STP in uncertainty environments.

Also, Ojha et al. (2010) and Xu and Tao (2012) published MOSTP in stochastic environment. Tao and Xu (2012) proposed multi-objective solid transportation problem in rough environment. In practice, we may come across a specific phenomenon that fuzziness and randomness simultaneously appear in STP problems (cf., Kwakernaak, 1978; Bhattacharya, 2007; and others). Then, the fuzzy random variable initiated by Kwakernaak (1978) is used to describe this type of uncertainty. So far, how to establish the optimisation models to cope with this type of uncertainty in the inventory problem is still a new and challenging work. Recently, some researchers discussed the STP problem in fuzzy random environments. However, the literatures about the studies on fuzzy random inventory problem are still lacking. Recently, a few research paper have published on MOSTPs in uncertain environment. Kundu et al. (2013), Ojha et al. (2011), Narayanamoorthy and Anukokila (2015) and Dewess (2014) developed a MOSTP in fuzzy environment. Pramanic et al. (2014) proposed a MOSTP in fuzzy and bi-fuzzy environments.

3 Preliminary knowledge about random fuzzy variables

Definition 1 [possibility space (Liu, 2004)]: Let \( \Theta \) be a non-empty set, and \( P(\Theta) \) be the power set of \( \Theta \). For each \( A \in P(\Theta) \), there is a non-negative number \( Pos(A) \), called its possibility, such that

1. \( Pos(\emptyset) = 0, \) \( Pos(\Theta) = 1 \)
2. \( Pos(\bigcup_k A_k) = \sup_k Pos(A_k) \) for any arbitrary collection \( A_k \) in \( P(\Theta) \).

The triplet \( (\Theta, P(\Theta), Pos) \) is called a possibility space, and the function \( Pos \) is referred to as a possibility measure. Then, a random fuzzy variable is firstly defined by Liu (2004) as a function from a possibility space to a collection of random variables.

Definition 2 [random fuzzy variable (Liu, 2004)]: A random fuzzy variable is defined as a function from the possibility space \( (\Theta, P(\Theta), Pos) \) to the set of random variables. An example of random fuzzy variables are given by Liu (2004) as follows:

Example 1 (Liu, 2004): Assume that \( \eta_1, \eta_2, \ldots, \eta_m \) are random variables and \( u_1, u_2, \ldots, u_m \) are real numbers in \([0,1]\) such that \( u_1 \lor u_2 \lor \cdots \lor u_m = 1 \). Then \( \tilde{\xi} \) is a random fuzzy variable expressed as

\[
\tilde{\xi} = \begin{cases} 
\eta_1 & \text{with possibility } u_1, \\
\eta_2 & \text{with possibility } u_2, \\
\vdots & \\
\eta_m & \text{with possibility } u_m.
\end{cases}
\]
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It should be noted here that \( \tilde{\xi}(i) = \tilde{\eta}_i, i = 1, 2, \cdots, m \) are regarded as functions from a possibility space \((\Theta, P(\Theta), \text{Pos})\) to a collection of random variables \( \Gamma \) if we define \( \Theta = \{1, 2, \cdots, m\}, \text{Pos}(i) = u_i, i = 1, 2, \cdots, m \) and \( \Gamma = \{\tilde{\eta}_1, \tilde{\eta}_2, \cdots, \tilde{\eta}_m\} \).

**Definition 3** [membership function of a random fuzzy variable (Liu, 2004)]: Let \( \tilde{\xi} \) be a random fuzzy variable on the possibility space \((\Theta, P(\Theta), \text{Pos})\). Then its membership function is derived from the possibility measure \( \text{Pos} \) by

\[
\mu(\tilde{\eta}) = \text{Pos}\{\theta \in \Theta | \tilde{\xi}(\theta)\}, \tilde{\eta} \in \Gamma
\]  

**Definition 4** [random fuzzy variable (Katagiri et al., 2012)]: Let \( \Gamma \) be a collection of random variables. Then, a random fuzzy variable \( \tilde{C} \) is defined by its membership function

\[
\mu_{\tilde{C}} : \Gamma \rightarrow [0, 1]
\]

**Example 2**: Assume that \( \tilde{\eta}_1, \tilde{\eta}_2, \cdots, \tilde{\eta}_m \) are random variables and \( u_1, u_2, \cdots, u_m \) are real numbers in \([0, 1]\) such that \( \max\{u_1, u_2, \cdots, u_m\} = 1 \). Then \( \tilde{\eta} \) is a random fuzzy variable and its membership function is expressed as

\[
\tilde{\xi}(\gamma) = \begin{cases} 
  u_1 & \text{if } \gamma = \tilde{\eta}_1, \\
  u_2 & \text{if } \gamma = \tilde{\eta}_2, \\
  \cdots \\
  u_m & \text{if } \gamma = \tilde{\eta}_m.
\end{cases}
\]

4 Multi-objective linear programming problems with random fuzzy variables

Consider the following random fuzzy multi-objective linear programming problems formulated as

\[
\begin{aligned}
\min & \quad \tilde{C}_l x, l = 1, 2, \cdots, k \\
\text{s.t} & \quad \tilde{A}_i(x, \tilde{\xi}) \leq \tilde{B}, i = 1, 2, \cdots, r \\
& \quad x \geq 0
\end{aligned}
\]  

where \( x \) is an \( n \) dimensional decision variable column vector.

When we formulate multi-objective programming problems as stochastic programming (Birge and Louveaux, 2011; Infanger, 2011), one of the most basic approaches is to assume that \( \tilde{c} = (\tilde{c}_{111}, \tilde{c}_{112}, \cdots, \tilde{c}_{11n}; \cdots, \tilde{c}_{n11}, \tilde{c}_{n12}, \cdots, \tilde{c}_{nmk}) \) is a random variable vector which has multivariate Gaussian random distribution.

In this paper, we assume that the mean of \( \tilde{c}_l \) is represented with an L-L fuzzy number \( \mu_{\tilde{c}_l} \) characterised by the membership function, is given by

\[
\mu_{\tilde{c}_l}^* (\tau) = \begin{cases} 
  L \left( \frac{m_{ij}^* - \tau}{\alpha_{ij}} \right) & \text{for } m_{ij}^* \geq \tau \\
  L \left( \frac{\tau - m_{ij}^*}{\alpha_{ij}} \right) & \text{for } m_{ij}^* < \tau
\end{cases}
\]
where the shape functions $L$ is a non-negative continuous function satisfying the following conditions:

a) $L(t)$ is non-increasing for any $t > 0$.

b) $L(t) = 1$

c) $L(t) = L(-t)$ for any $t \in \mathbb{R}$

d) There exists a $t^*_L$ such that $L(t) = 0$ for any $t$ larger than $t^*_L$.

The parameters $m_{i,j}^c$, $\alpha_{i,j}^c$ and $\beta_{i,j}^c$ are real constant values, and the values of $\alpha_{i,j}^c$ and $\beta_{i,j}^c$ represent left and right spreads of the fuzzy number $\tilde{M}_{i,j}$. Figure 1 illustrates an example of the membership function $\mu_{\tilde{M}_{i,j}}(\tau)$.

**Figure 1** An example of the membership function $\mu_{\tilde{M}_{i,j}}(\tau)$

\[ \mu_{\tilde{M}_{i,j}}(\tau) \]

Recently, Katagiri et al. (2012) and Pramanik et al. (2013) have developed random fuzzy multi-objective linear programming: optimisation of possibilistic value at risk (pVaR). Using possibility and necessity approach the above problem (26) can be expressed as

\[
\begin{align*}
&\min_{x} f_l, \quad l = 1, 2, \ldots, k \\
&\quad \text{subject to} \\
&\quad \text{Pos}\{\text{Prob}\{\tilde{C}_l \leq f_l\} \geq \tilde{\theta}_i^{obj}\} \geq \tilde{h}_i^{obj}, l = 1, 2, \ldots, k \\
&\quad \text{Nec}\{\text{Prob}\{\tilde{C}_l \leq f_l\} \geq \tilde{\theta}_i^{obj}\} \geq \tilde{h}_i^{obj}, l = 1, 2, \ldots, k \\
&\quad \text{Pos}\{\text{Prob}\{\tilde{A}x \leq \tilde{B}\} \geq \tilde{\theta}_i^{ext}\} \geq \tilde{h}_i^{ext}, i = 1, 2, \ldots, r \\
&\quad \text{Nec}\{\text{Prob}\{\tilde{A}x \leq \tilde{B}\} \geq \tilde{\theta}_i^{ext}\} \geq \tilde{h}_i^{ext}, i = 1, 2, \ldots, r \\
&\quad x \geq 0
\end{align*}
\] (6)

Degree of possibility of both objective and constraints can be depicted in Figures 2 and 3, respectively.

**Theorem 1:** Katagiri et al. (2012)

\[
\begin{align*}
\text{Pos}\{\text{Prob}\{\tilde{C}_l \leq f_l\} \geq \tilde{\theta}_i^{obj}\} & \geq \tilde{h}_i^{obj}, l = 1, 2, \ldots, k \quad \text{and} \\
\text{Nec}\{\text{Prob}\{\tilde{C}_l \leq f_l\} \geq \tilde{\theta}_i^{obj}\} & \geq \tilde{h}_i^{obj}, l = 1, 2, \ldots, k
\end{align*}
\]
is equivalently transformed into the condition

\[
\sum_{j=1}^{n} \{ m_{ij} - L^*(\hat{h}_{ij}^{\text{obj}}) \alpha_{ij} \} x_j + \Phi^{-1}(\hat{\theta}_{ij}^{\text{obj}}) \sqrt{x^t V_i^c x} \leq f_i \text{ and }
\]

\[
\sum_{j=1}^{n} \{ m_{ij} - L^*(1 - \hat{h}_{ij}^{\text{obj}}) \beta_{ij} \} x_j + \Phi^{-1}(\hat{\theta}_{ij}^{\text{obj}}) \sqrt{x^t V_i^c x} \leq f_i
\]

---

Figure 2  Degree of possibility \( \text{Pos}(\text{Prob}(\tilde{C}_i x \leq f_i) \geq \hat{\theta}_{ij}^{\text{obj}}) \geq \hat{h}_{ij}^{\text{obj}}) \)

![Figure 2](image)

---

Figure 3  Degree of possibility \( \text{Pos}(\text{Prob}(\tilde{A} x \leq \tilde{B}) \geq \hat{h}_{ij}^{\text{cat}}) \geq \hat{h}_{ij}^{\text{cat}}) \)

![Figure 3](image)

---

**Theorem 2:** Katagiri et al. (2012)

\[ \text{Pos}(\text{Prob}(\tilde{A} x \leq \tilde{B}) \geq \hat{\theta}_{ij}^{\text{cat}}) \geq \hat{h}_{ij}^{\text{cat}}, i = 1, 2, \ldots, r \]  
and  
\[ \text{Nec}(\text{Prob}(\tilde{A} x \leq \tilde{B}) \geq \hat{\theta}_{ij}^{\text{cat}}) \geq \hat{h}_{ij}^{\text{cat}}, i = 1, 2, \ldots, r \]

are equivalently transformed into the conditions

\[
\sum_{j=1}^{n} \{ m_{ij} - L^*(\hat{h}_{ij}^{\text{obj}}) \alpha_{ij} \} x_j + \Phi^{-1}(\hat{\theta}_{ij}^{\text{cat}}) \sqrt{x^t V_i^c x + (\sigma_i^t)^2} \leq m_i^b + L^*(\hat{h}_{ij}^{\text{cat}}) b_{ij}
\]

and

\[
\sum_{j=1}^{n} \{ m_{ij} - L^*(1 - \hat{h}_{ij}^{\text{obj}}) \alpha_{ij} \} x_j + \Phi^{-1}(\hat{\theta}_{ij}^{\text{cat}}) \sqrt{x^t V_i^c x + (\sigma_i^t)^2}
\]

\[
\leq m_i^b - L^*(1 - \hat{h}_{ij}^{\text{cat}}) b_{ij}
\]
Using Theorem 1 and Theorem 2, the above problem (6) can be written as

\[
\begin{align*}
\min x \\
\text{s.t.} \\
\end{align*}
\]

\[
\begin{align*}
\sum_{j=1}^{n} (m_{ij}^l - L^*(h_{ij}^{obj})\alpha_{ij}^l) x_j + \Phi^{-1}(\theta_{ij}^{obj}) \sqrt{x^TV_i^l x} & \leq f_i \\
\sum_{j=1}^{n} (m_{ij}^c - L^*(1 - h_{ij}^{obj})\beta_{ij}^c) x_j + \Phi^{-1}(\theta_{ij}^{ext}) \sqrt{x^TV_i^c x} & \leq f_i \\
\sum_{j=1}^{n} (m_{ij}^c - L^*(\hat{h}_{ij}^{obj})\alpha_{ij}^c) x_j + \Phi^{-1}(\hat{\theta}_{ij}^{ext}) \sqrt{x^TV_i^c x} & \leq \alpha_{ij}^c \sqrt{x^TV_i^c x} + (\sigma_{ij}^c)^2 \\
x \geq 0, i = 1, 2, \ldots, r
\end{align*}
\]

(7)

Theorem 3 (Xu, 2011): If \( \tilde{a}_r, \tilde{b}_r \) is triangular LR fuzzy variables, then the following expression are equivalent

\[
\text{Pos}\left\{ \sum_{j=1}^{n} \tilde{a}_r^T x \leq \tilde{b}_r \right\} \geq \theta_r
\]

\[
\Leftrightarrow b_r - \theta_r \alpha_r^T \geq a_r^T x \leq b_r + (\theta_r \beta_r^T x, r = 1, 2, \ldots, p
\]

5 Multi-objective programming problem in fuzzy environment

Let us consider, a fuzzy nonlinear multi-objective maximising problem where objective goal is only imprecise in nature, and contains no fuzzy parameters as below (if fuzzy parameters are appeared in the problem it can be replaced by equivalent GMIV):

\[
\begin{align*}
\text{Min} & \quad f_j(x), \quad j = 1, 2, \ldots, r \\
\text{Max} & \quad f_j(x), \quad j = r + 1, r + 2, \ldots, k \\
\text{subject to} & \quad x \in X
\end{align*}
\]

\[
X = \left\{ \begin{array}{l}
x = (x_1, x_2, \ldots, x_n)^T \\
x : g_r(x) \leq b_r, \quad r = 1, 2, \ldots, m \\
x_i \geq 0, \quad i = 1, 2, \ldots, n
\end{array} \right\}
\]

(8)

Here, objectives are imprecise in nature.

5.1 Interactive fuzzy satisfying method

We introduce the interactive fuzzy satisfied technique (FIST) proposed by Sakawa (1993). Sakawa et al. (1987) proposed interactive fuzzy satisfying (IFS) method to solve multi-objective nonlinear programming problem with fuzzy goals. Following Sakawa et al. (1987) an interactive approach can be developed to solve a constraint multi-objective nonlinear programming problem like (8). To do this, at first, membership function \( \mu_{f_i}(x) \) of each objectives \( f_i(x) \) \( i = 1, 2, \ldots, k \) has to be derived. For
this purpose the individual objective function $f_i(x)$ under the given constraints are considered and its minimum value $f_i^{\text{min}}$ and maximum value $f_i^{\text{max}}$ for $i = 1, 2, \ldots, k$ are determined. Depending upon these maximum and minimum values, DM specifies lower limit $L_i$ and upper limit $U_i$ of $\mu f_i(x)$ where $f_i^{\text{min}} \leq L_i \leq U_i \leq f_i^{\text{max}}$ for $i = 1, 2, \ldots, k$. Then DM constructs membership function $\mu f_i(x)$ for each of the minimisation type objective functions $f_i(x)$, $i = 1, 2, \ldots, r$, as below:

\[
\mu f_i(x) = \begin{cases}
1 & \text{for } L_i < f_i(x) \\
d_i(f_i(x)) & \text{for } L_i \leq f_i(x) \leq U_i \\
0 & \text{for } f_i(x) > U_i
\end{cases}
\]

where $d_i(f_i(x))$ is a strictly monotonic decreasing continuous function of $f_i(x)$, $i = r + 1, r + 2, \ldots, k$, membership functions $\mu f_i(x)$ takes the form:

\[
\mu f_i(x) = \begin{cases}
0 & \text{for } L_i < f_i(x) \\
d_i(f_i(x)) & \text{for } L_i \leq f_i(x) \leq U_i \\
1 & \text{for } f_i(x) > U_i
\end{cases}
\]

where $d_i(f_i(x))$ is a strictly monotonic increasing continuous function of $f_i(x)$ which may be linear or nonlinear. Different types of membership functions are presented below:

**Linear membership function:** Linear membership functions $\mu f_i(x)$ for minimisation and maximisation types objective function $f_i(x)$ are respectively:

\[
\mu f_i(x) = \begin{cases}
1 - \frac{f_i(x) - L_i}{U_i - L_i} & \text{for } L_i < f_i(x) \\
0 & \text{for } L_i \leq f_i(x) \leq U_i \\
1 & \text{for } f_i(x) > U_i
\end{cases}
\]  

(11)

and

\[
\mu f_i(x) = \begin{cases}
1 - \frac{U_i - f_i(x)}{U_i - L_i} & \text{for } L_i < f_i(x) \\
0 & \text{for } L_i \leq f_i(x) \leq U_i \\
1 & \text{for } f_i(x) > U_i
\end{cases}
\]  

(12)

After determination of different linear/nonlinear membership functions for each of the objective functions, DM specifies his/her choice of membership functions as well as reference levels of achievement of the membership functions (called reference membership values), for different objectives. According to the choice of membership functions for different objectives, let $\mu f_i, i = 1, 2, \ldots, k$ be the respective reference membership values of the DM’s. Then corresponding pareto optimal solution of the problem (2.35) can be obtained by solving the following min-max problem:

\[
\min_{x \in X} \max_{1 \leq i \leq k} (\mu f_i(x) - \mu f_i(x))
\]  

(13)

which is equivalent to

\[
\min_{x \in X} \alpha \quad \text{subject to } (\mu f_i(x) - \mu f_i(x)) \leq \alpha \quad \text{for } i = 1, 2, \ldots, k
\]  

(14)

where value of $\mu f_i$, and form of $\mu f_i(x)$ depend on DM’s choice.
6 Notations and assumptions

6.1 Notations

In this STP the following notations

- $m$ number of sources of the transportation problem
- $n$ number of destinations of the transportation problem
- $K$ number of conveyances, i.e., different modes of the transportation problem
- $O_i$ origins of the transportation problem
- $D_j$ destination of the transportation problem
- $E_k$ conveyances of the transportation problem
- $\tilde{a}_i$ random fuzzy amount of a homogeneous product available at $i$th origin
- $\tilde{b}_j$ random fuzzy demand at $j$th destination
- $\tilde{e}_k$ random fuzzy amount of product which can be carried by $k$th conveyance
- $r_{ijk}$ reliability of transportation reachable of item from from $i$th source to $j$th destination by means of the $k$th conveyance
- $\tilde{c}_{ijk}$ random fuzzy unit transportation cost from $i$th origin to $j$th destination by $k$th conveyance
- $\tilde{t}_{ijk}$ random fuzzy transportation time with respect to transportation activity from source $i$ to destination $j$ by conveyance $k$
- $x_{ijk}$ the amount to be transported from $i$th origin to $j$th destination by means of $k$th conveyance (decision variables)
- $\lambda_{ijk}$ amount of damaging item of the transportation problem from $i$th origin to $j$th destination by $k$th conveyance
- $R_s$ overall reliability of the transportation system.

6.2 Assumption

In this STP, the following assumptions are made.

1. Due to damageability of the units, the transported damaged amount is constant.

7 Formulation of MOSTP

7.1 Model in random fuzzy environment (Model 1)

We consider $m$ origins (or sources) $O_i$ ($i = 1, 2, \ldots, m$), $n$ destinations (i.e., demands) $D_j$ ($j = 1, 2, \ldots$) and $K$ conveyances $E_k$ ($k = 1, 2, \ldots, K$). $K$ conveyances, i.e., different modes of transport may be trucks, cargo flights, goods trains, ships, etc. Let $\tilde{a}_i$ be the random fuzzy amount of a homogeneous product available at $i$th origin, $\tilde{b}_j$ be the random fuzzy demand at $j$th destination and $\tilde{e}_k$ represents the random fuzzy...
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amount of product which can be carried by \( k \)th conveyance. The variable \( x_{ijk} \) represents the unknown quantity to be transported from origin \( O_i \) to destination \( D_j \) by means of \( k \)th conveyance. Then, we propose the mathematical model for the MOSTP with fuzzy resources, demands, conveyances and cost coefficients. One objective of the problem is to minimise the total transportation cost as follows:

\[
f_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{c}_{ijk} \ x_{ijk}
\]

(15)

If the transportation activity occurs, then transportation time will be spent. In this paper, we use \( \tilde{t}_{ijk} \) to denote the transportation time with respect to transportation activity from source \( i \) to destination \( j \) by conveyance \( k \). Then the total transportation time can be formulated as:

\[
f_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} y(x_{ijk}) \tilde{t}_{ijk}
\]

(16)

should be minimised, where

\[
y(x_{ijk}) = \begin{cases} 
1 & \text{for } x_{ijk} > 0 \\
0 & \text{for } x_{ijk} = 0 
\end{cases}
\]

(17)

From the discussion above, we develop mathematical formulations of objectives as follows:

\[
\min f_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{c}_{ijk} \ x_{ijk}
\]

(18)

\[
\min f_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} y(x_{ijk}) \tilde{t}_{ijk}
\]

(19)

\[
\max R_a = \prod_{i=1}^{m} \left[ 1 - \prod_{j=1}^{n} \prod_{k=1}^{K} (1 - r_{ijk}) x_{ijk} \right]
\]

(20)

As mentioned by Haley (1962), the constraints are divided into three types: source constraint, destination constraint and conveyance capacity constraint. Since the quantity from a source cannot exceed the supply capacity of products, we have

\[
\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq \bar{a}_i \quad i = 1, 2, 3, \ldots, m
\]

(21)
The quantity of product transported to a destination should be greater than its demand, that is

\[
\sum_{i=1}^{m} \sum_{k=1}^{K} (1 - \lambda_{ijk}) x_{ijk} \geq \tilde{b}_j \quad j = 1, 2, 3, \cdots, n
\]  

(22)

In addition, in order to ensure transportation safety, the transportation quantity of a conveyance should not exceed its capacity, then

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq \tilde{e}_k \quad k = 1, 2, 3, \cdots, K
\]  

(23)

It is natural to require the non-negativity of decision variable \(x_{ijk}\), that is

\[
x_{ijk} \geq 0 \quad \forall i, j, k.
\]  

(24)

It is noted that the Decision Maker (DM) and the modelling analyst are often different individuals. In the transportation problem, the DM is the manager of the transport enterprise, while the modelling analyst may be an expert in transportation problems, or a researcher in the enterprise. With the complexity of the feasible region, the DM may give an appropriately large region so that all the feasible solutions are included in it. The random fuzzy MOSTP can be written as

\[
\min \tilde{f}_1, \tilde{f}_2 = \left[ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{c}_{ijk} x_{ijk} \right], \left[ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{t}_{ijk}, y(x_{ijk}) \right]
\]

\[
\max R_k = \prod_{i=1}^{m} \left[ 1 - \prod_{j=1}^{n} \prod_{k=1}^{K} \left( 1 - r_{ijk} \right) x_{ijk} \right]
\]

s.t

\[
\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq \tilde{a}_i \quad i = 1, 2, \cdots, m
\]

\[
\sum_{i=1}^{m} \sum_{k=1}^{K} (1 - \lambda_{ijk}) x_{ijk} \geq \tilde{b}_j \quad j = 1, 2, \cdots, n
\]  

(25)

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq \tilde{e}_k \quad k = 1, 2, \cdots, K
\]

\[
x_{ijk} \geq 0 \quad \forall i, j, k.
\]

where \(\tilde{a}_i, \tilde{b}_j, \tilde{e}_k, \tilde{c}_{ijk}, \tilde{t}_{ijk}\) are random fuzzy variables. In order to construct a new model which was originally introduced in the framework of stochastic programming and/or financial engineering. However, this approach cannot be directly applied to (25) because
the objective function involves not only randomness but also fuzziness. Therefore, we consider a new decision making model optimising pVaR formulated as follows:

$$\begin{align*}
\min_{\bar{f}_1, \bar{f}_2} \\
\max_{x} \quad & R_x \\
\text{s.t.} \\
& \text{Pos}\{\text{Prob}\left(\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{e}_{ijk} x_{ijk} \leq \bar{f}_1 \right) \geq \tilde{\theta}_1^{\text{obj}} \} \geq \tilde{h}_1^{\text{obj}} \\
& \text{Neg}\{\text{Prob}\left(\sum_{i=1}^{m} \sum_{j=1}^{n} y(x_{ijk}) \bar{f}_2 \leq \bar{f}_2 \right) \geq \tilde{\theta}_2^{\text{obj}} \} \geq \tilde{h}_2^{\text{obj}} \\
& \text{Pos}\{\text{Prob}\left(\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq \tilde{\alpha}_i \right) \geq \tilde{\theta}_1^{\text{ext}} \} \geq \tilde{h}_1^{\text{ext}} \\
& \text{Pos}\{\text{Prob}\left(\sum_{i=1}^{m} \sum_{j=1}^{n} (1 - \lambda_{ijk}) x_{ijk} \geq \tilde{\beta}_j \right) \geq \tilde{\theta}_2^{\text{ext}} \} \geq \tilde{h}_2^{\text{ext}} \\
& \text{Neg}\{\text{Prob}\left(\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq \tilde{\epsilon}_k \right) \geq \tilde{\theta}_3^{\text{ext}} \} \geq \tilde{h}_3^{\text{ext}} \\
& x_{ijk} \geq 0
\end{align*}$$

(26)

From Theorem 3 and Theorem 4, (26) is equivalently transformed into

$$\begin{align*}
\min_{\bar{f}_1, \bar{f}_2} \\
\max_{x} \quad & R_x \\
\text{s.t.} \\
& \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{m}_{ijk} \tilde{e}_{ijk} - L^*(\bar{f}_1) \tilde{\alpha}_{ijk} x_{ijk} \\
& + \Phi^{-1}(\tilde{\theta}_1^{\text{ext}}) \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sigma_{ijk}^2 \tilde{e}_{ijk}^2} \leq \bar{f}_1 \\
& \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{m}_{ijk} \tilde{e}_{ijk} - L^*(1 - \tilde{h}_2^{\text{ext}}) \tilde{\beta}_{ijk} x_{ijk} \\
& + \Phi^{-1}(\tilde{\theta}_2^{\text{ext}}) \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sigma_{ijk}^2 \tilde{e}_{ijk}^2} \leq \bar{f}_2 \\
& \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} + \Phi^{-1}(\tilde{\theta}_3^{\text{ext}}) \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sigma_{ijk}^2} \leq m_i \tilde{\alpha}_i \\
& + L^*(\bar{h}_i) \tilde{\alpha}_i \\
& \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} - \lambda_{ijk} + \Phi^{-1}(\tilde{\theta}_4^{\text{ext}}) \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sigma_{ijk}^2} \leq m_j \tilde{\beta}_j \\
& + L^*(\bar{h}_j) \tilde{\beta}_j \\
& \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} + \Phi^{-1}(\tilde{\theta}_5^{\text{ext}}) \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sigma_{ijk}^2} \leq m_k \tilde{\epsilon}_k \\
& - L^*(1 - \bar{h}_k) \tilde{\epsilon}_k \tilde{\alpha}_k x_{ijk} \geq 0
\end{align*}$$

(27)

The above problem can be solved by fuzzy satisfied method (§4)
7.2 Particular cases

7.2.1 Model in random environment (Model 2)

In this random (stochastic) unbalanced solid transportation problem, may be formulated as

\[
\min \mathbf{f} = \left[ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{c}_{ijk} x_{ijk} \right], \quad \left[ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{f}_{ijk} y(x_{ijk}) \right]
\]

\[
\max R_s = \prod_{i=1}^{m} \left[ 1 - \prod_{j=1}^{n} \prod_{k=1}^{K} \left( 1 - r_{ijk} \right) x_{ijk} \right]
\]

s.t

\[
\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq \tilde{a}_i \quad i = 1, 2, \cdots, m
\]

\[
\sum_{i=1}^{m} \sum_{k=1}^{K} (1 - \lambda_{ijk}) x_{ijk} \geq \tilde{b}_j \quad j = 1, 2, \cdots, n
\] (28)

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq \tilde{c}_k \quad k = 1, 2, \cdots, K
\]

\[
x_{ijk} \geq 0 \quad \forall i, j, k.
\]

Here \( \tilde{a}_i, \tilde{b}_j, \tilde{c}_k, \tilde{C}_{ijk}, \tilde{t}_{ijk} \) are considered as a random variables. Using fuzzy programming algorithm (cf. Bhattacharya), the above objective functions reduces to

\[
\min \mathbf{f}_1 = \sum_{i=1}^{m} \theta_{i1} \sum_{j=1}^{n} \sum_{k=1}^{K} m_{\tilde{c}_{ijk}} x_{ijk} + \sum_{i=1}^{m} \theta_{i2} \sum_{j=1}^{n} \sum_{k=1}^{K} \sigma_{\tilde{c}_{ijk}}^2 x_{ijk}^2
\] (29)

\[
\min \mathbf{f}_2 = \sum_{i=1}^{m} \theta_{i3} \sum_{j=1}^{n} \sum_{k=1}^{K} m_{\tilde{t}_{ijk}} x_{ijk} + \sum_{i=1}^{m} \theta_{i4} \sum_{j=1}^{n} \sum_{k=1}^{K} \sigma_{\tilde{t}_{ijk}}^2 x_{ijk}^2
\] (30)

\[
\max R_s = \prod_{i=1}^{m} \left[ 1 - \prod_{j=1}^{n} \prod_{k=1}^{K} \left( 1 - r_{ijk} \right) x_{ijk} \right]
\] (31)

Using Chance constrained Programming technique, the above constraints can be written as

\[
\text{Prob} \left\{ \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq \tilde{a}_i \right\} \geq \delta_1 \quad i = 1, 2, 3, \cdots, m
\]

\[
\text{Prob} \left\{ \sum_{i=1}^{m} \sum_{k=1}^{K} (1 - \lambda_{ijk}) x_{ijk} \geq \tilde{b}_j \right\} \geq \delta_2 \quad j = 1, 2, 3, \cdots, n
\] (32)

\[
\text{Prob} \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq \tilde{c}_k \right\} \geq \delta_3 \quad k = 1, 2, 3, \cdots, K
\]

\[
x_{ijk} \geq 0 \quad \forall i, j, k.
\]
where $\delta_1, \delta_2, \delta_3$ are the confidence level of the availability, demand and conveyances respectively. Now from the first constraints of the equation (32), we get

$$
\text{Prob} \left[ \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq \tilde{a}_i \right] \geq \delta_1 \quad i = 1, 2, \ldots, m
$$

i.e.,

$$
\text{Prob} \left[ \sum_{j=1}^{n} \sum_{k=1}^{K} \frac{x_{ijk} - m\tilde{a}_i}{\sigma^2_{\tilde{a}_i}} \leq \frac{\tilde{a}_i - m\tilde{a}_i}{\sigma^2_{\tilde{a}_i}} \right] \geq \delta_1 \quad i = 1, 2, \ldots, m \ \forall i(33)
$$

Now taking $\beta_i = \frac{\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} - m\tilde{a}_i}{\sigma^2_{\tilde{a}_i}}$ and $\Phi_i = \frac{\tilde{a}_i - m\tilde{a}_i}{\sigma^2_{\tilde{a}_i}}$. The inequality (33) can be written as:

$$
\text{Prob} \left[ \beta_i \leq \Phi_i \right] \geq \delta_1 \quad \forall i. \quad (34)
$$

The equation (34) shows that $\Phi_i, \ i = 1, 2, \ldots, m$ be the standard normal variant by Reproductive property. So $\beta_i$ can be obtained from the table of the standard normal distribution. Thus the above equation (32) reduces to

$$
\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq m\tilde{a}_i + \Phi^{-1}(\delta_i) \sqrt{\frac{\sigma^2_{\tilde{a}_i}}{\tilde{a}_i}} \quad \forall \ \ i = 0, 2, \ldots, m \quad (35)
$$

Thus the stochastic linear programming problem of equation (32), can be stated as equivalent deterministic nonlinear programming problem as:

$$
\begin{align*}
\min \{ \tilde{f}_1, \tilde{f}_2 \} & \quad s.t \quad \\
\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq m\tilde{a}_i + \Phi^{-1}(\delta_i) \sqrt{\frac{\sigma^2_{\tilde{a}_i}}{\tilde{a}_i}} \quad \forall \ i \\
\sum_{j=1}^{n} \sum_{k=1}^{K} (1 - \lambda_{ijk})x_{ijk} \geq m\tilde{b}_j + \Phi^{-1}(\delta_j) \sqrt{\frac{\sigma^2_{\tilde{b}_j}}{\tilde{b}_j}} \quad \forall \ j \\
\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq m\tilde{c}_k + \Phi^{-1}(\delta_k) \sqrt{\frac{\sigma^2_{\tilde{c}_k}}{\tilde{c}_k}} \quad \forall \ k \\
x_{ijk} \geq 0 \quad \forall \ i, j, k.
\end{align*}
$$

7.2.2 Model in fuzzy environment (Model 3)

In this case, we consider the parameters are fuzzy in nature, the fuzzy model is given by

$$
\begin{align*}
\min \{ \tilde{f}_1, \tilde{f}_2 \} & = \left[ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{c}_{ijk} x_{ijk} \right], \left[ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{f}_{ijk} y(x_{ijk}) \right] \\
\max R_x = \prod_{i=1}^{m} \left[ 1 - \prod_{j=1}^{n} \prod_{k=1}^{K} (1 - r_{ijk})^{x_{ijk}} \right]
\end{align*}
$$
\[
\begin{align*}
\text{s.t} \quad & \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq \tilde{a}_i \quad i = 1, 2, \ldots, m \\
& \sum_{i=1}^{m} \sum_{k=1}^{K} (1 - \lambda_{ijk}) x_{ijk} \geq \tilde{b}_j \quad j = 1, 2, \ldots, n \\
& \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq \tilde{c}_k \quad k = 1, 2, \ldots, K \\
& x_{ijk} \geq 0 \quad \forall i, j, k.
\end{align*}
\]

We use the chance operator to deal with the fuzzy multi-objective model, and here the decision maker is supposed to be comparatively optimistic, so we adopted the Pos measure to measure the chance,

\[
\begin{align*}
\min_{x} \left[ f_1, f_2 \right], \max R_s \\
\text{s.t} \quad & \text{Pos} \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} c_{ijk} x_{ijk} \leq \tilde{f}_1 \right\} \geq \theta_1, \\
& \text{Pos} \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{t}_{ijk} y(x_{ijk}) \leq \tilde{f}_2 \right\} \geq \theta_2, \\
& \text{Pos} \left\{ \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq \tilde{a}_i \right\} \geq \delta_1 \quad i = 1, 2, \ldots, m \\
& \text{Pos} \left\{ \sum_{i=1}^{m} \sum_{k=1}^{K} (1 - \lambda_{ijk}) x_{ijk} \geq \tilde{b}_j \right\} \geq \delta_2 \quad j = 1, 2, \ldots, n \\
& \text{Pos} \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq \tilde{c}_k \right\} \geq \delta_3 \quad k = 1, 2, \ldots, K, x_{ijk} \geq 0 \\
& \forall i, j, k.
\end{align*}
\]

Using Theorem 3, the problem (37) can be written as

\[
\begin{align*}
\min_{x} \left[ \tilde{f}_1, \tilde{f}_2 \right], \max R_s \\
\text{s.t} \quad & \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} m_{1ijk} x_{ijk} - (1 - \theta_1) \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \alpha_{1ijk} x_{ijk} \leq \tilde{f}_1 \\
& \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} m_{2ijk} y(x_{ijk}) - (1 - \theta_2) \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \alpha_{2ijk} y(x_{ijk}) \leq \tilde{f}_2, \\
& \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq m_{1i} + (1 - \delta_1) \beta_{i} \quad i = 1, 2, \ldots, m \\
& \sum_{i=1}^{m} \sum_{k=1}^{K} (1 - \lambda_{ijk}) x_{ijk} \geq m_{2j} + (1 - \delta_2) \beta_{j} \quad j = 1, 2, \ldots, n
\end{align*}
\]
The optimum results in Tables 5 and 6 respectively. Let $8.2$ Output data for Model 1

Values of parameters involved in the random fuzzy constraints

Table 1 Values of parameters in objective functions

<table>
<thead>
<tr>
<th>$m_{ij}$</th>
<th>$\tilde{c}_{j111}$</th>
<th>$\tilde{c}_{j121}$</th>
<th>$\tilde{c}_{j211}$</th>
<th>$\tilde{c}_{j221}$</th>
<th>$\tilde{c}_{j112}$</th>
<th>$\tilde{c}_{j122}$</th>
<th>$\tilde{c}_{j212}$</th>
<th>$\tilde{c}_{j222}$</th>
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<tr>
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<td>1.6</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
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</tr>
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<td>0.88</td>
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<tr>
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Table 2 Values of parameters involved in the random fuzzy constraints

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<tr>
<th>$\tilde{a}_{i}$</th>
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<th>$\tilde{c}_{i}$</th>
<th>$\tilde{d}_{i}$</th>
<th>$\tilde{e}_{i}$</th>
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</tr>
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<td>3.2</td>
<td>4.2</td>
<td>4.8</td>
</tr>
<tr>
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<td>4.2</td>
<td>3.2</td>
<td>4.2</td>
<td>4.8</td>
</tr>
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<td>4.2</td>
<td>3.5</td>
<td>2.5</td>
<td>3.5</td>
<td>3.7</td>
</tr>
</tbody>
</table>

8 Numerical experiment

8.1 Input data for Model 1

Let us consider a multi-objective solid transportation problem with three origins, three destinations, three types of conveyances, three objectives and random fuzzy resources, demands, transported amounts and cost coefficients as random fuzzy numbers. Here $m = n = K = p = 2$, $\lambda_{011} = 0.15$, $\lambda_{121} = 0.13$, $\lambda_{211} = 0.14$, $\lambda_{221} = 0.12$, $\lambda_{112} = 0.14$, $\lambda_{122} = 0.13$, $\lambda_{212} = 0.11$, $\lambda_{222} = 0.13$.

8.2 Output data for Model 1

With these above input data, the equations (25) is solved using GRG and we present the optimum results in Tables 5 and 6 respectively. Let $h_i^{x_1} = 0.90$, $h_i^{x_2} = 0.70$, $L^*(t) = 1 - t$, then $L^*(h_i^{x_1}) = 0.10$, $L^*(h_i^{x_2}) = 0.30$, $\Phi^{-1}(1 - \delta_i) = -1.28$, $\delta_i = 1, 2$. The computation of $f_i^U$ and $f_i^L$ $(i = 1, 2)$, from the equation (27) is as follows:

$$f_i^U = 334, \quad f_i^L = 211, \quad f_i^U = 32, \quad f_i^L = 21, \quad R_i^U = 0.99, \quad R_i^L = 0.12,$$
Then we compute the following model to get the interactive satisfied solution,

\[
\begin{align*}
\mu_1(f_1(x)) &= \begin{cases} 
1 & \text{for } f_1(x) < 211 \\
\frac{334-f_1(x)}{344-211} & \text{for } 211 < f_1(x) < 334 \\
0 & \text{for } f_1(x) > 334 
\end{cases} \\
\mu_2(f_2(x)) &= \begin{cases} 
1 & \text{for } f_2(x) < 21 \\
\frac{23-f_2(x)}{32-21} & \text{for } 21 < f_2(x) < 32 \\
0 & \text{for } f_2(x) > 32 
\end{cases} \\
\mu_3(R_s(x)) &= \begin{cases} 
1 & \text{for } R_s(x) > 0.99 \\
0.12 & \text{for } 0.12 < R_s(x) < 0.99 \\
0 & \text{for } R_s(x) > 0.99 
\end{cases}
\end{align*}
\]

Then we compute the following model to get the interactive satisfied solution,

\[
\begin{align*}
\min_{x} & \quad \lambda \\
\text{s.t.} & \quad \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \left( m_{1ijk} - 0.1 \alpha_{1ijk} \right) x_{ijk} - 1.28 \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sigma_{1ijk}^2 x_{ijk}^2 } \\
& \quad \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \left( m_{2ijk} - 0.9 \beta_{2ijk} \right) x_{ijk} - 1.28 \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sigma_{2ijk}^2 y^2(x_{ijk}) } \\
& \quad \sum_{i=1}^{m} \left[ 1 - \prod_{j=1}^{n} \prod_{k=1}^{K} \left( 1 - r_{ijk} \right) \right] \geq R_s^L + \left( \mu_3 - \lambda \right) (R_s^U - R_s^L) \\
& \quad \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} (x_{ijk} - \lambda_{ijk}) - 1.28 \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sigma_{ijk}^2 (x_{ijk} - \lambda_{ijk})^2 } = m_{ijk} - 0.3 \beta_{ijk} \\
& \quad \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} (x_{ijk} - \mu_{ijk}) - 1.28 \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sigma_{ijk}^2 (x_{ijk} - \mu_{ijk})^2 } = m_{ijk} + 0.7 \alpha_{ijk} \\
& \quad x_{ijk} \geq 0
\end{align*}
\] (39)

After solving the model (39), we can get the satisfied solution of model (27), which are listed in Table 3.

The first line of Table 3 lists each reference value of membership function \( \mu_1(f_1) \), when the initialised membership function is 1, the value of objective function \( f_1(x) \), and its corresponding solution \( x \). If the decision maker hopes that improve \( f_2(x) \) on the basis of sacrifice \( f_1(x) \). We may consider reset the reference value of membership function \( (\mu_1, \mu_2, \mu_3) \), e.g., we set \( (\mu_1, \mu_2, \mu_3) = (0.9, 1, 0.9) \), or \( (\mu_1, \mu_2) = (1, 0.9, 0.9) \). The corresponding result are listed in the second and third lines. Suppose that when the reference value of membership function is
the decision maker is satisfied, then the interactive process is stopped, so we obtain the 0.9-Pr 0.8-Pos 0.9-Pos satisfied solution is $x^* = (24.91, 17.07, 16.54, 17.99, 18.16, 8.4518, 16, 12.87)^T$, and the corresponding value of objective function is $(f_1^*, f_2^*, R^*_s) = (303.659, 26.769, 0.835)$. 

Table 3: Employ the interactive fuzzy satisfied method based on possibility and necessity

<table>
<thead>
<tr>
<th>$\bar{\mu}_1$</th>
<th>$\bar{\mu}_2$</th>
<th>$\bar{\mu}_3$</th>
<th>$\bar{f}_1$</th>
<th>$\bar{f}_2$</th>
<th>$R_s$</th>
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<th>$\mu_2(\bar{f}_2)$</th>
<th>$\mu_3(R_s)$</th>
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<th>$x_{112}$</th>
<th>$x_{121}$</th>
<th>$x_{122}$</th>
<th>$x_{211}$</th>
<th>$x_{212}$</th>
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<td>0.24</td>
<td>0.791</td>
<td>0.237</td>
<td>23.91</td>
<td>18.07</td>
<td>16.54</td>
<td>0.99</td>
<td>0.696</td>
<td>18.16</td>
<td>8.45</td>
<td>18.16</td>
<td>12.87</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>0.9</td>
<td>308.376</td>
<td>23.615</td>
<td>0.99</td>
<td>0.24</td>
<td>0.791</td>
<td>0.237</td>
<td>23.91</td>
<td>18.07</td>
<td>16.54</td>
<td>0.99</td>
<td>0.696</td>
<td>18.16</td>
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<td>18.16</td>
<td>12.87</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
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<td>23.615</td>
<td>0.99</td>
<td>0.24</td>
<td>0.791</td>
<td>0.237</td>
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<td>0.696</td>
<td>18.16</td>
<td>8.45</td>
<td>18.16</td>
<td>12.87</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.9</td>
<td>1</td>
<td>308.376</td>
<td>23.615</td>
<td>0.99</td>
<td>0.24</td>
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<td>1</td>
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<td>0.696</td>
<td>18.16</td>
<td>8.45</td>
<td>18.16</td>
<td>12.87</td>
</tr>
</tbody>
</table>

8.3 Input data for Model 2 and Model 3

The values of $m, n, K, \lambda_{ijk}$ are same as in Model 1, only the from Table 1, it takes the values $a_i, b_j, c_k, C_{ijk}, t_{ijk}, \delta_1 = \delta_2 = \delta_3 = 0.7$ and $\theta_1 = \theta_2 = 0.9$.

Table 4: Optimum transported amounts and min. cost for different models via LINGO

<table>
<thead>
<tr>
<th>Model</th>
<th>Total transported amounts</th>
<th>Total vehicle resources demand</th>
<th>Min cost</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x_{111}</td>
<td>x_{112}</td>
<td>x_{121}</td>
<td>x_{122}</td>
<td>x_{211}</td>
<td>x_{212}</td>
<td>x_{221}</td>
<td>x_{222}</td>
</tr>
<tr>
<td>1</td>
<td>21.8, 3.36, 8.74, 1.06, 4.85, 0.73</td>
<td>726.05</td>
<td>90</td>
<td>51.86, 25.87, 24.07</td>
<td>24.18, 29.47</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>19.77, 5.89, 6.49, 2.15, 1.22, 8.02</td>
<td>728.57</td>
<td>80</td>
<td>49.84, 46.37, 24.02</td>
<td>24.4, 27.79</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>20.38, 13.18, 2.78, 0.24, 0.48, 9.09</td>
<td>746.98</td>
<td>90</td>
<td>51.57, 26.32, 26.87</td>
<td>22.74, 28.29</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>15.65, 12.51, 0.28, 0.66, 0.95, 13.24</td>
<td>766.87</td>
<td>85</td>
<td>50.54, 25.71, 24.46, 22.3</td>
<td>29.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sensitivity analysis for Model 1:

For some different values of the damageability parameters due to conveyance ($K$), the total received amount and total loss at destinations are presented in Table 5. Here the total despatched cost is 133.016 units.
Table 5  Optimum Transported Amounts and Min. cost for different models via MOGA

<table>
<thead>
<tr>
<th>Model</th>
<th>Transported amounts</th>
<th>Total Transported</th>
<th>Fulfilled</th>
<th>Min vehicle resources</th>
<th>demand cost</th>
<th>cost</th>
<th>(a₁, a₂)</th>
<th>(b₁, b₂, b₃)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x₁₁₁ x₁₂₁ x₁₃₁ x₂₁₁ x₂₂₁ x₂₃₁</td>
<td>x₁₁₂ x₁₂₂ x₁₃₂ x₂₁₂ x₂₂₂ x₂₃₂</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>21.8, 3.36, 8.74, 1.06, 4.85, 0.73</td>
<td>726.05</td>
<td>90</td>
<td>51.86, 25.87</td>
<td>24.07, 24.18</td>
<td>29.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.66, 5.5, 9.79, 2.95, 10.36, 2.92</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>19.77, 5.89, 6.94, 2.15, 1.22, 8.02</td>
<td>728.57</td>
<td>80</td>
<td>49.84, 26.37</td>
<td>24.02, 24.4</td>
<td>27.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.74, 10.94, 4.56, 4.13, 5.97, 4.89</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>3</td>
<td>20.38, 13.18, 2.78, 0.24, 0.48, 9.09</td>
<td>746.98</td>
<td>90</td>
<td>51.57, 26.32</td>
<td>26.87, 22.74</td>
<td>28.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.18, 0.9, 9.15, 2.49, 12.30, 1.72</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>15.65, 12.51, 0.28, 0.66, 0.95, 13.24</td>
<td>766.87</td>
<td>85</td>
<td>50.54, 25.71</td>
<td>24.46, 22.3</td>
<td>29.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.54, 5.7, 6.83, 3.63, 5.3, 1.93</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It may be noted that minimum times required for transportation are almost same in all cases. The result in Table 6 are observed with the change of damageability. If the percentage change of damageability of conveyances are increases then the total received amount decreases and the total loss at destination are increases.

Table 6  Changes in the objectives due to damageability for Model 1

<table>
<thead>
<tr>
<th>Breaking conveyance</th>
<th>Total received amount</th>
<th>Total loss at destinations ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ₁₁₂ (= (7%, 10%))</td>
<td>111.01</td>
<td>62.459</td>
</tr>
<tr>
<td>λ₁₁₂ (= (10%, 7%))</td>
<td>110.84</td>
<td>63.325</td>
</tr>
<tr>
<td>λ₁₁₂ (= (9%, 13%))</td>
<td>110.54</td>
<td>63.763</td>
</tr>
<tr>
<td>λ₁₁₂ (= (13%, 9%))</td>
<td>110.31</td>
<td>64.919</td>
</tr>
<tr>
<td>λ₁₁₂ (= (12%, 15%))</td>
<td>110.00</td>
<td>65.357</td>
</tr>
<tr>
<td>λ₁₁₂ (= (15%, 12%))</td>
<td>109.83</td>
<td>66.224</td>
</tr>
</tbody>
</table>

9 Conclusions

The MOSTP with damageable items has been explored in this paper. Here we minimise transportation cost and time and maximise total transportation reliability. We have considered all transpiration cost and demand, supplies, capacity of conveyances are assumed to be fuzzy random variables. For the first time, we have discussed the problem of determining fuzzy random criterion for the decision-makers. The transported items are likely to be damaged during transportation and damageability is different for different conveyances along different roots. Two special cases have been derived from the proposed model. The STP is formulated as a new decision making model optimising pVaR by incorporating the concept of value at risk into possibility and necessity measure theory. The reduced deterministic constrained problem is solved using GRG method. The model is illustrated with numerical examples and some sensitivity analyses are made on damageability. Some recommendations for future works are:
A MOSTP with reliability for damageable items

1 some additional sources of uncertainty in the STP problem may be considered in
the problem

2 it would be interesting to consider extensions to multi-criteria or multi-item STP

3 also extensions to fuzzy random programming problems with two decision-makers
under non-cooperative environments will be required in practice

4 this method can be used in other different areas such as portfolio distribution,
urban and regional planning, etc.

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