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Study of porous fin with temperature-dependent thermal conductivity and internal heat generation

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Abstract: In this paper, we consider mathematical model of heat transfer in rectangular porous fin with temperature-dependent thermal conductivity and internal heat generation. Governing equation and boundary conditions are presented in dimensionless form. In this paper, we consider three cases for temperature dependent thermal conductivity namely, constant, linear and exponential function of temperature. We have applied three different methods for solution of the problem, i.e., Legendre wavelet collocation method, finite difference method and moment method. In the particular case, we obtained the exact solution, it is compared with these three methods and found good agreement. Effect of various parameters like thermal conductivity, thermo-geometric properties, porosity, heat transfer rate and internal heat generation are presented in the form of tables and figures. The Legendre wavelet collocation method gives high accuracy. The novelty of the work is that a highly nonlinear problem is solved by a hybrid numerical method.

Keywords: wavelet collocation method; porous fin; thermal conductivity; internal heat generation; finite; moment.

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1 Introduction

Heat is thermal energy that arises in transit due to the temperature difference. The process by which energy is transferred in the form of heat is known as heat transfer. When a warmer object comes in contact with a cooler one, energy from the warmer object naturally moves towards the cooler one until both objects reach the same temperature. For releasing heat into the environment, fins are used. Fins are extended surfaces affixed to larger structures or bodies. The rate of heat transfer is contingent upon the surface area of the fin attached to the hot surface, with greater surface area results in a higher rate of heat transfer. The most commonly used material for manufacturing fins are aluminium alloys. Porous fins have gained popularity in recent time due to their simple design and their

effectiveness in various combined heat transfer applications. There are various types of fin shapes available for releasing heat into the environment, including rectangular, annular, elliptic, parabolic, etc. Basavarajappa et al. (2020) said that the applications of fins are increasing in a wide range of industries, including air conditioning, refrigeration, internal combustion engines, etc. Through the application of mathematical formulations, it becomes possible to develop new fin structures, improve fin quality, streamline material expenses and uphold the efficiency of fins.

Numerous researchers have investigated different issues related to fins or heat exchangers. Firstly, (Harper and Brown, 1923) presented an analytical solution to the fin problem. Kundu et al. (2012) investigation centered on evaluating the performance and conducting an optimal design of a porous fin. The Adomian Decomposition

method was employed to derive the solution. The study's findings indicated that heat transfer rates are notably higher in cases with exponential profiles featuring negative power factors when contrasted with rectangular profiles. Furthermore, they demonstrated that the fin's performance excelled when employing exponential profiles characterised by positive power factors in comparison to other profile shapes. Hatami and Ganji (2013) explore heat transfer rates specifically in the context of circular porous fins. The fourth order Runge-Kutta and the least square method was employed to derive the solution. Their analysis led to the conclusion that the results obtained through the least square method exhibited a notable degree of agreement. Hatami and Ganji (2014) conducted an investigation into the refrigeration efficiency and temperature profile in fully wet circular porous fins. The analysis involved the utilisation of both the least square method and the fourth-order Runge-Kutta method to derive the solution. Ma et al. (2017) utilised the spectral element method to calculate heat transfer involving radiation, convection, and conduction in porous fins with motion. The model involved the application of Chebyshev polynomials to define basic functions for individual elements and the solution region was partitioned into discrete non-intersecting sections by using finite element method meshing tools. Oguntala and Abd-Alhameed (2017) utilised the Haar wavelet collocation method to carry out a thermal performance of a porous fin characterised by the properties that vary with temperature. The study's findings indicate that as the fin's porosity increases, we see a significant improvement in both thermal performance and heat transfer from the fin. Khatami and Rahbar (2019) utilised the concepts and principles of the second law of thermodynamics to explore and evaluate the performance of porous fins. Arslanturk (2005) utilised the adomian decomposition method to deduce the temperature profile within a fin, which was represented as an infinite power series. Furthermore, he assessed the efficiency of the fin. Heat transfer in a moving fin subjected to convection and radiation with temperature dependent properties was conducted by (Aziz and Khani, 2011; Aziz and Bouaziz, 2011). They utilised the homotopy analysis method to derive the solution. Aziz and Torabi (2012) conducted a study involving the analysis of a convective-radiative fin characterised by the properties that varies with temperature. For the solution of this problem, they employed both the fourth and fifth-order Runge-Kutta-Fehlberg methods. Chang (2005) employed the adomian decomposition method to investigate the thermal attributes of a annular fin. Chiu (2002) utilised the adomian decomposition method to derive the temperature profile within the fin. Additionally, efficiency of the fin in cases where thermal conductivity exhibited a linear dependence on temperature was calculated. This method yields an analytical solution represented as an infinite power series. Pakdemirli and Sahin (2006) explored a nonlinear heat transfer fin problem, where thermal conductivity varies with temperature. They used Runge-Kutta algorithm for finding the solution. Singh et al. (2013) employed the wavelet collocation method to investigate the temperature

profile in a continuously moving fin with temperature dependent properties. They compared the outcomes of wavelet collocation approach to the exact results, they observed a remarkable similarity between the results. Also, they give the study of different parameters on surface heat loss. Kahalerras and Targui (2008) conducted a study focusing on improving heat transfer within double pipe heat exchangers by adding porous fins to the outer surface of the inner cylinder. They used finite element method for the solution. Chuan et al. (2015) presented the new design concept for micro-channel heat sinks, where traditional solid fins were replaced by porous fins with the aim of reducing pressure drops across the heat sink. Cuce and Cuce (2015) extensively analysed the thermal characteristics of straight fins, considering cases with constant and temperature dependent thermal conductivity. They used finite difference method for the solution. Bhanja et al. (2014) devised a unique analytical model to assess fin efficiency, providing insights into temperature profile and optimal design parameters for a porous moving fin experiencing heat loss through both convection and radiation. The adomian decomposition method was utilised to derive the solution. Remarkably, the study revealed that the moving porous fin exhibited significantly greater heat dissipation compared to the stationary fin.

The objective of this study is to authenticate a mathematical model and simulation technique for analysing the temperature profile in a rectangular porous fin. The investigation concentrates on factors, i.e., internal heat generation and thermal conductivity, which vary with temperature. We explore three different situations concerning temperature dependent thermal conductivity, i.e., constant, linear and exponential function for temperature. To address our research objective, we employ three distinct methods: Legendre wavelet collocation, finite difference and moment. In particular scenario, we compute an exact results, compared it with the outcomes of all these methods and found good agreement. Effect of various parameters like thermal conductivity, thermo-geometric properties, porosity, heat transfer rate and internal heat generation are presented in the form of tables and figures.

2 Formulation of problem

Figure 1 displays a visual representation of a rectangular porous fin. The fin possesses dimensions, including a length L , a thickness t , and is positioned in an environment where convective heat transfer takes place on both surfaces at temperature T_{∞} . Oguntala and Abd-Alhameed (2017) considering the porous media as uniform, isotopic, and immersed in a uniphase fluid. The physical attributes of both the solid and the fluid are treated as constants except the variable density of the liquid, which could impact the buoyancy term. The fluid and porous media within the domain are in a state of thermodynamic equilibrium, while any surface radiative transfers and non-Darcian impacts are considered insignificant. The temperature change within the fin is limited to a one-dimension, implying that it varies

along its length while staying unchanged over time. It is assumed that the tip of the fin is isothermal. Using Darcy's model and its assumptions, the governing equation for the heat transfer in the fin is given as:

$$\frac{d}{dx} \left[k_{eff}(T) \frac{dT}{dx} \right] - \frac{h(T - T_\infty)}{t} - \frac{\rho c_p g \beta' K (T - T_\infty)^2}{tv_f} + q_a(T) = 0, \quad (1)$$

and the specified boundary conditions (BCs) are as follows:

$$x = L, \quad T = T_b, \quad (2)$$

$$x = 0, \quad \frac{dT}{dx} = 0. \quad (3)$$

Here, we focus on thermal conductivity and internal heat generation. These factors are influenced by temperature, which are described as:

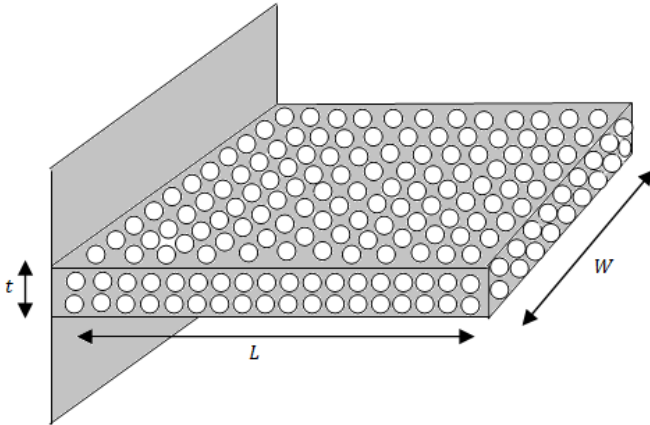
$$k_{eff}(T) = k_a f \left(\frac{T - T_0}{T_b - T_\infty} \right), \quad (4)$$

$$q_a(T) = q_a [1 + \psi(T - T_\infty)], \quad (5)$$

In this study, we consider thermal conductivity as:

- constant, i.e., $k_{eff}(T) = 1$
- linear, i.e., $k_{eff}(T) = k_a [1 + \lambda(T - T_\infty)]$
- exponential, i.e., $k_{eff}(T) = k_a e^{\lambda(T - T_\infty)}$.

Figure 1 Rectangular porous fin geometry



Putting equations (4) and (5) into equation (1), we get

$$\frac{d}{dt} \left[f \left(\frac{T - T_0}{T_b - T_\infty} \right) \frac{dT}{dx} \right] - \frac{h(T - T_\infty)}{k_a t} - \frac{\rho c_p g \beta' K (T - T_\infty)^2}{k_a t v_f} + q_a [1 + \psi(T - T_\infty)] = 0, \quad (6)$$

Instigating dimensionless variables and parameters:

$$X = \frac{x}{L}, \quad \theta = \frac{T - T_\infty}{T_b - T_\infty},$$

$$Ra = \left(\frac{\beta' g T_a t^3}{v_f^2} \right) \left(\frac{\rho c_p v_f}{k_{eff,a}} \right),$$

$$Q = \frac{qv_f t}{\rho c_p g \beta' K (T_b - T_\infty)^2},$$

$$M^2 = \frac{hL^2}{k_{eff,a} t}, \quad Da = \frac{k}{t^2},$$

$$S_h = \frac{Ra Da \left(\frac{L}{t} \right)^2}{k_{eff,a}}, \quad \beta = \lambda(T_b - T_\infty),$$

$$\gamma = \psi(T_b - T_\infty), \quad (7)$$

using equation (7), the dimensionless form of the model equation (6) subjected to the BCs equations (2)–(3) come out to be:

$$f(\theta) \frac{d^2 \theta}{dX^2} + f'(\theta) \left(\frac{d\theta}{dX} \right)^2 - M^2 \theta - S_h \theta^2 + S_h Q (1 + \gamma \theta) = 0, \quad (8)$$

the conditions at the boundaries are:

$$X = 1, \quad \theta = 1, \quad (9)$$

$$X = 0, \quad \frac{d\theta}{dX} = 0. \quad (10)$$

3 Solution of the problem

In this section, we present the exact result for a specific case, along with numerical solutions achieved through the utilisation of three methods namely the Legendre wavelet collocation, finite difference and moment.

3.1 Exact solution

For the validity of the numerical method, in a particular case, we compute the exact result. We put $f(\theta) = 1$ and $S_h = 0$ in equation (8), we get

$$\frac{d^2 \theta}{dX^2} - M^2 \theta = 0, \quad (11)$$

and BCs are provided in equations (9) and (10).

The exact solution of equation (11) is:

$$\theta = \frac{\cosh MX}{\cosh M}. \quad (12)$$

3.2 Operational matrix of integration

The matrix that represents the integration operational properties of the Legendre wavelet is given by:

$$\psi_{n,m} = \begin{cases} \sqrt{(m+1/2)2^{k/2}} P_m \\ (2^k X - \hat{n}), & \frac{\hat{n}-1}{2^k} \leq X \leq \frac{\hat{n}+1}{2^k}, \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

where $n = 1, 2, \dots, 2^{k-1}$, $m = 0, 1, \dots, S-1$ and k is the positive integer. Here, $P_m(X)$ signifies the Legendre polynomial of degree m , and $\psi(X)$, as defined in equation (13), is derived as follows:

$$\int_0^X \psi(t)dt = P\psi(X), \in [0, 1),$$

where P is the operational matrix of order $2^{k-1}M \times 2^{k-1}M$ and $k = 1$, which was investigated by Razzaghi and Yousefi (2001).

3.3 Legendre wavelet collocation method

Let

$$\theta'' = C^T \psi(X), \quad (14)$$

integrating equation (14) twice from 0 to X , and using BC , we obtain

$$\theta'(X) = C^T P\psi(X), \quad (15)$$

and

$$\theta(X) = 1 - C^T P^2\psi(1) + C^T P^2\psi(X), \quad (16)$$

putting equations (14), (15) and (16) in equation (8), we get

$$\begin{aligned} & f(\theta)[C^T \psi(X)] + f'(\theta)[C^T P\psi(X)]^2 \\ & - M^2[1 - C^T P^2\psi(1) + C^T P^2\psi(X)] \\ & - S_h[1 - C^T P^2\psi(1) + C^T P^2\psi(X)]^2 \\ & + S_h Q[1 + \gamma(1 - C^T P^2\psi(1) + C^T P^2\psi(X))] = 0, \end{aligned} \quad (17)$$

as $\theta(X)$ is an approximation of the results of the equation (17). We choose n collocation points denoted as $(X_i, i = 1, 2, 3, \dots, n)$, such that the residuals $R(X, c_1, c_2, \dots, c_n)$ become zero. It is essential to note that the count of these collocation points and coefficients should be identical.

For temperature dependent thermal conductivity we apply three cases in equation (17), we get:

Case 1 Where thermal conductivity remains constant, the substitution of $f(\theta) = 1$ into equation (17) produces the following outcome:

$$\begin{aligned} & [C^T \psi(X)] - M^2[1 - C^T P^2\psi(1) \\ & + C^T P^2\psi(X)] - S_h[1 - C^T P^2\psi(1) \\ & + C^T P^2\psi(X)]^2 + S_h Q[1 + \gamma(1 \\ & - C^T P^2\psi(1) + C^T P^2\psi(X))] = 0. \end{aligned} \quad (18)$$

Case 2 Where thermal conductivity remains linear, the substitution of $f(\theta) = 1 + \beta\theta$ into equation (17), produces the following outcome:

$$\begin{aligned} & [1 + \beta(1 - C^T P^2\psi(1) \\ & + C^T P^2\psi(X))][C^T \psi(X)] \\ & + \beta[C^T P\psi(X)]^2 - M^2[1 - C^T P^2\psi(1) \\ & + C^T P^2\psi(X)] - S_h[1 - C^T P^2\psi(1) \\ & + C^T P^2\psi(X)]^2 + S_h Q[1 + \gamma(1 \\ & - C^T P^2\psi(1) + C^T P^2\psi(X))] = 0. \end{aligned} \quad (19)$$

Case 3 Where thermal conductivity remains exponential, we put $f(\theta) = e^{\beta\theta}$ in equation (17), we get

$$\begin{aligned} & e^{\beta\theta}[C^T \psi(X)] + \beta e^{\beta\theta}[C^T P\psi(X)]^2 \\ & - M^2[1 - C^T P^2\psi(1) + C^T P^2\psi(X)] \\ & - S_h[1 - C^T P^2\psi(1) + C^T P^2\psi(X)]^2 \\ & + S_h Q[1 + \gamma(1 - C^T P^2\psi(1) + C^T P^2\psi(X))] \\ & = 0. \end{aligned} \quad (20)$$

Solving the system of equations (18), (19) and (20) separately for cases 1, 2 and 3 respectively, by using Newton-Raphson method. This system consists of nine nonlinear algebraic equations, which are evaluated by utilising nine Legendre wavelet basis functions and collocation points in the range (0, 1). The unknowns C_i were obtained and the dimensionless temperature is calculated from the equation (16).

3.3.1 Algorithm (programming working procedure)

- Step 1 Define all of the constants.
- Step 2 Define $P, \psi(x), \psi(1)$.
- Step 3 $C = [c_1, c_2, \dots, c_n]$.
- Step 4 Define $F = R(X, c_1, c_2, \dots, c_n) = 0$.
- Step 5 Using collocation points as $X = 0.1, 0.2, \dots, 0.9$ in step 4, we get, $f = [f_1, f_2, \dots, f_9]$, where $f_i = f(C)$ and $C = [c_1, c_2, \dots, c_9]$.
- Step 6 We get nine nonlinear equations in nine unknowns as follows, $f(c_1, c_2, \dots, c_9) = 0$.
- Step 7 Find Jacobian function, $J = \text{jacobian}(f, C)$.
- Step 8 Using Newton-Raphson method, $C = C(0) - J^{-1}f$, where $C(0)$ are initial values of C .
- Step 9 Putting C again in step 3 and repeat steps 3 to 9 until we get the correct value of C , i.e., $f(C) = 0$.
- Step 10 Compute required temperature $\theta(X) = 1 - C^T P^2\psi(1) + C^T P^2\psi(X)$.

3.4 Convergence analysis

Let $\theta(X)$ be the exact result and $\theta_m(X)$ be the approximate result of the system of equations (8)–(10), i.e.,

$$\theta(X) = \sum_{m=0}^{\infty} b_m \psi_{1m}(X),$$

and

$$\theta_m(X) \approx \sum_{m=0}^{M-1} b_m \psi_{1m}(X),$$

thus, the error is defined as:

$$E_m(X) = \theta(X) - \theta_m(X),$$

$$E_m(X) = \sum_{m=0}^{\infty} b_m \psi_{1m}(X) - \sum_{m=0}^{M-1} b_m \psi_{1m}(X),$$

$$E_m(X) = \sum_{m=M}^{\infty} b_m \psi_{1m}(X),$$

$$E_m(X) = \sum_{m=M}^{\infty} \langle \theta(X), \psi_{1m} \rangle \psi_{1m}(X),$$

$$\|\theta(X) - \theta_m(X)\|^2 = \sum_{m=M}^{\infty} |\langle \theta(X), \psi_{1m}(s) \rangle|^2.$$

The fact that $\|E_m(X)\|^2 = \sum_{m=0}^{\infty} |\langle \theta, \psi_{1m}(X) \rangle|^2 < \infty$. This implies that the error approaches zero.

Hence

$$\lim_{m \rightarrow \infty} \|E_m(X)\|^2 \rightarrow 0.$$

This shows that the error tends to zero as the number of Legendre wavelet basis function tends to infinite.

3.5 Finite difference method

Han et al. (2005) previously highlighted the use and validity of the finite element technique to examine the heat flow in fins for constant thermal properties. Dukkipati (2011) and Jain et al. (2007) applied finite difference method in different linear and nonlinear problems. We used the central difference approximation in equation (8) to discretise X co-ordinate and switching up the terminology, we get

$$f(\theta_i)(\theta_{i+1} - 2\theta_i + \theta_{i-1}) + f'(\theta_i) \left(\frac{\theta_{i+1} - \theta_{i-1}}{2} \right)^2 - M^2 \theta_i h^2 - S_h \theta_i^2 h^2 + S_h Q h^2 (1 + \gamma \theta_i) = 0, \quad (21)$$

Equation (21) is valid for interior nodes, i.e., $(i = 2, 3, \dots, n-1)$. From the given BC's first order derivative replaced by central difference approximation, we get $\theta_2 = \theta_0$, the fin's base temperature and at last node, i.e., $X = 1, \theta = 1$. Thus, we get nonlinear equations for nodes, i.e., $(i = 1, 2, 3, \dots, n)$ from equation (21). Thus, we have:

- when $i = 1$

$$f(\theta_1)(\theta_2 - 2\theta_1 + \theta_0) + f'(\theta_1) \left(\frac{\theta_2 - \theta_0}{2} \right)^2 - M^2 \theta_1 h^2 - S_h \theta_1^2 h^2 + S_h Q h^2 (1 + \gamma \theta_1) = 0, \quad (22)$$

- when $i = 2$

$$f(\theta_2)(\theta_3 - 2\theta_2 + \theta_1) + f'(\theta_2) \left(\frac{\theta_3 - \theta_1}{2} \right)^2 - M^2 \theta_2 h^2 - S_h \theta_2^2 h^2 + S_h Q h^2 (1 + \gamma \theta_2) = 0, \quad (23)$$

- $\vdots$

- when $i = n$

$$\theta_n = 1 \quad (24)$$

We solve these nonlinear equations with the help of Newton-Raphson method.

3.6 Moment method

In this meathod, we consider a trial solution with constants, which satisfy the BS's. Substituting this trial solution in equation (8), we get the residual $R(X)$. The weight function is selected from a family of polynomials, such as:

$$W_i(X) = X^i, \quad i = 0, 1, 2, \dots, n \quad (25)$$

the expression for the moment method is formulated as follows:

$$\int_X W_i(X) R(X) dX = 0, \quad (26)$$

using equation (25) into equation (26), we get

$$\int_X X^i R(X) dX = 0, \quad (27)$$

next, we compute the residual using MATLAB software and determine the temperature profile within the fin.

4 Result and discussion

The present dimensionless model equations (8)–(10) is a boundary value problem with a system of nonlinear differential equations. To determine a solution for this model, we employed three distinct techniques, i.e., *LWCM*, *MM*, and *FDM*.

These methods were utilised to approximate the temperature profile within a rectangular porous fin. In particular scenario, we compute an exact result, it is compared with these three methods and found good agreement as shown in Table 1. For computation of the results, we employ the subsequent reference values: $\gamma = 0.2$, $\beta = 0.0$, $S_h = 0.0$, $Q = 0.4$, and $M = 0.3$. From Table 1, we observed that results of these methods are well matched with exact results. To check the maximum accuracy among these methods, we compute error analysis as shown in Table 2. An observation reveals that, the error in *LWCM* is less as compared to *MM* and *FDM*. Which shows the validity of *LWCM*. Further, we compute numerical results using the different numbers of wavelet basis function as shown in Table 3. From this table, we concluded that by increasing the number of wavelet basis function in our computation, accuracy increases. The percentage error for the 3, 6 and 9 basis function is presented in Table 4. It has been observed that the highest accuracy is achieved in nine basis function. Hence, further computation have been carried out by using 9 Legendre wavelet basis functions in

LWCM. Table 5 represents the comparative analysis of different parameters on temperature profile in fin for cases 1, 2 and 3. In this table, we compute results for $X = 0.0, 0.4$ and 0.8 . It has been observed that, the temperature in porous fin for case 3 is highest for each parameter as compared to the cases 2 and 1. It indicates that, case 1 is more efficient then other cases for cooling process.

Figure 2 Consequences of thermal conductivity parameter on temperature profile in fin (see online version for colours)

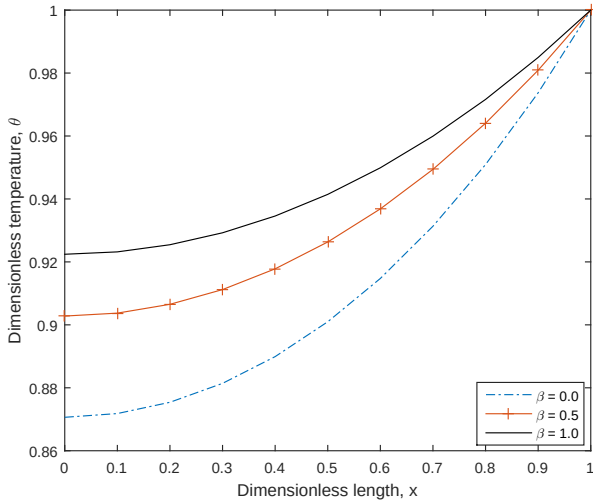
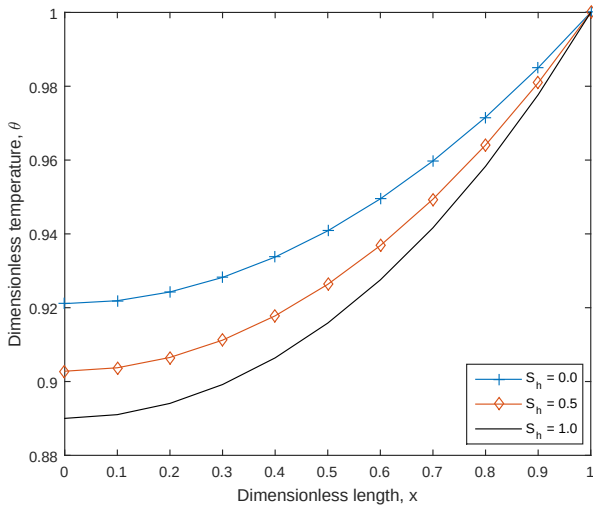


Figure 3 Impact of the porosity parameter on temperature profile in fin (see online version for colours)



The formulation of temperature dependent thermal conductivity and internal heat generation, were discussed previously in Section 3 for rectangular porous fin. We explore three different situations concerning temperature dependent thermal conductivity, i.e., constant, linear and exponential function for temperature. Effect of various parameters like thermal conductivity (β), thermo-geometric (M), porosity (S_h), heat transfer rate (Q) and internal heat generation (γ) are showcased within Figures 2–9. The reference values for parameters are taken as $\beta = 0.5$, $S_h = 0.5$, $\gamma = 0.5$, $M = 0.5$ and $Q = 0.5$. Figure 2 display

the consequences of thermal conductivity parameter (β) on the temperature profile in fin for case 2. From this figure, we found that the temperature in fin goes up with higher thermal conductivity values. Figure 3 displayed the impact of the porosity parameter (S_h) on the temperature profile within the fin for case 2. We noticed that as the value of S_h rises, the temperature within the fin drops. This finding indicates that the fin's temperature tends to decrease with lower Darcy and Rayleigh number or higher effective thermal conductivity ratio. Figure 4 show the consequences of a internal heat generation parameter (γ) on the fin's temperature profile in case 2. We observed that the temperature within the fin goes up with higher values of γ . Figure 5 displays that how thermo-geometric parameter (M) affect the fin temperature profile in case 2. We concluded that as the value of M rises, temperature within the fin drops.

Figure 4 Consequences of internal heat generation parameter on temperature profile in fin (see online version for colours)

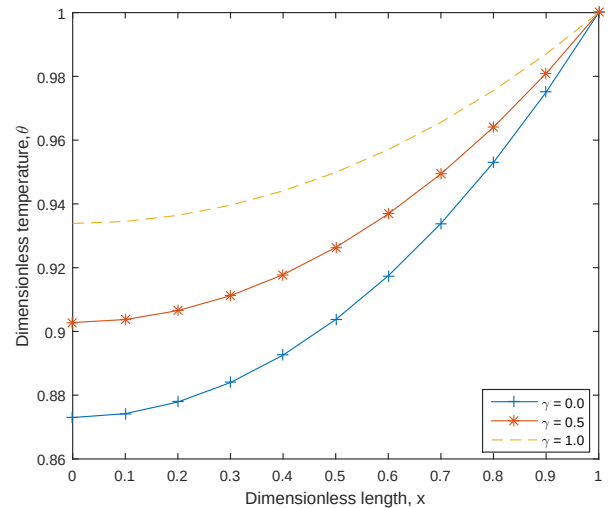


Figure 5 The impact of thermo-geometric parameter on temperature profile in the fin (see online version for colours)

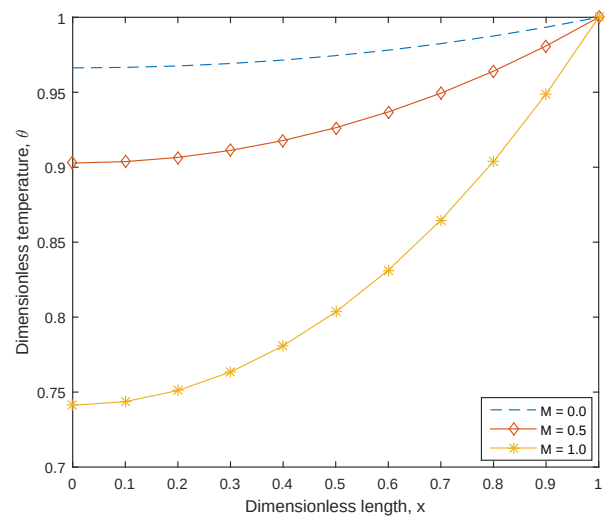


Table 1 A comparative analysis of exact results with *LWCM*, *FDM* and *MM* in case 1

X	<i>Exact</i>	<i>LWCM</i>	<i>FDM</i>	<i>MM</i>
0.0	0.956627911900248	0.956627911900249	0.956628107692020	0.956627909831415
0.1	0.957058426747764	0.957058426747753	0.957058620610012	0.957058439455950
0.2	0.958350358782735	0.958350358782722	0.958350546851783	0.958350380263701
0.3	0.960504870831200	0.960504870831203	0.960505049233111	0.960504879922477
0.4	0.963523902099437	0.963523902099422	0.963524066941668	0.963523889896722
0.5	0.967410169919377	0.967410169919378	0.967410317284990	0.967410144945963
0.6	0.972167172194391	0.972167172194380	0.972167298133904	0.972167151859637
0.7	0.977799190547633	0.977799190547621	0.977799291072906	0.977799187428294
0.8	0.984311294175793	0.984311294175771	0.984311365252331	0.984311305651169
0.9	0.991709344411717	0.991709344411725	0.991709381990155	0.991709354180129
1.0	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000

Table 2 Error analysis of exact results with *LWCM*, *FDM* and *MM* in case 1

X	Percentage error		
	<i>LWCM</i>	<i>FDM</i>	<i>MM</i>
0.0	1.0445×10^{-13}	2.04669×10^{-5}	2.16263×10^{-7}
0.1	1.14844×10^{-12}	2.02561×10^{-5}	1.3278×10^{-6}
0.2	1.35543×10^{-12}	1.96242×10^{-5}	2.2415×10^{-6}
0.3	3.12086×10^{-13}	1.85738×10^{-5}	9.4651×10^{-7}
0.4	1.55554×10^{-12}	1.710×10^{-5}	1.26647×10^{-6}
0.5	1.03286×10^{-13}	1.5233×10^{-5}	2.58147×10^{-6}
0.6	1.13058×10^{-12}	1.29545×10^{-5}	2.09169×10^{-6}
0.7	1.22626×10^{-12}	1.02808×10^{-5}	3.19016×10^{-7}
0.8	2.23328×10^{-12}	7.22094×10^{-6}	1.1658×10^{-6}
0.9	8.06043×10^{-13}	3.78926×10^{-6}	9.8501×10^{-7}
1.0	0	0	0

This indicates that, the fin's temperature goes down with higher effective thermal conductivity value or shorter fin lengths. Figure 6 depicts the impact of heat transfer rate (Q) on the temperature profile within the fin for case 2. Our observation reveals that as the value of Q rises, temperature within the fin also rises. This suggests that the fin's temperature goes up as both the kinematic viscosity and fin thickness rises, or as both the specific heat and coefficient of thermal expansion decrease. Figure 7 depicts the consequences thermal conductivity parameter (β) on temperature profile in fin for cases 1, 2 and 3. From this figure, we concluded that the temperature in fin rises with higher thermal conductivity values. Notably, the temperature distribution for case 3 is maximum as compared to cases 2 and 1. Consequently, in terms of the cooling process, case 1 demonstrates greater efficiency than case 3. Figure 8 displays the combined effect of thermal conductivity parameter (β) and thermo-geometric parameter (M) on temperature profile in fin for case 2. We observe that as the value of β and M rises, temperature in fin increases and decreases respectively. In this figure, temperature goes down rapidly with higher values of M . Figure 9 presented the effect of thermo-geometric parameter (M) on temperature profile in fin for cases 1, 2 and 3, when $X = 0.0, 0.4$ and 0.8 . We observe that as the value of X rises, temperature in fin also rises for cases 1, 2 and 3. Temperature is lowest when thermal conductivity

is constant as compared to linear and exponential function of temperature. Figure 10 presented the effect of different Legendre wavelet basis function with exact results on temperature profile in fin. We observed that, by increasing the number of wavelet basis, accuracy increases.

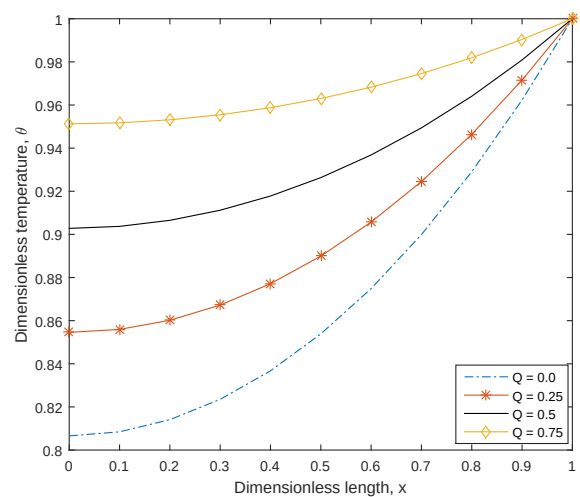
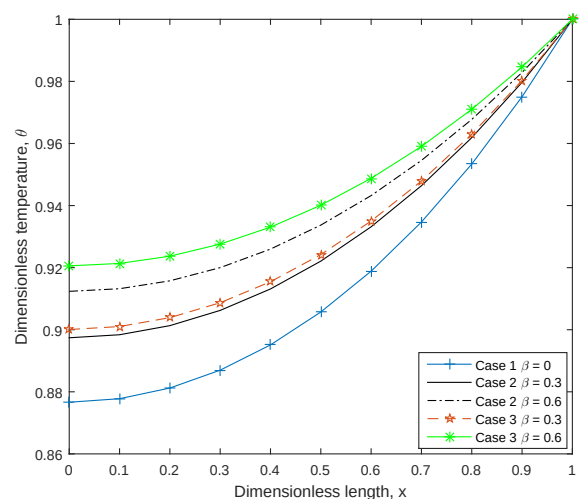
Figure 6 Outcomes of heat transfer rate on temperature profile in the fin (see online version for colours)**Figure 7** Impact of thermal conductivity parameter on temperature profile in fin for cases 1, 2 and 3 (see online version for colours)

Table 3 Comparative analysis of exact result with different Legendre wavelet basis functions in case 1

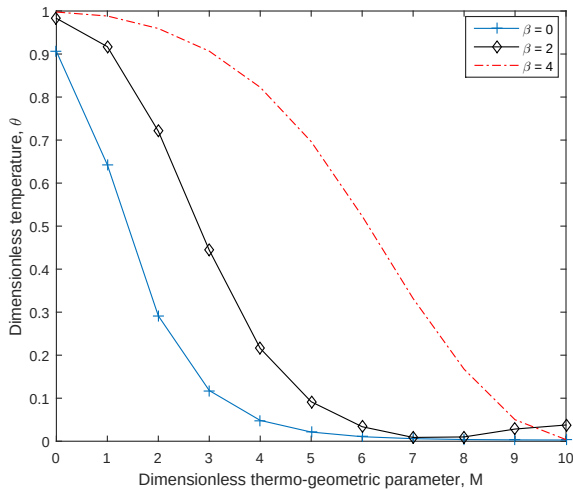
X	Exact	3 LW basis function	6 LW basis function	9 LW basis function
0.0	0.956627911900248	0.956691463388555	0.956627911640626	0.956627911900249

Table 4 Error analysis of exact solution with different Legendre wavelet basis functions in case 1

X	Percentage error		
	3 LW basis function	6 LW basis function	9 LW basis function
0.0	$6.6432818357576 \times 10^{-3}$	$2.7139286586035 \times 10^{-8}$	$1.044502998222 \times 10^{-13}$

Table 5 The comparative analysis of S_h , Q , M and γ on temperature profile in fin for cases 1, 2 and 3

Cases	X	S_h		Q		M		γ	
		0.0	0.5	0.0	0.5	0.0	0.5	0.0	0.5
Case 1	0.0	0.88682	0.87063	0.74643	0.87063	0.95401	0.87063	0.83281	0.87063
	0.4	0.90461	0.88991	0.78415	0.88991	0.96095	0.88991	0.85756	0.88991
	0.8	0.95872	0.95092	0.90363	0.95092	0.98272	0.95092	0.93628	0.95092
Case 2	0.0	0.92418	0.90668	0.82107	0.90668	0.96675	0.90668	0.87939	0.90668
	0.4	0.93622	0.92102	0.84868	0.92102	0.97188	0.92102	0.89789	0.92102
	0.8	0.97255	0.96538	0.93382	0.96538	0.98770	0.96538	0.95520	0.96538
Case 3	0.0	0.93119	0.91412	0.83599	0.91412	0.96934	0.91412	0.88903	0.91412
	0.4	0.94215	0.92741	0.86151	0.92741	0.97409	0.92741	0.90618	0.92741
	0.8	0.97514	0.96830	0.93972	0.96830	0.98869	0.96830	0.95901	0.96830

Figure 8 Combined consequences of thermal conductivity parameter (β) and thermo-geometric parameter (M) on temperature profile in fin (see online version for colours)

5 Conclusions

In this paper, we consider a mathematical model and simulation technique for analysing heat transfer in a porous fin with temperature-dependent thermal conductivity and internal heat generation. The entire analysis has been done in dimensionless form. To address our research objective, we employ three distinct methods: *LWCM*, *MM* and *FDM*. Results are presented in the form of figures and tables. The main significant outcomes are summarised as: when, we increase the values of thermal conductivity, heat transfer rate and internal heat generation,

temperature in fin also increases. If the values of porosity and thermo-geometric parameter goes up, temperature in fin goes down. Temperature in case 1 is noticeably lower in comparison to cases 2 and 3. This observation implies that case 1 exhibits enhanced efficiency in the context of the cooling process within the fin. We also observed that, by increasing the number of wavelet basis, accuracy in results increases. In a particular case, results of *LWCM*, *MM* and *FDM* are compared with the exact result and found that *LWCM* gives high accuracy in comparison to *FDM* and *MM*. Hence the present method *LWCM* is applicable for highly nonlinear fin problem. It can be used in the engineering industry to improve the quality of fins.

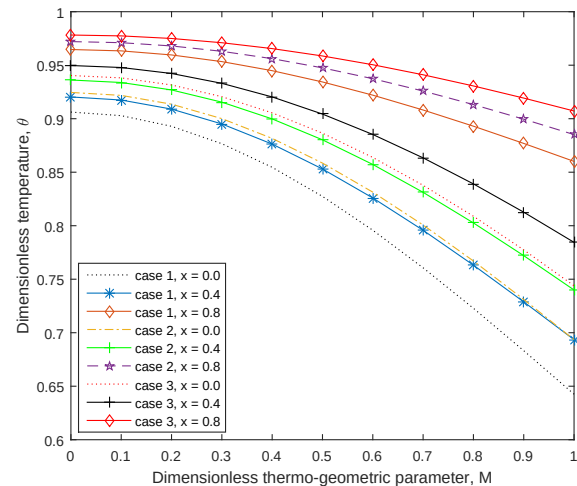
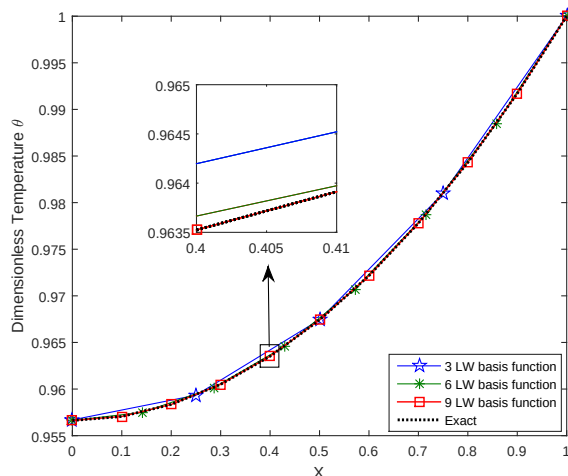
Figure 9 Impact of thermo-geometric parameter on temperature profile in fin for cases 1, 2 and 3 (see online version for colours)

Figure 10 Outcomes of different Legendre wavelet basis function with exact results on temperature profile in fin (see online version for colours)



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Nomenclature

β	Thermal conductivity parameter
P	Fin perimeter (m)
β'	Coefficient of thermal expansion (K^{-1})
K_{eff}	Effective thermal conductivity ratio (W/mK)
T	Local fin temperature (K)
T_b	Fin base temperature (K)
T_∞	Sink temperature for convection (K)
t	Thickness of the fin (mm)
h	Heat transfer coefficient (W/m^2K)
L	Length of the fin (m)
g	Gravity constant (m/s^2)
c_p	Specific heat ($J/kg - K$)
K	Permeability of the porous fin (m^2)
k_a	Thermal conductivity at the base ($Wm^{-1}k^{-1}$)
q	Internal heat generation (W/in^3)
x	Axial length measured from fin tip (m)
Da	Darcy number (m^2)
ν	Kinematic viscosity (m^2/s)

<i>Dimensionless parameter</i>	
θ	Temperature in fin
Ra	Rayleigh number
X	Length of the fin
Q	Heat transfer rate per unit area
γ	Internal heat generation
S_h	Porosity
M	Thermo-geometric parameter

<i>Abbreviation</i>	
$LWCM$	Legendre wavelet collocation method
FDM	Finite difference method
MM	Moment method
$BC's$	Boundary conditions
