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# A multi-objectives optimisation model for the joint design of statistical process control and engineering process control

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**Abstract:** Statistical process control and engineering process control are two methodologies used for process control and improvement. These technologies have existed independently of one another. Consequently, this research aims to simultaneously design statistical process control and engineering process control utilising multi-objectives optimisation. In this research, statistical and economic criteria are used to construct statistical process control and engineering process control jointly. To solve the developed model, an effective heuristic method is proposed. A numerical example is used to illustrate the significance of combining the two techniques. The results showed that the proposed solution could obtain the Pareto efficient solutions. This will help decision-makers to select the best solution based on their preferences. In addition, the findings indicated that the expected income values range between \$172.0839 and \$177.2175, and the Taguchi cost values vary between \$4.469333 and \$7.907547. On the other hand, the power values range between 0.91373 and 1. Moreover, the results revealed that as the Taguchi cost increases the expected income will increase and the power will decrease. Furthermore, sensitivity analysis is performed to determine the effect of variables in the model. The sensitivity analysis showed that the power of the chart decreases as the value of sigma is raised. [Received: 31 July 2023; Accepted: 26 November 2023]

**Keywords:** statistical process control; SPC; engineering process control; EPC; multi-objectives; control charts; process monitoring.

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## 1 Introduction

Statistical process control (SPC) is identified as one of the most efficient first-rate innovative procedures for increasing productivity for industrial processes. It was developed by Shewhart (1931) in the 1920s. Although SPC has been employed in discrete manufacturing environments to decrease error in the product/process, SPC processes do not prioritise real-time automatic outcome rectification. Therefore, engineers in system control utilised engineering process control to observe and correct errors in the process. The main purpose of statistical process control is to identify and remedy the assignable reasons that cause the process to fall out of control. This process can be applied in cases when the errors are independent and correlated. The SPC can be described as the use of statistical methods or tools to manage processes. These tools and tactics can assist in monitoring process behaviour, discovering troubles in interior systems, and finding proper solutions for production issues. Moreover, SPC employs statistical control charts for separating the assignable and common sources of variation. Control charts applications involve the use of several specialised and behavioural decision-making techniques which are exceptionally noteworthy within the program of SPC and designing this methodology plays a critical work in moving forward the firms' normal quality and efficiency.

Even though statistical process control is commonly employed in order to reduce process variability, it is not essentially the most optimal technique in various scenarios, especially the process mean drifts (Messina, 1992). To tackle these types of processes, engineering process control can be used efficiently. Engineering process control (EPC) is a general strategy for system optimisation and process improvement. It deals with the manufacturing process as an input-output system in which the input factors are adjusted to achieve the process target. The process output will be an estimation of the process factors and variables that require control. EPC is typically used in the control of continuous production processes to control the flexible factors and variables to keep the problem output near the target. It offers an instant response that counteracts changes in the balance of a process and implements corrective actions aiming to reduce the gap between the preferred target and the output. A possible approach is to predict the output variation from the target that would occur in case no control actions were considered and then act in order to eliminate this deviation. Proper feedback and feedforward control strategies can make process control actions possible by indicating when and by how much a process adjustment is needed.

SPC is primarily used to monitor and control processes to ensure consistent product or service quality. Organisations can improve product quality and customer satisfaction by continuously measuring and analysing process data. Furthermore, it aids in lowering the costs associated with defects and rework, as well as providing insights into process performance and assisting organisations in optimising their processes. Companies can reduce waste, improve resource allocation, and increase productivity by understanding how a process behaves statistically. EPC, on the other hand, seeks to maintain consistent and stable processes. This is critical in industries where even minor variations can have significant consequences, such as chemical manufacturing or semiconductor production. It focuses on optimising processes to achieve specific performance objectives. This includes improving efficiency, reducing cycle times, and increasing output. Furthermore, EPC is motivated by the desire to increase productivity and throughput. Organisations can produce more output with the same resources by carefully controlling and fine-tuning processes. As a result, combining both techniques is critical to achieving the best results.

The existing literature reveals a significant gap in the design of integrating statistical process control and engineering process control in the case of multi-objectives. The majority of the previous studies focused on the design of SPC and EPC integration in the single objective problems. The literature emphasises the critical need for advancements in the design of an integrated statistical process control and engineering process control in the case of multi-objectives. Addressing this gap will significantly contribute to more efficient and effective control strategies, ultimately leading to improved quality, cost savings, and overall process optimisation the industries. Consequently, the objective of this paper is to design statistical process control and engineering process control jointly using multi-objective optimisation. Specifically, the purpose of this research is to propose a multi-objective design model for an integration of integrated of this paper is to identify relevant objectives for the multi-objectives model, and SPC and EPC. In this model, three objective functions are developed. These functions include maximisation of the expected income, minimisation of the Taguchi cost, and minimisation of the power of control process. In addition, the proposed model is an NP-hard problem. This problem is a class of computational problems that are at least as hard as the hardest problems in NP (non-deterministic polynomial time). These problems are challenging to solve because they are not known to have polynomial-time algorithms (Papadimitriou and Steiglitz, 1982). Thus, this paper proposes an effective heuristic algorithm to solve the proposed model. Finally, sensitivity analysis is performed to identify the effect of the model parameters on the proposed model. In this research, the performance of the proposed model is evaluated using a numerical study.

The remainder of the paper is structured as follows: Section 2 presents the literature review followed by the problem statement in Section 3. Section 4 presents the proposed mathematical model followed by a numerical example in Section 5. The solution methodology outlined in Section 6, followed by results and analysis in Section 7. Finally, Section 8 summarises the finding the paper.

## 2 Literature review

The first use of single objective for the design of X-bar control chart proposed by Duncan (1956). Duncan used an economic model for considering a single assignable cause of the production process. Also, Lorenzen and Vance (1986) compared their model to Duncan (1956) with changing average run length (ARL) of in-control and out-of-control system instead of probabilities of types I and II. The authors developed different cost functions for the design of statistical and economic control charts. Moreover, Banerjee and Rahim (1988) proposed a model by considering a Weibull failure mechanism. Taguchi et al. (1989) developed the economic loss that occurs due to the deviation of quality characteristic represents approximately a quadratic function. Moreover, Saniga (1989) considered the economic design and statistical design addressing three objective functions. These objectives include minimisation out of control average time to signal, optimisation of the sample interval and sample size and control limits without considering the economic principles, maximisation of the average time to signal state the process in control. Evans and Emberton (1991) presented bi-criterion process control chart design based on multi-criteria decision making.

Jiang and Tsui (2000) established an economic model for automatic process control (APC) controlled process monitoring by the SPC. The author proposed an economic loss-based criterion, the average quality cost (AQC) criterion, for evaluating SPC chart for monitoring the mechanism managed by APC. It was found that the AQC criterion usually conforms to ARL criterion even if APC control behaviour balances the process change significantly. Furthermore, the AQC applies a separate weight proportionate to the average shift transitions to each run-length likelihood, providing more economic insights than the ARL.

Moreover, Montgomery (2001) displayed both the deviation and mean of the quality characteristic by using Shewhart's S and X-bar control charts widely. However, Chen and Yang (2002) investigated several possible causes of economic X-bar control charts with Weibull in the control state. Duffuaa et al. (2004) indicated that combining SPC with APC is beneficial for process control. Gulbay et al. (2004) presented the direct fuzzy approach, a new approach to fuzzy control charts based on fuzzy transformation algorithms. Linguistic information was converted into numerical values before the control limits are determined. Both control limits and sample values are thus numeric. Yu (2005) developed an economic model for variable sampling for continuous-flow process of X-bar control chart. Zarandi et al. (2007) considered fuzzy values for economic parameters and used genetic algorithm for optimising and apply adaptive neuro-fuzzy inference system to interpret the model. A numerical example is used to validate the proposed model. Vommi and Seetala (2007) presented a model to minimise the cost function using a new approach in the economic statistical design of X-bar control chart. Genetic algorithm is used to solve the proposed model.

Carolan et al. (2010) investigated the X-bar control chart's economic design, sampling intervals, and changeable and adaptive sample size. For sample intervals with continuously varying intervals, a two-stage approach was proposed. In the economic design, the findings of X-bar charts with discretely adjustable parameters are contrasted to those of X-bar charts with continuously changing sample intervals. It demonstrated that the proposed continuously variable sampling interval method is more cost-effective than that of the variable form. Sun and Wang (2010) proposed an integrated SPC and EPC economic design methodology, which analysed the effect of special cause and the economic design of the integration of SPC and APC based on quality statistical properties. The objective function was to minimise the operational cost. Saif et al. (2011) introduced a combined APC and SPC model for monitoring processes and optimisation with fuzzy logic interaction. Also, Akram (2011) proposed process improvement through the use of integrated statistical and automatic processes control approaches. The main focus of SPC was on process tracking, while APC's focus was on process change.

Safaei et al. (2012) generated the Pareto optimal solution of an X-bar control chart using the non-dominated sorting genetic algorithm-II (NSGA-II). In the economic design of the X-bar control chart, the authors combine intangible external costs and the Taguchi loss function. The results showed the superiority of the proposed method, since the detection capacity of the control chart can be enhanced by a small increase in the hourly expected cost. Park et al. (2012) investigated the average run length (ARL) which is determined by processes using APC and SPC. The findings indicated that ARL might not be suitable as it does not take into account the difference in run length. Yang et al. (2012) found the optimal design of S and X-bar control charts by using multi-objectives particle swarm optimisation algorithm (MOPSO).

Faraz and Saniga (2013) used genetic algorithm for optimising two objective functions for economic statistical control chart. Bashiri et al. (2013) investigated the economic-statistical design of a MEWMA control chart with multiple objectives. The objective functions include the likelihood of type I error, the power of the X-bar control chart, and the average time to signal (ATS). To obtain the Pareto optimal control chart design solutions, a multi-objective genetic algorithm was proposed. Bashiri et al. (2013) extended the work of Bashiri et al. (2013) by including ATS and cost function to the model. A multi-objectives genetic algorithm was used to get the optimal design of X-bar control chart with multiple assignable causes. Lupo (2014) considered random process shifts and Taguchi function to improve the multi-objectives design of X-bar control chart by applying mixed integer models. Moreover, Mobin et al. (2015) used NSGA-II to generate the Pareto optimal frontier of an X-bar control chart. The economic use of a control chart includes the design of the optimum parameters for the sample size, the frequency of sampling and the chart control limits. Morabi et al. (2015) proposed a design of X-bar chart using particle swarm algorithm to optimise a multi-objectives control charts with fuzzy process parameters. The objective functions include estimated hourly cost, average run length in control, and control chart detection capacity. The decision variables of the model are the frequency of sampling, control limits, and sample size. Safaei et al. (2015) designed control charts to track the service and production processes. The economic statistical design (ESD) of the X-bar control chart was discussed using a robust optimisation method that takes into account interval estimates of unknown parameters. To obtain the robust scheme of the control chart, a heuristic algorithm was developed. Numerical and simulation studies indicate that the proposed X-bar control chart provides practitioners with a stronger methodology and more accurate solutions. Moghaddam et al. (2016) presented an economic design for multi-objectives of aggregate count of adjustment control chart. Also, with an emphasis on physics-based methodologies, the authors proposed a multi-objective optimisation methodology for integrated radar-radiometer soil moisture estimate. Tavana et al. (2016) utilised two methods to overcome the fact that the method used for single-objective optimisation is not appropriate when a multi-objective optimisation problem is to be solved as it may contradict each other. The multi-objective particle swarm optimisation approach is used to tackle a variety of optimisation problems.

Asghar (2018) presented the modified average time to signal (AATS) control chart with a multi-objective economic statistical design of VSS; the estimated cost per hour as the economic objective and $T^2$ as the statistical objective. The authors proposed a multiple assignable cause for the shift process using multi-objectives genetic algorithm for economic design of control chart and $T^2$ control chart. According to Aslam et al. (2018), traditional statistics are used to supervise the process once the observations are identified from the sample or population. When there is uncertainty, neutrosophic statistics (NS) is used. This work is used to create a control chart that investigates failure-censored (type-II) reliability. This study also showed that the Weibull distribution is used when NS produces asymmetry.

John and Singhal (2019) devised a method that integrates statistical process control (SPC) with engineering process control (EPC). Design, methodology, and strategy – Pulp bleaching was a chemical process that removed cellulose from impurities found in cooked wood chips. Because the responses were naturally associated, dynamic regression was employed to model them. The best chemical dose that maximised both pulp brightness and viscosity was then estimated using a fuzzy optimisation technique. Imanian et al. (2020) applied control charts to the managed process output by using automated process control SPC and control charts were used to track and regulate the managed pressure drilling (MPD) process. The results demonstrated that applying SPC to the APC performance will increase confidence in controlling the process in the presence of random and special causes and under different operating conditions.

Katebi and Khoo (2021) presented an economic statistical model for the DSVSIT2 chart. The average number of false alarms (ANF) and adjusted average time to signal (AATS) criterion are used to assess statistical performance of the proposed approach. To cope with the AATS, the Wald's identity technique was used. The authors integrated T2 control charts of the double and variable sample interval (DSVSI) to generate an economic statistical design (ESD). Moon et al. (2022) developed a production system based on a fundamental economic-production framework, considering factors like carbon emissions, storage limitations, and unit production costs dependent on demand. The study found that production facilities with higher reliability produced fewer faulty products compared to less reliable ones. To find an approximate solution, the authors utilised a geometric programming method due to the inclusion of a power-function in the model.

García et al. (2022) demonstrated that it is vital to monitor the process by means of statistical methods in order to guarantee instability occurs in a process. The results indicated that applying statistical process control to observe and classify the unusual patterns is a solution to tack unusual patterns. Kumar (2022a) introduced an enhanced version of the traditional Elman neural network (ENN) and demonstrated its application in identifying the behaviour of complex systems with time delays in nonlinear plants. Interestingly, this model only requires two inputs: the delayed value of the system and the current value of an external signal, regardless of the system's order (which might often be unknown). To ensure stability, the MRENN model adjusts its parameters using equations derived from Lyapunov-stability criteria. Based on simulation results, the MRENN model consistently demonstrates superior performance compared to other established models. Hu et al. (2022) discussed the main problems that arise in one-sided EWMA charts when the sample exceeds the target level. The authors examined the efficiency of the modified one-side EWMA (MOEWMA) charts to observe a normally distributed process. The control chart's steady-state run length properties were controlled using Monte-Carlo simulation. Kumar (2022b) introduced a novel neural network architecture for the purpose of identifying nonlinear dynamical systems. The proposed model is derived from the traditional pi-sigma neural network (PSNN) and incorporates an extra layer recognised as the context layer, which consists of context nodes. Pi-sigma networks use a multiplication element in their final layer, which indirectly gives them the ability to function like higher-order networks and simultaneously simplifies their network structure. To adjust the weights of the CLRPSNN model, a learning procedure has been developed by integrating the back-propagation (BP) algorithm with the Lyapunov-stability method.

In sum, APC can more control individual process parameters, but it is unable to manage the distribution for those parameters. It also can not anticipate the controlled output process and the control state. Since SPC and APC have different strengths, SPC will acquire the ability to balance their weaknesses and carry more control effects from other strong points. Process control could be maintained by combining the SPC and APC systems. Intelligent techniques such as artificial neural networks, fuzzy control, artificial intelligence, and expert systems can be used depending on how the control object differs. The majority of the integration strategies described in the literature, however, employ APC approaches to process regulation and SPC strategies for monitoring. On the other hand, other derived SPC controllers based on the APC loop knowledge and applied their use has not been investigated. Previous studies have focused on integration of SPC and APC in the case of single objective. These studies used several methods and charts such as Shewart control chart, EWMA control chart, MMSE-controlled process, proportional-integral controller, fuzzy logic and Taguchi quality loss function. Table 1 summarises the previous studies related to the current work.

**Table 1** A summary of related works

| *Reference* | *Objective* | | | *Solution technique* | | *Model integration* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *Cost* | *Revenue* | *Other* | *Exact* | *Heuristic* | *SPC* | *EPC* | *SPC and EPC* | *SPC and EPC and TQE* | *Others* |
| Lorenzen and Vance (1986) | ✓ | | | ✓ | | | | | | ✓ |
| Banerjee and Rahim (1988) | ✓ | | | ✓ | | | | | | ✓ |
| Saniga (1989) | ✓ | | | ✓ | | | | | | ✓ |
| Jiang and Tsui (2000) | | | ✓ | ✓ | | ✓ | | | | |
| Duffuaa et al. (2004) | ✓ | | | ✓ | | | | | ✓ | |
| Gulbay et al. (2004) | ✓ | | | ✓ | | | | | | ✓ |
| Yu (2005) | ✓ | | | ✓ | | | | | | ✓ |
| Zarandi et al. (2007) | ✓ | | ✓ | ✓ | | | | | | ✓ |

**Table 1** A summary of related works (continued)

| *Reference* | *Objective* | | | *Solution technique* | | *Model integration* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *Cost* | *Revenue* | *Other* | *Exact* | *Heuristic* | *SPC* | *EPC* | *SPC and EPC* | *SPC and EPC and TQE* | *Others* |
| Vommi and Seetala (2007) | ✓ | | | ✓ | | | | | | ✓ |
| Sun and Wang (2010) | ✓ | | | ✓ | | | | ✓ | | |
| Carolan et al. (2010) | ✓ | | | ✓ | | | | | | ✓ |
| Saif et al. (2011) | | | ✓ | ✓ | | | | ✓ | | |
| Park et al. (2012) | ✓ | | | ✓ | | | | ✓ | | |
| Yang et al. (2012) | ✓ | | ✓ | | ✓ | | | | | ✓ |
| Faraz and Saniga (2013) | ✓ | | | | ✓ | | | | | ✓ |
| Bashiri et al. (2013) | ✓ | | ✓ | | ✓ | | | | | ✓ |
| Lupo (2014) | ✓ | | ✓ | ✓ | | ✓ | | | | |
| Morabi et al. (2015) | ✓ | | ✓ | | ✓ | | | | | ✓ |
| Safaei et al. (2015) | ✓ | | ✓ | | ✓ | | | | | ✓ |
| Moghaddam et al. (2016) | ✓ | | ✓ | ✓ | | | | | | ✓ |
| John and Singhal (2019) | ✓ | | ✓ | ✓ | | | | ✓ | | |
| Katebi and Khoo (2021) | ✓ | | | ✓ | | | | | | ✓ |
| García et al. (2022) | | | ✓ | ✓ | | | | | | ✓ |
| Our paper | ✓ | ✓ | ✓ | | ✓ | | | ✓ | | |

From Table 1, it is clear that there is a lack of the combination of engineering process control and statistical process control has not been considered in multi-objective optimisation. Consequently, the main purpose of this paper is to design statistical process control (SPC) and engineering Control (EPC) jointly using multi-objectives optimisation. Three objective functions are developed, namely; minimising the total quality cost per product, maximising the expected net income per hour, and maximising the detection power. To solve the developed model, a heuristic algorithm was proposed. The proposed algorithm is designed, coded, and tested properly. In this paper, a numerical example was used to validate the proposed model and evaluate the proposed algorithm.

## 3 Problem statement

This section presents the joint design of SPC and EPC. SPC and EPC have been identified as two different approaches used to assess quality improvement and process adjustments. These techniques, which have appeared independently of one another, are used widely across many industries. SPC investigates and eliminates the causes of variation, whereas EPC is frequently used to reduce variation by making regular online modifications to some inputs to the process. First, both techniques were believed to be antagonistic to one another, until it was found that the methods are complementary. SPC design entails determining sample size, sampling frequency, and control chart limits. The determination of the design parameters is made based on statistical or economic objectives. The statistical design of SPC takes into consideration the average run length (ARL), type I and type II errors. EPC was developed with the aims to control processes and make the quality characteristic remains closer to the target. In the design of EPC the following items are determined: The loss cost due to product characteristic ($A$), production measurement cost ($B$), production cost adjustment ($C$), production tolerance ($\Delta$), present production measurement interval ($n_0$), present production control limit ($D_0$). The aim of the economic design of EPC is to determine the optimal parameters that include the sampling frequency and the limits of the production control such that the total quality cost will be minimised, whereas the expected net income per hour and the detection power will be maximised. In the joint design of SPC and EPC, it is required to identify all relevant constraints as they relate to SPC and EPC design. The constraints will be used to develop the multi-objectives optimisation model together with an efficient algorithm to solve the model. The use of the model will be demonstrated by a realistic example.

## 4 The mathematical model

The joint design of SPC and EPC is developed in this section using a multi-objective optimisation model.

### 4.1 Model notation and assumption

Table 2 defines the acronyms used throughout the paper.

**Table 2** Notations

| *Parameters* | |
|---|---|
| $E(A)$ | The expected net income per hour. |
| $E(C)$ | The expected net cost per cycle. |
| $E(T)$ | The expected cycle length. |
| $W_t$ | Average time taken to find the assignable cause after a point has been found to fall outside the control chart limits. |
| $B$ | Cost per measurement of the production characteristics. |
| $C$ | Cost per adjustment. |
| $D$ | Adjustment or control limit. |

**Table 2** Notations (continued)

| | |
|---|---|
| *Parameters* | |
| $1/\lambda$ | The average time for occurrence of the assignable cause. |
| $\lambda$ | Assignable cause follows Poisson process with rate $\lambda$. |
| $V_0$ | Net income per hour of operation in the in-control state. |
| $V_1$ | Net income per hour of operation in the out-of-control state. |
| $a_1$ | Fixed component of sample cost. |
| $a_2$ | Variable components of sample cost. |
| $a_3$ | The cost of finding an assignable cause. |
| $a_3'$ | The cost of investigating a false alarm. |
| $a_4$ | The hourly penalty cost associated with production in the out of control state |
| $\alpha$ | The probability of false alarm (type-I error). |
| $\beta$ | The probability of type-II error. |
| $P$ | The probability that the assignable cause will be detected. P = =1-β |
| $g$ | A constant used to estimate the average time of sampling, inspection, evaluation and plotting for each sample ($gn$). |
| $\sigma$ | Standard deviation. |
| $\tau$ | The expected time of occurrence of the assignable cause within the interval between two samples. |
| $P(s)$ | The detection power of the economical control chart (after out of control status). |
| $p_L$ | The lower bound of the economical control chart detection power. |
| $\alpha_U$ | The upper bound of the type-I error. |
| $l$ | Time lag of measurement. |
| $\sigma_m^2$ | Measurement error. |
| $\Phi(z)$ | The probability density function of the standardised normal distribution. |
| $\delta$ | The number of standard deviations σ in the shift of process mean $\mu_0$. |
| $\Delta$ | Tolerance of the product characteristic. |
| $A$ | In-plant cost of reworking or scrapping a unit that falls outside of tolerance interval. |
| $\bar{u}$ | Predicted average number of product between successive adjustment. |
| $u_0$ | Current average number of products (unit)between successive adjustment. |
| $D_0$ | Current adjustment or control limit. |
| *Decision variables* | |
| $h$ | Interval between samples. |
| $k$ | The control limit within terms of standard deviation. |
| $n$ | Sample size. |

In this model development, the following assumptions are addressed:

1 the process starts in statistical control status

2 a fixed shift in the process can only be caused by one particular kind of assignable causes

3 the process that can be attributed has a Poisson process with a rate of $\lambda$, and the duration of control time has an exponential random variable with a rate of $1/\lambda$ hours

4 the sampling data has a normal distribution.

## *4.2 Multi-objective model for the design of the X-bar control chart*

In this paper, the process mean changes from $\mu_0$ to $\mu_0 + \delta + \sigma$, $\delta$ is known, when an assignable cause occurs. $W_t$ is the time it takes between detecting an out-of-control point and restoring the process to in-control status and even if there is a detection of an out-of-control point the process is not fixed. Four measurable performances can be obtained by considering the mentioned assumptions. Firstly, on account of the Poisson occurrence of the assignable cause, $\frac{1}{\lambda}$ is the expected time interval that the process remains in statistical control status. Secondly, equation (1) shows the error in each sampling [probability of false alarm, $\alpha(s)$]. While the process is in control state, it can be defined as the reciprocal of the ARL. Thirdly, equation (2) represents the conditioned expectation time of assignable cause that occurs between every two succeeding samples (provided the assignable cause takes place in the respective range). Moreover, the time interval between succeeding samples is denoted by $h$. Finally, equation (3) denotes the detection power $P(s)$ after the process enters in out-of-control status, for each sample. Where $\frac{1}{P}$ denotes the expected number of samples prior to investigating the mean shift and $\Phi(z)$ is the standard normal probability density function.

$$\alpha(s) = 2\int_{-\infty}^{-k} \Phi(z)dz \tag{1}$$

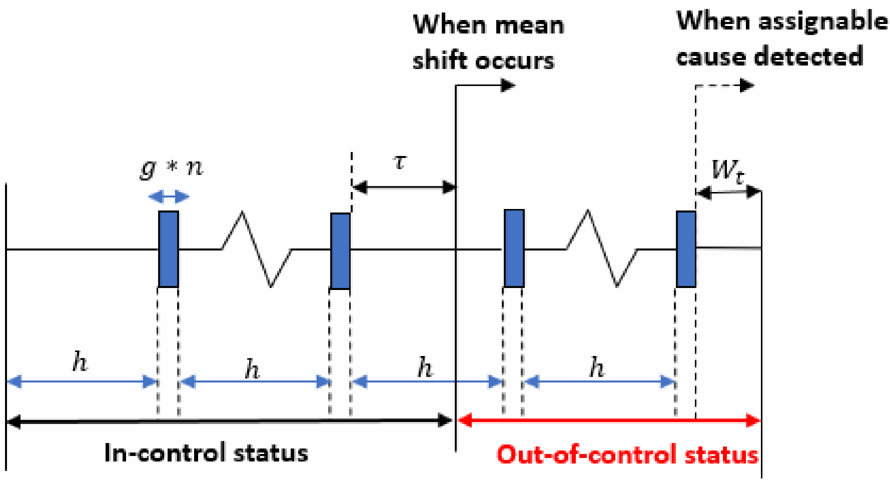
$$\tau = \frac{\int_{jh}^{(j+1)h} \lambda e^{-\lambda t}(t - jh)dt}{\int_{jh}^{(j+1)h} \lambda e^{-\lambda t} dt} = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(-e^{-\lambda h})} \tag{2}$$

$$p(s) = \int_{-\infty}^{-k-\delta\sqrt{n}} \Phi(z)dz + \int_{k-\delta\sqrt{n}}^{\infty} \Phi(z)dz \tag{3}$$

Various types of operating measures can be found in the review including the measures as shown above. For example, an estimate of the number of samples while the position of the process is in a state of statistical control. The estimate is evaluated to be $\frac{1}{\lambda h}$, and therefore the number of false alarms expected in this period can be derived by $\frac{\alpha(s)}{\lambda h}$. The analysis of the entire process cycle and both statuses: in-control and out-of-control are easily understood from Figure 1. $g$ is defined as the mean time elapsed, when every sample goes through inspection, evaluating, plotting and sampling. This constant $g$ is proportionate to $n$ (sample size). Hence $g \times n$ is known as the time delay in this phase. In Fig. 1, the time elapsed between each successive sample is denoted by h. The expected duration of occurring the assignable cause over the course of two samples is defined as $\tau$.

The duration of dealing with the assignable cause and returning the process to the status of in-control can be defined by a constant $W_t$.

**Figure 1** Single process cycle (see online version for colours)




## 4.3 Model objective functions

In this paper, three objective functions are addressed. The first objective is to minimise the total quality cost. The second objective is to maximise the expected net income per hour, and the third one is to maximise the detection power. These three objective functions are formulated as follows:

*Objective 1*

$$L = \frac{B}{n} + \frac{C}{\bar{u}} + \frac{A}{\Delta^2}\left[\frac{D^2}{3} + \frac{D^2}{\bar{u}}\left(\frac{n+1}{2} + l\right) + \sigma_m^2\right] \tag{4}$$

where the first term shows the diagnosis cost per product and the second term shows the adjustment cost per product, last term consists of the loss when process in-control and out of control, additional to that measurement error respectively.

*Objective 2*

$$E(A) = \frac{E(C)}{E(T)} = V_0 - \frac{(a_1 + a_2 n)}{h} - \frac{a_4\left(\frac{h}{1-\beta} - \tau + gn + W_t\right) + a_3 + \frac{a_3' \alpha e^{-\lambda h}}{1 - e^{-\lambda h}}}{\frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn + W_t} - Br - \frac{C}{D} \tag{5}$$

First term is the total revenue for each operating hour in the state of control $V_0$, the second term $(a_1 + a_2 n)$ represent the collection cost of a sample size n, the penalty cost of performing in an out-of-control condition for one hour is represented by the third term $a_4$,

$a_3$ is the cost of exploring an assignable cause, fourth term is the investigation cost of a false alarm is $a_3'$, fifth term $B$ represent the cost per measurement and last term represents the cost per adjustment over the control limit $D$.

*Objective 3*

$$p(s) = \int_{-\infty}^{-k-\delta\sqrt{n}} \Phi(z)dz + \int_{k-\delta\sqrt{n}}^{\infty} \Phi(z)dz \tag{6}$$

where $\Phi(z)$ is calculated using either the typical normal cumulative distribution function table or using below formula:

$$\Phi(z) = \int_{-\infty}^{Z} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du \tag{7}$$

### 4.4 Model constrains

In the proposed multi-objective model, the decision variables are sample size ($n$), the time interval between successive sample ($h$), the control- limit width in term of standard deviation $\sigma$ ($k$) and adjustment or control limit ($D$). The decision variables in the control chart constrains are $n$, $h$, $k$ and $D$ denote by $s = (n, h, k, D)$, complex multi-objective decision-making methods using computer algorithm can be applied to select the best compromise solution for the previous mathematical model.

In the multi-objective economic design of the X-bar control chart four decision variables ($n$, $h$, $k$, $D$) must satisfy also following context:

$$n_{\min} \le n \le n_{\max} \tag{8}$$

The sample size ($n$) range is restricted between two values one of them will be the minimum and other will be the maximum.

$$h_{\min} \le h \le h_{\max} \tag{9}$$

$h$ (the sampling time interval) has a range set between two values and the unit of $h$ is hour.

$$k_{\min} \le k \le k_{\max} \tag{10}$$

$k$ is the control limit width range with standard deviation $\sigma$.

$$D_{\min} \le D \le D_{\max} \tag{11}$$

Adjustment or control limit range will be between two values.

$$p(s) \ge P_L \tag{12}$$

Represents the lower bound of the detection power.

$$\alpha(s) \le \alpha_U, \qquad \forall s = (n, h, k, D) \tag{13}$$

Represents the upper bound on the type-I error

$\alpha_U$ the type-I error's upper bound.

$P_L$ the detecting power's lower bound

$p(s)$ shows the power of control chart and $\alpha$ type I error or the probability of false alarm.

$n \in \mathbb{Z}^+$ is discrete and $h$, $k$ and $D$ are continuous variable such as $h, k, D \in \mathbb{R}^+$.

### *4.4.1 Adjustment or control limit (parameter D)*

In case of a deviation of a quality characteristic from the target, then the process is adjusted to make the quality characteristic be closer to the target. In the USA (Taguchi, 1985) in case if there was a deviation from the quality target but non-defective produced then no actions were taken, they are mainly focusing in minimising the control cost. On the other hand, in Japan (Taguchi, 1985), they focus to minimise the deviation of the quality characteristic from the target. Generally, two factors lead to the optimal solution, which are the cost-emphasis and quality-emphasis methods. In feedback control system, measuring the quality characteristic of the products produced by a process. If the result of the measured quality characteristic differs from the target, then the process condition will be adjusted to make the quality characteristic close to the target. Adjustment and measurement of the process conditions is costly, and for the feedback control system there were a small number of factories using the quality control which is based on economic matter.

## 5 Numerical example

A numerical example is used to show the proposed model's applicability. This example was obtained from Montgomery (2009) and Taguchi et al. (1989). The fixed cost of collecting a sample is expected to be \$1 ($a_1$). The variable cost of sampling is expected to be \$0.10 per item ($a_2$), while measuring and recording the wall thickness of an item ($g$) takes approximately 1 minute (0.0167 h). The shift ($\delta$) is around two standard deviations in magnitude. Process shifts happen at random, roughly once every 20 hours of operation. For the period the process is under control, an exponential distribution with parameter $\lambda = 0.05$ is a realistic model. The average investigation time for an out-of-control signal is one hour ($W_t$). The investigation of an action signal that leads to the removal of an assignable cause costs \$25 ($a_3$), whereas the investigation of a false alarm costs \$50 ($a_3'$).

The manufacturer predicts that operating in an out-of-control state for one hour will cost \$100 ($a_4$) [Castillo et al., (1996), pp.467–474]. Loss due to a defective piece \$6 ($A$), measurement cost \$1.5 ($B$), adjustment cost \$12 ($C$), time lag is 3 unit ($l$), net income per hour of operation in the in-control state is 200 ($V_0$), observed average adjustment interval ($\mu_0$) 180 unit, current control limit 3 μm ($D_0$), tolerance limit 10 μm ($\Delta$). The amble size is between 2 and 20, the frequency is between 2 and 4, whereas adjustment or control limit is between 2 and 3. On the other hand, it is assumed that $\alpha(s) \le 0.05$ and $p(s) \ge 0.7$.

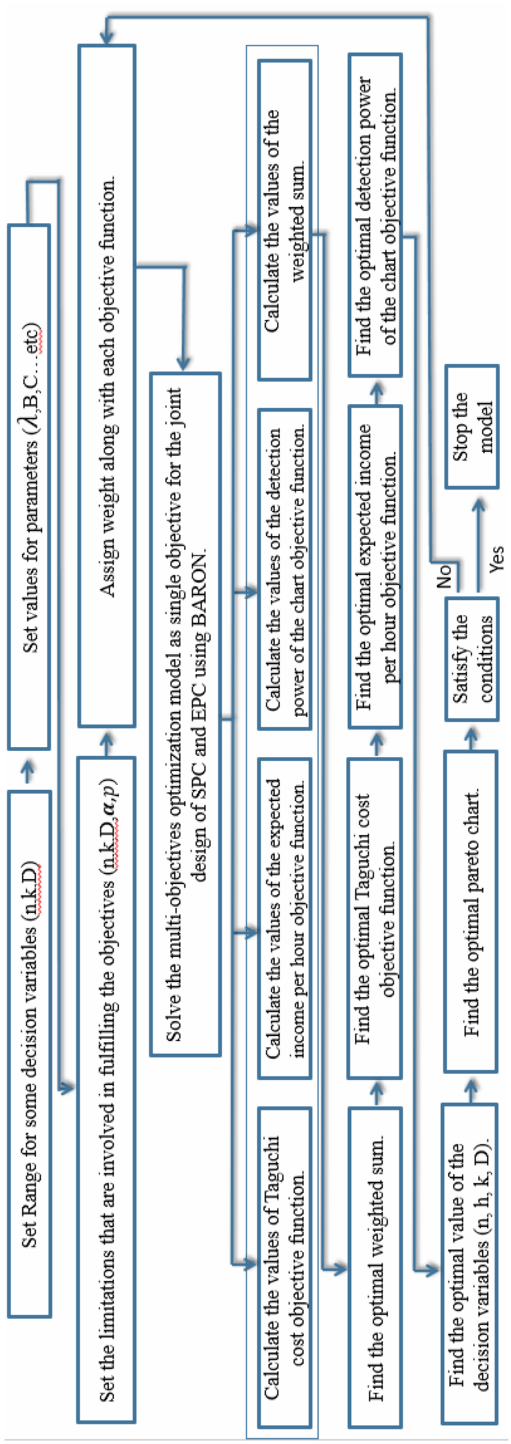
## 6 Solution methodology

In this paper, the developed model is NP hard model. Therefore, an exact algorithm is expected to take a long time to obtain the optimal solution. In this regard, an efficient algorithm has been proposed to solve the model. Taking advance of information the authors know about the characteristics of the problem and the ranges of the optimal solution, an exhaustive search algorithm is proposed to obtain a near optimal solution. The proposed algorithm combines an exact algorithm embedded in GAMS software with model constraint relaxation. It begins by establishing a range for each decision variable. The values of some decision variables, which represent complex terms, are then determined. These values must fall within the range of values for this variable. The implementation of this approach or strategy will contribute significantly to simplifying the complexity of the problem at hand. After that, the model will be solved using an exact algorithm to obtain the optimal solution. The proposed algorithm is capable of exploring all possible solutions and obtaining the Pareto optimal solutions. Figure 2 illustrates the steps of the proposed algorithm for solving the developed multi-objective optimisation model. It consists of the following main steps:

1. Set a range for the decision variables ($n$, $k$, $D$).
2. Set values for parameters ($\lambda$, $B$, $C$, …, etc.).
3. Set the limitations that are involved in fulfilling the objectives ($n$, $k$, $D$, $\alpha$, $p$).
4. Assign weight for each objective function.

   Note that the summation of all objective function weights must be equal to 1.
5. For the joint design of SPC and EPC, use BARON to solve the multi-objective optimisation model as a single objective.
   a. calculate the values of Taguchi cost objective function
   b. calculate the values of the expected income per hour objective function
   c. calculate the values of the detection power of the chart objective function
   d. calculate the values of the weighted sum.
6. Find the optimal weighted sum.
7. Find the optimal Taguchi cost objective function.
8. Find the optimal expected income per hour objective function.
9. Find the optimal detection power of the chart objective function.
10. Find the optimal value of the decision variables (n, h, k, D).
11. Find the optimal Pareto solution.

**Figure 2** The proposed solution algorithm (see online version for colours)



## 7 Results and analysis

The developed algorithm is used to solve the integrated multi-objective optimisation model for the integrating the design of SPC and EPC by using BARON solver embedded in GAMS software. As the problems in this model are factorable in the branch-and-reduce algorithm solver (BARON).

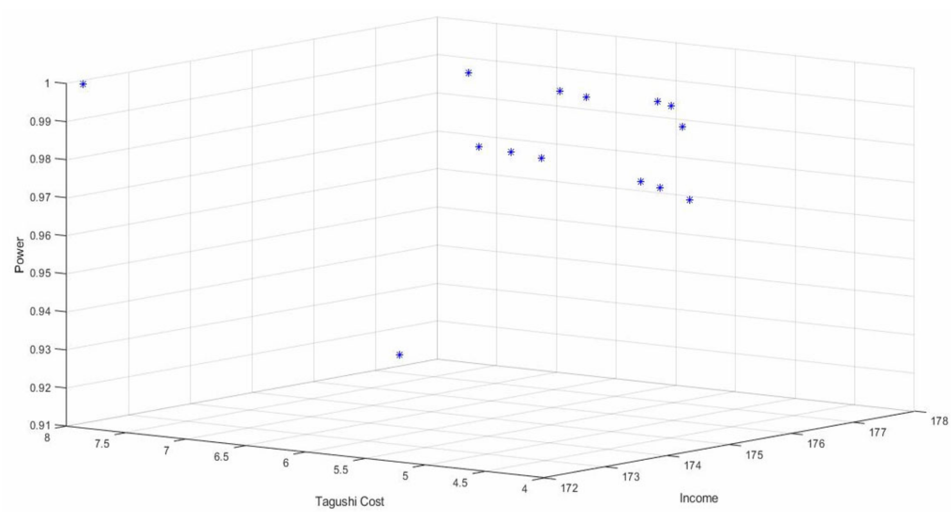
**Table 2** Weighted sum method results

| # | *Weights* | | | *Optimal obj* | *Taguchi cost* | *Income* | *Power* | *Variable* | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *W1* | *W2* | *W3* | | *obj1* | *obj2* | *obj3* | *N* | *K* | *D* | *h* |
| 1 | 0.1 | 0.8 | 0.1 | 0.10536 | 7.18868 | 177.1105 | 0.97128 | 4 | 2.1 | 3 | 0.55864 |
| 2 | 0.1 | 0.7 | 0.2 | 0.11285 | 7.18868 | 177.1105 | 0.97128 | 4 | 2.1 | 3 | 0.55865 |
| 3 | 0.1 | 0.6 | 0.3 | 0.12035 | 7.18868 | 177.1105 | 0.97128 | 4 | 2.1 | 3 | 0.55865 |
| 4 | 0.2 | 0.7 | 0.1 | 0.18134 | 6.84776 | 176.9725 | 0.97128 | 4 | 2.1 | 2.9 | 0.55865 |
| 5 | 0.3 | 0.6 | 0.1 | 0.21289 | 5.31651 | 176.1105 | 0.97128 | 4 | 2.1 | 2.4 | 0.55864 |
| 6 | 0.4 | 0.5 | 0.1 | 0.19480 | 4.53644 | 175.3961 | 0.97126 | 4 | 2.1 | 2.1 | 0.55864 |
| 7 | 0 | 1 | 0 | 0.00000 | 7.90755 | 177.2175 | 0.91373 | 3 | 2.1 | 3 | 0.52192 |
| 8 | 1 | 0 | 0 | 0.00001 | 4.46933 | 172.0839 | 0.99929 | 7 | 2.1–4 | 2.1 | 0.84769 |
| 9 | 0 | 0 | 1 | 0.00000 | 7.90755 | 172.0839 | 1 | 20 | 2.1 | 2.1–3 | 0.84769 |
| 10 | 0.8 | 0.1 | 0.1 | 0.04554 | 4.47384 | 174.9776 | 0.99744 | 6 | 2.1 | 2.1 | 0.60673 |
| 11 | 0.1 | 0.1 | 0.8 | 0.04996 | 4.46933 | 174.7496 | 0.99929 | 7 | 2.1 | 2.1 | 0.62689 |
| 12 | 0.7 | 0.1 | 0.2 | 0.04627 | 4.47384 | 174.9776 | 0.99744 | 6 | 2.1 | 2.1 | 0.60673 |
| 13 | 0.7 | 0.2 | 0.1 | 0.08629 | 4.49264 | 175.2004 | 0.99115 | 5 | 2.1 | 2.1 | 0.58480 |
| 14 | 0.2 | 0.1 | 0.7 | 0.04973 | 4.46933 | 174.7496 | 0.99929 | 7 | 2.1 | 2.1 | 0.62689 |
| 15 | 0.2 | 0.6 | 0.2 | 0.18426 | 6.51840 | 176.8247 | 0.97128 | 4 | 2.1 | 2.8 | 0.55865 |
| 16 | 0.6 | 0.3 | 0.1 | 0.12490 | 4.49264 | 175.2004 | 0.99115 | 5 | 2.1 | 2.1 | 0.58481 |
| 17 | 0.6 | 0.1 | 0.3 | 0.04699 | 4.47384 | 174.9776 | 0.99744 | 6 | 2.1 | 2.1 | 0.60673 |
| 18 | 0.6 | 0.2 | 0.2 | 0.08856 | 4.49264 | 175.2004 | 0.99115 | 5 | 2.1 | 2.1 | 0.58480 |
| 19 | 0.2 | 0.2 | 0.6 | 0.09266 | 4.47384 | 174.9776 | 0.99744 | 6 | 2.1 | 2.1 | 0.60673 |
| 20 | 0.3 | 0.1 | 0.6 | 0.04916 | 4.47384 | 174.9776 | 0.99744 | 6 | 2.1 | 2.1 | 0.60675 |
| 21 | 0.1 | 0.3 | 0.6 | 0.11104 | 6.17717 | 176.4702 | 0.99116 | 5 | 2.1 | 2.7 | 0.58481 |
| 22 | 0.9 | 0.05 | 0.05 | 0.02343 | 4.47384 | 174.9776 | 0.99744 | 6 | 2.1 | 2.1 | 0.60675 |
| 23 | 0.05 | 0.05 | 0.9 | 0.02616 | 4.46933 | 174.7496 | 0.99929 | 7 | 2.1 | 2.1 | 0.62689 |
| 24 | 0.5 | 0.3 | 0.2 | 0.12717 | 4.49264 | 175.2004 | 0.99115 | 5 | 2.1 | 2.1 | 0.58481 |
| 25 | 0.5 | 0.2 | 0.3 | 0.09049 | 4.47384 | 174.9776 | 0.99744 | 6 | 2.1 | 2.1 | 0.60673 |
| 26 | 0.5 | 0.4 | 0.1 | 0.16127 | 4.53644 | 175.3961 | 0.97126 | 4 | 2.1 | 2.1 | 0.55864 |
| 27 | 0.5 | 0.1 | 0.4 | 0.04771 | 4.47384 | 174.9776 | 0.99744 | 6 | 2.1 | 2.1 | 0.60675 |
| 28 | 0.3 | 0.5 | 0.2 | 0.19839 | 5.04493 | 175.8931 | 0.97128 | 4 | 2.1 | 2.3 | 0.55864 |
| 29 | 0.2 | 0.5 | 0.3 | 0.17960 | 5.86717 | 176.2993 | 0.99116 | 5 | 2.1 | 2.6 | 0.58481 |
| 30 | 0.1 | 0.5 | 0.4 | 0.12006 | 7.17736 | 176.9147 | 0.99116 | 5 | 2.1 | 3 | 0.58481 |
| 31 | 0.4 | 0.1 | 0.5 | 0.04844 | 4.47384 | 174.9776 | 0.99744 | 6 | 2.1 | 2.1 | 0.60675 |
| 32 | 0.1 | 0.4 | 0.5 | 0.11711 | 7.17736 | 176.9147 | 0.99116 | 5 | 2.1 | 3 | 0.58481 |
| 33 | 0.3 | 0.2 | 0.5 | 0.09194 | 4.47384 | 174.9776 | 0.99744 | 6 | 2.1 | 2.1 | 0.60673 |

**Table 3** Domination check

| # | *Weights* | | | *Optimal obj* | *Taguchi cost* | *Income* | *Power* | *Variable* | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *W1* | *W2* | *W3* | | *obj1* | *obj2* | *obj3* | *N* | *K* | *D* | *H* |
| 1 | 0.1 | 0.8 | 0.1 | 0.10536 | 7.18868 | 177.1105 | 0.97128 | 4 | 2.1 | 3 | 0.55864 |
| 2 | 0.2 | 0.7 | 0.1 | 0.18134 | 6.84776 | 176.9725 | 0.97128 | 4 | 2.1 | 2.9 | 0.55865 |
| 3 | 0.3 | 0.6 | 0.1 | 0.21289 | 5.31651 | 176.1105 | 0.97128 | 4 | 2.1 | 2.4 | 0.55864 |
| 4 | 0.4 | 0.5 | 0.1 | 0.19480 | 4.53644 | 175.3961 | 0.97126 | 4 | 2.1 | 2.1 | 0.55864 |
| 5 | 0 | 1 | 0 | 0.00000 | 7.90755 | 177.2175 | 0.91373 | 3 | 2.1 | 3 | 0.52192 |
| 6 | 0 | 0 | 1 | 0.00000 | 7.90755 | 172.0839 | 1 | 20 | 2.1 | 2.1-3 | 0.84769 |
| 7 | 0.8 | 0.1 | 0.1 | 0.04554 | 4.47384 | 174.9776 | 0.99744 | 6 | 2.1 | 2.1 | 0.60673 |
| 8 | 0.1 | 0.1 | 0.8 | 0.04996 | 4.46933 | 174.7496 | 0.99929 | 7 | 2.1 | 2.1 | 0.62689 |
| 9 | 0.7 | 0.2 | 0.1 | 0.08629 | 4.49264 | 175.2004 | 0.99115 | 5 | 2.1 | 2.1 | 0.58480 |
| 10 | 0.2 | 0.6 | 0.2 | 0.18426 | 6.51840 | 176.8247 | 0.97128 | 4 | 2.1 | 2.8 | 0.55865 |
| 11 | 0.1 | 0.3 | 0.6 | 0.11104 | 6.17717 | 176.4702 | 0.99116 | 5 | 2.1 | 2.7 | 0.58481 |
| 12 | 0.3 | 0.5 | 0.2 | 0.19839 | 5.04493 | 175.8931 | 0.97128 | 4 | 2.1 | 2.3 | 0.55864 |
| 13 | 0.2 | 0.5 | 0.3 | 0.17960 | 5.86717 | 176.2993 | 0.99116 | 5 | 2.1 | 2.6 | 0.58481 |
| 14 | 0.1 | 0.4 | 0.5 | 0.11711 | 7.17736 | 176.9147 | 0.99116 | 5 | 2.1 | 3 | 0.58481 |

**Figure 3** Optimal points of the objectives function (see online version for colours)




Different weights are used for each objective to find the optimal point for each weight. Basically, when the weight is close to 1, the optimal result for the respective objective and when it is close to zero it will give the worst result. A total of 33 experiments were tried for different weights for the objectives functions in Table 2 and result at the end with the Pareto optimal points obtained by performing a domination check as shown in Table 3. It can be observed from the results in Table 2 that the optimal value of the weighed sum is between 0 and 1. The value of interval between samples ($h$) is between 0.5 and 0.85. While the power objective function values are between 0.913732 and 1. However, the values of the expected income objective function fall between 172.0839 and 177.2175. Also, the values of the Taguchi cost objective function are in the range of 4.469333 and 7.907547. These observations can also see from the Figure 3. The comparison between detection power objective function and the expected net income objective function is shown in the Figure 4, while Figure 5 describes the relation between detection power objective function and Taguchi cost objective function. Moreover, Figure 6 illustrate the relation of Taguchi cost with the expected net income objectives functions vary in proportion. Figure 6 shows that both Taguchi cost and proportional to the net income objective function and after the expected net income exceeds 176 then it increases rapidly.

**Figure 4** Power vs. income (see online version for colours)

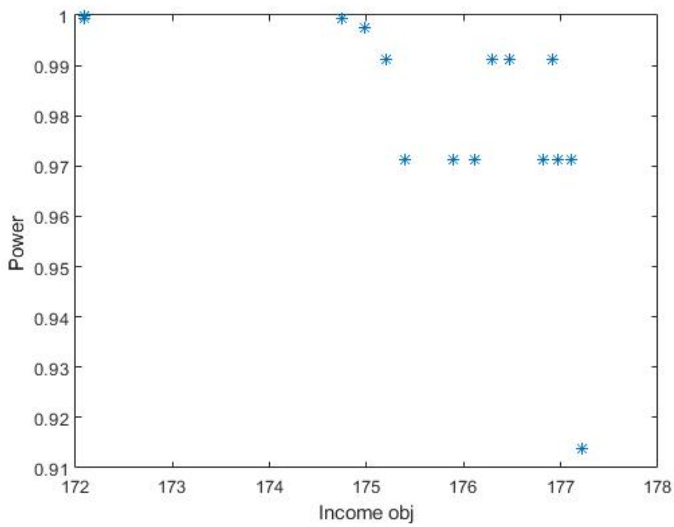
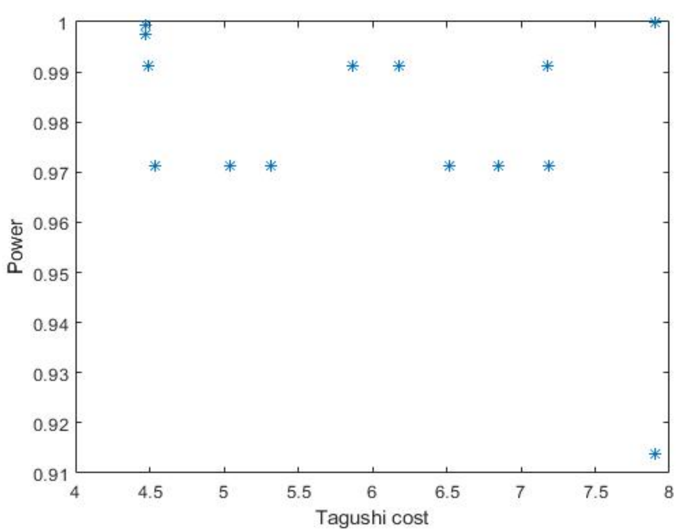



**Figure 5** Power vs. Taguchi cost (see online version for colours)



Alpha has no proportional relation with the expected income per hour (objective function) and the value of interval between samples ($h$), such that when alpha decrease then the expected income per hour will increase and the value of interval between samples ($h$) increase. However, when beta decrease then the power of chart will increase.

**Figure 6** Taguchi cost vs. income (see online version for colours)

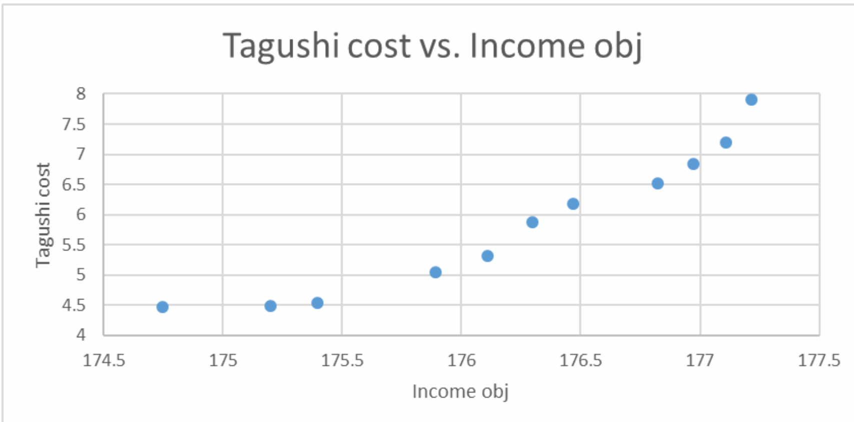
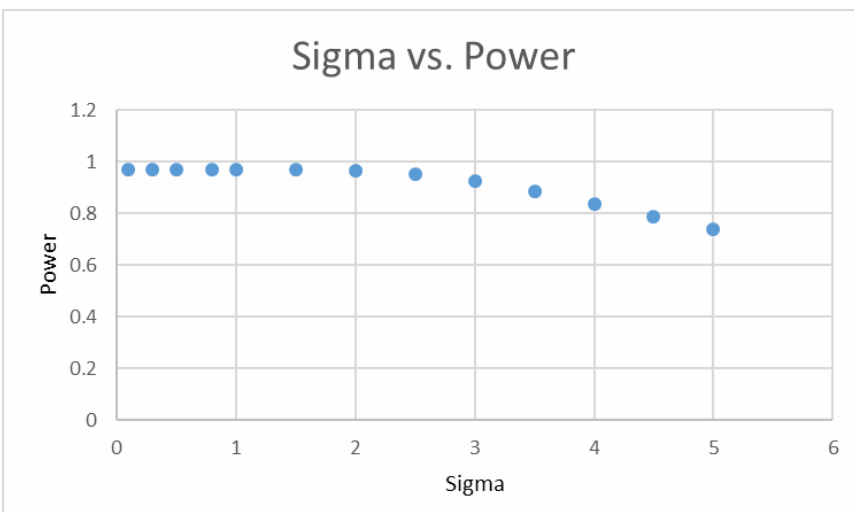



**Figure 7** Sigma vs. power (see online version for colours)




## 7.1 *Sensitivity analysis*

In this section, we will analyse the parameters and see how it will affect the objectives. Considering the impact of sigma (standard deviation) on the power of the chart. As it can be seen in Figure 7, as the value of sigma increases the value of the power is reduced. This is because sigma is inversely proportional to power. In addition, it is clear how Taguchi cost is affected when changing in-plant cost as well as the measurement cost as shown in Figures 8 and 9. It can be seen as in-plant cost and measurement cost are increased Taguchi cost function is increased as well. This is understandable as in-plant cost (cost of rework or scrap) falls outside the tolerance unit and when that is increased it will increase total cost function. Furthermore, when the cost per measurement of the product characteristic increases then the total cost function will increase because the cost of measurement is part of the total cost function. Finally, as it is shown in Figures 10 and 11, the expected net income objective function is affected by the penalty cost and the required average time to find the assignable cost. It can be realised as the expected net income objective function decreases as the average time for determining the assignable and penalty costs increase.

**Figure 8** In-plant cost vs. Taguchi-cost (see online version for colours)

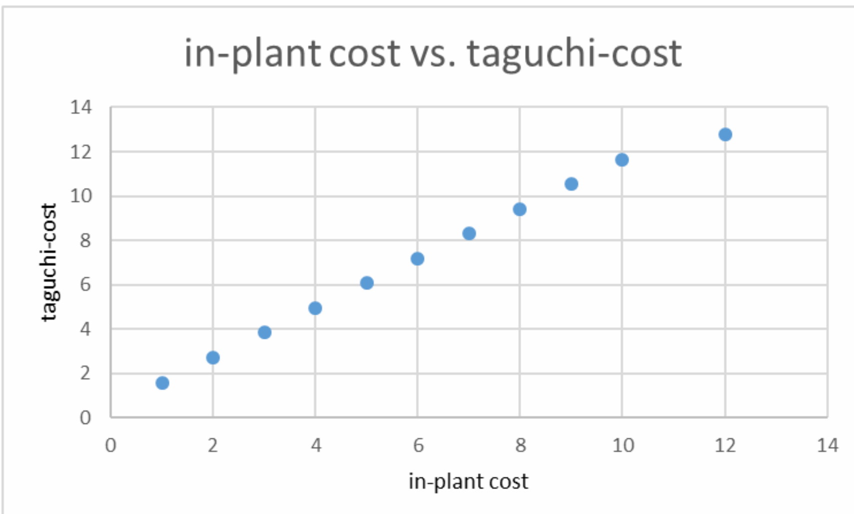



**Figure 9** Measurement cost vs. Taguchi-cost (see online version for colours)

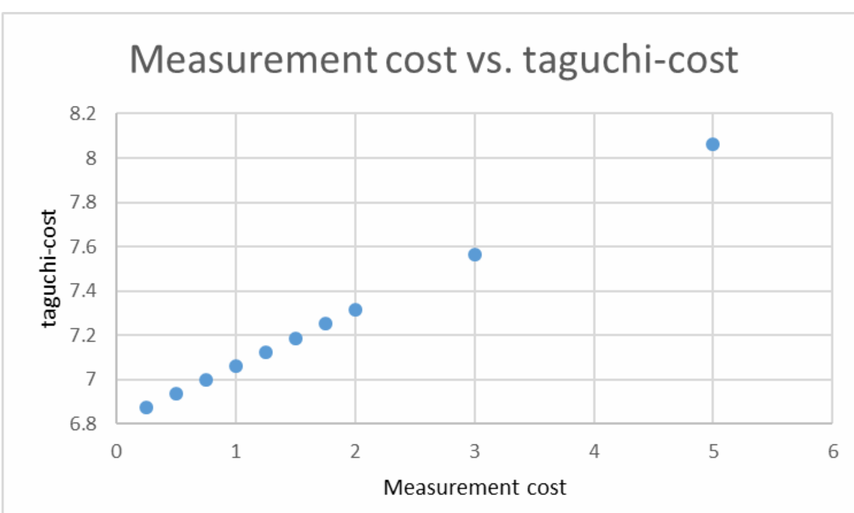


**Figure 10** Average time to find assignable cause vs. Income (see online version for colours)

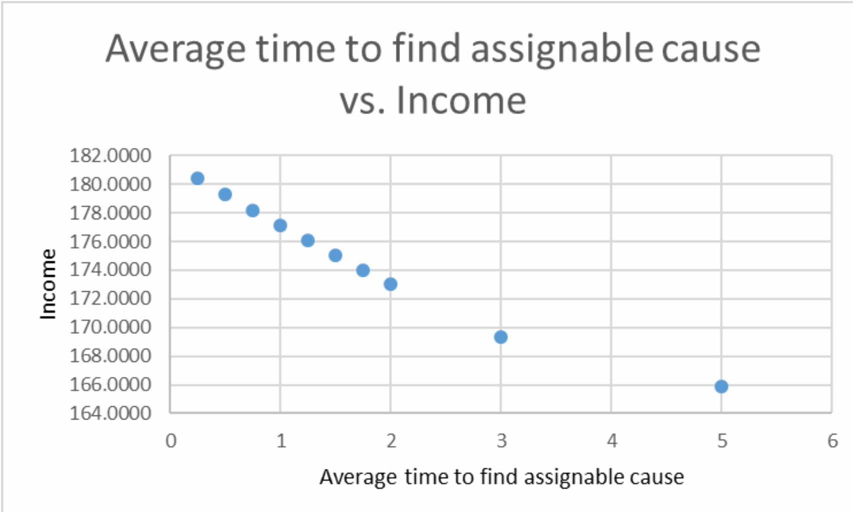
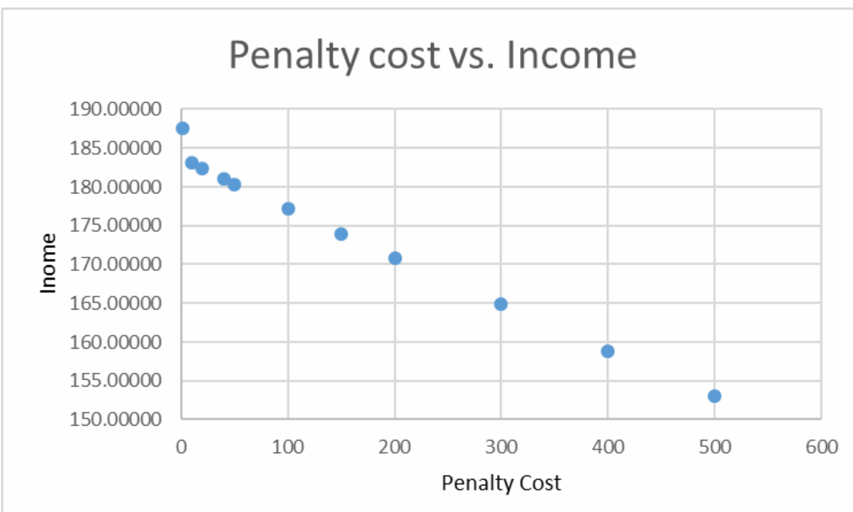



**Figure 11** Penalty cost vs. income (see online version for colours)




## 8 Conclusions

This paper proposed a hybrid process control model integrating SPC and EPC via a multi-objectives optimisation. The model combines the power of SPC and EPC to detect out of control states and control processes. A numerical example is provided to clarify in the study to clarify the application of the integrated two process control techniques. The results showed that the proposed algorithm is able to obtain the Pareto efficient solutions for the muri-objective model. In addition, the findings indicated that the expected income values range between $172.0839 and $177.2175, and the Taguchi cost values vary between $4.469333 and $7.907547. On the other hand, the power values range between 0.91373 and 1. Moreover, the results revealed that as the Taguchi cost increases the expected income will increase and the power will decrease. Furthermore, sensitivity analysis is performed to determine the effect of variables in the model. The sensitivity analysis showed that the power of the chart decreases as the value of the process variance increases. To assess the performance of the designed integrated process control techniques sensitivity analysis is conducted. The algorithm showed that the designed technique is robust.

This study can be expanded by introducing different objective functions to cover more aspects of quality. To accomplish this, the algorithm proposed in this study may need to be refined further. One viable method for improving the algorithm is to use meta-heuristic techniques to achieve global solutions. Furthermore, conducting real-world case studies and exploring its practical applications could improve the model's applicability and effectiveness. Another possibility for expanding the model is to address uncertainties within the multi-objective framework, such as scenarios in which the costs associated with type I and II errors are unknown or uncertain.

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