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A two-echelon sustainable supply chain model for third-party logistics and warehouse financing using multi factor-dependent demand under revenue sharing and consumer segmentation

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Abstract: In this model, we consider that customer demand depends on multi-factors such as quality, price, and logistics service level, and the manufacturer can decide the product's quality degree. Further, we develop this model by combining supply chain management with two financial approaches. Namely, third-party logistics and warehouse financing practices developed. This study aims to understand the relationship between supply chain management and third-party logistics under revenue sharing and consumer segmentation. This paper is developed under two models. Model I analyses the impact of an integrated two-echelon SC without market segmentation. Similarly, model II analyses the impact of an integrated two-echelon SC with market segmentation. The revenue-sharing contract is a type of agreement that specifies how earnings and losses are shared among partners in a supply chain. This paper investigates the manufacturer's and retailer's profits in a supply chain management system with third-party logistics and warehouse financing services. Moreover, we obtain the optimal solutions for the given decision variables by classical optimisation methods.

Keywords: third-party logistics; TPL; warehouse financing; revenue sharing; consumer segmentation; multi-factor sensitive demand.

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1 Introduction

The purpose of this research is to find the maximum profit of the two-echelon supply chain management (SCM) with quality, logistic service level, and price-dependent demand under third-party logistics (TPL) and warehouse financing services. The TPL companies offer comprehensive and even one-stop solutions for supply chain (SC) in packaging, warehousing, transportation. The role of logistics in SCM is one of the significant improvements in industrial development. TPL assist in supplying the right product to the right consumers in the correct quantity. The TPL acts as an essential role in distribution services. It involves transporting goods from the distributor's storage facilities to other parts of the SC or customers. Drayage, cross-docking, intermodal transport, brokerage, drop deck, and lowboy transport, full container load, less than container load, truckload, and less than truckload are all examples of distribution services. In a market world, the TPL have four different kinds of sources for distributors. The first TPL source is a *standard supplier*: the TPL company only offers basic logistics at this stage. The *service developer* is the second source of TPL: it provides support in IT, security, and production planning. The third source of TPL is the *customer adapter*, which takes over and runs the current logistics service of the partner business. *Customer developer* is the final source of TPL: it refers to TPL taking on the position of the entire logistics department rather than just providing services. The revenue-sharing contract plays a crucial role in the SC coordination contracts. A joint venture, development, management, operation, or similar agreement or arrangement to share revenues, profits, losses, and costs is referred to as a revenue sharing contract. The following factors are used in the revenue sharing contract: party's rights, management and auditing, costs and revenue share, timeframe, intellectual property, dispute resolution, liability, and termination. It entails associated financial partners sharing operating profits and losses. It can also function as a profit-sharing system, ensuring that each entity is fairly compensated for its efforts. This proposed model is used to obtain the total profit, TPL service level, and retailer's optimal order quantity. Warehouse financing is a method of financial support to the inventories. It is a loan to a business, manufacturer, or processor made by a financial institution. On the other side, the customer's demand is assumed to be quality and price-oriented demand rates. Customer segmentation is a way to separate customers into categories based on the features shared by those customers.

1.1 Framework of the paper

This paper is framed as follows: Section 2 summarises the relevant literature. The notations and assumptions are classified in Section 3. Section 4 described the basic

cost and price components of the two-echelon system. Sections 5.1 (model I) and 5.2 (model II) are designed for the profitability of an integrated sustainable two-echelon SC system. A numerical experiment and managerial outcomes are defined in Section 6. Section 7 provides the conclusion and future plan of the study.

2 Literature survey

In this section, we address four types of literature that are relevant to our proposed model. The first focuses on optimum methods for the SC under the financial side of the TPL. The second is to concentrate on warehouse financing. The third assumes to focus on price, quality, and logistical dependent demands on the demands of a customer. Discussing the significance and role of the revenue sharing contract and consumer segmentation is the final aspect of this literature.

2.1 Third-party logistics financing

In 2019, Hua et al. presented financing and ordering strategies of an SCM under the option contract. Huang et al. (2019a) established the optimal financing service strategies of SC by a TPL company. Also, Huang et al. (2019b) extended the effect of transportation costs on capital-constrained SC's operations under the TPL financing service. Yu and Xiao (2017) determined a cold SC with service level decision in fresh agro-products and logistics outsourcing. In a constrained SC, Zhan et al. (2019) described the rebate contract to identify the value of trade credit. In 2020, Zhang et al. developed the financial operations of TPL with logistics service level and price-dependent demand in a capital-constrained SC. TPL service companies must ensure every customer service related to acceptance of orders and their execution or problems. These results are related to customer satisfaction with TPL, service level, and their quality with revenue sharing for maximum profit.

2.2 Warehouse financing

In 2012, the impact of maintaining storable product risks and the financial hedge was discussed by Kouvelis et al. (2013) and also, Kouvelis et al. (2018) extended the integrated SCM and financial hedging: A dynamic mean-variance analysis. Lin et al. (2018) extended a SC coordination in confirming warehouse financing. Marchi et al. (2020) described the economic lot size models for hedging stocks using different coordination policies under warehouse financing and financial contracts. Steinrcke and Albrecht (2018) studied an SC network and financial planning with capital market's imperfection.

2.3 Multi-factor dependent demand (quality, price and logistic service)

Cao et al. (2019) investigated an inventory model for financing preferences and performances for an emission-dependent SC model. In 2014, Giri and Sharma addressed a SC model for manufacturer's pricing strategy and advertising cost-dependent demand. Malleeswaran and Uthayakumar (2020) developed an integrated SC model for price

discount in backorder and price-dependent demand. Malleeswaran and Uthayakumar (2021) formulated the consignment model for reducing waste items and GHG considering promotional efforts dependent demand. Malleeswaran and Uthayakumar (2022) addressed a sustainable reworking model for green and environmental sensitive demand under discrete ordering cost reduction. Qian (2014) explored a supplier selection problem with the price, delivery time, and service level-dependent demand.

2.4 Revenue sharing and consumer segmentation

Cachon and Lariviere (2005) developed an SC model with revenue sharing contracts: strength and limitations. Ji et al. (2020) introduced revenue sharing contracts for production decisions, cap set, and cap-and-trade regulation. In 2018, Lin and Xiao examined the necessity of retailer credit guarantee in SC with push and pulled contract. Seifbarghy et al. (2015) constructed an SC model under customer segmentation with price and quality-dependent demand. In 2019, Taleizadeh et al. introduced a SC model with market segmentation and quantity discount policies.

Consumer or market segmentation helps increase the chances of people engaging with their ad or content. In a market, the regional, geographical, psychographic, and behavioural are the factors to improve the efficiency of market or customer segmentation.

Table 1 Types and uses of market or consumer segmentation

<i>Types</i>	<i>Uses</i>
Geographic	Greater company focus and depending on the location
Demographic	It depends on the income, occupation, etc.
Psychographic	Position in society, targeted communication, lifestyle, and personality type are all factors to consider
Behavioural	It depends on the intensity of the product, price sensitivity and technology adoption

The revenue sharing contract stipulates that the manufacturer charges the retailer a reduced wholesale price and shares a part of the merchant's revenue; even if no returns are permitted, everyone in the SC will benefit from this arrangement. There are many advantages for considering the revenue sharing contract; a few of them are listed below:

- 1 the profit-sharing plans are helpful to increase efficiency
- 2 it provides an atmosphere of accountability
- 3 people are encouraged to buy into the company vision
- 4 it has the potential to save the company money and offering plans for competitive advantage.

3 Notations and assumptions

3.1 Notations

The proposed model is developed by using the notations listed below.

3.1.1 Manufacturer's side

S_c^m	Setup cost (%).
C'_m	Carrying cost.
P_r^m	Production rate.
M_c	Manufacturing cost.
f_i^m	Financial interest rate.
P_c^m	Production cost (\$/unit).
I_c^m	Initial capital cost (\$/unit).
(C'_m, p)	Physical component made by C'_m .
(C'_m, f)	Financial component made by C'_m .
W_p^m	Wholesale price for each unit of product (\$/kWh).

3.1.2 Retailer's side

r_c	Retail cost.
r_p	Retail price.
O_c^r	Ordering cost.
C_r	Carrying cost.
f_i^r	Financial interest rate.
I_c^r	Initial capital cost (\$/unit).

3.1.3 TPL and warehouse financing

L_s^p	TPL price per unit.
C^{TPL}	TPL cost per unit (\$/unit).
δ_1	Price sensitivity coefficient.
η	Capital cost affect the discount.
δ_2	Logistics service sensitivity coefficient.
α	Coefficient for the liquidity of the goods.
η_o	Capital cost does not affect the discount.

ψ	Share of the inventory level used as guarantee.
γ	Elasticity of the capital cost w.r.t. the stock level.

3.1.4 Consumer segmentation

q_d^m	The product's quality degree.
C_b^q	Cost for reach a basic quality degree.
τ	Total population of potential consumers.
μ	Sensitivity elasticity parameter for price ($\mu \geq 1$).
C_i^q	The cost of enhancing the quality degree by one unit.
ω	Sensitivity for quality degree ($0 < \omega < 1$, $\omega + 1 < \mu$).
ξ	Retailer's share from revenue while making revenue sharing ($0 < \xi < 1$).

3.1.5 Decision variables

Q	Order quantity
n	Number of shipments
L_s^l	TPL service level per visit
L_s^c	TPL service cost per visit
$\checkmark$	Denotes the number of optimal solutions.

3.2 Assumptions

In order to develop this study, we consider some basic assumptions which are listed below.

- 1 A single manufacturer-single retailer SC is considered by a centralised approach with TPL and warehouse financing under revenue sharing contract.
- 2 The customers demand is depends on the retail price and logistics service level.

$$D' = \tau(r_p)^{-\mu} + \delta_1 L_s^l \quad (1)$$

where ($\mu \geq 1$) and $\delta_1 > 0$.

- 3 The logistics service level cost is $L_s^c = \frac{\delta_2 (L_s^l)^2}{2}$ (Yu and Xiao, 2017).

4 Model description

This section develops the mathematical models for retailer's and manufacturer's profit functions. Further, this paper is developed under two models: model I develops the profit function without market segmentation, and model II develops the profit function with

market segmentation. Model II consists of two cases: SC profit under quality-oriented demand of customers and SC profit under price-oriented customer demand. Each model has the solution methodology to calculate the optimal values of the decision variables and the maximum profit of the three objective functions.

4.1 Retailer's revenue sharing

The revenue sharing contract, the retailer and manufacturer preplanned the share of total revenue (ξ) of the retailer. The retailer's profit ($0 < \xi < 1$) function of revenue sharing is in equation (2) (Cachon and Lariviere, 2005).

$$\Pi_r^{rs} = D' (\xi r_p - W_p^m r_c) \quad (2)$$

For, D' refer equation (1)

4.2 Warehouse financing

Except for a minor service fee, this warehouse financing can provide interest-free financing (η_o) (Lin et al., 2018).

The capital cost for the scenario with warehouse financing is

$$\eta' = \eta_o + \frac{\eta}{\alpha \gamma \psi} \quad (3)$$

where γ denotes the elasticity of the capital cost w.r.t. the stock level.

The total profit function of warehouse financing is

$$\Pi_r^{wf} = \frac{D'}{Q} \left(O_c^r + \frac{S_c^m}{n} \right) - \frac{QC'_m}{2} \left\{ n \left(\left(1 - \frac{D'}{P_r^m} \right) + \frac{2D'}{P_r^m} - 1 \right) - \frac{C_r Q}{2} \right\} \quad (4)$$

where, $C'_m = C'_{m,f} + C_{m,p}$ and $C'_{m,f} = P_c^m \eta'$.

4.3 Retailer's profit with TPL

The subleader of this case becomes TPL. At the initial stage of the sales period, the TPL offers financing ($W_p^m Q - I_c^r$) to the retailer with interest rate f_i^m . Moreover, the retailer's product transportation service price L_s^p is made by TPL.

The retailer must return the principal and interest at the end of the sales period is $(W_p^m Q - I_c^r)(1 + f_i^r)$ with logistics service fee $L_s^p Q$.

Thus, the retailers profit function with TPL is

$$\Pi_r^{tpl} = QD' (L_s^p - C^{TPL}) + f_i^r (W_p^m - I_c^r) - L_s^c D' \quad (5)$$

where $L_s^c = \frac{\delta_2 (L_s^l)^2}{2}$.

4.4 Manufacturer's profit with TPL

The TPL offers financing $(M_c Q - I_c^m)$ to the capital-constrained manufacturer with interest rate f_i^m . Similarly, the retailer's product transportation service price L_s^p is made by TPL. The manufacturer is responsible for repaying the principle and interest from the end of the sales time. $((M_c Q - I_c^m)(L_s^p + f_i^m))$ with logistics service fee $L_s^p Q$.

Thus, the manufacturers profit function with TPL is

$$\Pi_m^{tpl} = QD' L_s^p + f_i^m (M_c Q - I_c^m) \quad (6)$$

5 Methodology

5.1 Model 1: the profitability of manufacturer and retailer without market segmentation

This section develops the profit functions for the proposed integrated two-echelon sustainable SC system. The cost components are described as below. Moreover, the objective functions for this paper is considered into three ways. There are two models developed.

5.1.1 Retailer's annual profitability

The annual expected profit function of the retailer is formulated by adding equations (2), (4) and (5), which is defined by

$$\begin{aligned} \Pi_{TPL}^r &= \Pi_r^{rs} + \Pi_r^{wf} + \Pi_r^{tpl} \\ \Pi_{TPL}^r &= D' (\xi r_p - W_p^m r_c) + \frac{D'}{Q} \left(O_c^r + \frac{S_c^m}{n} \right) \\ &\quad - \frac{QC'_m}{2} \left\{ n \left(\left(1 - \frac{D'}{P_r^m} \right) + \frac{2D'}{P_r^m} - 1 \right) - \frac{C_r Q}{2} \right\} \\ &\quad + QD' (L_s^p - C^{TPL}) + f_i^r (W_p^m - I_c^r) - L_s^c D' \end{aligned} \quad (7)$$

5.1.2 Manufacturer's annual profitability

From the equations (1) to (4) and (6), the annual profit function for the manufacturer is derived as follows,

$$\Pi_{TPL}^m = QD' (L_s^p - C^{TPL}) + f_i^m (M_c Q - I_c^m) - L_s^c D' \quad (8)$$

5.1.3 Objective function

Now, we constructed the total centralised SC's profit function without market segmentation is [from equations (7) and (8)]

$$\Pi_{TPL}^m(n, Q, L_s^l) = \Pi_{TPL}^r + \Pi_{TPL}^m$$

$$\begin{aligned}
\Pi_{TPL}^m(n, Q, L_s^l) = & D' (\xi r_p - W_p^m r_c) + \frac{D'}{Q} \left(O_c^r + \frac{S_c^m}{n} \right) \\
& - \frac{QC'_m}{2} \left\{ n \left(\left(1 - \frac{D'}{P_r^m} \right) + \frac{2D'}{P_r^m} - 1 \right) \right\} \\
& - \frac{C_r Q}{2} + QD' (L_s^p - C^{TPL}) + f_i^r (W_p^m - I_c^r) - L_s^c D' \\
& + QD' (L_s^p - C^{TPL}) \\
& + f_i^m (M_c Q - I_c^m) - L_s^c D'
\end{aligned}$$

It can be rewritten as,

$$\begin{aligned}
\Pi_{TPL}^m(n, Q, L_s^l) = & D' (\xi r_p - W_p^m r_c) + \frac{D'}{Q} \left(O_c^r + \frac{S_c^m}{n} \right) \\
& - \frac{QC'_m}{2} \left\{ n \left(\left(1 - \frac{D'}{P_r^m} \right) + \frac{2D'}{P_r^m} - 1 \right) \right\} \\
& - \frac{C_r Q}{2} + 2QD' (L_s^p - C^{TPL}) - 2D' L_s^c \\
& + f_i^m (M_c Q - I_c^m) + f_i^r (W_p^m - I_c^r)
\end{aligned} \tag{9}$$

Here, $C'_m = C'_{m,f} + C_{m,p}$ and $C'_{m,f} = P_c^m \eta'$, for η' use equation (3).

5.1.4 Solution procedures

Partially differentiating w.r.t. L_s^l by equation (9),

$$\begin{aligned}
\frac{\partial \Pi_{TPL}^m(n, Q, L_s^l)}{\partial L_s^l} = & \delta_1 (\xi r_p - W_p^m r_c) + \frac{1}{Q} \left(O_c^r + \frac{S_c^m}{n} \right) - \frac{nQC'_m}{2P_r^m} + \frac{2}{P_r^m} \\
& + 2\delta_1 (Q(L_s^p - C^{TPL})) - \delta_2 L_s^l
\end{aligned}$$

where $\sigma_1 = (\xi r_p - W_p^m r_c) + \frac{1}{Q} \left(O_c^r + \frac{S_c^m}{n} \right) - \frac{nQC'_m}{2P_r^m} + \frac{2}{P_r^m}$.

That is,

$$\frac{\partial \Pi_{TPL}^m(n, Q, L_s^l)}{\partial L_s^l} = \delta_1 (\sigma_1 + 2Q(L_s^p - C^{TPL})) - \delta_2 L_s^l \tag{10}$$

Differentiating partially w.r.t. L_s^l by equation (10),

$$\frac{\partial^2 \Pi_{TPL}^m(n, Q, L_s^l)}{\partial L_s^{l^2}} = -\delta_2 < 0 \tag{11}$$

For obtaining the logistics service level of the inventory (L_s^l), we need to equate equation (10) by zero.

$$L_s^l = \frac{\delta_1 (\sigma_1 + 2Q(L_s^p - C^{TPL}))}{\delta_2} \tag{12}$$

Partially differentiating w.r.t. Q by equation (9),

$$\begin{aligned} \frac{\partial \Pi_{TPL}^m(n, Q, L_s^l)}{\partial Q} &= \frac{-D'}{Q^2} \left(O_c^r + \frac{S_c^m}{n} \right) - \frac{C'_m}{2} \left\{ n \left(\left(1 - \frac{D'}{P_r^m} \right) + \frac{2D'}{P_r^m} - 1 \right) \right\} \\ &\quad - \frac{C_r}{2} + 2D' (L_s^p - C^{TPL}) + f_i^m M_c + f_i^r W_p^m \end{aligned}$$

where

$$\begin{aligned} \sigma_2 &= n \left(\left(1 - \frac{D'}{P_r^m} \right) + \frac{2D'}{P_r^m} - 1 \right) \\ \sigma_3 &= 2D' (L_s^p - C^{TPL}) + f_i^m M_c + f_i^r W_p^m \end{aligned}$$

That is,

$$\frac{\partial \Pi_{TPL}^m(n, Q, L_s^l)}{\partial Q} = \frac{-D'}{Q^2} \left(O_c^r + \frac{S_c^m}{n} \right) - \frac{1}{2} ((C'_m \sigma_2) - C_r) + (\sigma_3) \quad (13)$$

Differentiating partially w.r.t. Q by equation (13),

$$\frac{\partial^2 \Pi_{TPL}^m(n, Q, L_s^l)}{\partial Q^2} = \frac{2D'}{Q^3} \left(O_c^r + \frac{S_c^m}{n} \right) > 0 \quad (14)$$

For obtaining the order quantity of the retailer (Q), we need to equate to zero by equation (13)

$$Q = \sqrt{\frac{D' \left(O_c^r + \frac{S_c^m}{n} \right)}{\sigma_3 - \frac{1}{2} (\sigma_2 C'_m - C_r)}} \quad (15)$$

5.2 Model II: the profitability of manufacturer and retailer with market segmentation

According to this study, the return policy decisions are compatible with product quality and pricing strategies. In this proposed model, the customer's demand is classified into two groups such as quality and price-oriented demand of customers.

5.2.1 Case 1: centralised SC profit in quality-oriented demands of customers

Assuming (λ) is the % of the potential quality-oriented customers. The quality-oriented demand of customers is represented in equation (16). Furthermore, it is defined from equation (1),

$$D_1' = \lambda (\tau(r_{p_1})^{-\mu_1} (q_{d_1}^m)^{\omega_1}) + \delta_1 L_s^l \quad (16)$$

where, r_{p_1} , $q_{d_1}^m$ represents the price, quality degree of the products for this case and $0 < q_{d_1}^m < 1$.

5.2.2 Objective function

The total profit function of case 1 of model II is written as follows,

$$\begin{aligned} \Pi_{TPL}^{msq}(n, Q, L_s^l) = & D_1' (\xi r_p - W_p^m r_c) + \frac{D_1'}{Q} \left(O_c^r + \frac{S_c^m}{n} \right) \\ & - \frac{QC'_m}{2} \left\{ n \left(\left(1 - \frac{D_1'}{P_r^m} \right) + \frac{2D_1'}{P_r^m} - 1 \right) \right\} \\ & - \frac{C_r Q}{2} + 2QD_1' (L_s^p - C^{TPL}) \\ & - 2D_1' L_s^c + f_i^m (M_c Q - I_c^m) + f_i^r (W_p^m - I_c^r) \end{aligned} \quad (17)$$

where $C'_m = C'_{m,f} + C_{m,p}$; $C'_{m,f} = P_c^m \eta'$, for η' use equation (3). And also, $(\mu_1 \geq 1)$, $(0 < \omega_1 < 1, \omega_1 + 1 < \mu_1)$, and $\delta_1 > 0$.

5.2.3 Solution procedures

Differentiating partially w.r.t. L_s^l by equation (17),

$$\frac{\partial \Pi_{TPL}^{msq}(n, Q, L_s^l)}{\partial L_s^l} = \lambda \delta_1 (\sigma_1 + 2Q(L_s^p - C^{TPL})) - \delta_2 L_s^l \quad (18)$$

Equation (19) represents the value of logistics service level (L_s^l),

$$L_s^l = \frac{\lambda \delta_1 (\sigma_1 + 2Q(L_s^p - C^{TPL}))}{\delta_2} \quad (19)$$

Differentiating partially w.r.t. Q by equation (17),

$$\frac{\partial \Pi_{TPL}^{msq}(n, Q, L_s^l)}{\partial Q} = \frac{-D_1'}{Q^2} \left(O_c^r + \frac{S_c^m}{n} \right) - \frac{1}{2} ((C'_m \sigma_2) - C_r) + (\sigma_3) \quad (20)$$

where

$$\begin{aligned} \sigma_2 &= n \left(\left(1 - \frac{D_1'}{P_r^m} \right) + \frac{2D_1'}{P_r^m} - 1 \right) \\ \sigma_3 &= 2D_1' (L_s^p - C^{TPL}) + f_i^m M_c + f_i^r W_p^m \end{aligned}$$

Differentiating partially w.r.t. Q by equation (20),

$$\frac{\partial^2 \Pi_{TPL}^{msq}(n, Q, L_s^l)}{\partial Q^2} = \frac{2D_1'}{Q^3} \left(O_c^r + \frac{S_c^m}{n} \right) > 0 \quad (21)$$

For obtaining the order quantity of the retailer (Q) so that, we need to equate equation (20) by zero.

$$Q = \sqrt{\frac{D_1' \left(O_c^r + \frac{S_c^m}{n} \right)}{\theta_2 - \frac{1}{2} (\theta_1 C'_m - C_r)}} \quad (22)$$

where

$$\begin{aligned} \theta_1 &= n \left(\left(1 - \frac{D_1'}{P_r^m} \right) + \frac{2D_1'}{P_r^m} - 1 \right) \\ \theta_2 &= 2D_1' (L_s^p - C^{TPL}) + f_i^m M_c + f_i^r W_p^m \end{aligned}$$

5.2.4 Case 2: centralised SC profit in price oriented demands of customers

Assuming $(1 - \lambda)$ is the % of the potential price-oriented customers. The price-oriented demand of customer is represented in equation (23) and it is derived from equation (1),

$$D'_2 = (1 - \lambda) (\tau(r_{p_2})^{-\mu_2} (q_{d_2}^m)^{\omega_2}) + \delta_1 L_s^l \quad (23)$$

where, r_{p_2} , $q_{d_2}^m$ represents the price, quality degree of the products for this case and $0 < q_{d_2}^m < 1$.

5.2.5 Objective function

The total profit function of case 2 of model II is written as follows,

$$\begin{aligned} \Pi_{TPL}^{msp}(n, Q, L_s^l) = & D'_2 (\xi r_p - W_p^m r_c) + \frac{D'_2}{Q} \left(O_c^r + \frac{S_c^m}{n} \right) \\ & - \frac{QC'_m}{2} \left\{ n \left(\left(1 - \frac{D'_2}{P_r^m} \right) + \frac{2D'_2}{P_r^m} - 1 \right) \right\} \\ & - \frac{C_r Q}{2} + 2QD'_2 (L_s^p - C^{TPL}) - 2D'_2 L_s^c \\ & + f_i^m (M_c Q - I_c^m) + f_i^r (W_p^m - I_c^r) \end{aligned} \quad (24)$$

where $(\mu_2 \geq 1)$, $(0 < \omega_2 < 1, \omega_2 + 1 < \mu_2)$ and $\delta_1 > 0$.

5.2.6 Solution procedures

Differentiating partially w.r.t. L_s^l by equation (24),

$$\frac{\partial \Pi_{TPL}^{msp}(n, Q, L_s^l)}{\partial L_s^l} = (1 - \lambda) \delta_1 (\sigma_1 + 2(Q(L_s^p - C^{TPL}))) - \delta_2 L_s^l \quad (25)$$

Equation (26) represents the optimal value of logistics service level (L_s^l),

$$L_s^l = \frac{(1 - \lambda) \delta_1 (\sigma_1 + 2Q(L_s^p - C^{TPL}))}{\delta_2} \quad (26)$$

Differentiating partially w.r.t. Q by equation (24),

$$\frac{\partial \Pi_{TPL}^{msp}(n, Q, L_s^l)}{\partial Q} = \frac{-D'_2}{Q^2} \left(O_c^r + \frac{S_c^m}{n} \right) - \frac{1}{2} ((C'_m \sigma_2) - C_r) + (\sigma_3) \quad (27)$$

Differentiating partially w.r.t. Q by equation (27),

$$\frac{\partial^2 \Pi_{TPL}^{msp}(n, Q, L_s^l)}{\partial Q^2} = \frac{2D'_2}{Q^3} \left(O_c^r + \frac{S_c^m}{n} \right) > 0 \quad (28)$$

For obtaining the order quantity of the retailer (Q) so that, we need to equate equation (27) by zero.

$$Q = \sqrt{\frac{D'_2 \left(O_c^r + \frac{S_c^m}{n} \right)}{\theta_4 - \frac{1}{2} (\theta_3 C'_m - C_r)}} \quad (29)$$

where

$$\theta_3 = n \left(\left(1 - \frac{D'_2}{P_r^m} \right) + \frac{2D'_2}{P_r^m} - 1 \right)$$

$$\theta_4 = 2D'_2 (L_s^p - C^{TPL}) + f_i^m M_c + f_i^r W_p^m$$

6 Numerical examples

We have developed two kinds of testing problems (TP1, TP2) for calculating the maximum profit values of model I (9), and model II (17) and (24). Table 2 represents the numerical input values of the parameters. The optimal values of the decision variables are denoted by n^* , Q^* , L_s^{l*} , and L_s^{c*} .

Table 2 Numerical input parameters and its values

Parameters	TP1	TP2	Parameters	TP1	TP2
S_c^m	400	600	ψ	100%	80%
τ	3,000	4,500	η_o	5%	8%
P_r^m	9	15	η	15%	10%
(C'_m, f)	$P_c^m \eta'$	$P_c^m \eta'$	C^{TPL}	3	5
(C'_m, p)	7.5	4.5	ξ	0.6	0.4
M_c	3	5	L_s^p	15	10
f_i^m	0.03	0.05	I_c^r	400	300
I_c^m	10	15	λ	0.7	0.5
W_p^m	20	10	q_{d1}^m	0.3	0.7
r_c	2	4	q_{d2}^m	0.2	0.5
r_p	50	35	μ	1.6	2.3
O_c^r	100	200	ω	0.1	0.3
C_r	2	5	C_b^q	2	4
f_i^r	0.03	0.05	C_i^q	4	8
δ_1	3	2	δ_2	5	2
P_c^m	5	10			

6.1 List of optimal profit values for model I

Table 3 List of optimal solutions for TP1

Parameters	n^*	Q^*	L_s^{l*}	L_s^{c*}	Π_{TPL}^r	Π_{TPL}^m	Π_{TPL}^{rm}
$\gamma = 25$	2	24.56	3.76	21.21	147.67	289.43	546
	3	22.56	4.35	28.28	157.37	299.43	579
	4	28.45	5.12	39.32	164.23	316.45	658✓
$\gamma = 50$	1	25.24	3.84	22.12	148.24	278.62	537
	2	26.35	4.2	26.46	153.75	292.3	551
	3	29.47	4.87	35.57	157.81	306	565✓

Table 3 List of optimal solutions for TP1 (continued)

<i>Parameters</i>	n^*	Q^*	L_s^{l*}	L_s^{c*}	Π_{TPL}^r	Π_{TPL}^m	Π_{TPL}^{rm}
$\alpha = 15$	5	28.35	4.32	27.99	161.23	289	559
	6	30.63	5.1	39.01	164.59	296.27	576
	7	31.23	5.4	43.74	170	303	588✓
$\alpha = 30$	3	28.77	3.96	23.52	151.32	288.25	539
	4	29.53	4.43	29.43	162.72	298.42	561
	5	30.84	5.21	40.71	171.39	310	593✓
$P_c^m = 5$	7	25.28	3.27	16.03	146.23	273.2	528.2
	8	26.62	3.78	21.43	168.24	289.75	542.7
	9	28.39	4.19	26.33	172.92	301.63	564✓
$P_c^m = 8$	10	28.62	3.93	23.16	149.25	274.1	535
	11	31.67	4.34	56.50	157	291.63	557
	12	33.2	4.87	35.57	176.35	308.72	582✓

Table 4 List of optimal solutions for TP2

<i>Parameters</i>	n^*	Q^*	L_s^{l*}	L_s^{c*}	Π_{TPL}^r	Π_{TPL}^m	Π_{TPL}^{rm}
$\gamma = 20$	5	26.23	2.89	8.35	142.31	281.25	523.76
	6	27.63	3.43	11.76	156.25	294.38	538.94
	7	28.65	3.98	15.84	168.12	311.12	565.32✓
$\gamma = 40$	9	31.23	4.05	16.40	157.21	289.31	532.12
	10	31.65	4.31	18.57	169.11	301.25	551.75
	11	32	4.89	23.91	178.52	312.39	579.30✓
$\alpha = 25$	3	25.13	2.12	4.49	140.31	271.37	520.89
	4	25.66	2.78	7.73	153.82	286.53	535.36
	5	26.35	3.13	9.79	167.42	297.26	552.03✓
$\alpha = 50$	6	26.65	3.18	10.11	147.23	273.12	528.21
	7	27.12	3.4	11.56	158.42	291.6	542.45
	8	27.93	3.95	15.60	169.10	311.2	569.47✓
$P_c^m = 4$	2	23.17	2.83	8	143.32	269.11	525
	3	23.91	3.21	10.3	156.45	283.14	540.01
	4	24.22	3.76	14.14	168.31	299.05	561.06✓
$P_c^m = 7$	7	25.61	2.76	7.62	146.21	272.10	531.18
	8	25.94	3.75	14.06	154.27	296.02	556.20
	9	26.27	4.07	16.56	169.13	304.23	578.03✓

6.2 List of optimal profit values for model II

6.2.1 Optimal solutions for quality oriented demand of customers

Table 5 List of optimal solutions for TP1

<i>Parameters</i>	n^*	Q^*	L_s^{l*}	L_s^{c*}	Π_{TPL}^{msq}
$\mu_1 = 1.8$	1	35.23	4.16	25.96	572.25
	2	35.82	4.81	34.70	589.44
	3	36.21	5.1	39.01	604.23✓

Table 5 List of optimal solutions for TP1 (continued)

<i>Parameters</i>	n^*	Q^*	L_s^{l*}	L_s^{c*}	Π_{TPL}^{msq}
$\mu_1 = 2.3$	3	35.89	4.45	29.7	583.34
	4	36.24	4.78	34.27	598.41
	5	36.69	5.5	45.37	612.91 [✓]
$\omega_1 = 0.5$	2	32.42	4.23	26.83	592.68
	3	33.13	4.64	32.29	617.58
	4	33.9	5.9	52.21	636.81 [✓]
$\omega_1 = 1$	5	32.83	4.29	27.61	598.37
	6	33.44	4.82	34.85	620.00
	7	34	5.53	45.87	643.25 [✓]

Table 6 List of optimal solutions for TP2

<i>Parameters</i>	n^*	Q^*	L_s^{l*}	L_s^{c*}	Π_{TPL}^{msq}
$\mu_1 = 1.8$	6	36.15	4.67	21.84	583.17
	7	36.34	4.81	23.14	591.36
	8	36.52	5.02	25.20	614.19 [✓]
$\mu_1 = 2.3$	3	35.84	4.25	18.06	583.34
	4	35.96	4.48	20.07	601.05
	5	36.14	5.12	26.21	629.63 [✓]
$\omega_1 = 0.5$	8	36.35	4.83	23.33	595.25
	9	36.68	5.15	26.52	614.79
	10	36.81	5.69	32.38	645.55 [✓]
$\omega_1 = 1$	11	36.93	4.97	24.70	604.76
	12	37.27	5.23	27.35	660.00
	13	37.49	5.58	31.14	673.15 [✓]

6.2.2 Optimal solutions for price oriented demand of customers

Table 7 List of optimal solutions for TP1

<i>Parameters</i>	n^*	Q^*	L_s^{l*}	L_s^{c*}	Π_{TPL}^{msp}
$\mu_2 = 2.5$	2	32.42	4.25	18.06	578.91
	3	32.89	4.38	19.18	595.64
	4	33.08	4.67	21.81	611.25 [✓]
$\mu_2 = 3.5$	5	32.69	4.30	18.49	584.22
	6	32.97	4.52	20.43	603.45
	7	33.19	4.84	23.42	618.32 [✓]
$\omega_2 = 0.6$	4	32.79	4.32	18.66	588.29
	5	33.12	4.57	20.88	607.31
	6	33.67	4.92	24.21	625.19 [✓]
$\omega_2 = 1.2$	6	32.83	4.38	19.18	592.71
	7	33.22	4.71	22.18	637.89
	8	33.77	4.98	24.8	656.81 [✓]

Table 8 List of optimal solutions for TP2

<i>Parameters</i>	n^*	Q^*	L_s^{l*}	L_s^{c*}	Π_{TPL}^{msp}
$\mu_2 = 2.5$	1	33.16	3.98	15.84	579.25
	2	33.64	4.08	16.65	597.41
	3	34.12	4.74	22.47	616.38 [✓]
$\mu_2 = 3.5$	4	34.56	4.82	23.23	589.26
	5	34.62	4.95	24.50	615.75
	6	34.81	4.75	22.56	633.65 [✓]
$\omega_2 = 0.6$	7	34.21	4.52	20.43	599.62
	8	34.87	4.85	23.52	639.24
	9	35.03	5.06	25.60	657.58 [✓]
$\omega_2 = 1.2$	10	35.26	5.13	26.31	633.22
	11	35.56	5.46	29.81	678.97
	12	35.93	5.76	33.18	698.35 [✓]

6.3 Managerial outcomes

- 1 Nowadays, marketing has become digitalised; for such cases, customer segmentation is the source of income for the manufacturer.
- 2 There are several factors to keep TPL for the logistics companies and manufacturers. The most things is to save time and money and provide quality products for the customers.
- 3 The logistics service level depends on the demand. It helps to enhance the company's name and sales in a market.
- 4 The significance of the revenue sharing contract is helpful to identify the maximum profit for manufacturer and retailer in both with and without market segmentation cases.

7 Conclusions

This two-echelon sustainable inventory model with TPL and warehouse financing system concludes that SC participant's optimal decisions and profits increase with logistics service sensitivity coefficient and consumer segmentation. We have constructed the objective functions and solution methodologies for the models I and II. In this paper, we discussed the fundamental managerial uses of TPL and warehouse financing in the SCM. Further, the revenue sharing contract is the factor in evaluating the problem and obtaining the maximum profit for both manufacturer and retailer. Customer segmentation is another key to increasing sales and profit in a market. In this proposed model, we introduced the customer demand into two parts: quality and price-oriented demand of customers. Based on the numerical solutions, we conclude that the SC's maximum profit occurs in the customer's price-oriented demand for both testing problems 1 and 2.

7.1 Future plans

To extend the factors of TPL in the automobile industry and resource management systems. To analyse the role of customer segmentation in machine learning and airline industries. To identify the impact of revenue sharing contract in newsboy-newsvendor problem and pricing mechanism under demand uncertainty.

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