



Asian J. of Management Science and Applications

ISSN online: 2049-8691 - ISSN print: 2049-8683

<https://www.inderscience.com/ajmsa>

Hierarchical production planning system for a plastic container manufacturer with a combined processing-assembly and hybrid flow shop

Chihiro Hayashi

DOI: [10.1504/AJMSA.2025.10073484](https://doi.org/10.1504/AJMSA.2025.10073484)

Article History:

Received:	18 April 2024
Last revised:	02 August 2024
Accepted:	04 August 2024
Published online:	01 October 2025

Hierarchical production planning system for a plastic container manufacturer with a combined processing-assembly and hybrid flow shop

Chihiro Hayashi

Department of Business Administration,
Faculty of Business Administration,
Ryukoku University,
67 Tsukamoto-cho, Fukakusa,
Fushimi-ku, Kyoto 612-8577, Japan
Email: c-hayashi@biz.ryukoku.ac.jp

Abstract: This research presents a basic study on the optimisation of a hierarchical production planning system used by a plastic container manufacturer with a combined processing-assembly and hybrid flow shop. To optimise hierarchical production planning in a two-stage manufacturing process, a mathematical approach was used for decoration and assembly. First, a hierarchical production plan utilised for the two processes was formulated using the safety inventory setting and demand forecast priority parameters, which increased or decreased production capacity as controllable variables, and inventory and stockout as criteria. A mathematical analysis was then performed to minimise the inventory, which restricted the analysis to the condition in which no stockouts occurred. Production planning based on the relationship between parameters and inventory shown in the analysis provides an effective means of addressing the real process problem of minimising stockouts while preventing excess inventory.

Keywords: hierarchical production planning system; plastic container manufacturer; processing-assembly flow shop; hybrid flow shop.

Reference to this paper should be made as follows: Hayashi, C. (2025) 'Hierarchical production planning system for a plastic container manufacturer with a combined processing-assembly and hybrid flow shop', *Asian J. Management Science and Applications*, Vol. 8, No. 2, pp.160–184.

Biographical notes: Chihiro Hayashi is a Lecturer at the Ryukoku University, Faculty of Business Administration and has a Doctor of Engineering from Tokyo Metropolitan University. His main research areas are production management and management information data analysis. He is a member of the Japan Industrial Management Association, the Japan Society for Production Management, and the Japan Society for Management Information.

This paper is a revised and expanded version of a paper entitled 'Hierarchical production planning for a plastic container manufacturer – basic research for scheduling optimization in a combined flow shop of processing-assembly and hybrid types' presented at the 7th Asian Conference of Management Science and Applications, Okinawa, Japan, 15–17 December 2023.

1 Introduction

This study aims to optimise a hierarchical production planning system using a plastic container manufacturer with a combined processing-assembly flow shop and a hybrid flow shop production configuration.

Narasimhan and Mangiameli (1987) defined a hybrid flow shop as a manufacturing system in which products flow in one direction, consisting of different types of production processes. For example, a combination of several production processes and continuous production is a typical hybrid flow shop. Ohba et al. (1995) analysed the effects of lot sizing and input sequencing in the front-end process of lot production on the work-in-process inventory quantity before the back-end process in a hybrid flow shop, where one cutting machine was used in the front-end process, and multiple machines for packaging the cut film were used in the back-end process. Sun et al. (1999) described a processing-assembly flow shop as a flow shop in which assembly operations cannot start until the processing of all components is complete. They proposed an approximate solution that minimises the maximum dwell time in a processing-assembly flow shop consisting of multiple processing machines and one assembly machine. However, Tsubone and Tanaka (1990) described a hierarchical production planning system as a form of production planning in which a production plan with a long planning period for determining production capacity and production levelling, as well as a production plan for each item unit with a short planning cycle, are structured as a series of decision-making processes in a hierarchical structure. This study develops a hierarchical production planning system model for a two-stage manufacturing process consisting of part machining and final product assembly.

The plastic container manufacturing process used in this study combined two types of production. Because the manufacturing of the back-end process depends on the supply of parts from the front-end process, scheduling is required to increase both customer delivery date compliance and the back-end process utilisation rate. The details of the target process and product configuration are described in Section 2. However, this problem can be considered an optimisation problem for hierarchical production planning in a multistage process. However, it is difficult to comprehensively optimise a production plan for a complex process.

These study first attempts to optimise a hierarchical production plan for a two-stage manufacturing process using a mathematical approach, targeting two of the more important processes in the target process as basic research. The actual process being assessed is a combined processing-assembly and hybrid flow shop that employs hierarchical production planning. Because no engineering studies dealing with such a process can be found, formulating a production plan and analysing it through a mathematical approach is useful for the optimal scheduling of hierarchical production planning in a multi-stage process. Specifically, two processes were targeted: the decoration process, in which the moulded bottles were printed or painted, and the assembly process, in which the final product was assembled. The hierarchical production planning used in the two processes is formulated with the safety inventory setting parameter and demand forecast priority parameter, which increase or decrease the production capacity as controllable variables and the actual inventory quantity and stockout quantity as criteria. In addition, the process is a nonlinear model in which the inventory is reset to zero when stockouts occur, and the stockout quantity is not added to

the requirements, which forms the basis for production planning. Therefore, cases in which stockouts do not occur are separated from those in which stockouts occur.

2 Target processes and product composition

The target company manufactured various plastic containers to fill detergents, food, beverages, and other products. Major food products include extrusion-blow-moulded products, tubes, and lightweight thin-walled containers. The company has multiple production facilities in Japan and overseas. While each facility can produce different products, many factories use a combination of processing-assembly flow shops and hybrid flow shop production methods.

Figure 1 presents a schematic of the process flow of the subject company. The target process comprises processes with their own inspection functions, such as moulding, decoration, assembly, and packaging/shipping. For example, when manufacturing push-nozzle shampoo and conditioner containers, different moulds are used to mould not only the base bottle but also the nozzle and pump that pushes the bottle out and the cap that holds the bottle and nozzle in place. Figure 2 shows the relationship between each process and product composition. Parts moulded in the moulding process are sometimes used as they are, but major parts, such as bottles, are sent to the decoration process. In the decoration process, some parts are painted or printed with the same specifications, whereas others are sent to the next process because different parts with different decorations are applied to the same moulded part. Parts from the moulding and decoration processes manufactured in lot production were assembled according to the specifications of the assembly process to complete the final product. Thus, the target process is a hybrid flow shop in which the moulding and decoration processes are produced in different lot sizes and a processing-assembly flow shop in which the final product is assembled after the parts made in these processes are assembled. To analyse these complex processes step-by-step, this study focuses on two essential processes: decoration and assembly.

Figure 1 Schematic diagram of process flow (see online version for colours)

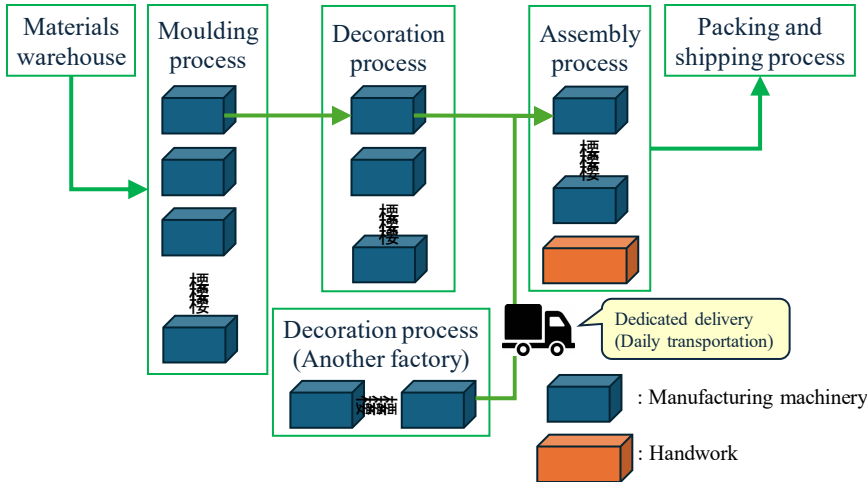
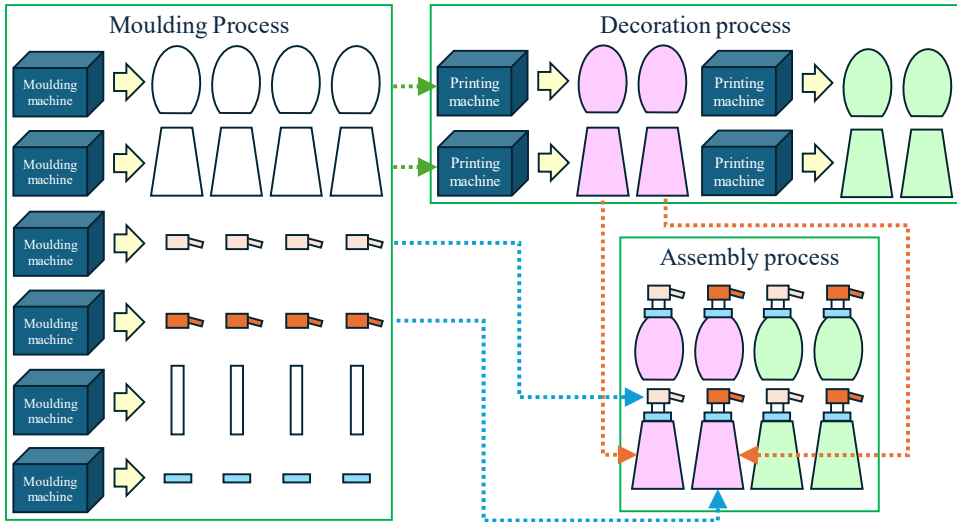


Figure 2 Relationship between each process and product composition (see online version for colours)



The target process is a make-to-order production system; however, some users receive orders and offers in half to one-month cycles, whereas others receive orders at intervals of several months or do not receive offers at all. There are many cases of sudden demand fluctuations for user-branded products, such as frequent stockouts of hand soaps and other products owing to infectious diseases, rapid increases in product recognition through social networking services, and sluggish sales of new beverage products, making stable order announcements and demand forecasts difficult for many products. Although it is desirable to have an ample work-in-process inventory for the processing-assembly flow shop, where work cannot be started until all parts are in stock, the risk of excess inventory is high. However, when stockouts occur, they do not remain as back orders but as lost orders, leading to a loss of business opportunities. Therefore, they must handle short planning cycles that prioritise the most recent demand forecasts. However, factories also want to prioritise long-term production planning to level out production as much as possible, and this trade-off relationship is a real problem. Therefore, there is a need for optimal production scheduling for each process, such as determining how much safety inventory should be provided and how much priority should be given to the most recent demand forecasts.

3 Previous study

In a previous study on hierarchical production planning, Takeda and Kuroda (2000) described an inventory minimisation problem with the delivery date setting and delivery date achievement rate as parameters for a multi-stage production and inventory system when the processing times differed for each product. Yanagawa et al. (2006) proposed a method for determining the safety factor that optimises the safety inventory of finished products using a simulation analysis of the total cost. In addition, Tamura (2016) presented an algorithm for determining the optimal inventory level at each inventory

point through a simulation analysis using the permissible stockout rate, stockout cost, and storage cost as parameters for a multi-step process. However, these studies focused on setting inventory quantities and focused on simulation-based methods. Hayashi et al. (2021) conducted a study on three-stage hierarchical production planning consisting of semi-annual, monthly, and daily plans; however, this study targeted a single bottleneck process, and there is no study on hierarchical production planning for a hybrid flow shop, such as the target process.

Previous studies on hybrid flow shops include Tsubone et al. (1993), Ohba et al. (1995) (as described in Section 1), Uetake et al. (1995) and Suzuki et al. (1997). These studies applied single-stage production planning and demonstrated the effects of the setup frequency and input sequencing rules by formulating front-end process utilisation rates and inventory quantities. Suzuki and Tsubone (1999) extended these studies to hierarchical production planning consisting of upper and lower plans; however, these were not optimisation studies based on a mathematical approach but rather heuristic approaches using simulations.

In a study on a processing-assembly flow shop, Sun et al. (1999) proposed an approximate solution method to minimise the maximum dwell time for the machine-specified make span minimisation scheduling problem in which the machining machines in the front-end process were specified for cases in which the number of parallel machines in the front-end process was up to five. Nagasawa and Morisawa (2002) considered a parallel flow line consisting of two lines with two processes instead of a single parallel machine in the front-end process. In contrast, Tsuchimoto et al. (2005) considered a machine-independent multifunctional parallel machine that can process flexibly. They proposed an approximate solution method for the make span minimisation scheduling problem, similar to Sun et al. (1999). Although all these studies have proposed highly accurate solution methods, they are mainly algorithmic improvements, which are difficult to understand when considering practical applications, such as the target process; therefore, more concrete proposals are required.

Furthermore, there are many studies on inventory and stockouts. For example, Lee and Kim (2000) presented an economic order point model under uncertain demand in the procurement period. Arizono et al. (2014) proposed an inventory model for a simultaneous method of determining economic order quantity and order point. However, these are optimisation problems for linear models that treat stockouts as backorders and cannot be applied to nonlinear models, such as the one used in this study, in which a stockout directly results in a lost order.

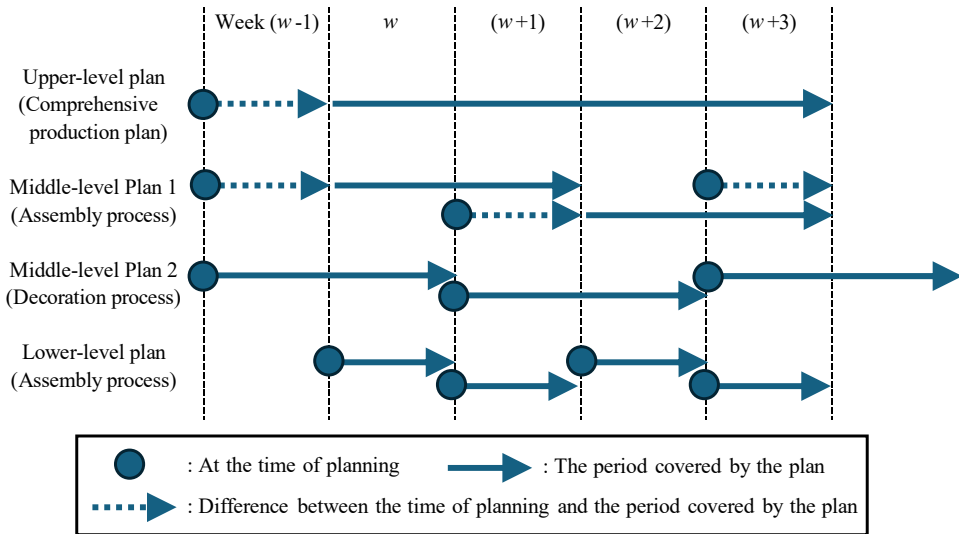
In this study, a mathematical approach was proposed to optimise hierarchical production planning for a combined processing-assembly flow shop and a hybrid flow shop. The model is nonlinear and varies in production capacity rather than inventory and order quantities, aiding planners in making decisions during actual production processes.

4 Hierarchical production planning

As discussed in Section 2, this study focuses on two of the most important processes shown in the schematic diagram of the process flow in Figure 1: decoration and assembly. In the assembly process, production scheduling is performed using hierarchical production planning, which consists of three stages: upper-level planning for monthly production capacity, middle-level planning that reviews the production capacity every L

weeks, and lower-level planning that determines the production quantity for each week. In addition, the decoration process draws up a production plan immediately after the middle-level plan of the assembly process is drawn up so that the parts can be supplied to the assembly process without excess or shortage. Because the middle-level plan for the assembly process and the production plan for the decoration process are created simultaneously, the former is called middle-level Plan 1 and the latter is called middle-level Plan 2. See Appendix at the end of this report for the various symbols used in this section and thereafter.

Figure 3 Timing of each planning stage (in case of $L = 2$) (see online version for colours)



The upper-level plan determines the comprehensive production capacity of the assembly process for the following month. This plan establishes an approximate operational structure. At this stage, detailed production quantities for each product type have not yet been planned; rather, the number of personnel, operating days, material requirements, etc. are estimated. In the middle-level Plan 1 for the assembly process, based on the production capacity determined in the upper-level plan, the production capacity for each bottle type is planned every two weeks, considering the more recent forecasted demand. In middle-level Plan 2, for the decoration process, the number of production lots to supply decorated bottles without excess or deficiency is planned against the approximate production capacity of the assembly process calculated in middle-level Plan 1. While middle-level plans are based on the production capacity and number of production lots for each bottle type, the final lower-level plan is based on the production plan for each product type. In addition, although demand and inventory are planned based on estimated quantities, production is calculated based on the most recent inventory and actual demand. As described above, the target process adopts hierarchical production planning, in which detailed production planning is not performed at a time; rather, the plan is moved to more detailed planning in stages to make various preparations.

Figure 3 shows the timing of each planning stage. The upper and middle-level Plan 1 were drawn up one week prior to the target period because the lead time of the decoration

process needs to be considered. The upper-level plan was planned monthly, and the middle-level Plan 1 was planned every L -weeks. Middle-level Plan 2 is also made every L weeks and is made immediately after middle-level Plan 1 is made. A lower-level plan was created at the end of each week by applying the most recent inventory or stockout quantity to the production plan for the following week. The relationships between months, periods, and weeks are shown in Figure 4. For week w , the month number of the upper-level plan is expressed as $m = \left\lceil \frac{w}{4} \right\rceil$ and the period number of middle-level Plan 1 is expressed as $l = \left\lceil \frac{w}{L} \right\rceil$. The period covered by middle-level Plan 1 can be set arbitrarily to $L = 1, 2, 4$, but in this study, $L = 2$ was used to match the actual situation.

Figure 4 Relationship among month, period, and week (in case of $L = 2$)

Month	1				2				3				...
Period	1		2		3		4		5		6		...
Week	1	2	3	4	5	6	7	8	9	10	11	12	...

Month	$m-1$				m				$m+1$				...
Period	$l-2$		$l-1$		l		$l+1$		$l+2$		$l+3$		...
Week	$w-4$	$w-3$	$w-2$	$w-1$	w	$w+1$	$w+2$	$w+3$	$w+4$	$w+5$	$w+6$	$w+7$	...

The final product types are denoted as products (b, n) . In this study, bottle type b was defined as a group of products assembled using bottle b produced during the decoration process, with shampoos and conditioner bottles as examples. Next, nozzle type n is defined as a product assembled using the same bottle b but with different product specifications owing to the use of different nozzles. Although, strictly speaking, this is not a combination of the two, as there are various product types, for better understanding, this study defines it as described above, and the case where three or more parts are combined is left for future work.

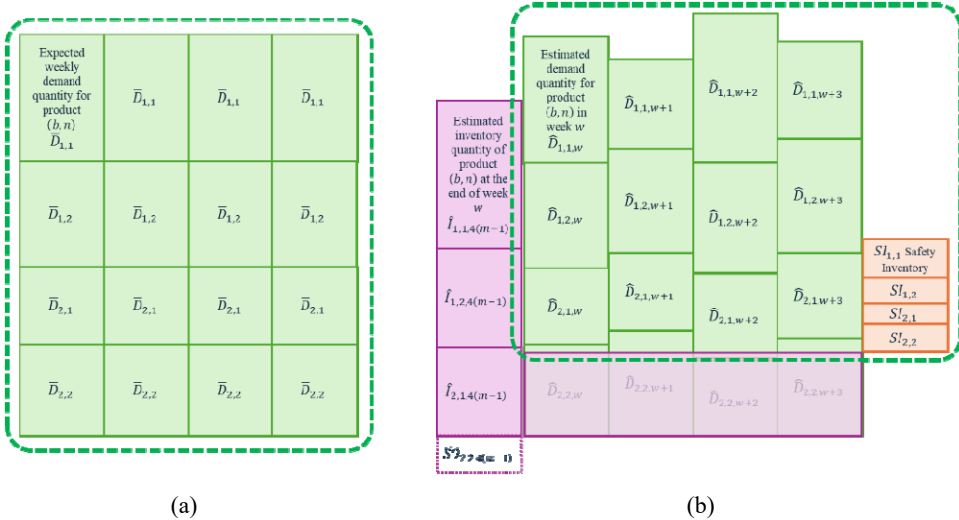
4.1 Upper-level plan

In the upper-level plan, which is a comprehensive production plan, the production plan for the following month is drawn up for all products for production levelling, with a planning period of four weeks to one month. The planning timing was not at the end of each month but one week before the end of the month, considering the lead time of the decoration process, as shown in Figure 3.

Figure 5 shows an image of comprehensive production planning. The height of each rectangle in the figure represents the quantity of demand and inventory. It is the same as that in Figure 12. The horizontal lengths are unified to represent one week's worth of production, but only with respect to the demand quantity, and have no meaning with respect to the inventory quantity. The dotted line in Figure 5(a) represents the production capacity based only on the expected demand without considering the safety inventory and the expected weekly demand for products (b, n) , $\bar{D}_{b,n}$, which represent the total expected demand for all products within the planning period of four weeks. The dotted line in Figure 5(b) represents the production capacity obtained by subtracting the estimated

inventory quantity at the end of the previous period, $\hat{I}_{b,n,4(m-1)}$, from the sum of the safety inventory $SI_{b,n}$ and the estimated demand for product (b, n) in week w , $\hat{D}_{b,n,w}$, during the planning period. In this case, even if the estimated stockout quantity $\widehat{SO}_{b,n,4(m-1)}$ is expected, it is not included in the comprehensive production plan because stockouts are treated as lost orders, and the plan is a rough monthly plan. Calculation methods for the safety inventory and various estimated quantities are described in this study. Using the demand forecast priority parameter γ ($0 \leq \gamma \leq 1$), the planner decides whether to give priority to Figure 5(a), which gives priority to the factory's demand for long-term production levelling, or to Figure 5(b), which gives priority to the most recent actual inventory quantity and demand forecast. Stable factory operations can be expected when the former priority is given. However, there is a risk of not responding to sudden increases or decreases in demand. If the latter is given priority, a production plan that responds to increases and decreases in demand can be expected; however, it becomes difficult to manage operations, such as responding to worker shifts and idle production facilities. The planner decides on γ at quarterly or semi-annual intervals, considering both the convenience of the factory and the views of the sales side.

Figure 5 Comprehensive production planning (see online version for colours)



The comprehensive production capacity for month m , CP_m , as determined by this upper-level plan is shown in equation (1).

$$\begin{aligned}
 CP_m = & 4(1-\gamma) \sum_{b \in B} \sum_{n \in N_b} \bar{D}_{b,n} \\
 & + \gamma \sum_{b \in B} \sum_{n \in N_b} \left(SI_{b,n} - \hat{I}_{b,n,4(m-1)} + \sum_{w \in M} \hat{D}_{b,n,w} \right)
 \end{aligned} \quad (1)$$

where B is the set of bottle types, N_b is the set of nozzle types within bottle type b , and M ($= \{w | 4(m-1) + 1 \leq w \leq 4m\}$) is the symbol for the set of week number w in month m .

The expected weekly demand for products (b, n) , $\bar{D}_{b,n}$, is averaged per week based on the actual shipments for the previous year. The forecast demand quantity, $\hat{D}_{b,n,w}$, is the monthly demand forecast obtained through daily sales activities and marketing and is converted into a weekly quantity. The safety inventory of product (b, n) , $SI_{b,n}$, is determined by the planner using the safety inventory setting parameter ε ($\varepsilon \geq 0$) based on the expected demand, $SI_{b,n} = \varepsilon \cdot \bar{D}_{b,n}$. Therefore, it is important to have a safety inventory to cover sudden increases in demand. However, if this setting is excessively large, there is a risk of excess inventory in the event of decreased demand. ε is also an important parameter for responding to demand fluctuations, as is the demand forecast priority parameter γ . Planners reset the value of ε every quarter or semi-annually while checking for changes in the actual inventory and stockouts.

The estimated inventory quantity of products (b, n) , $\hat{I}_{b,n,w}$, is the forecasted value one week after the time of planning and is calculated from the actual inventory quantity at that time, $I_{b,n,w-1}$, the already determined production quantity for that week $P_{b,n,w}$, and the estimated demand quantity $\hat{D}_{b,n,w}$ using equation (2).

$$\hat{I}_{b,n,w} = \max\{P_{b,n,w} + I_{b,n,w-1} - \hat{D}_{b,n,w}, 0\} \quad (2)$$

The weekly production quantity $P_{b,n,w}$ is shown in the lower-level plan in Subsection 4.3, and the actual inventory quantity $I_{b,n,w-1}$ is shown in the criteria in Subsection 4.4, as described below.

4.2 Middle-level plan

In the middle-level plan, a production plan for the assembly process, as in middle-level Plan 1, was first drawn based on the upper-level plan. Immediately thereafter, a production plan for the decoration process as middle-level Plan 2 is drawn up to ensure that there is no excess or shortage of decorated work-in-process inventory to satisfy middle-level Plan 1.

4.2.1 Middle-level Plan 1 for assembly process

Because the abovementioned upper-level plan is roughly planned monthly, the middle-level Plan 1 for the assembly process considers the inventory quantity of each bottle type and the forecasted demand quantity for each $L = 2$ weeks to create a production plan that is more in line with the actual situation.

First, as shown in Figure 6, the comprehensive production capacity of month m , CP_m , planned in the upper-level plan, is equally distributed for each week. The capacities corresponding to L weeks are calculated as the estimated production capacity of all bottle types in period l , $EP_l \left(= \frac{CP_m L}{4} \right)$.

Next, as a separate measure from the abovementioned estimated production quantity EP_l , the planner calculates the requirement quantity of all bottle types in period l , SR_l , every L weeks, as shown in Figure 7. The quantity required for bottle type b for period l , $r_{b,l}$, is obtained by adding the safety inventory $SI_{b,n}$ and the estimated demand $\hat{D}_{b,n,w}$ for period l , and then subtracting the estimated inventory at the end of the previous period

$\hat{I}_{b,n,L(l-1)}$, or by adding the estimated stockout quantity $\widehat{SO}_{b,n,L(l-1)}$. SR_l is the sum of $r_{b,l}$, and is calculated using equations (3) and (4).

$$r_{b,l} = \max \left\{ \sum_{n \in N_b} \left(SI_{b,n} L - \hat{I}_{b,n,L(l-1)} + \widehat{SO}_{b,n,L(l-1)} + \sum_{w \in Pe} \hat{D}_{b,n,w} \right), 0 \right\} \quad (3)$$

$$SR_l = \sum_{b \in B} r_{b,l} \quad (4)$$

where $Pe (= \{w | L(l-1) + 1 \leq w \leq L \cdot l\})$ is the symbol for the set of week number w in period l . Even if there is an excess inventory, the requirement quantity $r_{b,l}$ is always non-negative because there is usually no inventory to dispose of. In addition, middle-level Plan 1 was the basis of middle-level Plan 2 for the decoration process, which was one week ahead of the processing lead time for the decoration process. Thus, the inventory and stockout quantities at the end of the previous period in equation (3) are the estimated values. The estimated inventory quantity at the end of the previous period, $\hat{I}_{b,n,L(l-1)}$ was obtained using equation (2). The estimated stockout quantity $\widehat{SO}_{b,n,L(l-1)}$ is calculated by equation (5) using the actual inventory quantity at the time of planning $I_{b,n,L(l-1)-1}$, the production quantity $P_{b,n,L(l-1)}$, and the estimated demand quantity $\hat{D}_{b,n,L(l-1)}$, as is $\hat{I}_{b,n,L(l-1)}$.

$$\widehat{SO}_{b,n,w} = \max \{ \hat{D}_{b,n,w} - P_{b,n,w} - I_{b,n,w-1}, 0 \} \quad (5)$$

As in equation (2), the calculation methods for $P_{b,n,w}$ and $I_{b,n,w-1}$ are described in the next and subsequent sections. In the aforementioned upper-level plan, the estimated stockouts are not included because they are treated as lost orders, and the plan is a rough monthly plan. In contrast, in the middle-level Plan 1, the estimated stockout quantity $\widehat{SO}_{b,n,w}$ is added to the requirement quantity SR_l . This is because $\widehat{SO}_{b,n,w}$ is only an estimated value, and there is a gap in the demand forecast so that it can respond to cases in which the actual demand is postponed to the next week or later. However, this does not mean that stockouts are treated as back orders but as a buffer for production capacity.

Figure 6 Estimated production capacity of all bottle types in the period l , EP_l (see online version for colours)

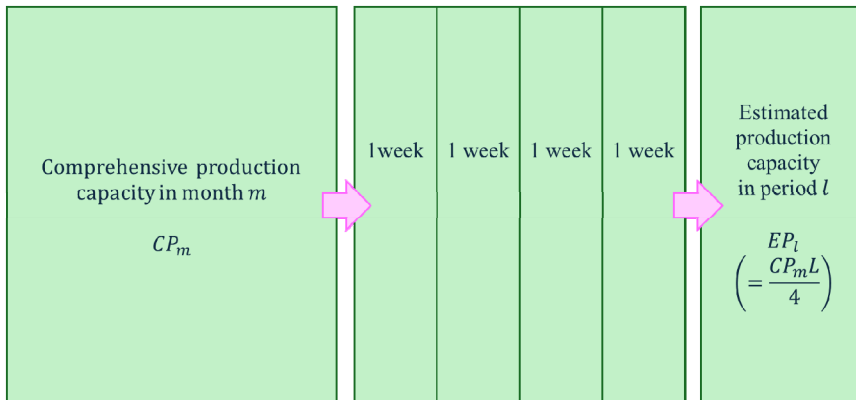
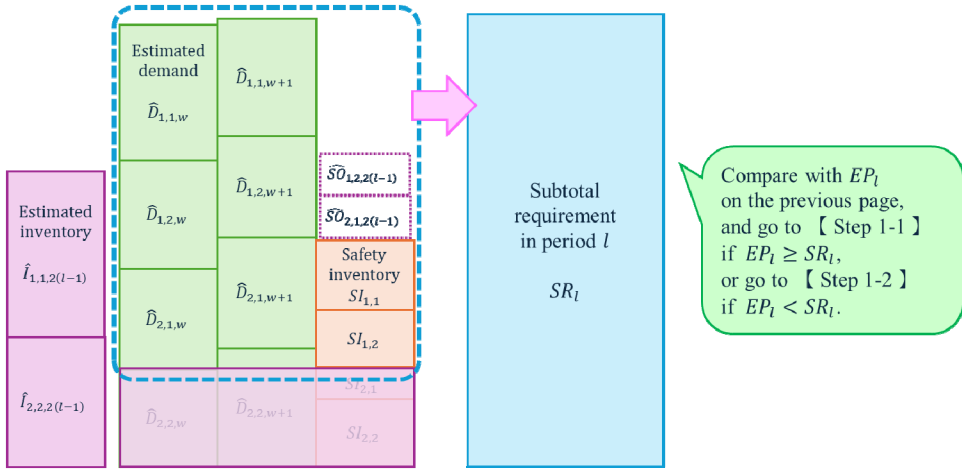
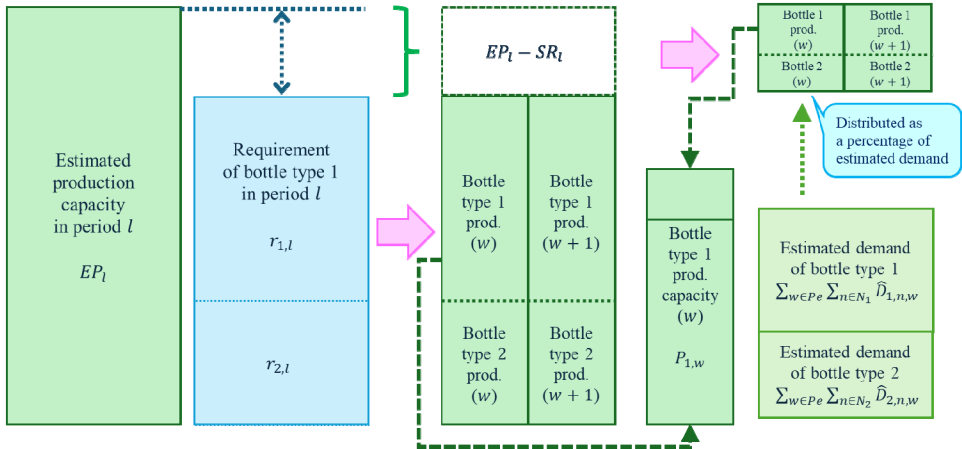


Figure 7 Requirement quantity of all bottle types in period l , SR_l (see online version for colours)

Finally, the production capacity of each bottle type during periods l , $P_{b,w}$, is determined. However, the process differs depending on the results of the comparison between the estimated production capacity EP_l and the required quantity of all bottle types in period l , SR_l . If EP_l is greater than SR_l , it is considered a case of surplus production capacity, so $P_{b,w}$ is drawn according to Step 1-1; if EP_l is less than SR_l , it is considered that the demand cannot be met, so $P_{b,w}$ is drawn according to Step 1-2.

Figure 8 Production capacity of bottle type b in week w , $P_{b,w}$, according to Step 1-1 (see online version for colours)

Step 1-1: $EP_l \geq SR_l$

If the estimated production capacity in period l , EP_l , which is obtained from the upper-level plan, is larger than the required quantity of all bottle types in period l , SR_l , then it is estimated that there is probably sufficient capacity even after producing the estimated demand and safety inventory quantities. Therefore, as shown in Figure 8, the

planner first assigns the required quantity of bottle type b in period l , $r_{b,l}$, to the production capacity. Furthermore, the difference between EP_l and SR_l is distributed by the ratio of the estimated demand quantity by bottle types $\hat{D}_{b,n,w}$ in the period under consideration, and added to the production capacity. The production capacity of bottle type b in week w , $P_{b,w}$, is the equal distribution of the above capacity on a weekly basis and is expressed by equation (6).

$$P_{b,w} = \frac{1}{L} \left\{ r_{b,l} + \frac{\sum_{w \in Pe} \sum_{n \in N_b} \hat{D}_{b,n,w}}{\sum_{w \in Pe} \sum_{b \in B} \sum_{n \in N_b} \hat{D}_{b,n,w}} (EP_l - SR_l) \right\} \quad (6)$$

Step 1-2: $EP_l < SR_l$

In this step, the estimated production capacity EP_l is less than the required quantity for all bottle types in period l , SR_l . Therefore, the estimated demand quantity or safety inventory was probably unsatisfactory. In this case, because a shortage in the production capacity is expected, measures to increase production should be implemented. However, it is difficult to increase the estimated production capacity EP_l set in the upper-level plan, even at this point, because of the process progress before the decoration process, securing workers, and other reasons. Therefore, as shown in Figure 9, the planner reserves production capacity by prioritising replenishment from EP_l to the estimated stockout quantity at the end of the previous period, $\widehat{SO}_{b,n,L(l-1)}$. The remainder is distributed in the ratio of the required quantity by bottle type $r_{b,l}$ to SR_l and added to the production capacity. The production capacity of bottle type b in week w , $P_{b,w}$, is an equal distribution of the above capacity on a weekly basis and is expressed by equation (7).

$$P_{b,w} = \left[\begin{array}{l} \frac{1}{L} \left\{ \frac{r_{b,l}}{SR_l} \left(EP_l - \sum_{b \in B} \sum_{n \in N_b} \widehat{SO}_{b,n,L(l-1)} \right) \right. \\ \left. + \sum_{n \in N_b} \widehat{SO}_{b,n,L(l-1)} \right\} \quad \left(EP_l \geq \sum_{b \in B} \sum_{n \in N_b} \widehat{SO}_{b,n,L(l-1)} \right) \\ \frac{1}{L} \cdot \frac{r_{b,l}}{SR_l} EP_l \quad \left(EP_l < \sum_{b \in B} \sum_{n \in N_b} \widehat{SO}_{b,n,L(l-1)} \right) \end{array} \right] \quad (7)$$

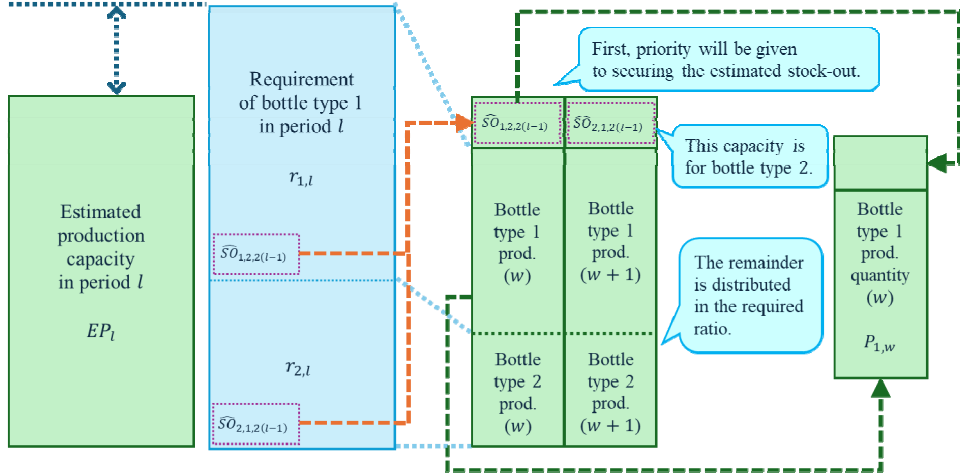
The production capacity of bottle type b in week w , $P_{b,w}$, determined in Step 1-1 or Step 1-2 above, was applied to middle-level Plan 2 in the decoration process and the lower-level plan in the assembly process as middle-level Plan 1 in the assembly process.

4.2.2 Middle-level Plan 2 for decoration process

Immediately after drawing up middle-level Plan 1 for the assembly process, middle-level Plan 2 was drawn to supply bottles from the decoration process to the assembly process without excess or shortage based on the production capacity of bottle type b in week w , $P_{b,w}$. Because the decoration process involves many production types, owing to the characteristics of the processing machine, the number of production lots was determined instead of the production quantity. As shown in Figure 3, the period covered by middle-level Plan 2 is L weeks, starting one week before middle-level Plan 1, and the

decoration process begins immediately after the plan is drawn. Therefore, the planning time of the middle-level Plan 2 can be expressed as the $(l \cdot L - 3)$ weekend. Since the middle-level Plan 2 for each week is a plan for every L weeks corresponding to the week number w of the middle-level Plan 1, the week number here is defined as $x (= \{w - 1 | w \in Pe\})$.

Figure 9 Production capacity of bottle type b in week w , $P_{b,w}$, according to Step 1-2 (see online version for colours)



First, find the total value of the production capacity by bottle type $P_{b,w}$ for the assembly process determined in the middle-level Plan 1 within period l . From that value, subtract the $(l \cdot L - 3)$ weekend's decorated work-in-process inventory quantity $WIP_{b,l-L-3}$ or add the shortage of decorated work-in-process inventory quantity $S_WIP_{b,l-L-3}$, to calculate the requirement quantity of decoration process corresponding to the assembly process in period l . This requirement quantity is distributed to the weekly requirement by dividing it by the period L covered by middle-level Plan 1 and is further converted to the number of lots by dividing it by the lot size. Thus, the number of decoration process production lots for bottle type b in each week, $PL_{b,x}$ is determined using equation (8).

$$PL_{b,x} = \left\lceil \frac{\sum_{w \in Pe} P_{b,w} - WIP_{b,l-L-3} + S_WIP_{b,l-L-3}}{\varphi \cdot L} \right\rceil \quad (8)$$

The lot size φ ($\varphi > 0$) of the decoration process is a parameter set to reduce the amount of setup and changeover work, such as the time-consuming cleaning and replacement of ink and graphics. The larger φ is, the less setup work is required, but an excessive lot size leads to excess inventory and less flexibility in switching between different types of products. However, if φ is too small, the response to demand fluctuations is improved, but the setup work increases. Therefore, φ is a parameter that must be set appropriately.

The decorated work-in-process of bottle type b for the x weekend $WIP_{b,x}$, used in equation (8) is calculated from the work-in-process of the previous weekend $WIP_{b,x-1}$, the production quantity $\varphi \cdot PL_{b,x}$ and the actual production quantity for each bottle type for the following week $\sum_{n \in N_b} P_{b,n,x+1}$, which is the paid-out quantity to the assembly

process, as shown in equation (9). If the paid-out quantity exceeds the quantity of $WIP_{b,x-1}$ and $\varphi \cdot PL_{b,x}$, $WIP_{b,x} = 0$ due to a shortage in work-in-process inventory, the decorated in-process inventory shortage of bottle type b for the x weekend, $S_WIP_{b,x}$, calculated by equation (10), is generated instead.

$$WIP_{b,x} = \begin{cases} WIP_{b,0} & (x = 0) \\ \max \left(WIP_{b,x-1} + \varphi \cdot PL_{b,x} - \sum_{n \in N_b} P_{b,n,x+1}, 0 \right) & (x > 0) \end{cases} \quad (9)$$

$$S_WIP_{b,x} = \max \left(\sum_{n \in N_b} P_{b,n,x+1} - WIP_{b,x-1} - \varphi \cdot PL_{b,x}, 0 \right) \quad (10)$$

If $S_WIP_{b,x}$ occurs, it is added to the numerator on the right side of equation (8) to replenish the shortage.

4.3 Lower-level plan

In the lower-level plan, the production quantity for each product for the following week was determined after checking the most recent inventory and stock-out situation at the end of each week.

First, the quantity required for bottle type b in week w , $R_{b,w}$, is calculated. $R_{b,w}$ is the sum of product (b, n) requirements for week w , $r_{b,n,w}$, obtained by subtracting the actual inventory at the end of the previous week $I_{b,n,w-1}$ from the safety inventory $SI_{b,n}$ and the estimated demand for product (b, n) in week w , $\hat{D}_{b,n,w}$, using equations (11) and (12).

$$r_{b,n,w} = \max \left(\hat{D}_{b,n,w} + SI_{b,n} - I_{b,n,w-1}, 0 \right) \quad (11)$$

$$R_{b,w} = \sum_{n \in N_b} r_{b,n,w} \quad (12)$$

Equations (11) and (12) are similar to equations (3) and (4) of middle-level Plan 1, except that equation (11) uses the actual inventory quantity and does not add the quantity of stockouts to the requirement quantity. In particular, with regard to the latter, in the middle-level Plan 1, the estimated stockout quantity is included in the plan; however, in the lower-level plan, if a stockout quantity occurs, it is not treated as a backorder but as a genuine lost order; therefore, the stockout quantity is not added to the requirement calculation. Thus, this is a nonlinear model.

Based on the above, the production quantity of products (b, n) in week w , $P_{b,n,w}$, is determined.

Again, as in middle-level Plan 1, the process differs depending on the comparison results between the required quantity $R_{b,w}$ and the production capacity of bottle type b in week w , $P_{b,w}$. If $P_{b,w}$ determined in the middle-level Plan 1 is greater than or equal to $R_{b,w}$, it is a case of excess production capacity; therefore, the plan is drawn according to Step 2-1. If $P_{b,w}$ is less than $R_{b,w}$, the demand cannot be met; therefore, the plan is drawn according to Step 2-2.

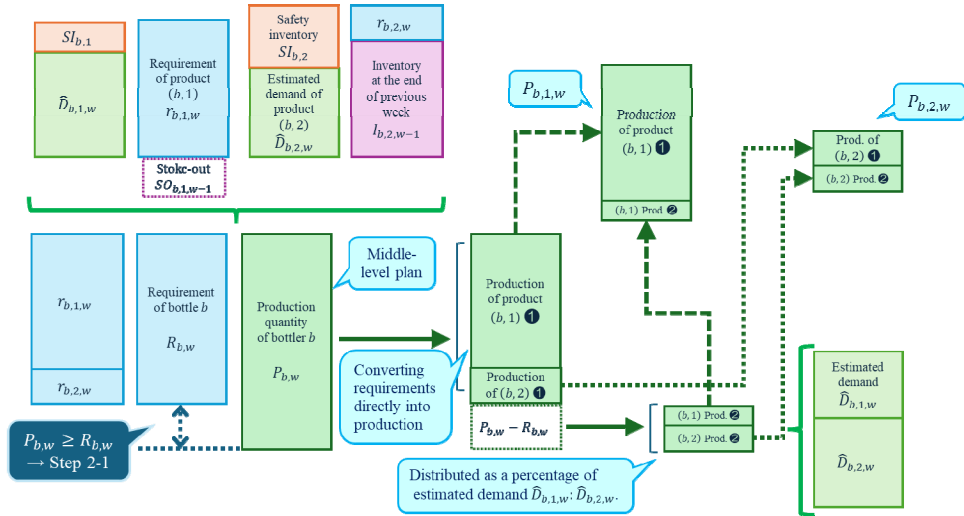
Step 2-1: $P_{b,w} \geq R_{b,w}$

Suppose the production quantity for bottle type b in week w , $P_{b,w}$, is greater than the required quantity $R_{b,w}$. In that case, it is probable that there will still be sufficient capacity

after the required quantity is produced. Figure 10 shows the production quantities $P_{b,n,w}$ when this step was followed. In the upper left part of the figure, the estimated demand quantity $\hat{D}_{b,n,w}$, safety inventory quantity $SI_{b,n}$, actual inventory quantity of the previous weekend $I_{b,2,w-1}$ and estimated stockout quantity $SO_{b,1,w-1}$ are shown, and the required quantity of each product $r_{b,n,w}$ is obtained. This differs from the middle-level Plan 1 in that stockouts are not included in the requirement quantity. In the lower left part of the figure, the total requirements $R_{b,w}$ are compared with the production quantity $P_{b,w}$ determined in the middle-level Plan 1 for the assembly process, and it is shown that $P_{b,w} \geq R_{b,w}$. In this case, the quantity required for each product $r_{b,n,w}$ is allocated to the production quantity, and the surplus production quantity is distributed in the ratio of the estimated demand quantity $\hat{D}_{b,n,w}$ and added to the production quantity. The production quantity of product (b, n) in week w , $P_{b,n,w}$, is determined in this manner and expressed by equation (13).

$$P_{b,n,w} = r_{b,n,w} + \frac{\hat{D}_{b,n,w}}{\sum_{n \in N_b} \hat{D}_{b,n,w}} (P_{b,w} - R_{b,w}) \quad (13)$$

Figure 10 Production quantity of product (b, n) in week w , $P_{b,n,w}$, according to Step 2-1 (see online version for colours)



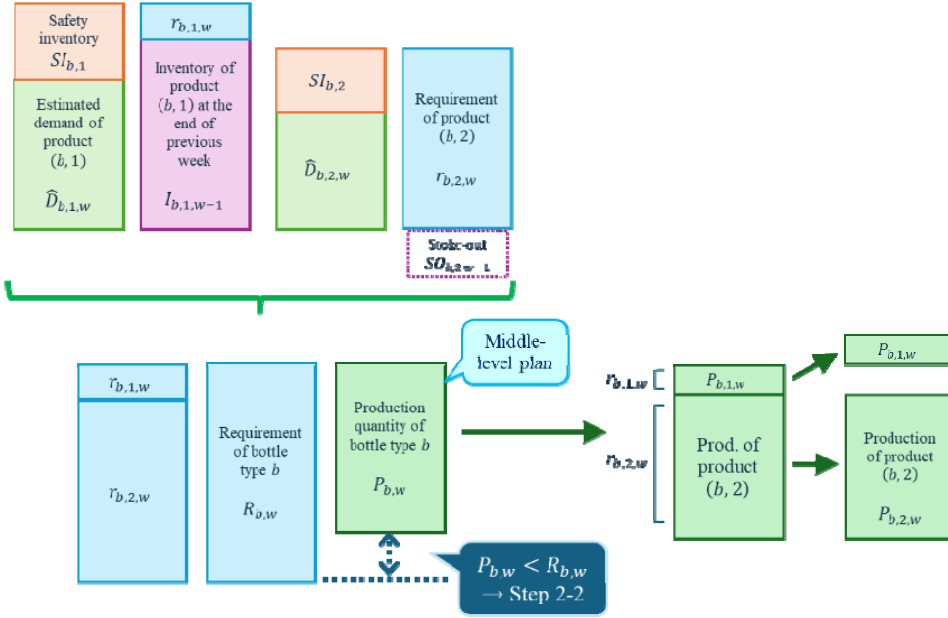
Step 2-2: $P_{b,w} < R_{b,w}$

In this step, the production quantity $P_{b,w}$ was less than the required quantity $R_{b,w}$, as shown in Figure 11. Therefore, the production quantity $P_{b,w}$ is simply distributed as the ratio of the requirement quantity for each product $r_{b,n,w}$ to the total requirement quantity $R_{b,w}$. This process determines the production quantity of products (b, n) in week w , $P_{b,n,w}$, which is expressed by equation (14).

$$P_{b,n,w} = \frac{r_{b,n,w}}{R_{b,w}} P_{b,w} \quad (14)$$

In this case, as in Step 1-2 in Subsection 4.2.1, the production quantity is expected to be insufficient; therefore, it is necessary to take measures to increase the production quantity. However, it is difficult to increase production quantities suddenly because production begins immediately after planning.

Figure 11 Production quantity of product (b, n) in week w , $P_{b,w}$, according to Step 2-2 (see online version for colours)



4.4 Criteria

The criteria for the target process were the actual inventory and stockout quantities. As shown in Figure 12, these are determined by the actual demand quantity $D_{b,n,w}$ arriving at the production quantity for each product in each week $P_{b,n,w}$, as determined by the hierarchical production plan described above. If the value obtained by subtracting the actual demand quantity $D_{b,n,w}$ from the inventory of the previous weekend plus the production quantity $P_{b,n,w}$ is positive, this is the actual inventory quantity of product (b, n) for the w weekend, $I_{b,n,w}$, as shown in equation (15). However, if the value is negative, the inventory quantity is reset to zero, and this quantity is counted as the stockout quantity of product (b, n) for the w weekend $SO_{b,n,w}$, as shown in equation (16).

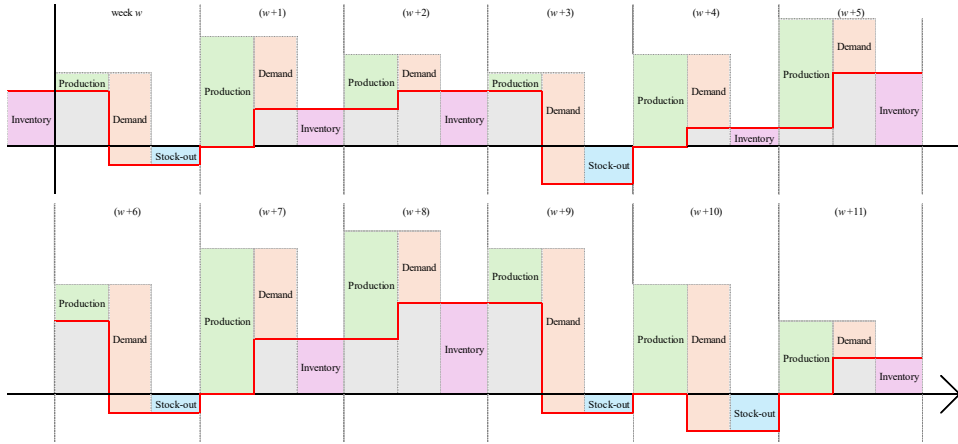
$$I_{b,n,w} = \begin{cases} I_{b,n,0} & (w = 0) \\ \max(I_{b,n,w-1} + P_{b,n,w} - D_{b,n,w}, 0) & (w > 0) \end{cases} \quad (15)$$

$$SO_{b,n,w} = \max(D_{b,n,w} - I_{b,n,w-1} - P_{b,n,w}, 0) \quad (16)$$

where $I_{b,n,0}$ is the default inventory quantity of product (b, n).

The results of these criteria were fed back into the production plan for the following week, and the planning cycle continued.

Figure 12 Image of change in inventory and stockout quantities of product (b, n)
(see online version for colours)



5 Mathematical analysis

In Section 4, a hierarchical production plan is formulated for the target process, which represents a combined production form of a processing-assembly flow shop and a hybrid flow shop. Both the actual inventory quantity and stockout quantity are calculated from the actual inventory quantity at the end of the previous week, the production quantity of the week, and the actual demand quantity. Furthermore, it is shown that the target process is a nonlinear model in which the inventory is reset to zero when stockouts occur. The stockout quantity is not added to the requirement quantity, which is the basis for production planning.

The optimisation of this model can be viewed as a problem of simultaneously minimising the sum of the actual inventory quantities of products (b, n) for each week $I_{b,n,w}$ and the sum of the stockout quantities $SO_{b,n,w}$ which are the criteria. This can be expressed as in equation (17), using the index k to represent the week.

$$\sum_{b \in B} \sum_{n \in N_b} \sum_{k=1}^w I_{b,n,k} + \sum_{b \in B} \sum_{n \in N_b} \sum_{k=1}^w SO_{b,n,k} \rightarrow \min \quad (17)$$

where $I_{b,n,w}$ and $SO_{b,n,w}$ are obtained for each product from the actual inventory quantity on the previous weekend $I_{b,n,w-1}$, the production quantity for weeks $P_{b,n,w}$ and the actual demand quantity $D_{b,n,w}$ as shown in equations (15) and (16) in Subsection 4.4. In particular, for $P_{b,n,w}$ calculated from equation (13) or equation (14) in Subsection 4.3, the estimated inventory quantity $\hat{I}_{b,n,w}$ or estimated stockout quantity $\hat{SO}_{b,n,w}$ is used, for example, $r_{b,l}$ for the requirement of bottle type b in period l in equation (3) which is required in the process of calculating $P_{b,n,w}$. $P_{b,n,w}$ is calculated differently at each stage of hierarchical production planning based on a comparison of the production capacity of bottle type b in week w , $P_{b,w}$, and the required quantity $R_{b,w}$. In addition, the calculation methods for the estimated inventory and stockout quantities depend on whether a stockout occurred on the previous weekend. Accordingly, it is difficult to solve the minimisation problem in equation (17) comprehensively, and it is necessary to consider

various conditions individually. Hence, the minimisation problem is divided into two cases: one in which stockouts occur and the other in which they do not. In this study, the mathematical analysis of equation (18) was only performed when stockouts did not occur.

$$\sum_{b \in B} \sum_{n \in N_b} \sum_{k=1}^w I_{b,n,k} \rightarrow \min \quad (18)$$

A mathematical analysis of the occurrence of stockouts will be undertaken in future research.

5.1 Assumptions

In this section, assumptions are made for the optimisation problem in equation (18), which is mathematically analysed in the next section.

Actual demand is random owing to many sudden demand fluctuations for user-branded products, as well as seasonal fluctuations. However, for the purpose of this study, the following assumptions are made to provide a basic understanding of the problem.

- Assume that the actual demand capacity of products (b, n) in week w , $D_{b,n,w}$, and the forecast demand $\hat{D}_{b,n,w}$ are the same as the expected weekly demand for product (b, n) , $\bar{D}_{b,n}$, such that $D_{b,n,w} = \hat{D}_{b,n,w} = \bar{D}_{b,n}$.
- The demand forecast priority parameter γ can be set to $0 \leq \gamma \leq 1$, but $0 < \gamma \leq 1$ because setting $\gamma = 0$, which does not consider the most recent demand estimation, is out of line with the actual situation.

Additionally, the following is assumed to restrict the analysis to cases in which no stockouts occur.

- Assume that $I_{b,n,w-1} + P_{b,n,w} - D_{b,n,w} \geq 0$ always holds and $SO_{b,n,w} = \widehat{SO}_{b,n,w} = 0$.
- Assume that there is no shortage of decorated work-in-process inventory required to produce an assembly process.

Furthermore, a safety inventory is considered for the initial setting and inventory forecasting as follows:

- To check the effect of the safety inventory setting parameter ε on the actual inventory quantity $I_{b,n,w}$, the initial inventory quantity of product (b, n) , $I_{b,n,0}$, and the estimated inventory quantity $\hat{I}_{b,n,w}$ are set independently of the safety inventory, so that assume $I_{b,n,0} = \hat{I}_{b,n,w} = \bar{D}_{b,n}$.
- Though the safety inventory setting parameter ε is a parameter that can be set for $\varepsilon \geq 0$, since it is impractical to have no safety inventory at all, $\varepsilon > 0$ is assumed.
- The demand forecast priority parameter γ and the safety inventory setting parameter ε are practically reviewed quarterly or semi-annually. However, in this section, they are assumed to be constant values to clarify their effects on production quantity and inventory quantity.

Finally, for middle-level Plan 1 of the assembly process, it is assumed that the target period is as follows, according to the actual situation.

- The period covered by middle-level Plan 1 was $L = 2$, and the estimated production quantity of bottle type b in each period, EP_l is expressed as $EP_l = \frac{CP_m}{2}$.

5.2 Mathematical analysis in the case of no stockouts

The minimisation problem dealt with in this study is $\sum_{b \in B} \sum_{n \in N_b} \sum_{k=1}^w I_{b,n,k} \rightarrow \min$, where the actual inventory quantity $I_{b,n,w}$ of product (b, n) at the w weekend is calculated using equation (15), described in Subsection 4.4. Here, the assumptions are applied from Subsection 5.1, where $I_{b,n,w-1} + P_{b,n,w} - D_{b,n,w} \geq 0$ is assumed to be no stockouts, $I_{b,n,0} = \bar{D}_{b,n}$ is the default inventory quantity of product (b, n) , and $D_{b,n,w} = \bar{D}_{b,n}$ is the demand quantity for each product in each week. With this application, equation (19) can be obtained from a gradual equation using equation (15) for $w > 0$.

$$I_{b,n,w} = \sum_{k=1}^w P_{b,n,k} - (w-1)\bar{D}_{b,n} \quad (19)$$

The second term $(w-1)\bar{D}_{b,n}$ on the right-hand side of equation (19) is a given value consisting of the week number w and the expected weekly quantity demanded of products (b, n) , $\bar{D}_{b,n}$. Therefore, the first term $\sum_{k=1}^w P_{b,n,k}$ on the right-hand side is focused on based on the production quantity of product (b, n) in week w , $P_{b,n,w}$.

The production quantities of products (b, n) in week w , $P_{b,n,w}$, consisting of equations (13) and (14) in Subsection 4.3, can be transformed into equation (20) by applying the following assumptions:

$$P_{b,n,w} = \begin{cases} r_{b,n,w} + \sum_{n \in N_b} \frac{\bar{D}_{b,n}}{\bar{D}_{b,n}} (P_{b,w} - R_{b,w}) & (R_{b,w} \leq P_{b,w}) \\ \frac{r_{b,n,w}}{R_{b,w}} P_{b,w} & (R_{b,w} > P_{b,w}) \end{cases} \quad (20)$$

This equation (20) consists of the production capacity of bottle type b in week w , $P_{b,w}$, the requirement quantity $R_{b,w}$, the requirement quantity of product (b, n) in week w , $r_{b,n,w}$, and the expected weekly demand $\bar{D}_{b,n}$. $P_{b,w}$ which consists of equations (6) and (7) in Subsection 4.2.1, can be transformed into equation (21) by applying the assumptions in Subsection 5.1 to equation (1) in Subsection 4.1, equations (3), (6) and (7) in Subsection 4.2.1, and also using equation (4).

$$P_{b,w} = \frac{4 + \varepsilon\gamma - \gamma}{4} \sum_{n \in N_b} \bar{D}_{b,n} \quad (21)$$

The requirement for bottle type b in week w , $R_{b,w}$, is obtained using equation (12), and the requirement quantity of products (b, n) , $r_{b,n,w}$, on which $R_{b,w}$ is based, can be converted to equation (22) by applying the following assumptions to equation (11).

$$r_{b,n,w} = \begin{cases} (1 + \varepsilon + w)\bar{D}_{b,n} - \sum_{k=1}^{w-1} P_{b,n,k} & \left\{ (1 + \varepsilon + w)\bar{D}_{b,n} < \sum_{k=1}^{w-1} P_{b,n,k} \right\} \\ 0 & \left\{ (1 + \varepsilon + w)\bar{D}_{b,n} < \sum_{k=1}^{w-1} P_{b,n,k} \right\} \end{cases} \quad (22)$$

Using equations (21), (22), (12), the production quantities $P_{b,n,w}$ in equation (20) were calculated.

To obtain the production quantity of product (b, n) in week w , $P_{b,n,w}$, in equation (20), the week number w is substituted stepwise into each of the following equations: the production capacity of bottle type b in week w , $P_{b,w}$, in equation (21); the requirement quantity of product (b, n) in week w , $r_{b,n,w}$, in equation (22); and the requirement quantity of bottle type b in week w , $R_{b,w}$, in equation (12). With $P_{b,n,w}$ now known, the actual inventory of products (b, n) on the w weekends w $I_{b,n,w}$ in equation (19) can also be known.

The calculation results for cases $w = 1, 2, 3$ are shown in equations (23)–(25).

$$I_{b,n,1} = \frac{4 + \varepsilon\gamma - \gamma}{4} \bar{D}_{b,n} \quad (23)$$

$$I_{b,n,2} = \frac{2 + \varepsilon\gamma - \gamma}{2} \bar{D}_{b,n} \quad (24)$$

$$I_{b,n,3} = \frac{4 + 3\varepsilon\gamma - 3\gamma}{4} \bar{D}_{b,n} \quad (25)$$

From the above, the actual inventory quantity of product (b, n) for the w weekend $I_{b,n,w}$ can be expressed by equation (26) using the safety inventory setting parameter ε , the demand forecast priority parameter γ , week number w and the expected weekly demand quantity of product (b, n) , $\bar{D}_{b,n}$.

$$I_{b,n,w} = \left\{ 1 - \frac{\gamma(1 - \varepsilon)}{4} w \right\} \bar{D}_{b,n} \quad (26)$$

From equation (26), when $\frac{\gamma(1 - \varepsilon)}{4} w = 1$, then $I_{b,n,w} = 0$, which is considered the optimal solution. However, $I_{b,n,w}$ cannot be simply set close to 0 due to constraints imposed by the actual process on the setting of the safety inventory parameter ε and the demand forecast priority parameter γ . These constraints are discussed in the following subsection.

5.3 Considerations on parameters to minimise inventory

Although under the assumptions of Subsection 5.1, where no stockouts are assumed to occur, in Subsection 5.2, the actual inventory quantity of product (b, n) on the w weekend $I_{b,n,w}$ is expressed by the safety inventory setting parameter ε , the demand forecast priority parameter γ , the week number w , and the expected weekly demand quantity of product (b, n) , $\bar{D}_{b,n}$, using the equation (26).

In equation (26), if $I_{b,n,w} = 0$, then $\gamma(1-\varepsilon) = \frac{4}{w}$, and the safety inventory setting parameter ε and the demand forecast priority parameter γ must be changed whenever the week number w changes. There is also the constraint $\frac{\gamma(1-\varepsilon)}{4}w \leq 1$, since the actual inventory $I_{b,n,w}$ is never negative. At first glance, one might think that it is sufficient to change the parameters each time so that $\gamma(1-\varepsilon) = \frac{4}{w}$ is satisfied, but this is not realistic because changing the parameters every week is the opposite of the approach expected by the factory to level production. The demand forecast priority parameter γ is a parameter that determines whether priority should be given to responding to ever-changing demand situations or levelling production. The safety inventory parameter ε is used to set safety inventory as a buffer against demand fluctuations. Clearly, setting either parameter too low is not a good decision. Therefore, the planning period should first be set at an interval, e.g., a quarter or semi-annual period. Next, the degree to which the demand forecast priority parameter γ should be set is determined by considering the actual situation. Then, to set the safety inventory setting parameter ε so that $\frac{\gamma(1-\varepsilon)}{4}w$ is as close to 1 as possible is proposed as a realisable solution that contributes to inventory minimisation.

Thus, it is vital to formulate a hierarchical production plan for the target process, conduct a mathematical analysis to minimise inventory and consider controllable parameters that match the actual situation.

6 Conclusions

This study aims to optimise a hierarchical production planning system using a plastic container manufacturer with a combined processing-assembly flow shop and hybrid flow shop. As part of this study, one of the issues faced by the target process was focused on. Because the manufacturing of the back-end process depends on the supply of parts from the front-end process, scheduling is necessary to increase both customer on-time delivery and the back-end process utilisation rate. In this study, the hierarchical production planning in a two-stage manufacturing process was attempted to be optimised using a mathematical approach targeting two of the most essential processes among the target processes.

Section 2 describes the target process and the product configuration. Section 3 describes previous related studies and clarifies the issues faced by the target process and the approaches to solving them. In Section 4, the hierarchical production planning of the target process is formulated, and a nonlinear model of the combined production form of a processing-assembly flow shop and a hybrid flow shop, which has not been addressed in previous studies, is presented. Fundamentally, one of the key contributions of this study is the formulation of demand forecast priority, safety inventory, and lot size, which have been set empirically by planners, in terms of controllable parameters: γ , ε , and φ . A mathematical analysis to minimise the inventory, which is one of the criteria, is described in Section 5. The demand forecasting priority parameter was set by considering the

constraints of the production facilities and worker shift problems of the target process. However, the safety inventory setting parameter must also consider the inventory storage location. Therefore, production planning based on the relationship between these parameters and the inventory quantity, as shown in Section 5, is an effective way to prevent excess inventory while minimising stockouts. In this study, a production planning method was formulated for an actual manufacturing process under limited conditions based on the assumption that stockouts did not occur and was analysed using a mathematical approach, thereby helping to solve the optimisation problem of hierarchical production planning in a multi-stage process.

Although this study used a mathematical approach for analysis under the limited condition that stockouts do not occur, further analysis is needed because stockouts can occur in reality. Therefore, it is necessary to clarify the relationship between controllable parameters and the quantities of inventory and stockouts. For example, to reduce stockouts to zero, it is necessary to determine the sizes of the set parameters for the safety inventory. In addition, because frequent stockouts are not realistic, it is necessary to mathematically determine the inventory transition when stockouts occur in a given week, considering the results of this study and the probability of stockouts. Furthermore, although this study was conducted mathematically for constant demand, numerical and convolutional integrals can also be considered for stochastically varying demand. First, consider the case where only the lower-level plan changes stochastically. Then, consider an analysis where the middle-level plan changes stochastically in a stepwise manner, and so on. After considering these mathematical approaches in depth, simulations were conducted using the model formulated in this study. To verify the results of the analysis conducted using the above mathematical approach, the relationship between the controllable parameters and criteria was clarified. Other issues to be addressed in future studies include ‘allocation of workers to parallel machines in consideration of skill differences’ and ‘demand forecasting in consideration of seasonal variations and sudden demand’.

Acknowledgements

In writing this paper, many valuable suggestions and guidance were provided by Dr. Hisashi Yamamoto, Professor Emeritus of Tokyo Metropolitan University. Deepest gratitude is expressed to him for his support. Valuable comments and suggestions were also provided by the anonymous reviewers during the review of this paper. In addition, English editing was performed by Editage (<http://www.editage.jp>).

References

- Arizono, I., Nishikawa, Y., Kato, W. and Takemoto, Y. (2014) ‘Improvement of procedure for simultaneous decision of economic order quantity and economic reorder point’, *Journal of Japan Industrial Management Association*, in Japanese, Vol. 65, No. 2, pp.110–113, DOI: 10.11221/jima.65.110.
- Hayashi, C., Yamamoto, H., Ohba, M., Maruyama, Y. and Nakamura, Y. (2021) ‘Study of production seat booking system in a make-to-stock manufacturing process with fixed-lot-size production’, *Journal of Japan Industrial Management Association*, in Japanese, Vol. 72, No. 2, pp.110–122, DOI: 10.11221/jima.72.110.

- Lee, K. and Kim, J. (2000) 'An ordering point model with a linear backorder ratio under the uncertainty of lead-time demand', *Journal of Japan Industrial Management Association*, in Japanese, Vol. 51, No. 1, pp.35–42, DOI: 10.11221/jima.51.35.
- Nagasawa, H. and Morizawa, K. (2002) 'Heuristic scheduling in machining-assembly flowshop with parallel two-machine flow lines at machining stage', *Journal of Japan Industrial Management Association*, in Japanese, Vol. 53, No. 1, pp.37–46, DOI: 10.11221/jima.53.37.
- Narasimhan, S.L. and Mangiameli, P.M. (1987) 'A comparison of sequencing rules for a two-stage hybrid flow shop', *Decision Science*, Vol. 18, No. 2, pp.250–265, DOI: 10.1111/j.1540-5915.1987.tb01520.x
- Ohba, M., Uetake, T. and Tsubone, H. (1995) 'The impact of lot sizing and sequencing effect on manufacturing performance – a production scheduling system in a hybrid flow shop', *Journal of Japan Industrial Management Association*, in Japanese, Vol. 46, No. 3, pp.175–181, DOI: 10.11221/jimapre.46.3_175.
- Sun, X., Nishiura, A., Morizawa, K. and Nagasawa, H. (1999) 'Machine-fixed, machining-assembly flow shop scheduling', *Journal of Japan Industrial Management Association*, in Japanese, Vol. 50, No. 5, pp.283–289, DOI: 10.11221/jima.50.283.
- Suzuki, M. and Tsubone, H. (1999) 'The relationship between rescheduling and safety stock – a study on production scheduling system in a hybrid flow shop', *Journal of Japan Industrial Management Association*, in Japanese, Vol. 50, No. 1, pp.11–19, DOI: 10.11221/jima.50.11.
- Suzuki, M., Tsubone, H., Ohba, M. and Uetake, T. (1997) 'The relationship between inventory levels of final products and work-in-process level – a study on production scheduling system in a hybrid flow shop', *Journal of Japan Industrial Management Association*, in Japanese, Vol. 47, No. 6, pp.359–366, DOI: 10.11221/jima.47.359.
- Takeda, K. and Kuroda, M. (2000) 'Analysis of a multi-stage production/inventory system with an acceptable response time – in the case of products with different processing times', *Journal of Japan Industrial Management Association*, in Japanese, Vol. 51, No. 2, pp.132–142, DOI: 10.11221/jima.51.132.
- Tamura, T. (2016) 'On a performance of a simulation-based algorithm to optimize stock levels (deterioration of solution effected by inconstancy among data)' in *The 2016 JSME: Proceedings of the 2016 JSME Manufacturing Systems Division Conference*, The Japan Society of Mechanical Engineers, Chiba, Japan, in Japanese, DOI: 10.1299/jsmemsd.2016.205.
- Tsubone, H. and Tanaka, T. (1990) 'An analytical study of hierarchical production planning system', *Journal of Japan Industrial Management Association*, in Japanese, Vol. 40, No. 6, pp.384–389, DOI: 10.11221/jimapre.40.6_384.
- Tsubone, H., Ohba, M., Takamuki, H. and Miyake, Y. (1993) 'A production scheduling system in the hybrid flow shop – a case study', *Omega*, Vol. 21, No. 2, pp.2–11, DOI: 10.1016/0305-0483(93)90053-N.
- Tsuchimoto, K., Morizawa, K. and Nagasawa, H. (2005) 'List-based squeezing branch-and-bound algorithm for minimizing makespan in a machine-unfixed, machining-assembly flowshop scheduling problem', *Journal of Japan Industrial Management Association*, in Japanese, Vol. 56, No. 5, pp.358–365, DOI: 10.11221/jima.56.358.
- Uetake, T., Tsubone, H. and Ohba, M. (1995) 'A production scheduling system in a hybrid flow shop', *International Journal of Production Economics*, Vol. 41, Nos. 1–3, pp.395–398, DOI: 10.1016/0925-5273(94)00081-6.
- Yanagawa, D., Hatono, I. and Taura, T. (2006) 'Study on inventory cost minimization in multi-process production systems', in *The 49th JACC: Proceedings of the Japan Joint Automatic Control Conference*, The Japan Joint Automatic Control Conference, Kobe, Japan, pp.121–122, in Japanese, DOI: 10.11511/jacc.49.0.121.0.

Appendix

Notation

1 Subscripts and sets

- w : Week number.
- $m\left(\left\lceil\frac{w}{4}\right\rceil\right)$: Month number.
- $l\left(\left\lceil\frac{w}{L}\right\rceil\right)$: Period number in middle-level Plan 1 of the assembly process.
- L : Period covered by middle-level Plan 1 of the assembly process.
- $M (= \{w|4(m-1)+1 \leq w \leq 4m\})$: Set of week number w in month m .
- $Pe (= \{w|L(l-1)+1 \leq w \leq L \cdot l\})$: Set of week number w in period l .
- $b (\in B)$: Subscripts indicating bottle type.
- B : Set of bottle types.
- $n (\in N_b)$: Subscripts indicating nozzle type.
- N_b : Set of nozzle types within bottle type b .

2 External environment variables

- $D_{b,n,w}$: Actual demand for product (b, n) in week w .
- $\hat{D}_{b,n,w}$: Estimated demand for product (b, n) in week w .
- $\bar{D}_{b,n}$: Expected weekly demand for product (b, n) .

3 Controllable variables

- $\gamma (0 \leq \gamma \leq 1)$: Demand forecast priority parameter.
- $\varepsilon (\geq 0)$: Safety inventory setting parameter.
- $SI_{b,n}$: Safety inventory quantity of product (b, n) .
- $\varphi (> 0)$: Lot size parameter for the decoration process.

4 Calculated variables

- CP_m : Comprehensive production capacity in month m .
- EP_l : Estimated production capacity in period l .
- SR_l : Quantity required for all bottle types in period l .
- $r_{b,l}$: Quantity required for bottle type b in period l .
- $R_{b,w}$: Quantity required for bottle type b in week w .
- $r_{b,n,w}$: Quantity required for product (b, n) in week w .
- $P_{b,w}$: Production capacity of bottle type b in week w .
- $P_{b,n,w}$: Quantity of product (b, n) produced in week w .
- $I_{b,n,w}$: Actual inventory quantity of product (b, n) at w weekend.
- $I_{b,n,0}$: Default inventory quantity of product (b, n) .

- $\hat{I}_{b,n,w}$ Estimated inventory quantity of product (b, n) in week w .
- $SO_{b,n,w}$: Actual stockout quantity of products (b, n) at w weekend.
- $\widehat{SO}_{b,n,w}$: Estimated stockout quantity of product (b, n) in week w .
- $PL_{b,w}$: Number of lots produced during the decoration process for bottle type b in week w .
- $WIP_{b,w}$: Decorated work-in-process inventory of bottle type b in week w .
- $WIP_{b,0}$: Default quantity of bottle type b decorated work-in-process inventory.
- $S_WIP_{b,w}$: Decorated work-in-process inventory shortages for bottle type b in w weekend.