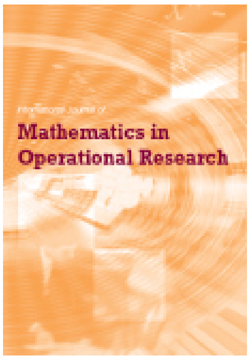

International Journal of Mathematics in Operational Research

ISSN online: 1757-5869 - ISSN print: 1757-5850
https://www.inderscience.com/ijmor

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DOI: 10.1504/IJMOR.2023.10059280

Article History:

Received:	24 June 2023
Last revised:	28 June 2023
Accepted:	14 July 2023
Published online:	19 February 2025

Fuzzy weak autocatalytic set for decision making and its application to the project manager selection

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Abstract: In decision-making procedures, the comparison of alternatives is often necessary to determine the most suitable option among a set of choices. One commonly used approach is pairwise comparison, where each alternative is compared to every other alternative based on certain criteria. FWACS method offers a way to represent the pairwise comparisons of alternatives as a directed graph with fuzzy edges. The FWACS is a technique used to address problems involving multiple criteria. In this study, a new ranking approach using FWACS is implemented on a previously published MCDM problem. The purpose of this implementation is to showcase the effectiveness of the FWACS method in addressing decision problems that involve multiple criteria.

Keywords: fuzzy weak autocatalytic set; FWACS; decision making; project manager selection; ranking; graph theory.

Reference to this paper should be made as follows: Mamat, S.S., Awang, S.R. and Ahmad, T. (2025) 'Fuzzy weak autocatalytic set for decision making and its application to the project manager selection', *Int. J. Mathematics in Operational Research*, Vol. 30, No. 1, pp.63–80.

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1 Introduction

When making decisions, it is common to compare various alternatives using pairwise comparisons. Each alternative is evaluated and the best one is selected. There are several decision making tools, such as simple additive method (SAW), the analytic hierarchy process (AHP), Viekriterijumsko Kompromisno Rangiranje (VIKOR), or the multi-criteria optimisation and compromise solution, the technique for the order of preference by similarity to an ideal solution (TOPSIS), preference ranking organisation method for enrichment of evaluations (PROMETHEE), elimination and choice translating reality (ELECTRE) and potential method (PM) (Sałabun et al., 2020; Taherdoost and Madanchian, 2023; French, 2023). Another technique that can be used to solve decision making problems is the fuzzy weak autocatalytic set (FWACS).

The origin of the FWACS theory can be traced back to a weak version of the autocatalytic set (ACS). Initially, ACS was a term used to describe a product or compound that accelerates a chemical reaction. It involves the use of a foreign substance, which speeds up the slow chemical reaction without being consumed by it. In general, an ACS refers to a group of entities, which can include people, molecules, or objects. Later, in 1998, Jain and Krishna presented the ACS as a directed graph (Ashaari et al., 2021).

A weak autocatalytic set (WACS) represents a weak form of ACS. A WACS is described as a non-loop subgraph that contains a vertex without any incoming link (Mamat et al., 2019). In other words, a WACS has a source vertex. As this means that an uncertainty may happen in a WACS, a new concept, FWACS, is thus created. Mamat et al. (2019) developed a new ranking algorithm for decision making purposes by integrating FWACS and PM.

This paper showcases the use of FWACS in multi-criteria decision making (MCDM) problem by implementing it in the selection of an aircraft, which is based on a problem presented by Torfi and Rashidi (2011). The paper is structured as follows: Section 2 introduces fundamental concepts and definitions that are necessary for this study. This section also explains the fuzzification of WACS, known as FWACS, and its algorithm. Section 3 outlines the ranking procedure using FWACS. The conclusions are presented in Section 4.

2 Preliminaries

This section elaborates on the different structures of WACS and FWACS, and also provides an overview of the ranking technique using FWACS.

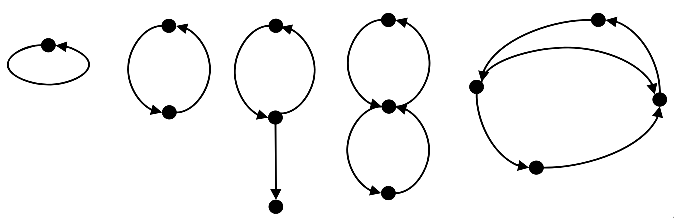
2.1 Autocatalytic set

The concept of autocatalysis was initially introduced in the field of chemistry to describe the catalytic interaction between molecules. Later, in 1998, Jain and Krishna formalised the concept of an ACS as a directed graph, with its vertices representing species and its edges representing catalytic interactions between them (Hordijk, 2013). The formal definition of an ACS is given as follows.

Definition 1 (Jain and Krishna, 1998, 1999): An ACS is a subgraph, each of whose vertices has at least one incoming link from vertices belonging to the same subgraph.

Some examples of ACSs are illustrated in Figure 1. The simplest ACS is a vertex with one-cycle.

Figure 1 Some examples of ACS



2.2 Weak autocatalytic set

The emergence of a weak form of ACS has led to the development of a new graph structure called WACS. The definition of WACS is given in Definition 2 and illustrated in Figure 2.

Definition 2 (Mamat et al., 2020): A WACS is a non-loop subgraph which contains a vertex with no incoming link.

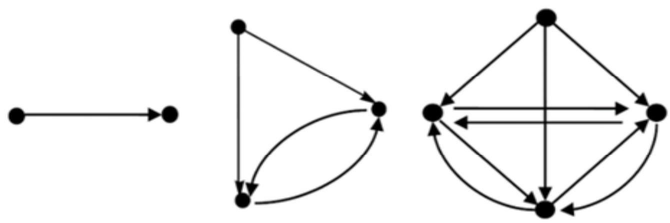
In their study, Mamat et al. (2020) established the properties of a WACS, as follows.

Theorem 1: Every WACS is a weakly connected graph.

Theorem 2: Every WACS must have at least one path that is not closed.

Additionally, a fuzzy graph is a representation of a crisp graph (Koam et al., 2020), and Mamat et al. (2020) proved that WACS is a special case of a fuzzy graph with the following theorem.

Figure 2 Some examples of WACS



Theorem 3: Every WACS is a fuzzy graph.

Furthermore, the introduction of fuzziness or uncertainties into WACS is discussed in the subsequent subsection.

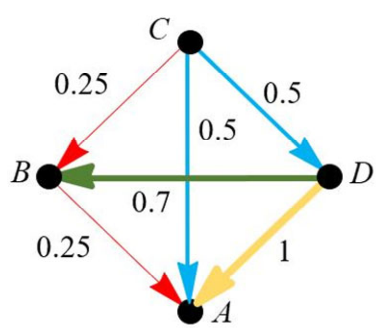
2.3 Fuzzy weak autocatalytic set

Mamat et al. (2020) introduced the concept of FWACS by integrating a fuzzy graph with a WACS. The formal definition of FACS is laid as follows.

Definition 3 (Mamat et al., 2020): A FWACS is a WACS in which every edge e_1 has a membership value, $\mu(e_i) \in [0, 1]$ for $e_i \in E$.

An example of a FWACS is shown in Figure 3, where the source vertex is represented by point *C*.

Figure 3 A FWACS (see online version for colours)



The membership values assigned to the edges of a FWACS determine their strengths, where higher values indicate stronger connections between vertices. As a result, the thickness and colour of an edge can be used to represent the strength of its connection. In their paper, Mamat et al. (2020) demonstrated that a WACS is a special case of fuzzy graph.

Theorem 4: Every FWACS is a fuzzy graph.

Therefore, it can be deduced to Theorems 5 and 6.

Theorem 5: Every WACS is a FWACS.

Theorem 6: Every FWACS is also a fuzzy graph.

2.4 Ranking by FWACS

This section outlines an algorithm for ranking using FWACS, with the input being the membership values of edges obtained through a pairwise comparison (Mamat et al., 2019). These membership values are used as entries for a $m \times 1$ matrix, denoted as F_μ. The orientation of edges is then listed in an incidence matrix, A. The procedure for ranking with FWACS is outlined as follows:

Step 1 Build a FWACS, $G = (V, F_\mu)$ for a given problem and determine the membership values for its edges. Set V as a set of vertices and F_μ is the fuzzy flow matrix, which represents the membership value of edges.

Step 2 Construct the incidence matrix, A. A $m \times n$ incidence matrix is given by

$$A_{\alpha,v} = \begin{cases} -1, & \text{if the edge } \alpha \text{ leaves } v \\ 1, & \text{if the edge } \alpha \text{ enter } v \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Step 3 Define Laplacian matrix, L. The Laplacian matrix $L = A^T A$ is with entries defined as

$$L_{i,j} = \begin{cases} -1, & \text{if there is an edge } (i, j) \text{ or } (j, i) \\ \deg(i), & \text{if } i = j \\ 0, & \text{else} \end{cases} \tag{2}$$

such that $\deg(i)$ is the degree of vertex i.

Step 4 Generate flow difference, D_μ. Let the flow difference be $D_\mu = A^T F$. The component of D_μ is determined as below.

$$\begin{aligned} D_\mu &= \sum\nolimits_{\alpha=1}^{m} A_{v.\alpha}^T F_\alpha \\ &= \sum\nolimits_{\alpha \text{ enters } v} F_\alpha - \sum\nolimits_{\alpha \text{ leaves } v} F_\alpha \end{aligned} \tag{3}$$

whereby D_μ is the difference between the total flow which enters v and the total flow which leaves v.

Step 5 Determine potential X. The potential X is a solution of the Laplacian system

$$LX = D_\mu \tag{4}$$

such that $\sum X_V = 0$ of its connected components.

Step 6 Check the consistency degree, $\beta < 12°$. The measure of inconsistency is defined as

$$Inc(F) = \frac{\|F - AX\|_2}{\|AX\|_2} \tag{5}$$

where $\|\cdot\|_2$ denotes two-norm and $\beta = \arctan(Inc(F))$ is the angle of inconsistency. The ranking is considered acceptable whenever $\beta < 12°$. If the matrix is not consistent, then it needs to be revised (Saaty, 2008).

Step 7 Determine the weight, w. The following equation is used to obtain the weight:

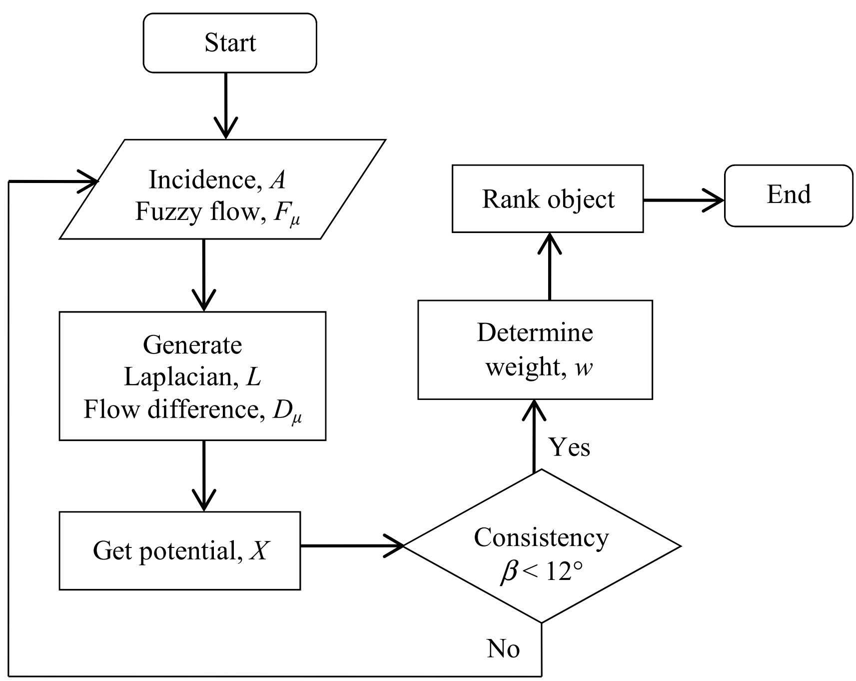
$$w = \frac{a^x}{\|a^x\|_1} \tag{6}$$

where $\|\cdot\|_1$ represents l_1-norm and parameter a is chosen to be 2 suggested by Čaklović and Kurdija (2017).

Step 8 Rank the objects with respect to their associated weights.

Figure 4 provides an illustration of the ranking procedure, which is outlined in the algorithm presented in Figure 5.

Figure 4 Flowchart for FWACS ranking



The next section provides an implementation of the FWACS ranking algorithm for a case study presented in Torfi and Rashidi (2011).

Figure 5 Algorithm for ranking by FWACS

Algorithm 1: Ranking with FWACS

Begin

Input: $A = (a_{ij})_{m\times n}$

$F = (f_1, f_2, f_3, \ldots, f_m)$

Output: $w = (w_1, w_2, w_3, \ldots, w_n)$

Procedure 1: [Define Laplacian, L]

$L = (l_{ij})_{n\times n}$

return L

Procedure 2: [Generate flow difference, D]

$D = (D_1, D_2, D_3, \ldots, D_n)$

return D

Procedure 3: [Get potential, X]

$X = (x_1, x_2, x_3, \ldots, x_n)$

return X

Procedure 4: [Consistency degree, β]

β

return β

Procedure 5: [Determine weight, w]

$w = (w_1, w_2, w_3, \ldots, w_n)$

return w

End

3 Implementation of case study

3.1 Overview of the case study

The Polband Construction Company needs to choose a project manager, and the selection process is depicted in Figure 6, consisting of four levels. The first level involves selecting a project manager, while the second level has four main criteria. The third level includes 23 sub-criteria, and the fourth level comprises ten applicants.

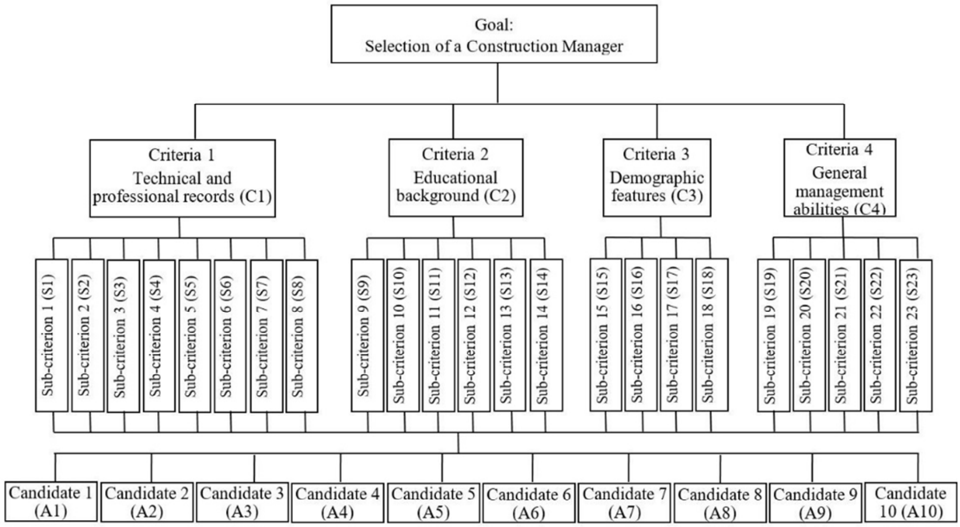
3.2 Ranking project managers by FWACS

To select the project manager, the criteria and sub-criteria on the second level need to be synthesised first, followed by the third level, and finally the main goal.

3.2.1 Second level: determination of criteria weights

The pairwise comparison matrix for the criteria is provided in Table 1. We first consider the second level. The FWACS for the criteria is illustrated in Figure 7, where $C1$ is more preferred than $C2$, while $C3$ and $C4$ are the least preferred.

Figure 6 Hierarchical problem of project managers

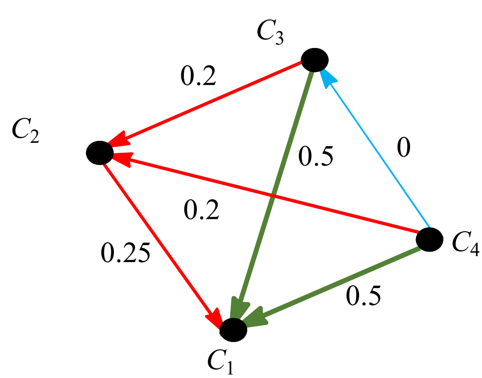


Source: Torfi and Rashidi (2011)

Table 1 Pairwise comparison matrix of criteria

Criteria	*C1*	*C2*	*C3*	*C4*	*AHP-FTOPSIS*	*Rank*	*PM*	*Rank*	*FWACS*	*Rank*
*C*1	1	2	3	3	0.463	1	0.5	1	0.307209	1
*C*2	1/2	1	2	2	0.232	2	0.25	2	0.258331	2
*C*3	1/3	1/2	1	1	0.153	3	0.125	3	0.217230	3
*C*4	1/3	1/2	1	1	0.153	3	0.125	3	0.217230	3

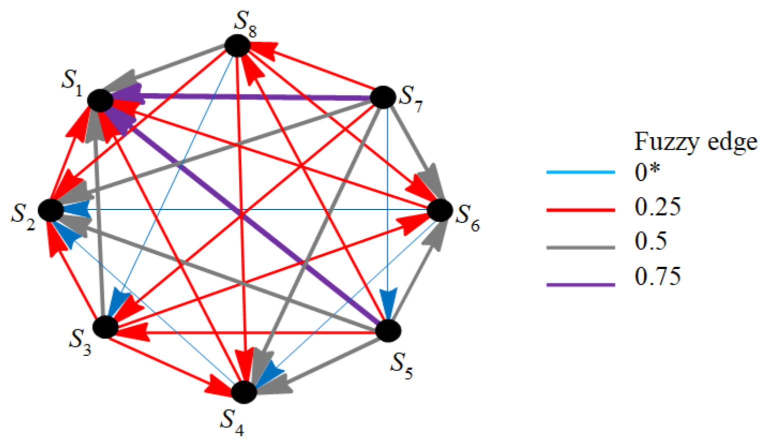
Figure 7 The FWACS criteria (see online version for colours)



3.2.2 Third level: sub-criteria

This section presents the potentials, X and weights, w for sub-criteria. The criterion $C1$ consists of eight sub-criteria. Figure 8 shows the FWACS sub-criteria for criterion $C1$.

Figure 8 The FWACS sub-criteria for *C*1 (see online version for colours)



The weights for sub-criteria are given in Table 2. Sub-criterion *S*1 has the highest weight. The smallest weight is assigned to *S*5 and *S*7.

Table 2 The *C*1 sub-criteria weights

Sub-criteria	*AHP-FTOPSIS*	*Rank*	*PM*	*Rank*	*FWACS*	*Rank*
*S*1	0.274	1	0.103553	1	0.041813	1
*S*2	0.137	2	0.073223	2	0.039645	2
*S*3	0.090	3	0.051777	3	0.037590	3
*S*4	0.137	2	0.073223	2	0.039645	2
*S*5	0.063	4	0.036612	4	0.035641	4
*S*6	0.137	2	0.073223	2	0.039645	2
*S*7	0.063	4	0.036612	4	0.035641	4
*S*8	0.090	3	0.051777	3	0.037590	3

3.2.3 Overall goal: decision table

After determining the weights of the criteria, the next step is to rank the applicants based on the overall criteria. This involves finding the composite flows by converting the information in the decision matrix in Table 2 into fuzzy values using a linear transformation. Torfi and Rashidi (2011) used five scales to indicate the difference among applicants with respect to each criterion. The scale of very low (VL) corresponds to a fuzzy membership value of 0, while a value of 1 is assigned to the scale of very high (VH). The linear transformation of the scale to the fuzzy membership value is summarised in Table 3.

Table 4 presents the fuzzy decision inputs for the ten applicants with respect to the criteria.

The pairwise composite flows between the applicants are calculated using equation (4) based on the fuzzy decision values from Table 3. The composite flow values are then used to compute the overall flows for each applicant using equation (3). The resulting flows for each applicant are presented in Table 5.

Table 3 Fuzzy membership value

Scale	*Membership function (Torfi and Rashidi, 2011)*	*Fuzzy membership value*
Very low (VL)	(0.00, 0.00, 0.25)	0
Low (L)	(0.15, 0.30, 0.45)	0.25
Medium (M)	(0.35, 0.50, 0.65)	0.5
High (H)	(0.55, 0.70, 0.85)	0.75
Very high (VH)	(0.75, 1.00, 1.00)	1

Table 4 Input for fuzzy decision

Applicants	*C1*	*C2*	*C3*	*C4*
*A*1	1	0	1	0.5
*A*2	0.5	0.5	0.25	0.75
*A*3	0.25	0.5	0.25	0.5
*A*4	0	1	0.75	0.5
*A*5	1	0.25	0.5	0.5
*A*6	1	0.5	0.25	0.75
*A*7	0	0	0.25	0
*A*8	0.5	1	0	1
*A*9	0.25	0.5	0.75	0.5
*A*10	1	0.75	0.25	0.75

Note: VL = 0, L = 0.25, M = 0.5, H = 0.75 and VH = 1.

Table 5 Fuzzy flow

Criteria	*Flow*				*Composite flow*
	C1	*C2*	*C3*	*C4*	
Relative weight	*0.307209*	*0.258331*	*0.217230*	*0.217230*	
Min./max.	*Min.*	*Max.*	*Min.*	*Max.*	
$F_{1,2}$	–0.5	0.5	–0.75	0.25	0.5
$F_{1,3}$	–0.75	0.5	–0.75	0	0.52249475
$F_{1,4}$	–1	1	–0.25	0	0.6198475
$F_{1,5}$	0	0.25	–0.5	0	0.17319775
$F_{1,6}$	0	0.5	–0.75	0.25	0.3463955
$F_{1,7}$	–1	0	–0.75	–0.5	0.3615165
$F_{1,8}$	–0.5	1	–1	0.5	0.7377805
$F_{1,9}$	–0.75	0.5	–0.25	0	0.41387975
$F_{1,10}$	0	0.75	–0.75	0.25	0.41097825
$F_{2,3}$	–0.25	0	0	–0.25	0.02249475
$F_{2,4}$	–0.5	0.5	0	–0.25	0.1198475
$F_{2,5}$	0.5	–0.25	0.25	–0.25	–0.3268022
$F_{2,6}$	0.5	0	0	0	–0.1536045

Table 5 Fuzzy flow (continued)

Criteria	*Flow*				*Composite flow*
	C1	*C2*	*C3*	*C4*	
Relative weight	*0.307209*	*0.258331*	*0.217230*	*0.217230*	
Min./max.	*Min.*	*Max.*	*Min.*	*Max.*	
$F_{2,7}$	–0.5	–0.5	0	–0.75	–0.1384835
$F_{2,8}$	0	0.5	–0.25	0.25	0.2377805
$F_{2,9}$	–0.25	0	0.5	–0.25	–0.0861202
$F_{2,10}$	0.5	0.25	0	0	–0.0890217
$F_{3,4}$	–0.25	0.5	0.5	0	0.09735275
$F_{3,5}$	0.75	–0.25	0.25	0	–0.349297
$F_{3,6}$	0.75	0	0	0.25	–0.1760992
$F_{3,7}$	–0.25	–0.5	0	–0.5	–0.1609782
$F_{3,8}$	0.25	0.5	–0.25	0.5	0.21528575
$F_{3,9}$	0	0	0.5	0	–0.108615
$F_{3,10}$	0.75	0.25	0	0.25	–0.1115165
$F_{4,5}$	–1	–0.75	–0.25	0	–0.4466497
$F_{4,6}$	–1	–0.5	–0.5	0.25	–0.273452
$F_{4,7}$	0	–1	–0.5	–0.5	–0.258331
$F_{4,8}$	0.5	0	–0.75	0.5	0.117933
$F_{4,9}$	0.25	–0.5	0	0	–0.2059677
$F_{4,10}$	1	–0.25	–0.5	0.25	–0.2088692
$F_{5,6}$	0	0.25	–0.25	0.25	0.17319775
$F_{5,7}$	–1	–0.25	–0.25	–0.5	0.18831875
$F_{5,8}$	–0.5	0.75	–0.5	0.5	0.56458275
$F_{5,9}$	–0.75	0.25	0.25	0	0.240682
$F_{5,10}$	0	0.5	–0.25	0.25	0.2377805
$F_{6,7}$	–1	–0.5	0	–0.75	0.015121
$F_{6,8}$	–0.5	0.5	–0.25	0.25	0.391385
$F_{6,9}$	–0.75	0	0.5	–0.25	0.06748425
$F_{6,10}$	0	0.25	0	0	0.06458275
$F_{7,8}$	0.5	1	–0.25	1	0.376264
$F_{7,9}$	0.25	0.5	0.5	0.5	0.05236325
$F_{7,10}$	1	0.75	0	0.75	0.04946175
$F_{8,9}$	–0.25	–0.5	0.75	–0.5	–0.3239007
$F_{8,10}$	0.5	–0.25	0.25	–0.25	–0.3268022
$F_{9,10}$	0.75	0.25	–0.5	0.25	–0.0029015

3.3 Comparison of AHP-TOPSIS, PM and FWACS

This section presents a comparison of the results obtained from AHP-TOPSIS, PM, and FWACS methods. The weights for the criteria are discussed in Section 3.3.1, along with the weights of the sub-criteria within each criterion. Section 3.3.2 discusses the ranking of applicants with their respective weights.

Table 6 Comparison of weights with respect to criteria

Criteria		AHP-FTOPSIS			PM			FWACS		
		Criteria weight	*Weight within criteria*	*Rank*	*Criteria weight*	*Weight within criteria*	*Rank*	*Criteria weight*	*Weight within criteria*	*Rank*
*C*1		0.463			0.5			0.307209		
	*S*1		0.274	1		0.103553	1		0.041813	1
	*S*2		0.137	2		0.073223	2		0.039645	2
	*S*3		0.090	3		0.051777	3		0.037590	3
	*S*4		0.137	2		0.073223	2		0.039645	2
	*S*5		0.063	4		0.036612	4		0.035641	4
	*S*6		0.137	2		0.073223	2		0.039645	2
	*S*7		0.063	4		0.036612	4		0.035641	4
	*S*8		0.090	3		0.051777	3		0.037590	3
*C*2		0.232			0.25			0.258331		
	*S*9		0.317	1		0.054307	1		0.046318	1
	*S*10		0.104	2		0.038401	2		0.042351	2
	*S*11		0.079	3		0.032291	3		0.040497	3
	*S*12		0.317	1		0.054307	1		0.046318	1
	*S*13		0.079	3		0.032291	3		0.040497	3
	*S*14		0.104	2		0.038401	2		0.042351	2
*C*3		0.153			0.125			0.217230		
	*S*15		0.429	1		0.034014	1		0.056371	1
	*S*16		0.214	2		0.031191	2		0.054288	2
	*S*17		0.214	2		0.031191	2		0.054288	2
	*S*18		0.143	3		0.028603	3		0.052283	3
*C*4		0.153			0.125			0.217230		
	*S*19		0.241	2		0.026656	2		0.044731	2
	*S*20		0.481	1		0.029069	1		0.046447	1
	*S*21		0.079	4		0.022415	4		0.041487	4
	*S*22		0.120	3		0.024444	3		0.043078	3
	*S*23		0.079	4		0.022415	4		0.041487	4

3.3.1 Criteria and sub-criteria weights

This section compares the results obtained from AHP, PM and FWACS. All methods have provided the same rank for criteria. The weights obtained from FWACS indicate that the weight for *C*1 is the highest, which is in agreement with Torfi and Rashidi (2011) and our calculated PM. The weights for *C*3 and *C*4 are the same. The rank and weights of the sub-criteria within the criteria and overall sub-criteria are given in Table 6 and Table 7, respectively.

For example, the comparison of sub-criteria *S*1 with respect to criteria *C*1 for all methods is highlighted in grey in Table 6. AHP assigns a weight of 0.274 to *S*1, while PM and FWACS assign weights of 0.103553 and 0.041813, respectively. AHP assigns a high weight to *S*1 compared to the other methods. However, the problem with such aggregation methods is that compensation between good scores on some criteria and bad scores on other criteria can occur, leading to the loss of detailed and often important information (Majumdar et al., 2021).

Table 7 Comparison of weights with respect to overall sub-criteria

Sub-criteria	*AHP-FTOPSIS*		*PM*		*FWACS*	
	Weight within sub-criteria	*Rank*	*Weight within sub-criteria*	*Rank*	*Weight within sub-criteria*	*Rank*
*S*1	0.127	1	0.109589	1	0.057749	1
*S*2	0.063	4	0.054795	2	0.048561	2
*S*3	0.042	5	0.027397	3	0.040834	3
*S*4	0.063	4	0.054795	2	0.048561	2
*S*5	0.029	8	0.013699	4	0.034337	4
*S*6	0.063	4	0.054795	2	0.048561	2
*S*7	0.029	9	0.013699	4	0.034337	4
*S*8	0.042	5	0.027397	3	0.040834	3
*S*9	0.073	2	0.109589	1	0.057749	1
*S*10	0.024	10	0.027397	3	0.040834	3
*S*11	0.017	12	0.013699	4	0.034337	4
*S*12	0.073	2	0.109589	1	0.057749	1
*S*13	0.017	12	0.013699	4	0.034337	4
*S*14	0.024	10	0.027397	3	0.040834	3
*S*15	0.065	3	0.054795	2	0.048561	2
*S*16	0.033	7	0.027397	3	0.040834	3
*S*17	0.033	7	0.027397	3	0.040834	3
*S*18	0.022	11	0.013699	4	0.034337	4
*S*19	0.037	6	0.054795	2	0.048561	2
*S*20	0.073	2	0.109589	1	0.057749	1
*S*21	0.012	13	0.013699	4	0.034337	4
*S*22	0.017	12	0.027397	3	0.040834	3
*S*23	0.012	13	0.013699	4	0.034337	4

3.3.2 Ranking applicants

Table 8 presents a comparison of the results obtained from different methods. The weights obtained from FTOPSIS are compared with those obtained from PM and FWACS. Additionally, the rankings of applicants with respect to the overall goal are included in Table 8. For the FWACS, applicant *A*1 is assigned the highest rank, while applicant *A*8 is assigned the lowest rank. The results obtained from FWACS suggest that applicant *A*1 is the most suitable candidate for the position of project manager at Polband Construction Company. The obtained preference order for applicants is also shown in Table 8.

Table 8 The rank of applicants

Applicants	*AHP-FTOPSIS*		*PM*		*FWACS*	
	Weight, w	*Rank*	*Weight, w*	*Rank*	*Weight, w*	*Rank*
*A*1	0.4565	1	0.263538	1	0.131438	1
*A*2	0.1846	7	0.065884	5	0.092941	7
*A*3	0.1020	9	0.050804	8	0.091503	8
*A*4	0.1946	6	0.030020	10	0.085532	9
*A*5	0.3430	2	0.186349	2	0.116569	2
*A*6	0.3091	4	0.132593	3	0.103382	3
*A*7	0.0456	10	0.060416	6	0.102304	4
*A*8	0.1544	8	0.039175	9	0.078818	10
*A*9	0.2452	5	0.060416	7	0.098658	6
*A*10	0.3297	3	0.110804	4	0.098856	5

To conclude, the final ranking of the applicants is reviewed in Table 9. It is observed that *A*1 has been ranked the highest by all three methods, indicating that *A*1 is the most suitable applicant for the position of project manager.

Table 9 Comparison between results obtained from FTOPSIS, PM and FWACS

Rank	*AHP-FTOPSIS*	*PM*	*FWACS*
1	*A*1	*A*1	*A*1
2	*A*5	*A*5	*A*5
3	*A*10	*A*6	*A*6
4	*A*6	*A*10	*A*7
5	*A*9	*A*2	*A*10
6	*A*4	*A*7	*A*9
7	*A*2	*A*9	*A*2
8	*A*8	*A*3	*A*3
9	*A*3	*A*8	*A*4
10	*A*7	*A*4	*A*8

3.4 Sensitivity analysis

Once an initial solution has been obtained using assigned criteria weights, sensitivity analyses were conducted to assess how changes in the relative importance of each criterion affect the overall priority of alternatives (Moktadir et al., 2018; Singh et al., 2020; Bagheri et al., 2021). This is a necessary step because evaluating the synthesis value of the ten applicants requires different levels of $C1$, $C2$, $C3$, and $C4$, depending on the importance of the criteria

To vary the local priority weights of subjective factors, Expert Choice 11 software was utilised. The performance graph displayed in Figure 9 illustrates how the applicants perform with respect to changes in the scenario for all parameters.

Figure 9 Performance sensitivity of ten applicants (see online version for colours)

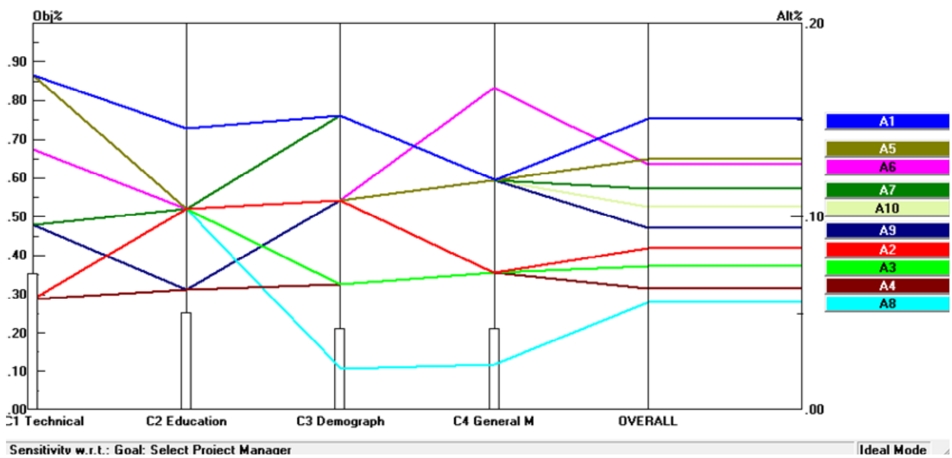
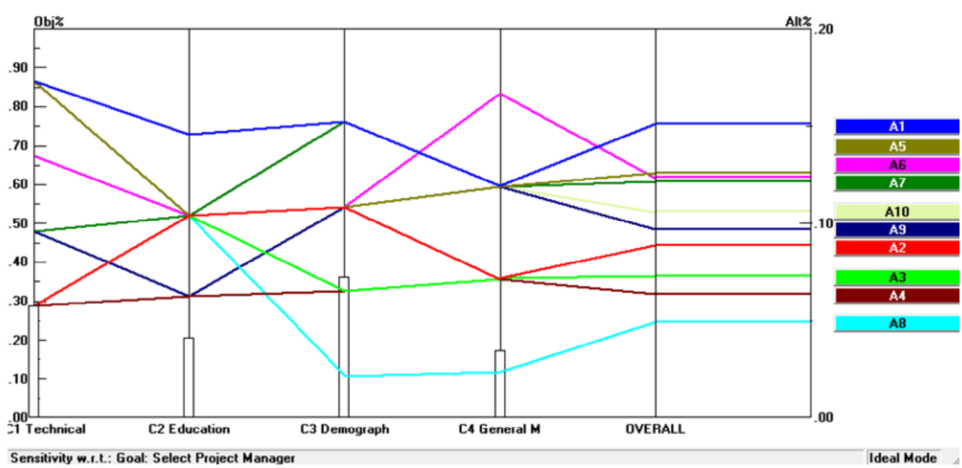


Figure 10 Performance sensitivity of applicants when $C3$ is increased by 15% (see online version for colours)



Performance sensitivity of applicants is performed when *C*3 is increased by 15% from its current level. However, the analysis reveals that the increase in *C*3 does not cause any change in the ranking of the applicants, as illustrated in Figure 10.

The applicants' performance sensitivity is evaluated by decreasing *C*1 to 25% and analysing its effect on the ranking, as shown in Figure 11. The decrease in *C*1 leads to a change in the ranking, with *A*7 (0.103 to 0.128) moving up to the second place and replacing *A*5 (from 0.116 to 0.112). The ranking of the remaining applicants remains unchanged.

Figure 11 Performance sensitivity of applicants when *C*1 is decreased to 25% (see online version for colours)

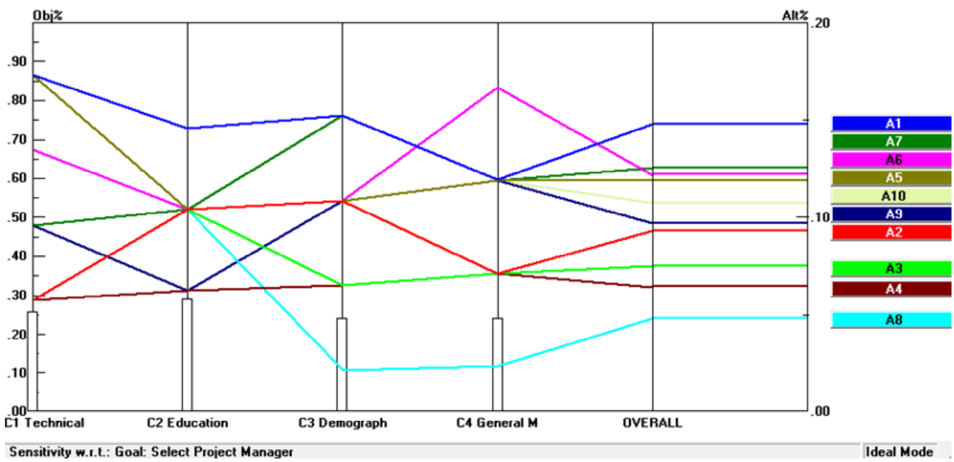
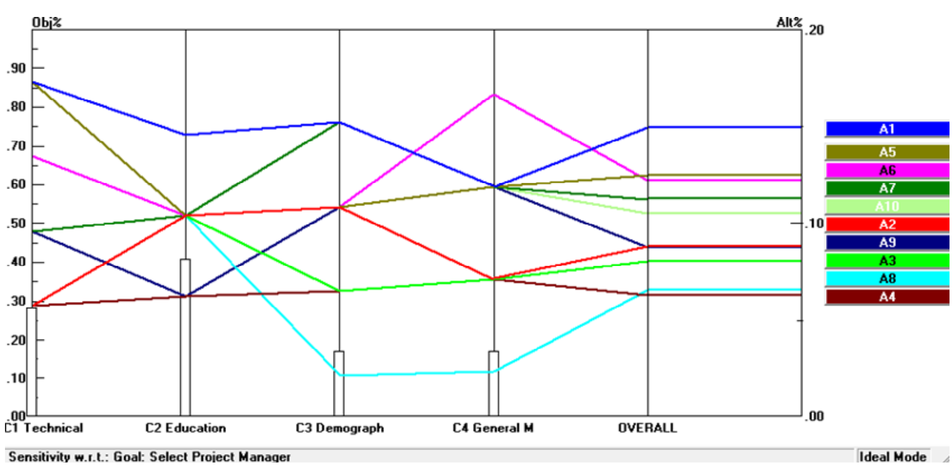


Figure 12 reveals that an increase in *C*2 to 40% increases the synthesis value of the applicant *A*8 from 0.078 to 0.087, which pushes *A*8 up from the last rank to the second last replacing *A*4 (0.085). This scenario results in a changed ranking of the *A*2 (from 0.093 to 0.100) that pushes the *A*9 (from 0.098 to 0.0.091) down from the sixth place.

Figure 12 Performance sensitivity of applicants when *C*2 is increased to 40% (see online version for colours)



Gradual changes were made to each criterion value during the sensitivity analysis, and the resulting rank order was observed. The analysis shows that the first rank (*A*1) remained unchanged even with changes to each criterion value. However, there was a rank reversal for the second and fourth ranks (*A*7 and *A*5, respectively) when *C*1 is reduced to 25%. Overall, the sensitivity analysis confirmed that the final decision was reliable and consistent.

The selection of a project manager for Polband Company was carried out using FWACS, and the results were compared to AHP-FTOPSIS from Torfi and Rashidi (2011) and our calculated PM. The comparison shows that FWACS is comparable to both PM and AHP-FTOPSIS.

4 Results and discussion

A novel ranking approach called FWACS is applied to the project manager selection problem by Torfi and Rashidi (2011). The FWACS is specifically designed to tackle the MCDM under conditions of uncertainty. By combining fuzzy logic and aggregation techniques, FWACS offers a comprehensive solution for ranking alternatives in complex decision scenarios. The study evaluates the performance of FWACS by comparing it against AHP-TOPSIS and PM. The findings from the evaluation reveal that FWACS yields comparable rankings to the other methods, indicating its effectiveness in ranking purposes.

Acknowledgements

The authors gratefully acknowledge the reviewers for their useful suggestions.

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