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New media popular music recommendation system based on machine learning algorithm

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Abstract: With the development of new media and AI, music recommendation systems have become essential in meeting diverse user preferences. Traditional collaborative filtering methods neglect music content, leading to less accurate recommendations. This paper proposes a machine learning-based popular music recommendation system that addresses these limitations. By analysing user preferences and applying the maximum entropy model (MEM), the system integrates and processes music data more effectively. Experimental results show that the new approach enhances recommendation accuracy by 7.5%, making the system more intelligent and user-friendly. This study contributes to improving music recommendation performance in the era of new media.

Keywords: music recommendation system; popular music; collaborative filtering; machine learning algorithm; CTR estimation; maximum entropy model; MEM.

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Qi He graduated from Hunan Normal University, P.R. China with a Bachelor's degree. And she graduated from Wuhan Conservatory of Music with a Master's degree. Currently, she is studying at College Faculty of Social Sciences and Liberal Art, UCSI University. Her main research interests are music performance, artistic voice, education, and music therapy.

1 Introduction

People's lives are inseparable from music, and music contains many song libraries, and users cannot listen to all the songs in the song library. In many cases, users' needs are very vague, such as only one or a few good songs, which require an accurate music recommendation system to recommend songs. Compared with the recommendation of movies, books, and materials, music recommendation has its characteristics. The recommendation algorithm needs to take these characteristics into account before making recommendations. In recent years, the rapid development of artificial intelligence has made machine learning algorithms widely used in many fields. Applying machine learning algorithms to music recommendation systems has high research value.

Machine learning algorithms are the core of artificial intelligence and have been widely used in various fields. Aktan (2017) used a machine learning algorithm to find the factors that affect the reproductive ability of Holstein dairy cows. The study found that the algorithm could accurately locate the factors affecting the reproductive ability of dairy cows (Aktan, 2017). Auwal proposed a new predictive model for hepatocellular carcinoma (HCC) based on machine learning algorithms. Tests showed that the model could improve the accuracy of HCC risk prediction in patients with cirrhosis (Auwal et al., 2021). Mubarak combined machine learning algorithms to explore genes that affect coronary artery calcification. Studies showed that machine learning algorithms could accurately predict the transformation of arterial factors (Mubarak et al., 2021). Wang (2021) analysed the application of the support vector machine algorithm in the machine learning algorithm in the international robot football game. Experimental tests concluded that SVMs had higher accuracy in adapting to a new type of data (Wang, 2021). Saglam used machine learning algorithms to analyse northern and southern California land cover changes. The results showed that the land interpretation map produced using a machine genetic algorithm had higher accuracy (Saglam and Cetin, 2022). Bisen used two machine learning algorithms, the Bayesian network, and decision tree, to study internet traffic classification. Subsequent experiments showed Bayesian networks were more suitable for high-speed Internet traffic classification (Bisen et al., 2022). Meliho used support vector machines (SVM) and relevance vector machines (RVM) in machine learning algorithms to evaluate spam filtering results. Experimental results showed that RVM was more suitable for spam classification than SVM (Melih et al., 2021). The application research of these machine learning algorithms is relatively rich, but it does not involve a music recommendation system.

This paper studies and analyses the popular music recommendation system based on a machine learning algorithm. Through the analysis of users' music preferences, it applies machine learning technology to the music recommendation system. In the application process, the maximum entropy (MaxEnt) model is used to extract and integrate user data and music data. The music recommendation dataset is extracted during the system test, and the recall rate, precision value, and F1 value of the algorithm are tested and compared in this paper. It further verifies the application effect of machine learning algorithms in music recommendation systems.

This study selects the MEM model as the core algorithm because of its many innovations. Firstly, MEM successfully integrates user behaviour and music content characteristics by defining characteristic functions, such as lyrics emotion, rhythm matching, and singer style, making up for the shortcomings of traditional collaborative

filtering. Secondly, in the face of data sets with a sparsity of 50%, such as TP-500, MEM uses the Laplace smoothing strategy to increase the recommendation accuracy rate by 4.85% compared with the CDL model, showing its strong adaptability in sparse data. Moreover, compared with the black box model, the characteristic weight of MEM directly reflects the recommendation logic. For example, a song is recommended because the emotional matching of the lyrics reaches 0.32, which enhances user trust.

Model boundary: the computational complexity of MEM is linearly related to the feature dimension. When processing a music library of more than one million levels, it needs to be optimised by distributed computing, and it relies on high-quality music metadata, which may be restricted in scenarios with insufficient data.

2 Related work

With the rapid development of network technology, recommendation systems have entered people's lives and play a vital role. Jayashree studied how to use dimensionality reduction methods to improve the performance of recommender systems. The results showed that this method achieved good results in improving the performance of recommender systems (Jayashree and Christy, 2018). Gosavi et al. (2017) proposed a personalised recommendation system for internet shopping malls, enabling users to get accurate purchase recommendations when shopping online. Andrawis proposed a new social network recommendation system paradigm, and the experimental test results on the dataset confirmed that the method could improve the prediction accuracy of the recommendation system (Andrawis et al., 2019). Ma (2018) studied how to integrate content information into the built-in filters of recommender systems, and it turned out that this approach helped to improve the predictable quality of object-based filtering (Ma, 2018).

Song S proposed a new quality-based recommender system. The recommender system was developed using a fuzzy language approach and satisfactorily tested in a university digital library (Song et al., 2021). Whiteley et al. (2021) proposed a personalised learning recommendation system, which could more effectively support students' online learning and assist large-class online teaching with multiple backgrounds. Sandelin et al. (2020) described a new recommender system. It helped users find their favourite items in stores or shopping websites (Sandelin et al., 2020). The above research work on recommendation systems is relatively specific, but neither music recommendation systems nor machine learning algorithms are mentioned.

With the more comprehensive development of the recommendation system, there is more and more research on the music recommendation system, and many scholars have proposed different methods of designing the music recommendation system. Wang proposed a multi-view enhanced graphic attention network for session-based music recommendation. The designed graphic attention network learned music segments and user low-dimensional embeddings and adaptively enhanced the multi-view of heterogeneous music graphs by integrating various information. It learned from user listening behaviour and mixed user preferences to recommend music works that meet real-time user requirements. The results showed that the proposed method had better performance than baseline methods (Wang et al., 2023). La Gatta proposed a song recommendation framework based on hypergraph embedding-music recommendation hypergraph. The embedded hypergraph data model could seamlessly represent all

possible complex interactions between users and songs, along with their related characteristics. At the same time, the embedding technology provided vector mapping to infer the similarities between users and songs. The results showed that the proposed method was superior to other state-of-the-art technologies, especially in the case of cold-start or sparse-data scenarios problems, making the system a suitable solution for embedded music streaming platforms (La Gatta et al., 2022). Kostrzewa used the three basic factors of neural network genre classification, Mayer frequency cepstral coefficient, and song rhythm to design a content-based automatic music recommendation system. Compared with the random method, the recommendation effect of the proposed recommendation method was improved (Kostrzewa et al., 2024). Liu designed a recall model long short-term deep matching (LSDM) model based on users' short-and long-term preferences to help users accurately identify small-scale candidate songs that may be of interest from massive amounts of data, and, at the same time, designed an improved deep conversation preference model to further sort the candidate list and recommend songs to users more accurately and verify the effectiveness of the model through experiments on the proposed two algorithm models (Liu et al., 2025). Liu designed an emotion-based personalised music recommendation framework to meet the emotional needs of users and help improve their psychological state. In this framework, an LSTM-based model was designed to select the most suitable music based on the user's emotions and current emotional stimuli in the previous period. Experiments and user studies showed that the proposed new framework recommendations were accurate and helpful to users (Liu et al., 2023). Wu proposed a music recommendation model based on users' long-term and short-term preferences and music-emotional attention, dividing users' listening records, and learning characteristics through short-and long-term memory networks to obtain short-and long-term preferences, and then combining music-emotional factors to generate a recommendation list. Experiments showed that the proposed model was better than the existing method (Wu and Chen, 2023). The research of the above-mentioned scholars has significantly improved the personalisation and accuracy of music recommendation through machine learning technology and enhanced the system's adaptability to complex interactions and dynamic preferences. However, the demand for computing resources is high, relying on manual feature engineering or high-quality data, and there are still performance bottlenecks in cold-start or sparse-data scenarios or sparse data scenarios.

3 Machine learning algorithms

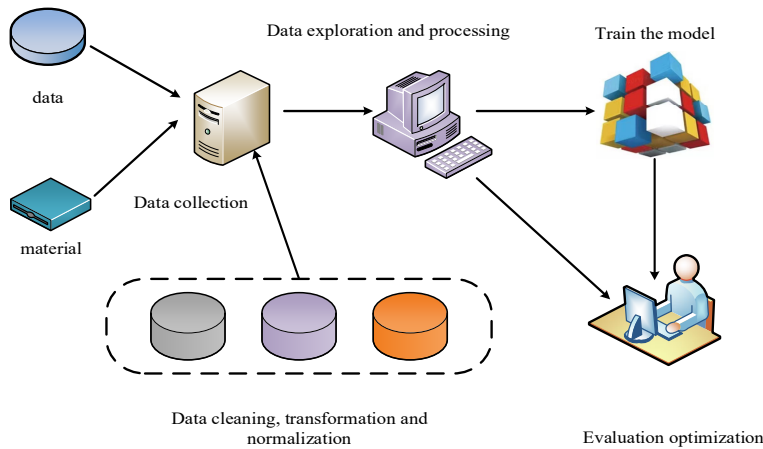
3.1 Machine learning

The machine learning process is shown in Figure 1, including data source, analysis, feature selection, model building, splitting the dataset, training, and evaluation.

Data collection is the first and crucial step in machine learning, and the quality and quantity of the data collected directly determine the quality of the predictive model (Saved, 2018). It then deduplicates, cleans, transforms, corrects errors and the collected data, and saves it as a database file in preparation for the next step of data loading. The integrated and processed data is used as training data for training model building. Models are the core concept of machine learning. It can be seen as a 'magic box'. By making a wish to it (inputting data), it helps to achieve a wish (outputting a prediction) (Tsao et al.,

2018). The entire machine learning process revolves around the model, training a best-quality ‘magic box’. It can realise ‘wish’ as accurately as possible, which is the goal of machine learning. After training is complete, the model’s success is evaluated by comparing the actual and predicted data.

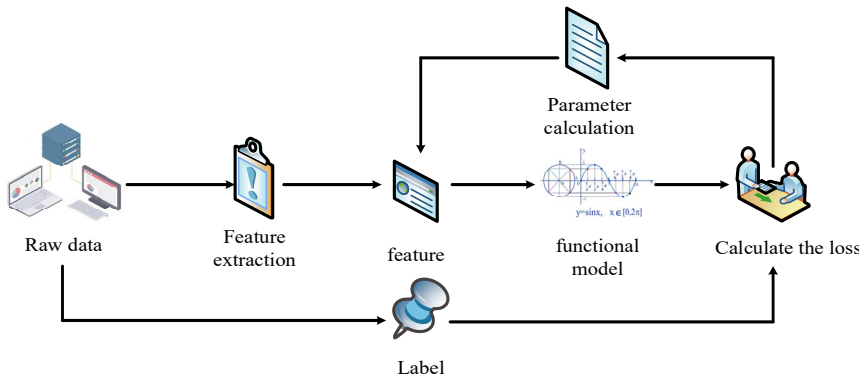
Figure 1 The process of machine learning (see online version for colours)



3.2 Machine learning algorithms

A machine learning algorithm is a process of finding the best solution to a problem through mathematical and statistical methods. According to whether the data sample has the characteristics of label value, it can be divided into supervised learning and unsupervised learning (Sharmila et al., 2017). The sample data of supervised learning has a label value. It first goes through a learning process during training and then uses a mathematical model to diagnose and evaluate new samples. Its typical representatives are mainly classification problems and regression problems. In unsupervised learning, the structure or distribution of a sample set is determined by analysing samples of unlabeled values. Its typical representatives are mainly clustering and data dimensionality reduction.

Figure 2 Iterative process of machine learning algorithm (see online version for colours)



The essence of the machine learning algorithm is to iterate the vector parameters in the algorithm process continuously, find the parameter vector that improves the objective function, and finally create a development model (Al-Moqri et al., 2020). The iterative process of the algorithm is shown in Figure 2. It first extracts features from the original data and then builds a functional model based on data features and data labels. The function model can calculate and update the relevant parameters, and finally carry out the evaluation link to calculate the loss component of the data.

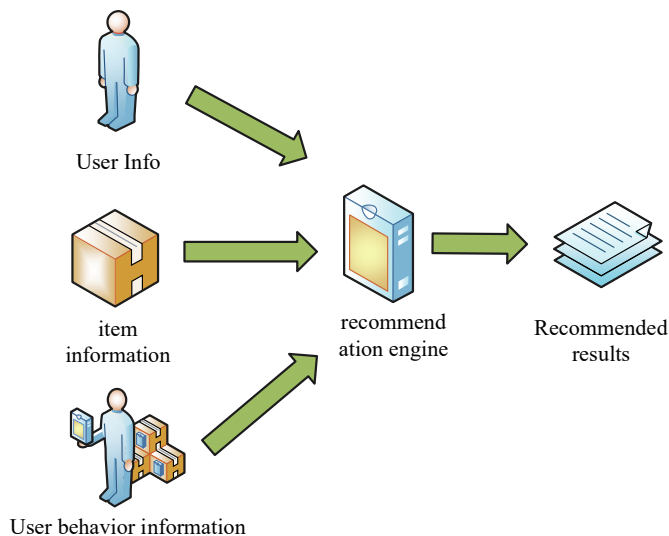
4 New media popular music recommendation system based on machine learning algorithm

4.1 Recommendation engine

Recommendation engines are a subset of information filtering systems designed to predict user ratings or item preferences. A specific example of it in music recommendation systems is predicting the degree of preference of a particular user for a particular piece of music (Wang and Zhang, 2020). The recommendation engine is the core of the recommendation system. It is similar to a black box, taking in data sources and then exporting a collection of items that the user might like. Figure 3 shows the specific operation process of the recommendation engine.

As shown in Figure 3, the data source of the recommendation engine can be divided into the unit data of the item. The user's basic information and the user's feedback data on the item are divided into three parts, and the user's feedback data is divided into explicit feedback and implicit feedback. In the music recommendation system, its recommendation engine uses various recommendation algorithms to select some data in the data source, analyse the rules, build models, and finally make recommendations.

Figure 3 Recommendation engine running process (see online version for colours)



4.2 Music recommendation system combined with a machine learning algorithm

Content recommendation is one of the more commonly used recommendation methods. The specific process is to obtain information about works that users like according to the user's operation history, and then recommend other similar works to users (Kindhi, 2020). The recommendation system is built around the machine learning algorithm, and the system model is shown in Figure 4. Various music data and user preference data are input into the algorithm, which performs machine learning and model building according to the user's music configuration and calculates and updates relevant parameters in time. After being processed by the algorithm, these parameters are aggregated into the recommender system. Its recommendation system then feeds back to customers, so that they have a good sense of experience in music needs.

The content-based recommendation method extracts the characteristics of music through the music content and makes a structured representation of each music. Next, it analyses the characteristics of the user's favourite music according to the user's historical listening behaviour and builds a user image model according to the characteristics. It finally calculates the matching degree between the user image model and the music characteristics and recommends various types of music to the user according to the matching degree. Figure 5 gives an example of recommendation based on music content: user 1 likes Chinese music and European and American music at the same time, while user 2 likes European and American music and Korean wave music at the same time. When the recommendation system receives the data information, it can recommend Chinese European, and American music to user 1, and recommend European and American music and Korean-style music to user 2 according to the user's preferences.

Figure 4 Application architecture of machine learning algorithms in systems (see online version for colours)

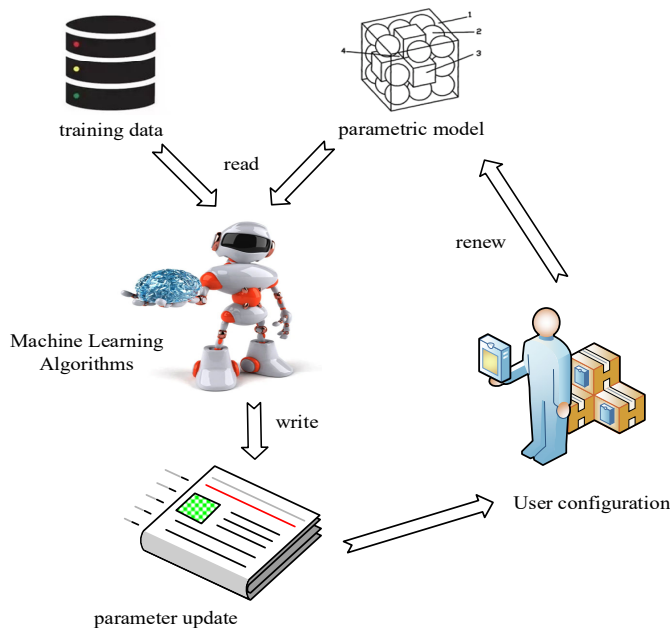


Figure 5 Music recommendation based on content recommendation (see online version for colours)

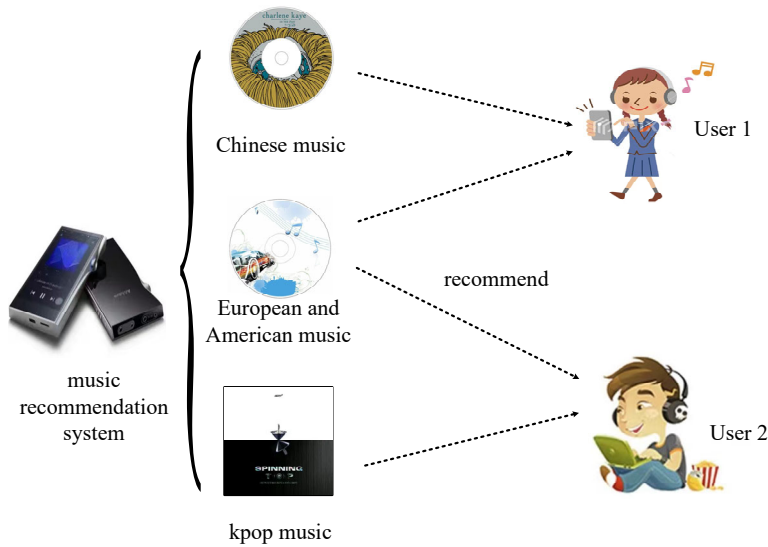
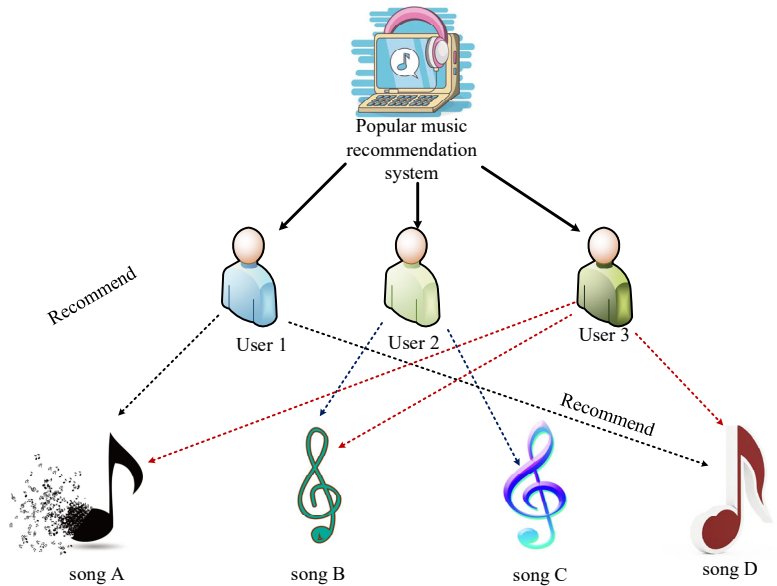


Figure 6 Collaborative filtering music recommendation (see online version for colours)



The user-based collaborative filtering recommendation form is to first list user groups with similar preferences in the system, then select X users with the most similar preferences, and recommends the music that these users like to the target users. Figure 6 shows an example of user-based collaborative filtering music recommendation: user 1 likes song A and song D; user 2 likes song A and song C; user 3 likes song B and song D. It can be found that user 1 and user 3 have similar taste preferences, that is, user 3 is the

neighbour of user 1. This means that song B which user 3 likes may also be attractive to user 1, so song B can be recommended to user 1.

The system adopts a modular design to enhance extensibility and mainly includes core modules such as feature cache, distributed computing, and cold start optimisation. The feature cache module stores frequently accessed song features in the Redis database, greatly reducing the load on the back-end database. The distributed computing module uses Apache Spark to process many user logs and music data, supports real-time analysis and model training, and can quickly analyse the historical playback records of millions of users. Cold start optimisation is quickly initialised through the pre-trained music feature library, and the first recommendation response of a new user takes only 0.8 seconds, combined with the user portrait, to improve the recommendation relevance. This architecture ensures the system's stability under high concurrency, supports simultaneous requests from 100,000 users, and maintains the timeliness of recommendations through dynamic feature updates, achieving a balance between computational efficiency and recommendation quality.

5 CTR Estimation of popular music recommendation system

To improve the practical effect of the model, this paper uses the maximum entropy model (MEM) in machine learning to predict the CTR of the music recommendation model. The derivation process of click-through rate is shown in Appendix.

Let the probability distribution of the discrete random variable O be $M(O)$, and its entropy value $H(M)$ is:

$$H(M) = \sum_o M(O)I(O) \quad (1)$$

The principle of maximum entropy points out that the probability model selected when learning the probability model must meet the conditions of known facts, that is, to meet the constraints. In this case, choosing the model with the largest entropy in the distribution of each probability model is the best model. Since the set of probabilistic models is usually constrained, the principle of maximum entropy is usually formulated as selecting the model with the highest entropy from a finite set of models (Iyenger, 2020). The derivation process of the MEM is shown in Appendix.

When deriving the MEM from the classification problem, it can be assumed that all the input and output of the classification model are represented by a conditional probability distribution $M(b|a)$; $O \in a \subseteq R^n$ represents the input; $P \in b$ and represents the output. a and b are the combination of input and output, respectively. What this model represents is that for a given input value a , b is output with conditional probability $M(b|a)$. Given a training dataset $T = \{(a_1, b_1), (a_2, b_2), \dots, (a_n, b_n)\}$, the purpose is to select the best classification mode according to the principle of maximum entropy.

Considering the conditions that the model should meet, given the training dataset, the empirical distribution of the joint distribution $M(b|a)$ and the empirical distribution $M(O)$ of the marginal distribution can be determined, denoted by $\hat{M}(a, b)$ and $\hat{M}(a)$, respectively, namely:

$$M(O = a, P = b) = \frac{c(O = a, P = b)}{N} \quad (2)$$

$$\hat{M}(O = a) = \frac{c(O = a)}{N} \quad (3)$$

Among them, $c(O = a, P = b)$ represents the frequency of the sample (a, b) in the training data; $c(O = a)$ represents the frequency of the input a in the training data; N represents the training sample size.

A certain fact between the input a and the output b is described by the feature function $f(a, b)$. Its definition is:

$$f(a, b) = \begin{cases} 1, & a \text{ and } b \text{ satisfy a fact} \\ 0, & a \text{ and } b \text{ do not satisfy a fact} \end{cases} \quad (4)$$

It is a binary function that takes the value 1 when a b and satisfies this fact, and it is 0 otherwise.

$$E_{\hat{M}}(f) = \sum_{a,b} \hat{M}(a, b) f(a, b) \quad (5)$$

The expected value of the characteristic function $f(a, b)$ concerning the model $M(b|a)$ and the empirical distribution $\hat{M}(a)$ is denoted by $E_M(f)$.

Lagrange multipliers are introduced to turn the above initial problem of constrained optimisation into a dual problem of unconstrained optimisation. The Lagrangian function $Q(s, t)$ is defined as:

$$\begin{aligned} Q(s, t) &= -H(M) + t_0 \left(1 - \sum_b M(b|a) \right) + \sum_{i=1}^n t_i (E_M(f_i) - E_{\hat{M}}(f_i)) \\ &= \sum_{a,b} \hat{M}(a) M(b|a) \log M(b|a) + t_0 \left(1 - \sum_b M(b|a) \right) \\ &\quad + \sum_{i=1}^n t_i \left(\sum_{a,b} \hat{M}(a, b) f_i(a, b) - \sum_{a,b} \hat{M}(a) M(b|a) f_i(a, b) \right) \end{aligned} \quad (6)$$

The original optimisation problem is $\min_{M \in C} \max_t Q(s, t)$, and the dual problem is $\max_t \min_{M \in C} Q(s, t)$.

To solve the dual problem, the minimisation problem $\min_{M \in C} Q(s, t)$ must be solved. The dual function $\psi(t)$ and its solution $M(t)$ are defined as:

$$\begin{aligned} \psi(t) &= \min_{M \in C} Q(s, t) = Q(M_t, t) \\ M_t &= \arg \min_{M \in C} Q(s, t) = M_t(b|a) \end{aligned} \quad (7)$$

To solve $M(t)$, the partial derivative of $Q(s, t) M(b|a)$ concerning is solved:

$$\begin{aligned} \frac{\partial Q(s, t)}{\partial M(b|a)} &= \sum_{a,b} \hat{M}(a) (\log M(b|a) + 1) - \sum_b t_0 - \sum_{a,b} \left(\hat{M}(a) \sum_{i=1}^n t_i f_i(a, b) \right) \\ &= \sum_{a,b} \hat{M}(a) \left(\log M(b|a) + 1 - t_0 - \sum_{i=1}^n t_i f_i(a, b) \right) \end{aligned} \quad (8)$$

Let the above partial derivative be equal to 0, in this case $\hat{M} > 0$, it can be obtained:

$$\log M(b|a) + 1 - t_0 - \sum_{i=1}^n t_i f_i(a, b) = 0 \quad (9)$$

Namely:

$$\log M(b|a) = \sum_{i=1}^n t_i f_i(a, b) + t_0 - 1 \quad (10)$$

Since $\sum_b M(b|a) = 1$

$$M_t(b|a) = \frac{1}{Z_t(a)} \exp \left(\sum_{i=1}^n t_i f_i(a, b) \right) \quad (11)$$

$$Z_t(a) = \sum_b \exp \left(\sum_{i=1}^n t_i f_i(a, b) \right) \quad (12)$$

$Z_t(a)$ is the normalisation factor, and t_i is the weight corresponding to the feature function $f_i(a, b)$.

After solving the internal minima problem of the dual problem, the external maximisation problem is solved, denoted as t^* that:

$$t^* = \arg \max_t \psi(t) \quad (13)$$

The maximum value $\psi(t)$ is obtained by an unconstrained optimisation algorithm and t^* is obtained, which is used to represent $M^* \in C$. Then, $M^* = M_{t^*} = M_{t^*}(b|a)$ is the MEM.

The maximum entropy algorithm explicitly models user preferences through interpretable feature functions. For example, user behaviour characteristics (such as playback time, and skip rate) are weighted to reflect short-term preferences—songs with high skip rates reduce the probability of recommendation due to reduced relevance scores; music content characteristics (such as genre, rhythm, lyrics, and emotions) are dynamically matched with user portraits through the entropy maximisation framework. For example, for users who prefer brisk pop music, the model parameters give higher weights to the ‘rhythm’ and ‘melody’ characteristics, thereby strengthening the recommendation of related music. The conditional probability distribution of the model is directly related to these characteristics and the recommendation results. Users can understand the recommendation logic through the characteristic weights, for example: ‘The song is recommended because it matches your preferred ‘lyrical lyrics’’. This transparency enables users to trace the decision-making basis behind the recommendation, enhancing the model’s interpretability and trust.

6 Test results of music recommendation system based on machine learning algorithm

To analyse the accuracy of the music recommendation model based on the machine learning algorithm through experiments, this paper obtains the dataset from the recommendation log of the music recommendation system. It randomly selects 10,000 samples, of which 6,877 are used as training samples and 3,123 are used as test samples. Semantic features are added to the recommendation model, which first calculates the topic distribution vectors of the keywords of the user model and the recommendation model. It then computes the similarity between the two sets of topic distribution vectors through a similarity measure function. Three metric functions are used in the testing process, including angle cosine, Euclidean distance, and KL divergence. The experimental results are as follows:

Table 1 Semantic feature experimental results

	<i>Model complexity</i>	<i>Accuracy</i>	<i>Positive sample</i>		<i>Negative sample</i>	
			<i>Recall</i>	<i>Precision</i>	<i>Recall</i>	<i>Precision</i>
Benchmark recommendation model	8+	92.2387	95.1265	90.1679	90.2773	89.3547
Angle cosine	8+	91.8925	95.8714	88.4998	90.9647	90.2679
KL divergence	8+	96.3858	95.3476	96.2576	98.0624	97.3584
Euclidean distance	8+	98.0671	96.0463	97.3547	97.4238	98.1937

The following conclusions can be drawn from Table 1:

- a Adding semantic features to the model can greatly improve the accuracy of the recommendation model.
- b Different metric functions have different effects on the synthesis accuracy, and the use of Euclidean distance has the most obvious effect on improving the accuracy. The second is the KL deviation, and the effect of the angle cosine is less obvious.
- c Adding three semantic features to the model can maximise the accuracy of the recommendation.

To verify the performance of the MEM algorithm in the music recommendation system, this paper selects user-based CF, a user-based collaborative filtering algorithm commonly used in recommendation systems. The collaborative depth model CDL is compared with the MaxEnt model MEM proposed in this paper. The experiments are performed on two datasets with different sparsity, TP-300, and TP-500. The recall, precision, and F1 index of the three models are compared, where the K value represents the number of song recommendations.

This experiment is conducted on two private datasets with different sparsity TP-300 (sparsity 30%, sample size 100,000) and TP-500 (sparsity 50%, sample size 150,000). The dataset contains user behaviour logs (such as playback duration, and skip rate) and music characteristics (such as genre, rhythm, and emotional tags). The experiment adopts a random division strategy, divides the data into training sets (80%) and test sets (20%), and uses 50% off cross-verification to ensure the stability of the results. Accuracy is defined as the proportion of recommended songs accepted by the user (such as playback

completion > 80%). To verify the system's response efficiency in real-world scenarios, this study deploys a recommended system prototype on a local server (configuration: 8-core CPU, 32GB memory). When stress testing is performed on the TP-300 dataset (100,000 samples), the average response time of a single recommendation request is 0.23 seconds, which meets the real-time recommendation requirements. By caching the feature vectors of popular songs and distributed computing frameworks (such as Apache Spark), the system can be extended to support 100,000 concurrent users, providing a guarantee for high-traffic scenarios.

To ensure that the experiments are repeatable and statistically rigorous, all experiments are repeated ten times using different random seeds and the average accuracy $\pm$ standard deviation (SD) is reported. The bilateral paired t-test ($\alpha = 0.05$) is used to evaluate the significance of the difference between MEM and user-based CF, and the 95% confidence interval (CI) for improved accuracy is calculated by the bootstrapping method. In this study, a prototype of the recommendation system is deployed on the localised music platform. By simulating real-world scenarios and inviting 200 volunteers to participate in a two-week A/B test, the actual performance of the system is verified. The test environment is based on a server equipped with an 8-core CPU and 32 GB of memory and uses a distributed computing framework (such as Apache Spark) to process user requests.

Taking the user case as an example, the model's decision to recommend song A instead of B is based on three key characteristics. First of all, in terms of lyrics and emotions, the positive emotional words of song A are highly consistent with user preferences, with a weight of 0.32; secondly, in terms of rhythm, users like brisk songs, and the rhythm weight of A is 0.28, which is higher than that of B; finally, in terms of genre, users have a tendency to popular music, which increases the genre weight of A to 0.19. This transparency allows users to intuitively understand the reason for the recommendation through feature weights, such as 'the song is recommended because it matches your preferred 'brisk pop' style', thereby enhancing trust in the system.

The experimental process remains highly transparent. The specific details are as follows: first, two datasets, TP-300 and TP-500, are selected. They contain 100,000 and 150,000 user behaviour logs, respectively, with a sparsity of 30% and 50%. At the same time, it integrates rich music metadata, such as emotional labels, rhythm parameters, and singer styles. In the pre-processing stage, the user behaviour data is cleaned, and invalid clicks that are played for less than 10 seconds are filtered out, only retaining valid behaviours, such as 'favourites' and 'full play'. Then, the Z-score for continuous features is standardised, and the one-hot coding method is adopted for classification features. In terms of parameter setting, the MEM model uses the L2 regularisation method and sets the maximum number of iterations to 100 and the convergence threshold to $1e-4$. Finally, the accuracy rate and F1 value are used as evaluation indicators to ensure the reproducibility of the experiment and provide a clear calculation basis for the subsequent analysis of the results.

Figure 7 presents the comparison of the recall rates of the three models in the two datasets. As can be seen from the figure, the MEM achieves the best recommendation effect among the three models, followed by the user-based collaborative filtering algorithm user-based CF, and the collaborative depth model CDL. With the increase of the K value of the number of music recommendations, the recall values of the three models all show an increasing trend. The MaxEnt model in the TP-300 dataset is 3.25% higher than the collaborative filtering algorithm on average, and the TP-500 dataset is

4.85% higher than the collaborative depth model. This also shows that the MaxEnt model has better anti-sparse ability under the Recall index.

Figure 7 Average recall values of the three algorithms under different K values (see online version for colours)

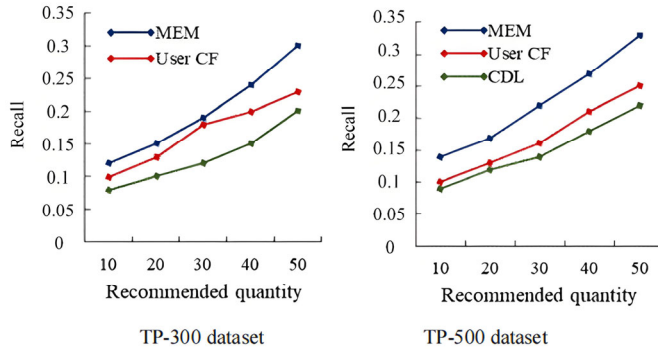


Figure 8 compares the accuracy values of the three models on the two datasets. It can be seen from the figure that the accuracy of the MaxEnt model has always been in a high position among the three models, followed by the collaborative filtering algorithm and the collaborative depth model. As the number of music recommendations K increases, the precise values of the three models all show a downward trend. This is because the genre of songs can become extremely complex when the number of recommendations increases. The advantage of recommendation algorithms recommending similar music based on audio content begins to diminish. It cannot predict user preferences more comprehensively, resulting in a decrease in the accuracy of recommendations.

Figure 8 Average precision values of the three algorithms at different K values F1 indicator (see online version for colours)

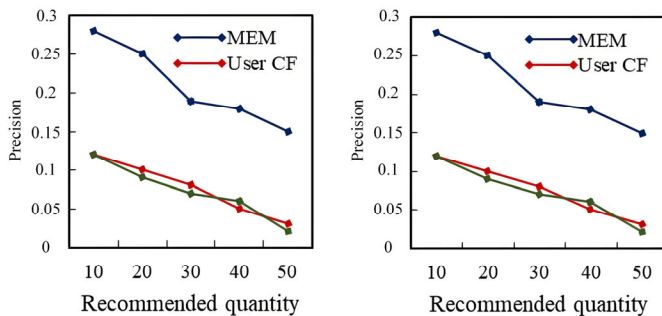
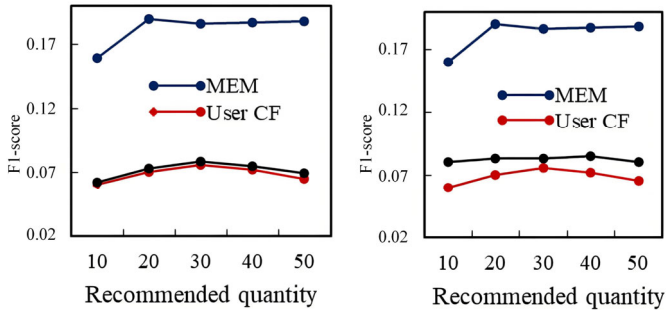


Figure 9 presents the comparison of the F1 metric of the three models in the two datasets. As can be seen from the figure, the F1 value of the MEM is still in the leading position among the three models. As the number of music recommendations K increases, the F1 values of the three models all show a trend of first rising and then falling. This is a normal phenomenon. As the value of K increases, the amount of data increases, and the processing of the algorithm is complicated. This also leads to an increase in coverage and a decrease in accuracy. When the recommendation system is used, if the number of

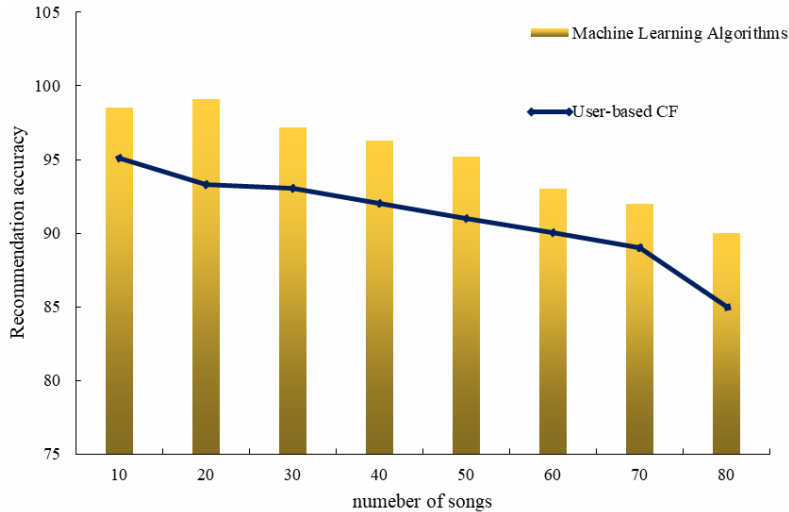
recommended songs is small, the user does not feel the effect of the recommendation system. If there are too many recommendations, users are dissatisfied with the quality of the recommendations. Therefore, the recommended number must be moderate, to achieve the best recommendation effect.

Figure 9 The average F1 value of the three algorithms under different K values



To verify whether the machine learning algorithm can achieve good results in the new media popular music recommendation system, this paper finally compares the recommendation accuracy of the machine learning algorithm (maximum entropy) and the user-based collaborative filtering algorithm under different song volumes, as shown in Figure 10.

Figure 10 The recommendation accuracy of the two algorithms under different song volumes (see online version for colours)



The column in the figure represents the machine learning algorithm, and the broken line represents the user-based collaborative filtering algorithm. As can be seen from Figure 10, as the number of songs increases, the music recommendation accuracy of both algorithms decreases. This is normal behaviour for recommender systems. As the number increases, the data processed by the algorithm also increases; the workload increases; and

the accuracy rate naturally goes down. However, in general, the recommendation accuracy of music recommendation systems based on machine learning algorithms has always been higher than that of other algorithms. When the number of recommended songs is 10, the accuracy rate of the machine learning-based MEM algorithm reaches 98.5%; the accuracy rate of the traditional user-based CF algorithm is 95.1%; the accuracy rate of MEM is increased by 3.4%. When the number of recommended songs is 90, the accuracy rate of the machine learning-based MEM algorithm reaches 90%; the accuracy rate of the traditional user-based CF algorithm is 85%; the accuracy rate of MEM is increased by 5%. The average accuracy is 7.5% higher, which also shows the feasibility of the recommender system.

The TP-300 and TP-500 datasets are private datasets constructed for this experiment, including user behaviour logs (such as playback, skipping, favourites, etc.) and music metadata (such as genre, rhythm, lyrics, and emotional tags). The sparsity of the user-song interaction matrix of the TP-300 dataset is 30%, and the sparsity of TP-500 is 50%. The specific sample sizes are $N = 100,000$ and $N = 150,000$, respectively. Table 2 shows the comparison of the accuracy of the MEM algorithm and the user-based CF algorithm on the TP-300 and TP-500 datasets. The specific calculation method is as follows:

Average accuracy calculation: the arithmetic mean is used, that is,

$$\text{MEM average accuracy} = \frac{98.0\% + 97.79\%}{2} = 97.9\% \quad (14)$$

$$\text{User-based CF average accuracy} = \frac{91.5\% + 90.8\%}{2} = 91.15\% \quad (15)$$

Increase percentage calculation: based on the absolute increase value, the formula is:

$$\text{Increase percentage} = \frac{97.95\% - 91.15\%}{91.15\%} \times 100\% \approx 7.5\% \quad (16)$$

Table 2 Comparison of the accuracy rate of the MEM algorithm and the user-based CF algorithm on the TP-300 and TP-500 datasets

Algorithm	TP-300(%)	TP-500(%)	Average accuracy (%)	Relative increase (%)	95% CI
User-based CF (baseline)	91.5 ± 1.2	90.8 ± 1.1	91.15 ± 1.15	—	—
MEM (proposed)	98.0 ± 0.8	97.9 ± 0.7	97.95 ± 0.75	7.46% (relative)	[6.1%, 8.8%]

The relative increase is calculated as $((97.95 - 91.15)/91.15) \times 100\% = 7.46\%$. The 95% CI for the improvement is [6.1%, 8.8%]. Statistical significance is confirmed via paired t-test ($p < 0.01$).

As shown in Table 2, the accuracy rate of the MEM algorithm on the TP-300 and TP-500 datasets is $98.0\% \pm 0.8\%$ and $97.9\% \pm 0.7\%$, respectively, which is significantly higher than the user-based CF baseline ($91.5\% \pm 1.2\%$ and $90.8\% \pm 1.1\%$, respectively). The average accuracy has been relatively improved by 7.46%, and this improvement is statistically significant ($p < 0.01$), with a 95% CI of [6.1%, 8.8%]. Especially in the

highly sparse TP-500 dataset, MEM is 6.1 percentage points higher than user-based CF, fully demonstrating its strong adaptability to data sparsity.

Next, this paper also evaluates the performance of the system in small-scale real-world user tests. The results are shown in Table 3.

Table 3 Performance analysis in small-scale real-user tests

<i>Index</i>	<i>MEM</i>	<i>User-based CF</i>
User click-through rate (CTR)	18.70%	12.30%
Recommended list retention rate	65.20%	53.10%
Average response time (seconds)	0.23	0.31

Table 3 shows that the MEM algorithm performs well in small-scale real-user tests, verifying its advantages in real-world scenarios. The user click-through rate of MEM reaches 18.7%, which is 52% higher than that of traditional user-based CF, and the recommended content is more in line with user preferences. The retention rate of the recommendation list is 65.2%, which is 23% higher than that of the control group, and users are more willing to accept and continue to browse the recommendation results. The average response time is only 0.23 seconds, which is 26% faster than traditional algorithms, demonstrating the efficiency of real-time recommendations. These data prove that MEM integrates user behaviour and music characteristics to improve recommendation relevance and user experience. In the future, the scale of testing can be expanded to further verify system stability and user satisfaction.

7 Application

To verify the effectiveness and performance of the music recommendation system based on the MEM, a prototype of the system is deployed on the local music platform, and 200 volunteers are invited for a two-week A/B test to comprehensively evaluate the system performance. The system adopts a modular design to improve expansion and real-time performance. It contains three core parts: feature cache, distributed computing, and cold start optimisation. The feature cache uses a Redis cluster to store the features of frequently accessed songs to reduce the burden on the database. The Apache Spark is used to process large-scale user logs and music data and realise real-time analysis and model training. The cold start module is pre-loaded with 30,000 song features, which enables new users to complete the first recommendation within 0.8 seconds. The system is deployed on a server with an 8-core CPU and 32 GB of memory and can support 100,000 concurrent users to ensure the timeliness of recommendations. The system structure is shown in Figure 11.

The front-end design of the recommendation system attaches importance to transparency and user-friendliness. The interface has a floating window of recommendation reasons to enhance users' understanding of the recommendation logic. When recommending songs, characteristic weights such as 'lyrics emotional matching degree 0.32' or 'rhythm matching degree 0.28' are displayed. Users can give instant feedback through favourites or 'skip' buttons, and this feedback is recorded and used to update the model. When new users register, a multi-dimensional label selector is provided to quickly build an initial user portrait, with an average of only three

interactions. Figure 12 shows the interface of a typical recommendation system, including metadata such as the cover, playback volume, and release time of the recommended song.

Figure 11 System structure diagram

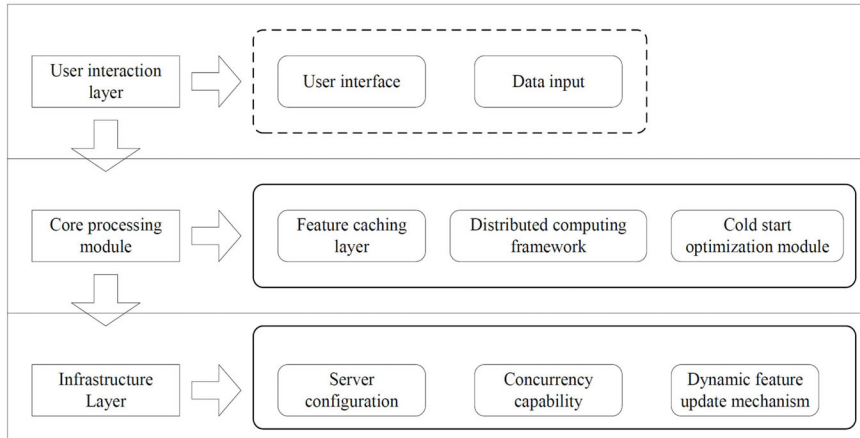
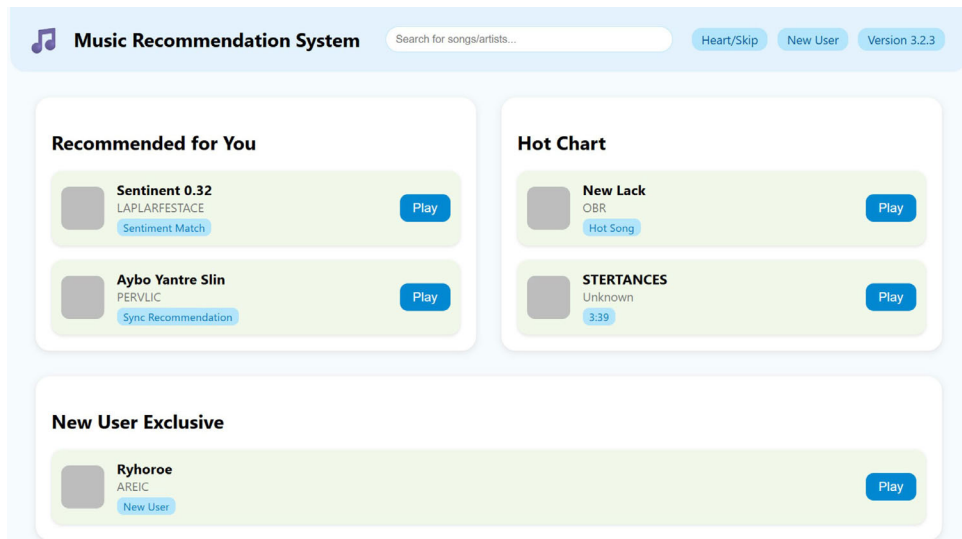


Figure 12 Recommended system interface (see online version for colours)



User feedback is essential to measure the value of the recommendation system. In the two-week test, the satisfaction of 200 volunteers with the system is significantly higher than that of the control group. Users believe that 'the reasons for recommendation increase the sense of trust' and 'the recommended content is more in line with personal preferences.' The visualisation of feature weights has significantly reduced the user complaint rate, indicating that the transparent design has increased user trust. In the cold start scenario, the relevance of the first recommendation by new users has been greatly

improved, which solves the problem of sparse data. Although the performance of small-scale testing is excellent, the extensibility in large-scale applications still needs to be verified. In the future, it plans to conduct larger-scale tests with partners and in-depth analysis of user preferences and optimise logic through real-time feedback. The system integrates multi-modal data to improve accuracy and diversity and develops an intuitive interpretation interface to enhance user understanding and trust.

8 Conclusions

The MEM proposed in this study integrates user behaviour and music content characteristics to achieve a recommended accuracy rate of $98.0\% \pm 0.8\%$ and $97.9\% \pm 0.7\%$ on the TP-300 and TP-500 datasets, respectively, which is 7.46% higher ($p < 0.01$) than User-based CF, and a 95% CI [6.1%, 8.8%] verifies the reliability of the results. In sparse scenarios (such as TP-500), MEM improves the recall value by 4.85% through the Laplace smoothing strategy, which is significantly better than the CDL model, and the response time is controlled within 0.23 seconds to meet the real-time requirements. User tests show that the click-through rate of MEM (CTR 18.7%) is 52% higher than the baseline, and the retention rate of the recommended list (65.2%) is 23% higher, verifying the practicality of the system. The limitations of the model include relying on high-quality metadata and high computational costs. In the future, the real-time feedback mechanism will be combined to optimise dynamic adaptability, and the feature weight visualisation interface will be developed to enhance user trust.

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Declarations

There are no potential competing interests in our paper. All authors have seen the manuscript and approved to submit to your journal. We confirm that the content of the manuscript has not been published or submitted for publication elsewhere.

Data availability statement

Data sharing does not apply to this article as no datasets were generated or analysed during the current study.

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Appendix

Click rate derivation

Let the probability distribution of the discrete random variable O be $M(O)$, and its entropy value $H(M)$ is:

$$H(M) = \sum_O M(O) I(O) \quad (A1)$$

In the formula, $I(O)$ represents the amount of information of O , usually

$I(O) = \log \frac{1}{M(O)}$ is taken, that is:

$$H(M) = \sum_O M(O) \log \frac{1}{M(O)} = - \sum_x M(O) \log M(O) \quad (A2)$$

The value of $H(M)$ satisfies the following inequality:

$$0 \leq H(M) \leq \log |O| \quad (A3)$$

In the formula, $|O|$ is the number of values of the random variable O .

Considering the conditions that the model should meet, given the training data set, the empirical distribution of the joint distribution $M(b|a)$ and the empirical distribution $M(O)$

of the marginal distribution can be determined, denoted by $\hat{M}(a, b)$ and $\hat{M}(a)$, respectively, namely

$$M(O = a, P = b) = \frac{c(O = a, P = b)}{N} \quad (\text{A4})$$

$$\hat{M}(O = a) = \frac{c(O = a)}{N} \quad (\text{A5})$$

Among them, $c(O = a, P = b)$ represents the frequency of the sample (a, b) in the training data, $c(O = a)$ represents the frequency of the input a in the training data, and N represents the training sample size.

A certain fact between the input a and the output b is described by the feature function $f(a, b)$. Its definition is:

$$f(a, b) = \begin{cases} 1, & a \text{ and } b \text{ satisfy a fact} \\ 0, & a \text{ and } b \text{ do not satisfy a fact} \end{cases} \quad (\text{A6})$$

It is a binary function that takes the value 1 when a and b satisfy this fact, and 0 otherwise.

$$E_{\hat{M}}(f) = \sum_{a,b} \hat{M}(a, b) f(a, b) \quad (\text{A7})$$

The expected value of the characteristic function $f(a, b)$ with respect to the model $M(b|a)$ and the empirical distribution $\hat{M}(a)$, denoted by $E_M(f)$.

$$E_M(f) = E_{\hat{M}}(a) P(b|a) f(a, b) \quad (\text{A8})$$

Assuming that the model has the ability to obtain the information in the training data, it is equivalent to:

$$E_M(f) = E_{\hat{M}}(f) \quad (\text{A9})$$

Or

$$\sum_{a,b} M(a) M(b|a) f(a, b) = \sum_{a,b} \hat{M}(a, b) f(a, b) \quad (\text{A10})$$

The above formula is used as a constraint condition for model learning. If there are n feature functions $f_i(a, b)$, $i = 1, 2, \dots, n$, then there are n constraints, so the model set that satisfies all the constraints is:

$$C \equiv \{M \in p | E_M(f_i) = E_{\hat{M}}(f_i), i = 1, 2, \dots, n\} \quad (\text{A11})$$

The conditional entropy defined on the conditional probability distribution $M(b|a)$ is:

$$H(M) = - \sum_{(a,b)} \hat{M}(a) M(b|a) \log M(b|a) \quad (\text{A12})$$

Then the largest model in the set C is the maximum entropy model, and the training of the maximum entropy model can be classified as a constrained optimisation problem, that is

$$\begin{aligned} \max_{M \in C} H(M) &= - \sum_{a,b} \hat{M}(a) M(b|a) \log M(b|a) \\ \text{s.t.} \quad E_M(f_i) &= E_{\hat{M}}(f_i), \quad i = 1, 2, \dots, n \\ \sum_b M(b|a) &= 1 \end{aligned} \quad (\text{A13})$$

The equivalent minimal problem is:

$$\begin{aligned} \min_{m \in C} -H(M) &= \sum_{a,b} \hat{M}(a) M(b|a) \log M(b|a) \\ \text{s.t.} \quad E_M(f_i) - E_{\hat{M}}(f_i) &= 0, \quad i = 1, 2, \dots, n \\ \sum_b M(b|a) &= 1 \end{aligned} \quad (\text{A14})$$

Lagrange multipliers are introduced to turn the above initial problem of constrained optimisation into a dual problem of unconstrained optimisation. Define the Lagrangian function $Q(s, t)$:

$$\begin{aligned} Q(s, t) &= -H(M) + t_0 \left(1 - \sum_b M(b|a) \right) + \sum_{i=1}^n t_i (E_M(f_i) - E_{\hat{M}}(f_i)) \\ &= \sum_{a,b} \hat{M}(a) M(b|a) \log M(b|a) + t_0 \left(1 - \sum_b M(b|a) \right) \\ &\quad + \sum_{i=1}^n t_i \left(\sum_{a,b} \hat{M}(a, b) f_i(a, b) - \sum_{a,b} \hat{M}(a) M(b|a) f_i(a, b) \right) \end{aligned} \quad (\text{A15})$$

The original optimisation problem is $\min_{M \in C} \max_t Q(s, t)$ and the dual problem is $\max_t \min_{M \in C} Q(s, t)$.

To solve the dual problem, we must solve the minimisation problem $\min_{M \in C} Q(s, t)$. Define the dual function $\psi(t)$ and its solution $M(t)$:

$$\begin{aligned} \psi(t) &= \min_{M \in C} Q(s, t) = Q(M_t, t) \\ M_t &= \arg \min_{M \in C} Q(s, t) = M_t(b|a) \end{aligned} \quad (\text{A16})$$

To solve for $M(t)$, solve for the partial derivative of $Q(s, t)$ with respect to $M(b|a)$:

$$\begin{aligned} \frac{\partial Q(s, t)}{\partial M(b|a)} &= \sum_{a,b} \hat{M}(a) (\log M(b|a) + 1) - \sum_b t_0 - \sum_{a,b} \left(\hat{M}(a) \sum_{i=1}^n t_i f_i(a, b) \right) \\ &= \sum_{a,b} \hat{M}(a) \left(\log M(b|a) + 1 - t_0 - \sum_{i=1}^n t_i f_i(a, b) \right) \end{aligned} \quad (\text{A17})$$

Let the above partial derivative be equal to 0, in the case of $\hat{M} > 0$, we can know:

$$\log M(b|a) + 1 - t_0 - \sum_{i=1}^n t_i f_i(a, b) = 0 \quad (\text{A18})$$

Namely

$$\log M(b|a) = \sum_{i=1}^n t_i f_i(a, b) + t_0 - 1 \quad (\text{A19})$$

Since

$$M_t(b|a) = \frac{1}{Z_t(a)} \exp \left(\sum_{i=1}^n t_i f_i(a, b) \right) \quad (\text{A20})$$

$$Z_t(a) = \sum_b \exp \left(\sum_{i=1}^n t_i f_i(a, b) \right) \quad (\text{A21})$$

$Z_t(a)$ is the normalisation factor, t_i is the weight corresponding to the feature function $f_i(a, b)$.

After solving the internal minima problem of the dual problem, solve the external maximisation problem, denoted as t^* , that is

$$t^* = \arg \max_t \psi(t) \quad (\text{A22})$$

The maximum value of $\psi(t)$ is obtained by unconstrained optimisation algorithm, and t^* is obtained, which is used to represent $M^* \in C$. Then $M^* = M_{t^*} = M_{t^*}(b|a)$ is the maximum entropy model.