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Model predictive control with constraints based on PSO and fuzzy logic applied to the control of coupled longitudinal-lateral dynamics of the autonomous vehicle

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Abstract: In this paper, a strategy for controlling the longitudinal and lateral dynamics of an autonomous vehicle is developed. This strategy is based on the model predictive control (MPC) with constraints combined with the LPV form. The three degrees of freedom (3DOF) model of the autonomous vehicle is used. The cornering stiffness is approximated by a fuzzy logic type, Takagi-Sugeno, with the aim of finally approximating the nonlinear lateral forces. In order to improve the systems performance, constraints for controller inputs and also for system outputs are defined. The MPC weights are determined using the particle swarm optimisation (PSO). The objective of this strategy is to follow the reference trajectory of the autonomous vehicle while reducing the lateral and longitudinal displacement error. The steering angle and the longitudinal acceleration are the control inputs, the outputs of this system

are the longitudinal velocity, the yaw angle, the longitudinal and lateral displacement. The system is multi-input and multi-output (MIMO) and has non-linear dynamics. Simulation results show some improvements over the literature.

Keywords: autonomous vehicles; MPC; model predictive control; MPC constraints; LPV system; PSO; particle swarm optimisation; MIMO system; nonlinear dynamic; path planning; fuzzy logic.

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Biographical notes: Rachid Aliko received his Master degree in 2009 from the University Sidi Mohamed Ben Abdellah, Faculty of Sciences, Fes Morocco. He is a PhD student in the same faculty. He has numerous publications in reputable international journals. His current research interests include control and modelling of vehicle, vehicle dynamics, robotics, super-twisting sliding mode control, model predictive control, fuzzy logic, neural networks, and particle swarm optimisation.

El Mehdi Mellouli obtained his doctorate in 2013 from Sidi Mohamed Ben Abdellah University, Faculty of Sciences, Fez Morocco. His doctorate focused on the modelling and control of nonlinear and multivariable systems using fuzzy logic and various control techniques. Since 2017, he has been a Professor at the National School of Applied Sciences, University of Fez Morocco. He has numerous publications in reputable international journals. His research has focused on the modelling and generation of the intelligent control law for autonomous systems, robotics, renewable energy systems and unknown dynamic systems.

El Houssaine Tissir received his High study Diploma (DES) and the state Doctorate from University Sidi Mohammed Ben Abdellah, Faculty of sciences, Morocco in 1992 and 1997, respectively. He is now a Professor at the University Sidi Mohammed Ben Abdellah. His research interests include robust control, singular systems, switched systems and systems with saturating actuators.

1 Introduction

Autonomous vehicles are one of the areas of interest for researchers. Artificial intelligence has been introduced to invent methods to improve the performances and to make the intelligent autonomous vehicle able to overcome the challenges caused by the driver, the infrastructure, or the vehicle itself. The exploitation of artificial intelligence domain makes the field of autonomous vehicle very advanced. The robust and nonlinear commands allow giving precise results and the autonomous vehicle becomes more efficient and has more resistance against disturbances. There are currently five levels of

autonomy. At level 1, the driver can delegate some of the tasks normally assigned to him, while he must perform all the other commands related to driving. At level 5, the vehicle is completely autonomous. Today high-speed autonomous driving is the main challenge for the researchers. The perception of the environment, the generation of trajectory and the control of the vehicle are the main stages for the autonomous vehicle to drive itself and follow the reference trajectory.

This document focuses specifically on the lateral and longitudinal control of the autonomous vehicle. The artificial intelligence allows improving the automatic steering of the autonomous vehicle and ensures the tracking of the reference trajectory. The control law, must be robust, can be applied to nonlinear systems, able to handle parameter variations and overcome disturbances. Adaptive cruise control of the autonomous vehicle based on sliding mode controller in Alika et al. (2024). A complex system used to produce three-dimensional (3D) components in Gandhi et al. (2013).

Fuzzy logic, particle swarm optimisation (PSO) and neural network are some of the most important methods in artificial intelligence, in Boumhidi et al. (2018) the fuzzy sliding mode control for three-tank system based on linear matrix inequality has been addressed. A modified sliding mode controller based fuzzy logic for a variable speed wind turbine has been made in Mellouli et al. (2017), the Direct adaptive fuzzy sliding mode controller without reaching phase for an uncertain three-tank-system has been elaborated in Mellouli and Boumhidi (2016), A Robust Adaptive Fuzzy Sliding Mode Controller for a Variable Speed Wind Turbine has been studied in Mellouli et al. (2015). In Kumar et al. (2023), the modified Linear Quadratic Gaussian (LQG) controller is designed and tuned to increase efficiency during an earthquake based on PSO. The neural network technique is used in nonlinear dynamic systems (Kumar et al., 2018; Kumar, 2022, 2023a, 2023b). An efficient prediction model based on a modified Elman neural network was developed in Gupta and Kumar (2023). Other interesting artificial intelligence methods are used. For example, three methods were tested in Kharola (2023) to control a triple inverted pendulum system mounted on a carriage, including proportional integral derivative (PID), adaptive neuro-fuzzy inference system (ANFIS), and networks neuronal (NN).

The integration of artificial intelligence techniques is becoming a necessity in the field of autonomous vehicles. Fuzzy logic, PSO and neural network are present in this area, an adaptive higher order sliding mode control by fuzzy logic to control the autonomous vehicle was realised in Alika et al. (2021), the optimisation of STSMC parameters by PSO to control the lateral dynamics of the vehicle has been studied in Alika et al. (2020). In Alika et al. (2023) a strategy for controlling the lateral dynamics of an autonomous vehicle using PSO and neural network (RBFNN) has been developed. In El Youssfi et al. (2020), a control H_∞ for T-S fuzzy vehicle lateral dynamics was used. In Mrazgua et al. (2021), a study of nonlinear active suspension systems (ASS) using a Takagi–Sugeno fuzzy approach. The lateral and longitudinal controls are the centre of interest of the searches now, Wang et al. (2019). Combined longitudinal and lateral control for automated lane guidance of full drive-by-wire vehicles has been studied in Song et al. (2015). A coupled longitudinal and lateral control strategy improving lateral stability for the autonomous vehicle has been proposed in Attia et al. (2012). The Suhail et al. (2023) proposes a control scheme for the design of cruise control for an autonomous battery electric vehicle (BEV), based on super twisting sliding mode control (STSMC). Disturbance Observer-Based Adaptive Sliding Mode Control for Autonomous Vehicles

has been made in Alika et al. (2022). Fault-tolerant-control for compensating actuator failures in ASS is proposed in Mrazgua et al. (2022).

Predictive control is an advanced automatic control technique. Its objective is to control complex industrial systems that have several inputs and outputs, where the simple control is insufficient. This technique is particularly interesting when the systems have significant delays, strongly oscillating, many disturbances, multivariable and nonlinear systems. Several systems use model predictive control (MPC), MPC (Benlahrache et al., 2017; Wijewardana et al., 2016; Bolognani et al., 2009). Using MPC, an experimental study was carried out to control the spatial temperature profile of a distributed parameter heating system in Mankad and Padhiyar (2023). The proposed Lyapunov-based MPC for nonlinear process with online-triggered linearised model is in Wu and Zhao (2023). Model predictive control and controller parameter optimisation of combustion instabilities has been used in Yazar et al. (2018). Autonomous racing using linear parameter varying-model predictive control (LPV-MPC) have been performed in Alcalá et al. (2020).

Predictive Active Steering Control for Autonomous Vehicle Systems has been studied in Falcone et al. (2007). MPC-based yaw and lateral stabilisation via active front steering and braking have been realised in Falcone et al. (2008). Model-predictive-control complex-path tracking for self-driving cars has been worked out in Farag, 2020). A LPV-MPC approach has been developed for the autonomous vehicle in Alcalá et al. (2019).

This work utilises LPV-MPC with fixed and variable inequality constraints. The parameters of this controller are optimised by PSO. The lateral forces are approximated through the cornering stiffness approximation by the fuzzy logic of type Takagi-Sugeno (T-S). The comparison of this proposed controller with the MPC controller used in Falcone et al. (2007) has been done and the results obtained by our proposed method are better. The simulation with MATLAB has given illustrations of the obtained results.

There are several advantages of this work, the use of MPC makes possible to control nonlinear systems with the presence of disturbances, and to put constraints on the speeds, accelerations and controls. The integration of logic fuzzy allows to control the system without knowing the lateral forces which are approximated by this technique. The integration of the artificial intelligence and the variety of choice of weights of MPC allows to show the best method.

The validation of the robustness and the precision of the various proposed techniques are seen from the illustrations presented in the simulation section.

1.1 Related works

In Alcalá et al. (2020), the authors have performed autonomous racing using LPV-MPC. The LPV method is employed to model vehicle dynamics and develop an LPV-model predictive controller (LPV-MPC) that can be calculated online with less computational cost. The optimal time problem is solved using an optimal offline trajectory planner. But they did not combine LPV-MPC with artificial intelligence techniques.

In Falcone et al. (2007), the authors have studied the Predictive Active Steering Control for Autonomous Vehicle Systems. Two approaches have been presented with different computational complexities. In the first approach, the MPC problem is formulated using a nonlinear vehicle model. The second approach is based on successive online linearisation of the vehicle model.

In Farag (2020), the authors have proposed a Model-predictive-control complex-path tracking for self-driving cars. A framework has been proposed to produce an MPC path tracker that provides a way to easily integrate multiple driving objectives by composing a multi-term cost function. Using this method, multiple state variables can be combined into one cost term.

1.2 Motivation

In the areas of autonomous research, the MPC controllers is rarely used and in all cases the use of the MPC remains limited to the treatment of lateral dynamics or longitudinal dynamics separately, which are simple systems and the dynamics are not coupled. On the other hand, MPC control is found with certain constraints on the system and not taking into consideration all the terms influencing the systems performances. The integration of artificial intelligence techniques (PSO, fuzzy logic, etc.) in this type of controller (MPC) does not exist in the literature.

In our work, the choice of the MPC for the nonlinear system that combines both lateral dynamics and longitudinal dynamics becomes from its robustness which makes possible to control complex and nonlinear systems. Integrating the PSO optimisation technique to automatically adjust the MPC controller points is more effective than choosing them randomly. And also, the integration of fuzzy logic in the MPC controller to approximate lateral forces that are considered unknown and non-linear is an added value to the autonomous vehicle system. These advantages motivate our work.

1.3 Novelities and contributions of this paper are as follows

- Our first method's novelty resides in the use of PSO to adjust the MPC weights. Moreover, the novelty lies in the way in which PSO is applied as an optimisation technique in the context of MPC for autonomous vehicle control. Such as PSO is used to optimise the weights or the parameters of the MPC controller. This is innovative because it allows automatic adjustment of control parameters, which is essential for achieving optimal control performance in dynamic and uncertain environments.
- Autonomous vehicles typically operate in complex and dynamic environments with nonlinear dynamics. For that, the second method proposed is MPC with constraints relies on fuzzy logic to control the coupled longitudinal and lateral dynamics of an autonomous vehicle. In this method, the novelty lies mainly in the integration of fuzzy logic within the MPC, in order to approximate and model the non-linear and unknown dynamics of the lateral forces. So the novelty is without knowing the lateral forces we can control this system by approximating these forces using fuzzy logic.
- MPC based on PSO and MPC based on fuzzy logic are tested and compared with MPC controller alone and with the MPC controller used in Falcone et al. (2007).
- Several constraints have been chosen appropriately such that each term has its own sub-objective in the cost function, and the proposed cost function is very comprehensive and includes all the terms that influence the system.

This integration of PSO and fuzzy logic into MPC, improves adaptability, robustness and the ability to handle complex and non-linear dynamics, making it a valuable addition to the field of autonomous vehicle control.

This paper is organised as follows. The dynamic model of the vehicle and the reference trajectory in Section 2 are represented. In Section 3, our MPC with constraints strategy, the integration of fuzzy logic for the approximation of the cornering stiffness and optimisation the weights of this control by PSO are carried out. The simulation results of the proposed controller are presented in Section 4. Section 5 covers the conclusion, remarks, and further researches.

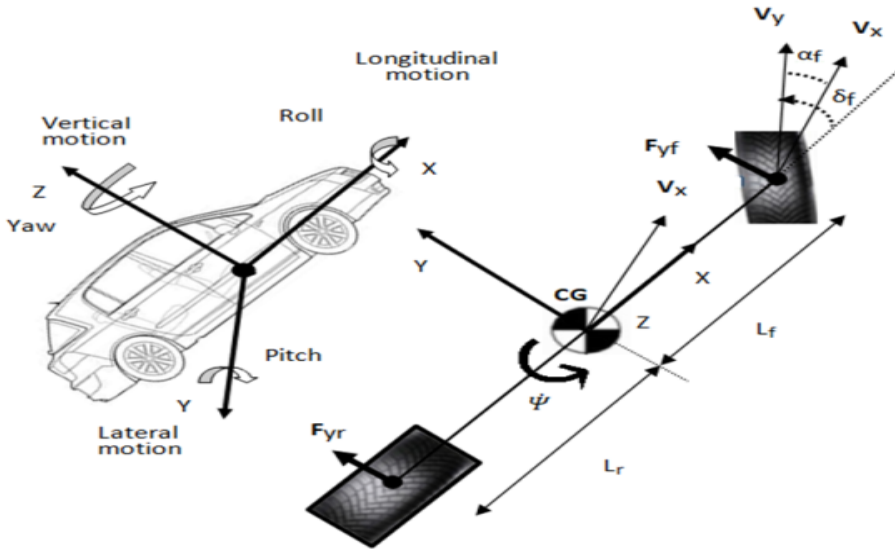
2 System modelling design

2.1 Vehicle dynamics model

An advanced model that describes the vehicle dynamics in both the lateral and longitudinal directions is used.

The dynamic bicycle model, shown in Figure 1, allows representing the lateral and the longitudinal behaviour of the autonomous vehicle (Falcone et al. (2007)). In this nonlinear model (Alcalá et al. (2020)), the vehicle is considered as symmetrical, and the sideslip angles are equals (same axle). The bicycle model is composed of lateral and longitudinal dynamics. The angles are considerable (slip, yaw and direction) which gives the non-linearity of the system. The longitudinal speed v_x is not constant in this system, it is variable in time between 20 m/s and 35 m/s and the dynamics of pitch and roll are neglected.

Figure 1 Bicycle nonlinear model vehicle



The differential equations describing its movement are given by,

$$\begin{cases} \dot{v}_x = a - \frac{F_{yf} \sin \delta_f}{m} - \mu mg + \dot{\psi} v_y \\ \dot{v}_y = \frac{F_{yr}}{m} + \frac{F_{yf} \cos \delta_f}{m} - \dot{\psi} v_x \\ \ddot{\psi} = \frac{F_{yf} \cos \delta_f L_f}{I_z} - \frac{F_{yr} L_r}{I_z} \end{cases} \quad (1)$$

with,

$$\begin{cases} F_{yf} = C_{\alpha f} \left(\delta_f - \frac{v_y}{v_x} - \frac{\dot{\psi} L_f}{v_x} \right) \\ F_{yr} = C_{\alpha r} \left(-\frac{v_y}{v_x} + \frac{\dot{\psi} L_r}{v_x} \right) \end{cases}$$

where v_x , v_y and $\dot{\psi}$ are the body frame velocities, linear in x , linear in y and angular, respectively. δ_f and a are the control actions, steering angle and longitudinal acceleration, respectively. The variables C_f and C_r represent the tyre stiffness coefficients for the front and rear wheels. m and I_z represent the vehicle mass and inertia and L_f and L_r are the distances from the centre of gravity to the front and rear wheel axes, respectively. μ and g are the friction coefficient and the gravity value, respectively.

The position and road geometry of the vehicle are shown in Figure 2. Such that the vehicle moves both in the longitudinal and lateral directions relative to an obstacle located at a distance in front of it.

The following equations determine the vehicle's position by the global coordinates (X, Y) based on (Rajamani, 2012) the local coordinates (x, y) , with ψ is the yaw angle, v_x is the longitudinal velocity and v_y is lateral velocity, X_0, Y_0 are the initial positions in (X, Y) .

$$\begin{cases} \dot{X} = v_x \cos \psi - v_y \sin \psi \\ \dot{Y} = v_x \sin \psi + v_y \cos \psi \end{cases}$$

Two inputs applied to our system, δ_f is the steering angle and a is the applied longitudinal acceleration, created by the engine when positive or by the brakes when negative. The net acceleration are a_x and a_y . So a and a_x are different.

$$a_x = \ddot{x} - \dot{\psi} \dot{y}, \quad a_y = \ddot{y} + \dot{\psi} \dot{x}$$

2.2 The LPV form of vehicle

Autonomous vehicle state space equations (1) can be converted to LPV form,

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

The state vector is $x = [v_x, v_y, \psi, \dot{\psi}, X, Y]^T$, the command vector is $u = [\delta_f, a]^T$.

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{\psi} \\ \dot{\ddot{\psi}} \\ \dot{X} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & A_{14} & 0 & 0 \\ 0 & A_{22} & 0 & A_{24} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & 0 \\ A_{51} & A_{52} & 0 & 0 & 0 & 0 \\ A_{61} & A_{62} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \psi \\ \ddot{\psi} \\ X \\ Y \end{bmatrix} + \begin{bmatrix} B_{11} & 1 \\ B_{21} & 0 \\ 0 & 0 \\ B_{41} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_f \\ a \end{bmatrix}$$

with,

$$\begin{aligned} A_{11} &= -\frac{\mu g}{v_x}, \quad A_{12} = \frac{C_{\alpha f} \sin \delta_f}{mv_x}, \quad A_{14} = v_y + \frac{C_{\alpha f} \sin \delta_f L_f}{mv_x}, \quad B_{11} = \frac{-C_{\alpha f} \sin \delta_f}{m}. \\ A_{22} &= \frac{-(C_{\alpha r} + C_{\alpha f} \cos \delta_f)}{mv_x}, \quad A_{24} = \frac{-(C_{\alpha f} \cos \delta_f L_f - C_{\alpha r} L_r)}{mv_x} - v_x, \quad B_{21} = \frac{C_{\alpha f} \cos \delta_f}{m}. \\ A_{42} &= \frac{-(C_{\alpha f} \cos \delta_f L_f - C_{\alpha r} L_r)}{I_z v_x}, \quad A_{44} = \frac{-(C_{\alpha f} \cos \delta_f L_f^2 - C_{\alpha r} L_r^2)}{I_z v_x}, \\ B_{41} &= \frac{C_{\alpha f} \cos \delta_f L_f}{I_z}. \\ A_{51} &= \cos \psi, \quad A_{52} = -\sin \psi. \\ A_{61} &= \sin \psi, \quad A_{62} = \cos \psi. \end{aligned}$$

Remarks,

- The system is nonlinear since matrix A depends on longitudinal velocity v_x , lateral velocity v_y , yaw angle ψ and steering angle δ_f and also matrix B depends on steering angle δ_f .
- The longitudinal velocity v_x must be different from zero so that certain values of matrix A do not become undefined.

The outputs of this system are the longitudinal velocity v_x , yaw angle ψ , longitudinal and lateral displacement (X, Y) respectively.

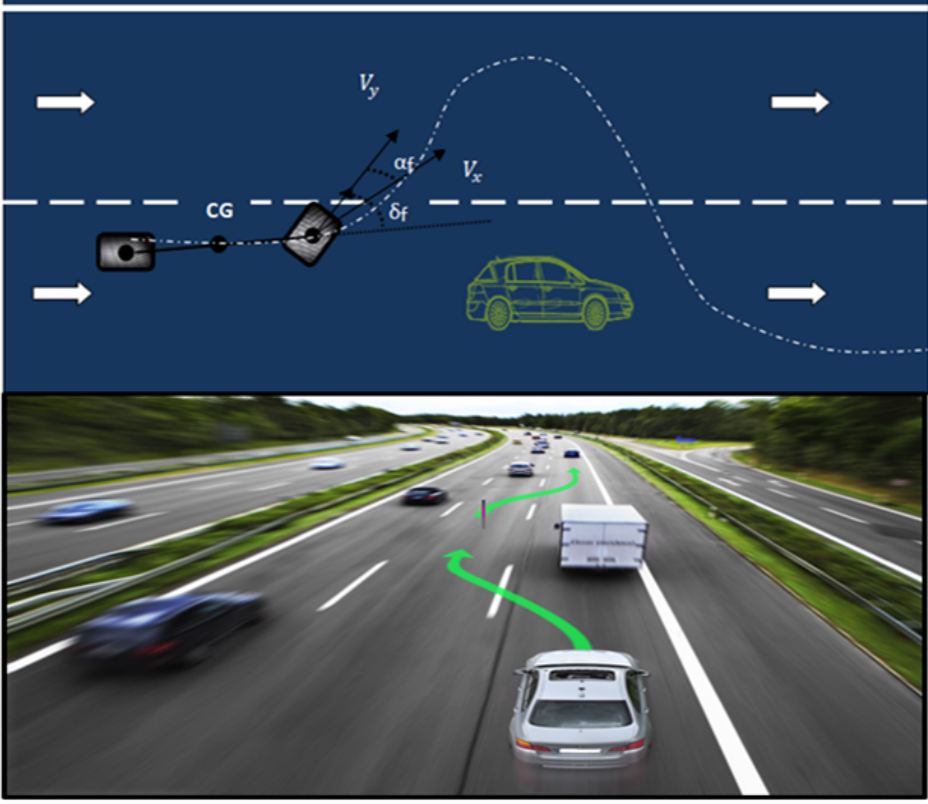
$$y = [v_x, \psi, X, Y]^T$$

with,

$$y = \begin{bmatrix} v_x \\ \psi \\ X \\ Y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \psi \\ \ddot{\psi} \\ X \\ Y \end{bmatrix}$$

and $D = 0$.

Figure 2 The autonomous vehicle avoids an obstacle located at a distance in front of it (see online version for colours)



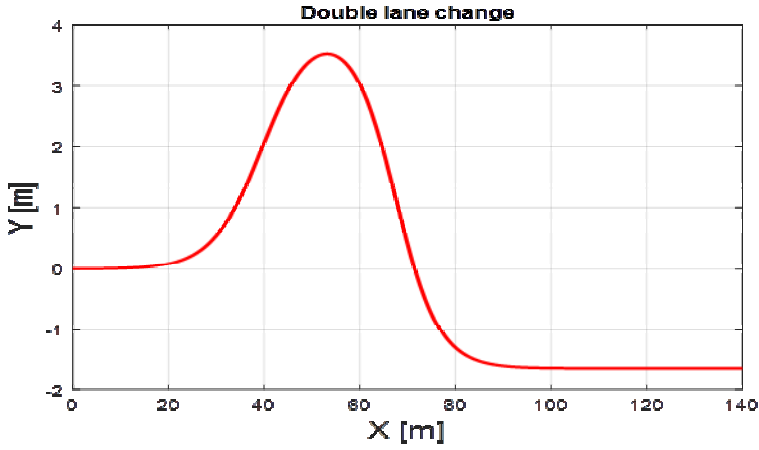
2.3 Trajectory planning

Several methods are used in the planning of autonomous vehicle trajectories, the use of mathematical functions by researchers is one of the most popular methods.

The double lane change manoeuvre is chosen. In order to avoid a collision, the trajectory is designed by a function (Falcone et al., 2007; Song et al., 2015) such that the reference lateral displacement Y_{ref} is a function of longitudinal position X , see Figure 3, the evolution of the trajectory is given by,

$$Y_{ref} = 2.025(1 + \tanh(z_1)) - 2.85(1 + \tanh(z_2)) \quad (2)$$

where, $z_1 = \left(\frac{2.4}{25}\right)(X - 27.19) - 1.2$, $z_2 = \left(\frac{2.4}{21.95}\right)(X - 56.46) - 1.2$.

Figure 3 Double lane change of reference path (see online version for colours)

2.4 Cornering stiffness approximation

Our model can represent the lateral forces as follows because the sideslip angle α is small,

$$F_{yf} = C_{\alpha f} \alpha_f \quad (3a)$$

$$F_{yr} = C_{\alpha r} \alpha_r \quad (3b)$$

$$\text{with } \alpha_f = \delta_f - \frac{v_y}{v_x} - \frac{\dot{\psi} L_f}{v_x}, \quad \alpha_r = -\frac{v_y}{v_x} + \frac{\dot{\psi} L_r}{v_x}$$

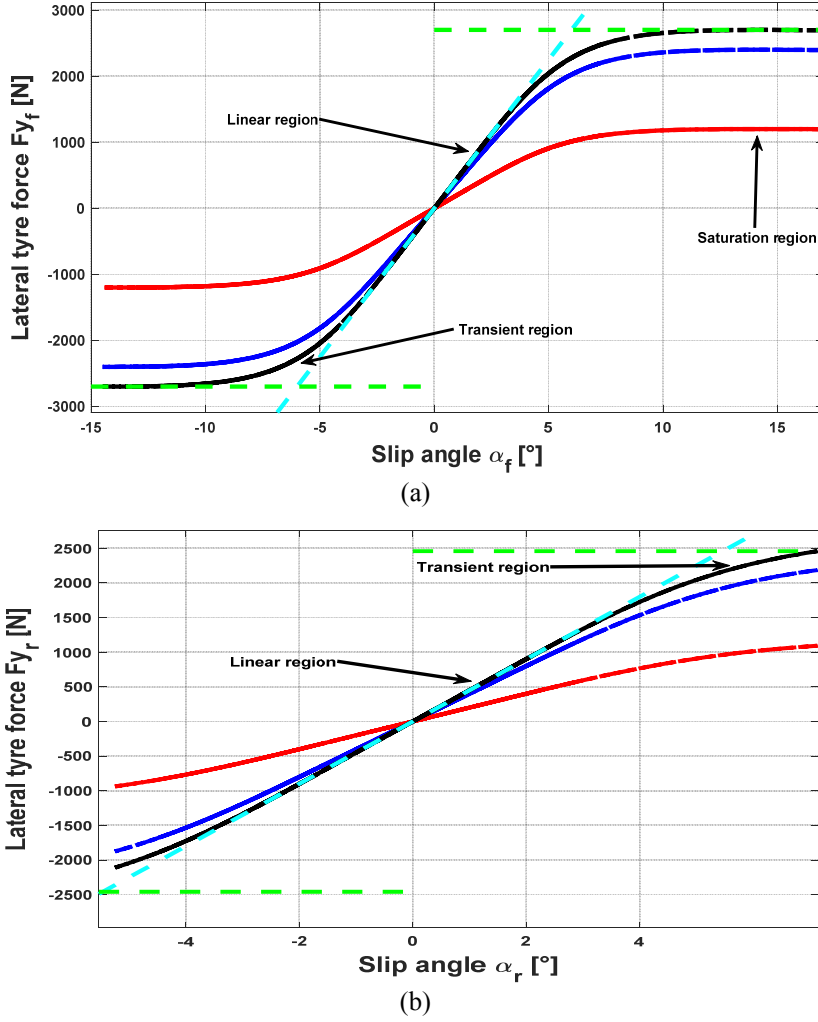
The lateral forces are really non-linear; the non-linearity of the lateral forces can be seen from Figure 4 and can be given by the Pacejka formula as follows (Atoui et al., 2020; Pacejka, 2006),

$$F_{yf} = c_3 \sin\left(c_2 \tan^{-1}\left(c_1 \alpha_f\right)\right)$$

$$F_{yr} = c_3 \sin\left(c_2 \tan^{-1}\left(c_1 \alpha_r\right)\right)$$

$$\text{with } \alpha_f = \delta_f - \tan^{-1}\left(\frac{v_y}{v_x} + \frac{\dot{\psi} L_f}{v_x}\right), \quad \alpha_r = -\tan^{-1}\left(\frac{v_y}{v_x} - \frac{\dot{\psi} L_r}{v_x}\right).$$

In our work, fuzzy logic is used to approximate $C_{\alpha f}$ and $C_{\alpha r}$, the front and rear cornering stiffness respectively. The nonlinear lateral forces are functions of the variation of sideslip angle $\alpha_f(t)$ and $\alpha_r(t)$, and the cornering stiffness $C_{\alpha f}(t)$ and $C_{\alpha r}(t)$. The nonlinear lateral forces are composed of two parts, the first part is a linear region and the second part is a nonlinear region see Figure 4. It can be noticed from Figure 4 and the equation (3) that if the angle sideslip $\alpha_f(t)$ ($\alpha_r(t)$) is large, the Cornering stiffness $C_{\alpha f}(t)$ ($C_{\alpha r}(t)$) is small and the reverse if $C_{\alpha f}(t)$ increases, $\alpha_f(t)$ decreases.

Figure 4 Lateral force (a) of front wheel, (b) of rear wheel (see online version for colours)


2.5 Logic fuzzy type Takagi-Sugeno (T-S)

The Takagi-Sugeno fuzzy system (Wiktorowicz and Krzeszowski, 2020) is the subject of our consideration,

$$F_{ts}(x, \theta) = \frac{\sum_{i=1}^R g_i(x) \mu_i(x)}{\sum_{i=1}^R \mu_i(x)}$$

where the consequences output is $g_i(x) = a_{i,0} + a_{i,1}x_1 + \dots + a_{i,n}x_n$ and

$\mu_i(x) = \prod_{j=1}^n \exp\left(-\frac{1}{2} \left(\frac{x_j - c_j^i}{\sigma_j^i}\right)^2\right)$, $i = 1, 2, \dots, R$. c_j^i and σ_j^i are the centre and the width of the membership function for the j th input and the i th rule respectively.

And also we can write $y = F_{ls}(x, \theta) = \theta^T \phi(x)$

where $\phi = [\zeta_1(x), \zeta_2(x), \dots, \zeta_R(x), x_1 \zeta_1(x), x_1 \zeta_2(x), \dots, x_1 \zeta_R(x), \dots, x_n \zeta_1(x), x_n \zeta_2(x), \dots, x_n \zeta_R(x)]^T$ and $\zeta_j(x) = \frac{\mu_j(x)}{\sum_{i=1}^R \mu_i(x)}, j = 1, 2, \dots, R$.

And $\theta = [a_{1,0}, a_{2,0}, \dots, a_{R,0}, a_{1,1}, a_{2,1}, \dots, a_{R,1}, \dots, a_{1,n}, a_{2,n}, \dots, a_{R,n}]^T$.

To update our estimate θ each time, the Weighted Recursive Least Squares method is used.

2.5.1 Weighted recursive least squares

In this section, recursive least squares method is introduced to tuning approximation structures where the parameters that will be tuned enter linearly. A ‘weighted recursive least squares’ (WRLS) algorithm (Maciel et al., 2021) consists in minimising the criterion,

$$J(\theta, G)|_{M=k} = \frac{1}{2} \sum_{i=1}^k \lambda^{k-i} (y(i) - \phi^T(x(i))\theta)^2$$

where $0 < \lambda \leq 1$ is called a ‘forgetting factor’ since it gives the most recent data higher weight in the optimisation (to see this, consider the effect of the term λ^{k-i} in the above summation).

Using a similar approach to the RLS case (Wu and Chen, 2021), shows that equations for WRLS are given by,

$$\theta(k) = \theta(k-1) + K(k)(y(k) - \phi^T(x(k))\theta(k-1))$$

$$K(k) = \frac{P(k-1)\phi(x(k))}{\lambda + \phi^T(x(k))P(k-1)\phi(x(k))}$$

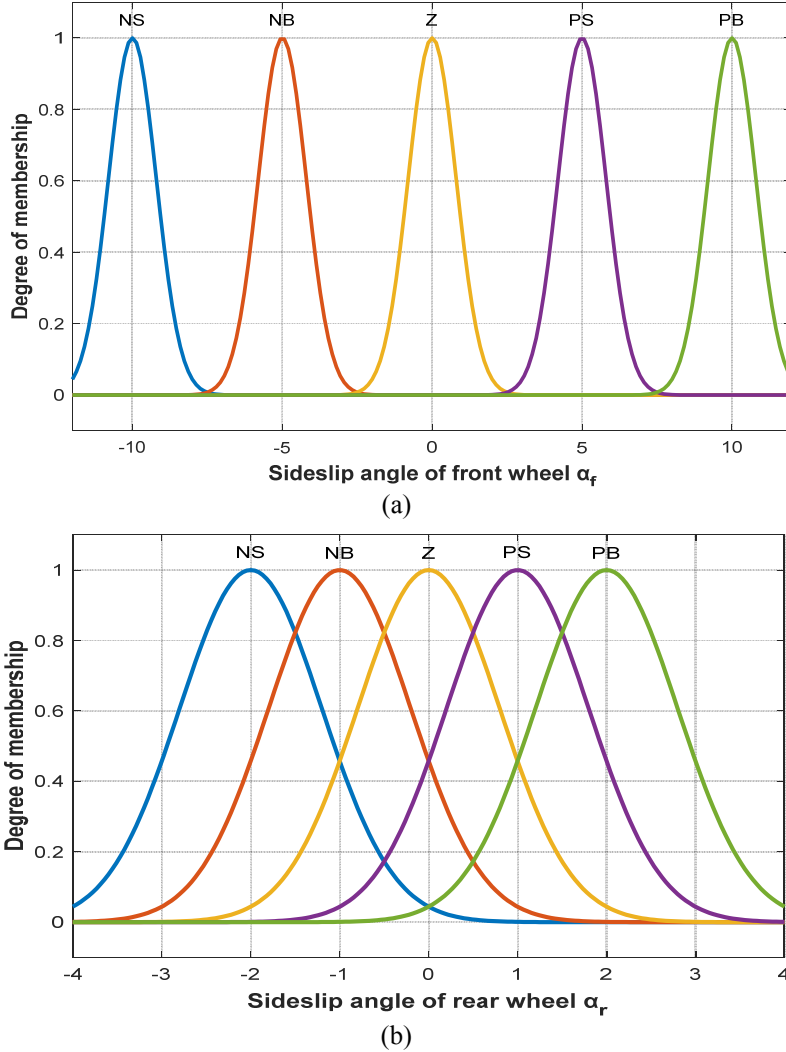
$$P(k) = \frac{1}{\lambda} (I - K(k)\phi^T(x(k)))P(k-1)$$

2.5.2 T-S fuzzy for approximation of lateral force

In this work, a Takagi-Sugeno (T-S) type fuzzy logic approach is used, it allows to generate numerical outputs. The membership functions are chosen as Gaussian functions and the defuzzification is performed by the centroid method. The cornering stiffness is estimated in order to approximate the nonlinear lateral forces.

Figure 5(a) represents the Gaussian membership functions chosen to represent the sideslip angle of front wheel α_f , the five functions are distributed over the universe of discourse between -15 and 15 . The five membership functions chosen of the Gaussian type to represent the Sideslip angle of rear wheel α_r are illustrated in Figure 5(b) and distributed over the universe of discourse between -4 and 4 .

Figure 5 (a) The Gaussian membership functions of α_f and (b) the Gaussian membership functions of α_r (see online version for colours)

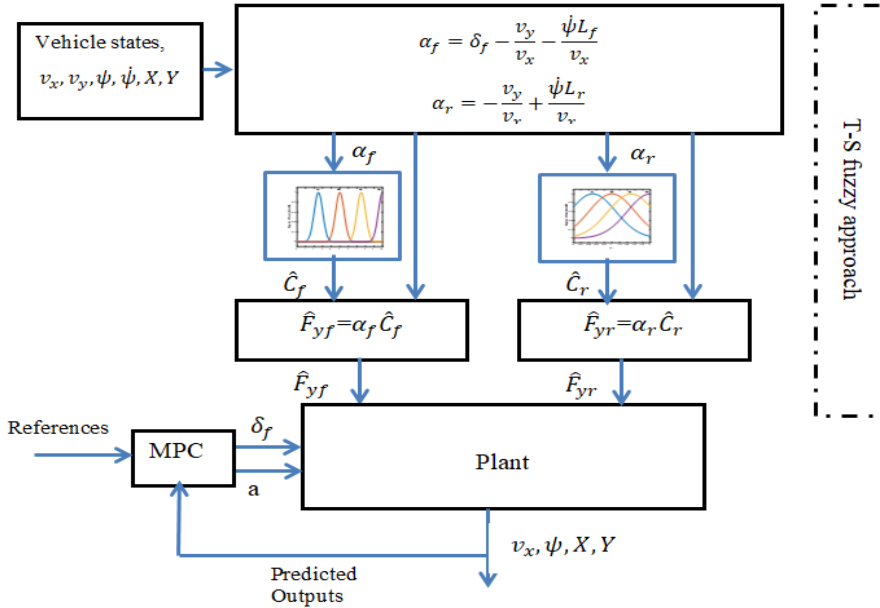


In Table 1, the different steps of the fuzzy logic approach in order to approximate the Cornering stiffness are given. In the first step, the sideslip angle front and rear of wheel were determined as fuzzy logic inputs. Second step, is the choice of type and number of membership functions. Third step is the choice of inference rules. The fourth step is to choose the defuzzification method, and the Centroid method has been selected. The last step, the numerical outputs giving the Cornering stiffness front C_f and rear C_r .

Figure 6 shows the approximation method of the lateral forces, the approximation is made by the Takagi-Sugeno type fuzzy logic approach. In the first step, the front cornering stiffness C_f and rear cornering stiffness C_r are approximated, and then used to calculate the front and rear lateral forces $\hat{F}_{yf}$ and $\hat{F}_{yr}$ respectively.

Table 1 Flowchart of proposed fuzzy logic observer for cornering stiffness (see online version for colours)

Crisp input	Input: Sideslip angle of front wheel $\alpha_f = \delta_f - \frac{v_y}{v_x} - \frac{\dot{\psi} L_f}{v_x} [-15, 15](\text{deg})$	Input: Sideslip angle of rear wheel $\alpha_r = -\frac{v_y}{v_x} + \frac{\dot{\psi} L_r}{v_x} [-4, 4](\text{deg})$
Fuzzification	Membership function: Gaussian function (NB, NS, Z, PS, PB) NB (Negative Big), NS (Negative Small), Z (Zero), PS (Positive Small), and PB (Positive Big)	
Fuzzy reasoning	Rule 1: If input is PS, then output is PB Rule 2: If input is PB, then output is PS Rule 3: If input is Z, then output is Z Rule 4: If input is NS, then output is NB Rule 5: If input is NB, then output is NS	
Defuzzification	Defuzzification: Centroid method	
Crisp output	Output: Cornering stiffness C_f	Output: Cornering stiffness C_r

Figure 6 Flowchart of estimation of cornering stiffness (see online version for colours)

2.6 Particle swarm optimisation

The PSO algorithm builds on particle intelligence (Kennedy and Eberhart, 1995).

This technique is inspired by a social model that represents the swarm theorem. The update of each particle is based on equation (4),

$$\begin{cases} v_i(j+1) = wv_i(j) + c_1r_1(p_i(j) - x_i(j)) + c_2r_2(p_g(j) - x_i(j)) \\ x_i(j+1) = x_i(j) + v_i(j+1) \end{cases} \quad (4)$$

where j is the number of iterations and the inertia weight is w . c_1 and c_2 are the random numbers between $[0,1]$. The velocity of the particle is represented by v_i and the position of the particle is x_i . p_i is a personal best and p_g is the global best.

Minimising a simple function with bounded constraints

It is possible to minimise a simple function of many variables with bounded constraints (Padhye et al., 2013), to help the PSO algorithm to quickly find the relevant values. The search interval should be defined by determining the minimum and maximum values of each variable for example, let lb be the Lower bound and ub the Upper bound such that $lb \leq x \leq ub$.

To find the optimal parameters q and r of the MPC, the objective function is minimised using the PSO algorithm. The goal in our work aims to reduce the lateral and longitudinal displacement errors so that the autonomous vehicle follows the reference path. The objective function is given by,

$$F_{obj} = \frac{1}{2} \Delta u_G^T \bar{H} \Delta u_G + [\tilde{x}_k^T r_G^T] \bar{F}^T \Delta u_G \quad (5)$$

$$\text{with} \quad \bar{H} = \left(\bar{C}_d^T \bar{Q} \bar{C}_d + \bar{R} \right), \quad \bar{F}^T = \begin{bmatrix} \hat{A}_d^T \bar{Q} \bar{C}_d \\ -\bar{T} \bar{C}_d \end{bmatrix}, \quad \bar{Q} = \begin{bmatrix} C_d^T \dot{Q} C_d & 0 & 0 \\ 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & C_d^T S C_d \end{bmatrix},$$

$$\bar{T} = \begin{bmatrix} \dot{Q} C_d & 0 & 0 \\ 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & S C_d \end{bmatrix}, \quad \bar{R} = \begin{bmatrix} R & 0 & 0 \\ 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & R \end{bmatrix}, \quad \dot{Q} = qQ \text{ and } R = rR.$$

The objective is to find q and r when the global minimum value of F_{obj} is found, according to equation (5). Let $J_{min}(q, r) = F_{obj}$, where q and r are the variables of this function, and have the same dimensions of q and r respectively.

3 Control strategy

The MPC is a robust controller, it is able to overcome the modelling uncertainties and disturbances.

3.1 Linear model predictive control

The objective of MPC is to predict the evolution of the vehicle model based on a sequence of future inputs at a finite horizon, by minimising a system cost under specified constraints. Figure 7(a) shows the MPC strategy with receding horizon control concept H_p . The cost function J to be minimised (Bolognani et al., 2009) is given by,

$$\underset{\Delta U, \tilde{X}}{\text{minimize}} J = \sum_{k=0}^{N-1} (\tilde{x}_k^T \bar{Q} \tilde{x}_k + \Delta u_k^T \bar{R} \Delta u_k) + \tilde{x}_N^T \bar{S} \tilde{x}_N$$

Subject to

$$-\frac{2\pi}{500} \leq \Delta \delta_f \leq \frac{2\pi}{500}$$

$$-0.1 \leq \Delta a \leq 0.1$$

$$-\frac{\pi}{5} \leq \delta_f \leq \frac{\pi}{5}$$

$$-2 \leq a \leq 4$$

$$1 \leq v_x \leq 35$$

$$-0.14v_x \leq v_y \leq 0.14v_x$$

where the decision vector variables are $\tilde{X} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_N)$, $\Delta U = (\Delta u_0, \Delta u_1, \Delta u_2, \dots, \Delta u_{N-1})$. The augmented state sequence and the variation of the input variables respectively are $\tilde{x} = [v_x \ v_y \ \psi \ \dot{\psi} \ X \ Y \ \Delta \delta_f \ \Delta a]^T$ and $\Delta u = [\Delta \delta_f \ \Delta a]^T$, N is the prediction horizons, $\bar{Q}$, $\bar{S}$ are 8×8 weighting matrices with respect to tracking errors of longitudinal velocity e_x , yaw angle e_ψ , global longitudinal position e_X , global lateral position e_Y , the change of steering angle and the change of longitudinal acceleration, and $\bar{R} = R$ is a 2×2 dimension with respect to the change of steering angle and the change of longitudinal acceleration of the control inputs, respectively.

Note that to ensure smooth trajectory following and to avoid jumping in the amplitude of the input variables δ_f and a in the transient periods, due to their high values compared to $\Delta \delta_f$ and Δa , the change of the variables inputs $\Delta \delta_f$ and Δa are used instead of inputs variables δ_f and a .

The Gradient of the cost function is given by,

$$\Delta J = \frac{1}{2} \Delta u^T \bar{H} \Delta u + f^T \Delta u$$

The control law is given by,

$$u_k = u_{k-1} + \Delta u_k^* \quad (6)$$

where $\Delta U_k^* = [\Delta u_k^*, \Delta u_{k+1}^*, \dots, \Delta u_{k+N-1}^*]^T$ is the sequence of optimal change of steering angle and the change of longitudinal acceleration, increments computed at time k . $\Delta u_k = [\Delta \delta_{fk} \ \Delta a_k]^T$ is the change of the input variables, Δu_k^* is the vector of the first

two elements of the first element of ΔU_k^* applied to the system. Therefore, the control law u_k is given by the above equation (6).

Assumption 1: *There exist positive definite weight matrices $\bar{Q}$, $\bar{S}$, and $\bar{R}$, that minimise the cost function J .*

Assumption 2: *There exist two inputs $\Delta u = [\Delta \delta_f \Delta a]^T$ at each instant $k = 1, \dots, N-1$, that are solutions of $\Delta J = 0$, which makes the system controllable at each time instant within this horizon, where N is the prediction horizon in MPC.*

A block diagram of MPC is represented by Figure 7(b). The operating principle of MPC is described as follows:

- 1 The dynamic model is used to predict the behaviour of the plant and future plant outputs, $y(k+i)$ for $i = 1, 2, \dots, H_p$, for a determined prediction horizon, H_p , at each instant k based on past and current inputs and outputs measurements up to instant k , and on the future control signal, $u(k+i)$, $i = 1, \dots, H_c-1$, where H_c is called control horizon.
- 2 The control signal, $u(k+i)$ is calculated by optimising the error between the predicted output signal and reference or target trajectory to keep the process as close as possible to the reference trajectory, $x(k+i)$.
- 3 The control signal $u(k)$ is implemented on the plant, the next sampling instant $y(k+1)$ is known.
- 4 Step 1 is repeated with the updated value and all sequences are brought up to date.

The nonlinear MPC relies on a nonlinear model and a quadratic objective function, and take into account nonlinear constraints to predict the behaviours/dynamics of the system.

It is assumed that the state vector noted $x(k)$ is available for measurement and the control vector noted $u(k)$ is the available current. The future control and the future state variables are denoted by:

$$u(k+1), u(k+2), \dots, u(k+H_c-1).$$

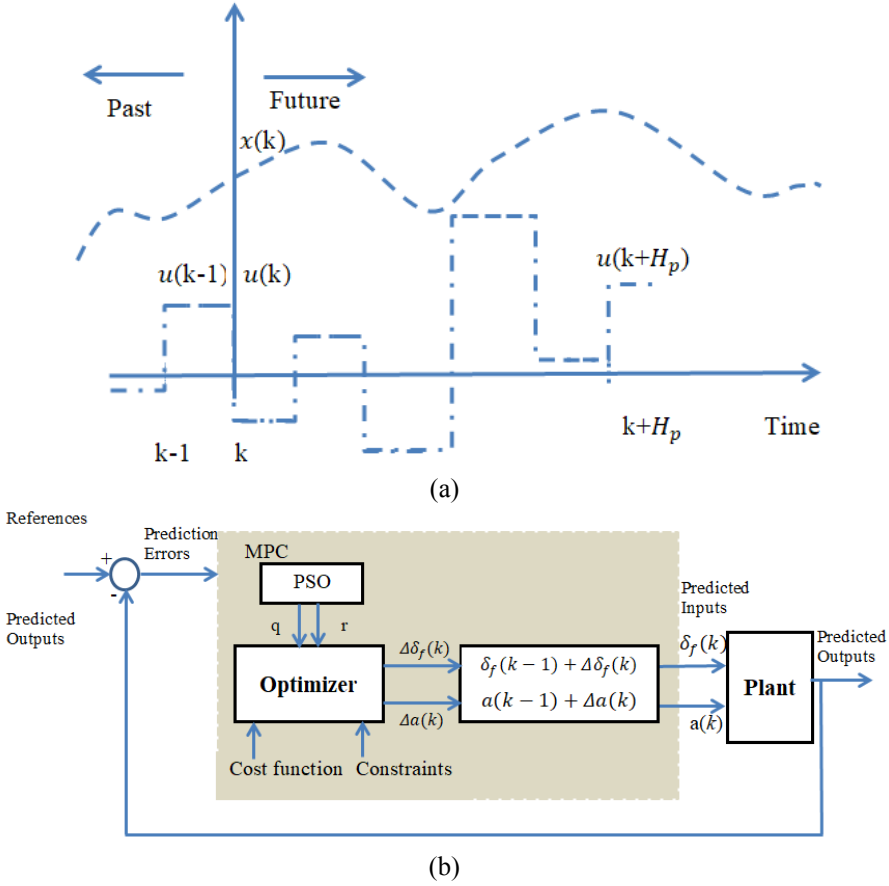
$$x(k+1), x(k+2), \dots, x(k+H_p).$$

with H_c and H_p are horizons control and prediction horizons respectively. In our study, we assume $H_c = H_p$.

The basic idea is to use the model to predict the response over a finite time horizon and use the predicted response to evaluate the objective function. The objective function is minimised to obtain the control. To implement the control law at the next instant of time, the measured states at the current time are used instead of the predicted states (Wijewardana et al., 2016).

3.2 Analysis of MPC combined with LPV and optimised by PSO

To synthesise the controller, we will adopt the strategy of MPC using a dynamic nonlinear vehicle bicycle model.

Figure 7 (a) Basic MPC strategy and (b) MPC general block diagram (see online version for colours)

The linear MPC is applied using LPV formulation. First, the car state space equations are converted to LPV form,

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

The MPC controller is a digital controller. For this it is necessary to discretise the system to get,

$$\begin{cases} x_{k+1} = A_d x_k + B_d u_k \\ y_k = C_d x_k + D_d u_k \end{cases}$$

with $D_d = 0$.

The cost function is,

$$J = \frac{1}{2} \sum_{i=0}^{N-1} (e_{k+i}^T q Q e_{k+i} + u_{k+i}^T r R u_{k+i}) + \frac{1}{2} e_{k+N}^T S e_{k+N}$$

with $e_{k+N} = [r_{k+N} - C_d x_{k+N}]$, $r_k = [v_{xref} \psi_{ref} X_{ref} Y_{ref}]^T$ is the vector references at time k , and $y_k = C_d x_k$ is the vector outputs, q and r are the optimisation parameters of PSO.

The augmented version of system is,

$$\begin{cases} \tilde{x}_{k+1} = \tilde{A}_d \tilde{x}_k + \tilde{B}_d \Delta u_k \\ y_k = \tilde{C}_d \tilde{x}_k \end{cases}$$

$$\text{with } \tilde{x}_k = [x_k \ \Delta u_{k-1}]^T, \ \tilde{A}_d = \begin{bmatrix} A_d & B_d \\ 0 & I \end{bmatrix}, \ \tilde{B}_d = \begin{bmatrix} B_d \\ I \end{bmatrix}, \ \Delta u_k = [\Delta \delta_{fk} \ \Delta a_k]^T, \ \tilde{C}_d = [C_d \ 0].$$

Remark: The values of these variable parameters are updated at each iteration taking into account the new values of the scheduling variables. A_d varies as a function of $\dot{x}$, $\dot{y}$, ψ , δ_f and B_d is variable as a function of δ_f .

The longitudinal-lateral coupled dynamic model of an autonomous vehicle is nonlinear due to the interaction of vehicle movements in both longitudinal and lateral directions at the same time (coupling of lateral and longitudinal dynamics), and additionally, to the complex relationship between the forces exerted on the tyres and slip angles. When the vehicle's tyres slip during cornering or braking, the forces they generate are not linear with respect to the slip angle.

Complex mathematical models are used to model and control the coupled longitudinal-lateral dynamics of an autonomous vehicle, which take these non-linearity into account. Our model contains matrices A_d and B_d which vary at each iteration and contains non-linear functions ($\sin \delta_f, \cos \delta_f$).

For the augmented version, our cost function:

$$J_1 = \frac{1}{2} \sum_{i=0}^{N-1} (e_{k+i}^T q Q e_{k+i} + \Delta u_{k+i}^T r R \Delta u_{k+i}) + \frac{1}{2} e_{k+N}^T S e_{k+N}$$

Subject to

$$-\frac{2\pi}{500} \leq \Delta \delta_f \leq \frac{2\pi}{500}$$

$$-0.1 \leq \Delta a \leq 0.1$$

$$-\frac{\pi}{5} \leq \delta_f \leq \frac{\pi}{5}$$

$$-2 \leq a \leq 4$$

$$1 \leq v_x \leq 35$$

$$-0.14v_x \leq v_y \leq 0.14v_x$$

with $e_{k+N} = [r_{k+N} - \tilde{C}_d \tilde{x}_{k+N}]$, $\Delta \delta_f$ and Δa are the change steering angle and the change longitudinal acceleration respectively. δ_f and a are the control actions, steering angle and longitudinal acceleration respectively. The steering angle and longitudinal

acceleration can take values within a bounded interval. v_x is the longitudinal velocity and v_y is the lateral velocity of vehicle. q and r are the parameters optimised by PSO.

The MPC formulation does not guarantee the global stability of the system, so the addition of the fixed and varying constraints to the optimisation problem becomes necessary.

The lateral speed varies according to the longitudinal speed for that we chose $v_y = 0.14v_x$.

The error e_{k+i} is replaced by its expression $e_{k+i} = [r_{k+i} - \tilde{C}_d \tilde{x}_{k+i}]$, J_1 becomes,

$$J_1 = \frac{1}{2} \sum_{i=0}^{N-1} \left[(r_{k+i} - \tilde{C}_d \tilde{x}_{k+i})^T q Q (r_{k+i} - \tilde{C}_d \tilde{x}_{k+i}) + \Delta u_{k+i}^T r R \Delta u_{k+i} \right] \\ + \frac{1}{2} (r_{k+N} - \tilde{C}_d \tilde{x}_{k+N})^T S (r_{k+N} - \tilde{C}_d \tilde{x}_{k+N}) \\ J_1 = \sum_{i=0}^{N-1} \left[\frac{1}{2} r_{k+i}^T q Q r_{k+i} - r_{k+i}^T q Q \tilde{C}_d \tilde{x}_{k+i} + \frac{1}{2} \tilde{x}_{k+i}^T \tilde{C}_d^T q Q \tilde{C}_d \tilde{x}_{k+i} + \frac{1}{2} \Delta u_{k+i}^T r R \Delta u_{k+i} \right] \\ + \frac{1}{2} r_{k+N}^T S r_{k+N} - \frac{1}{2} r_{k+N}^T S \tilde{C}_d \tilde{x}_{k+N} - \frac{1}{2} \tilde{x}_{k+N}^T \tilde{C}_d^T S r_{k+N} + \frac{1}{2} \tilde{x}_{k+N}^T \tilde{C}_d^T S \tilde{C}_d \tilde{x}_{k+N}$$

After some calculations, we get the cost function in quadratic form as (Wijewardana et al., 2016),

$$J_2 = \frac{1}{2} \tilde{x}_G^T \bar{\bar{Q}} \tilde{x}_G - r_G^T \bar{\bar{T}} \tilde{x}_G + \frac{1}{2} \Delta u_G^T \bar{\bar{R}} \Delta u_G$$

with $\tilde{x}_G = [\tilde{x}_{k+1}^T \tilde{x}_{k+2}^T \dots \tilde{x}_{k+N}^T]^T$, $r_G = [r_{k+1}^T r_{k+2}^T \dots r_{k+N}^T]^T$,

$$\Delta u_G = [\Delta u_k^T \Delta u_{k+1}^T \dots \Delta u_{k+N-1}^T]^T, \quad \bar{\bar{Q}} = \begin{bmatrix} C_d^T \dot{Q} C_d & 0 & 0 \\ 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & C_d^T S C_d \end{bmatrix}, \\ \bar{\bar{T}} = \begin{bmatrix} \dot{Q} C_d & 0 & 0 \\ 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & S C_d \end{bmatrix}, \quad \bar{\bar{R}} = \begin{bmatrix} \dot{R} & 0 & 0 \\ 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & R \end{bmatrix}, \quad \dot{Q} = qQ \text{ and } \dot{R} = rR.$$

To simplify the cost function J_2 , let (Benlahrache et al., 2017),

$$\tilde{x}_G = \bar{\bar{C}}_d \Delta u_G + \hat{\bar{A}}_d \tilde{x}_k$$

with

$$\bar{\bar{C}}_d = \begin{bmatrix} \tilde{B}_{d0} & 0 & \dots & \dots & 0 \\ \tilde{A}_{d1}\tilde{B}_{d0} & \tilde{B}_{d1} & \dots & \dots & \vdots \\ \tilde{A}_{d2}\tilde{A}_{d1}\tilde{B}_{d0} & \tilde{A}_{d2}\tilde{B}_{d1} & \tilde{B}_{d2} & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{A}_{dN-1}\tilde{A}_{dN-2} \dots \tilde{A}_{d1}\tilde{B}_{d0} & \tilde{A}_{dN-1}\tilde{A}_{dN-2} \dots \tilde{A}_{d2}\tilde{B}_{d1} & \dots & \dots & \tilde{B}_{dN-1} \end{bmatrix},$$

$$\hat{\hat{A}}_d = \begin{bmatrix} \tilde{A}_{d0} \\ \tilde{A}_{d1}\tilde{A}_{d0} \\ \vdots \\ \tilde{A}_{dN-1}\tilde{A}_{dN-2} \dots \tilde{A}_{d0} \end{bmatrix}.$$

Replacing $\tilde{x}_G = \bar{\bar{C}}_d \Delta u_G + \hat{\hat{A}}_d \tilde{x}_k$ in equation J_2 ,

$$J_2 = \frac{1}{2} \left(\bar{\bar{C}}_d \Delta u_G + \hat{\hat{A}}_d \tilde{x}_k \right)^T \bar{\bar{Q}} \left(\bar{\bar{C}}_d \Delta u_G + \hat{\hat{A}}_d \tilde{x}_k \right) - r_G^T \bar{\bar{T}} \left(\bar{\bar{C}}_d \Delta u_G + \hat{\hat{A}}_d \tilde{x}_k \right) + \frac{1}{2} \Delta u_G^T \bar{\bar{R}} \Delta u_G$$

$$J_2 = \frac{1}{2} \Delta u_G^T \bar{\bar{C}}_d^T \bar{\bar{Q}} \bar{\bar{C}}_d \Delta u_G + \frac{1}{2} \Delta u_G^T \bar{\bar{C}}_d^T \bar{\bar{Q}} \bar{\bar{C}}_d \hat{\hat{A}}_d \tilde{x}_k + \frac{1}{2} \tilde{x}_k^T \hat{\hat{A}}_d^T \bar{\bar{Q}} \bar{\bar{C}}_d \Delta u_G \\ + \frac{1}{2} \tilde{x}_k^T \hat{\hat{A}}_d^T \bar{\bar{Q}} \hat{\hat{A}}_d \tilde{x}_k - r_G^T \bar{\bar{T}} \bar{\bar{C}}_d \Delta u_G - r_G^T \bar{\bar{T}} \hat{\hat{A}}_d \tilde{x}_k + \frac{1}{2} \Delta u_G^T \bar{\bar{R}} \Delta u_G$$

After some calculations J_2 becomes,

$$J_3 = \frac{1}{2} \Delta u_G^T \bar{\bar{H}} \Delta u_G + \left[\tilde{x}_k^T r_G^T \right] \bar{\bar{F}}^T \Delta u_G$$

$$\text{with } \bar{\bar{H}} = \left(\bar{\bar{C}}_d^T \bar{\bar{Q}} \bar{\bar{C}}_d + \bar{\bar{R}} \right), \bar{\bar{F}}^T = \begin{bmatrix} \hat{\hat{A}}_d^T \bar{\bar{Q}} \bar{\bar{C}}_d \\ -\bar{\bar{T}} \bar{\bar{C}}_d \end{bmatrix}.$$

Remark: $\bar{\bar{H}}$ and $\bar{\bar{F}}^T$ are variables because A_d is a function of $\dot{x}$, $\dot{y}$, ψ , δ_f and B_d depends on δ_f .

The gradient of J_3 is,

$$\nabla J_3 = \bar{\bar{H}} \Delta u_G + \bar{\bar{F}} \begin{bmatrix} \tilde{x}_k^T \\ r_G^T \end{bmatrix}$$

$$\text{Let the gradient } \nabla J_3 = 0, \bar{\bar{H}} \Delta u_G + \bar{\bar{F}} \begin{bmatrix} \tilde{x}_k^T \\ r_G^T \end{bmatrix} = 0.$$

$$\Delta u_G = -\bar{\bar{H}}^{-1} \bar{\bar{F}} \begin{bmatrix} \tilde{x}_k^T \\ r_G^T \end{bmatrix}$$

The system is controlled by the MPC model using a linear variable parameter (LPV) modelling approach with linear inequality constraints. The MPC allows minimising the

Quadratic cost function, while at the same time respects the constraints. The MPC controller is used to find our most optimised control inputs.

MPC-LPV is a controller that does not need linearisation. The method is called the method of LPV systems.

Linearising leads to large angles, but the model in the controller box would just be an approximation of the model in the plant box.

LPV, however, will allow us to have the same continuous state space model in the planned box and in the controller box, and that makes LPV be more precise than linearisation. Furthermore, LPV is known to be more computationally efficient than linearisation. When using LPV, then the only difference between the controller and the plant is that after discretisation of the models more variable are computed in the state than in the controller. But for the continuous part, the models in both boxes plant and controller will be the same.

The relevant choice of the objective function and the determination of the minimum and maximum limits (Lower Bound, Upper Bound) of each parameter which can be optimised so that the execution of the algorithm becomes faster. The computational complexity of the proposed method, which combines MPC with fuzzy logic and PSO, increases when constraints are added, such as steering angle, change of steering angle, longitudinal acceleration, change of longitudinal acceleration, longitudinal speed and lateral speed. And also, when PSO and fuzzy logic are taken into account.

The choice of the type and number of membership functions, the inference rules and the defuzzification method are the keys of successful integration of fuzzy logic in MPC.

Our proposed method is better than traditional methods because MPC allows controlling nonlinear systems and this controller adapts to changing conditions. Furthermore, the inclusion of constraints guarantees safety, which is an essential requirement for autonomous vehicles. The integration of PSO and fuzzy logic algorithms improve the operation of this controller, and enhance trajectory tracking performance, which can result in smoother and more efficient vehicle control.

The steps of our proposed control strategy are illustrated in Figure 8. This algorithm shows the steps of applying a model predictive control with constraints combined with the LPV form, as well as the way in which fuzzy logic and PSO are integrated in the process of this system with the aim of obtaining the most optimized control inputs. This strategy is carried out in order to improve system performance.

Uncertainty, such as modelling errors, can pose challenges in current practices. The use of robust controller (MPC) based on PSO and fuzzy logic makes it possible to effectively manage these uncertainties.

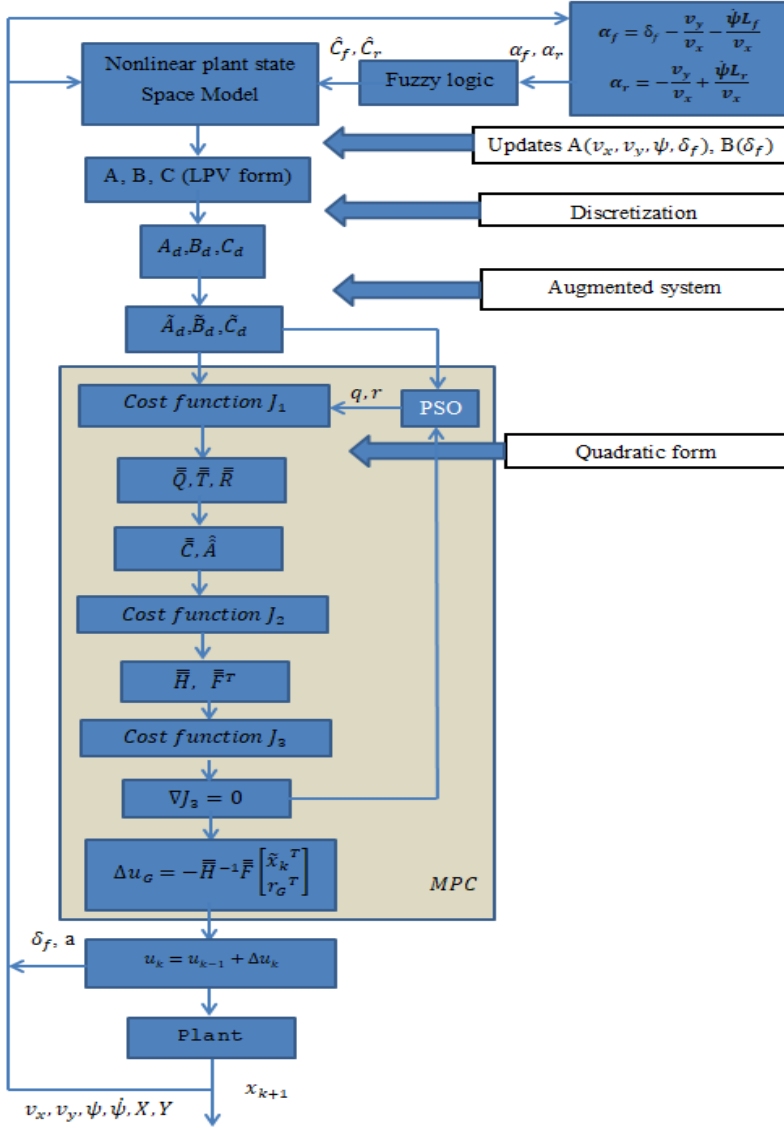
3.3 *Performance indicators*

An evaluation framework in various sectors relies on performance measures (error measures) as key components. Some of the most popular measures are presented.

Maximum absolute of errors

The maximum absolute error (MAE) is simply the largest absolute value of error observed during a simulation sequence (Hodson, 2022).

$$\mathbf{MAE} = \max_{i=1,\dots,n} |e_i|$$

Figure 8 Proposed control strategy algorithm (see online version for colours)

Mean squared error

Mean squared error (MSE) is the mean of the square of all of the error (Hodson, 2022).

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^n e_i^2$$

The efficiency of MPC with constraints based on PSO and fuzzy logic applied to the control of coupled longitudinal-lateral dynamics of the autonomous vehicle is compared using two main criteria:

- maximum absolute error (MAE)
- mean square error (MSE).

4 Simulation results and discussion

In this simulation, the predictive control model with constraints combined with the LPV form is applied to the longitudinal and lateral dynamics. The cornering stiffness are approximated by a fuzzy logic of the Takagi-Sugeno type, with the aim of finally approximating the nonlinear lateral forces. In order to improve the performance of the system, fixed and changing constraints for the inputs of the controller are defined, for system state variables, and for the outputs of the system. The MPC weights are determined using PSO. The MPC control parameters used in Falcone et al. (2007) are taken into account. The comparison is made between the MPC controller with and without optimisation of parameters. In the case of MPC with fuzzy logic, the comparison is made between the MPC controller with/without knowledge of the lateral forces, such that in the first case the approximation of these forces was made by fuzzy logic. The objective is to follow the reference trajectory of the autonomous vehicle while reducing the lateral and longitudinal displacement errors.

The fixed constraints are,

$$-\frac{2\pi}{500} \leq \Delta\delta_f \leq \frac{2\pi}{500}$$

$$-0.1 \leq \Delta a \leq 0.1$$

$$-\frac{\pi}{5} \leq \delta_f \leq \frac{\pi}{5}$$

$$-4 \leq a \leq 4$$

$$1 \leq v_x \leq 35$$

We can calculate the steering angle change so that the steering angle varies between $-\pi/5$ and $\pi/5$, so $|\Delta\delta_{f,max}| = 2\pi/500$ or 0.72 deg.

The changed constraints are,

$$-0.14v_x \leq v_y \leq 0.14v_x.$$

Remarks:

- The comfort zone for the longitudinal acceleration is between 6 m/s^2 and -10 m/s^2 see the g-g diagram in Fu et al. (2018) and the g-g-v diagram which represents the longitudinal acceleration as a function of a series of constant velocity places (Rice, 1973). The constraints for the longitudinal acceleration are chosen in this work between 4 m/s^2 and -2 m/s^2 in the comfort zone.
- The lateral speed varies as a function of the longitudinal speed for this we have chosen $v_y = 0.14 v_x$.

- The output vector is $y = [v_x \ \psi \ X \ Y]^T$ so the weights q and S in the cost function, are written as follows,

$$Q = \begin{pmatrix} Q_1(e_{v_x}) & 0 & 0 & 0 \\ 0 & Q_2(e_\psi) & 0 & 0 \\ 0 & 0 & Q_3(e_X) & 0 \\ 0 & 0 & 0 & Q_4(e_Y) \end{pmatrix}, S = \begin{pmatrix} S_1(e_{v_x}) & 0 & 0 & 0 \\ 0 & S_2(e_\psi) & 0 & 0 \\ 0 & 0 & S_3(e_X) & 0 \\ 0 & 0 & 0 & S_4(e_Y) \end{pmatrix}$$

with $e_{v_x} = v_x - v_{xref}$, $e_\psi = \psi - \psi_{ref}$, $e_X = X - X_{ref}$, and $e_Y = Y - Y_{ref}$, the weights Q_1 and S_1 adjust the error e_{v_x} , Q_2 and S_2 adjust the error e_ψ , Q_3 and S_3 adjust the error e_X , Q_4 and S_4 adjust the error e_Y .

- The control input is $\Delta u = [\Delta \delta_f \ \Delta a]^T$ and the weight R is written as follows,

$$R = \begin{pmatrix} R_1(\Delta \delta_f) & 0 \\ 0 & R_2(\Delta a) \end{pmatrix}$$

A comparison is made with Falcone et al. (2007) according to the longitudinal speeds mentioned in Table 2.

Table 2 MAE of the longitudinal, lateral and of the yaw angle according to the proposed methods and with different longitudinal speeds (see online version for colours)

v_x [ms]		10	15	19	21.5	
MAE	Y [m] (Falcone et al. (2007))	0.96	1.25	1.58	2.11	
	ψ [deg] (Falcone et al. (2007))	7.20	8.17	10.15	11.61	
	Y [m]	MPC Only	0.2673	0.2709	1.4878	1.8357
		MPC & PSO	0.1414	0.1559	0.5570	0.8669
		MPC & Fuzzy Logic	0.2649	0.2650	1.4367	1.9184
	ψ [deg]	MPC Only	1.1222	1.8355	10.5098	12.7423
		MPC & PSO	2.0801	1.1398	5.3601	6.5042
		MPC & Fuzzy Logic	1.1170	1.8166	10.5315	13.3921
	X [m]	MPC Only	1.0121	2.2564	2.1348	4.1267
		MPC & PSO	0.7633	0.9149	1.1719	1.2992
		MPC & Fuzzy Logic	1.0120	2.2653	2.1578	4.7077

Parameters:

- The parameters used in this simulation are, $T = 0.02$ s, $Hz = 10$, $\delta_f = 0.3$, $\delta_{f,min} = -36^\circ$, $\delta_{f,max} = 36^\circ$, $\Delta \delta_{f,min} = -0.72^\circ$, $\Delta \delta_{f,max} = 0.72^\circ$, $a_{min} = -4$, $a_{max} = 4$, $\Delta a_{min} = -0.1$, $\Delta a_{max} = 0.1$, $v_{x,min} = 1$, $v_{x,max} = 35$, $v_{y,min} = -0.14$, $v_{y,max} = 4.9$.

- The weights chosen in the case of using MPC with constraints only, and also in the case of using MPC with constraints & fuzzy logic are,

$$Q = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 11700 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 512 \end{pmatrix}, S = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 11700 & 0 & 0 \\ 0 & 0 & 80 & 0 \\ 0 & 0 & 0 & 210 \end{pmatrix}, R = \begin{pmatrix} 480 & 0 \\ 0 & 22.5 \end{pmatrix}.$$

- The parameters found by PSO are,

$$q = \begin{pmatrix} 50 & 0 & 0 & 0 \\ 0 & 1.305 & 0 & 0 \\ 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 5.2 \end{pmatrix}, r = \begin{pmatrix} 1 & 0 \\ 0 & 5.55 \end{pmatrix}.$$

Table 3 MSE of the longitudinal, lateral and of the yaw angle according to the proposed methods and with different longitudinal speeds (see online version for colours)

v_x [ms]		10	15	19	21.5	
MSE	Y [m] (Falcone et al. (2007))	0.0177	0.0256	0.0303	0.0541	
	ψ [deg] (Falcone et al. (2007))	0.39	0.41	0.42	0.68	
	Y [m]	MPC Only	0.0162	0.0113	0.1929	0.3232
		MPC & PSO	0.0040	0.0041	0.0317	0.0693
	ψ [deg]	MPC & Fuzzy Logic	0.0156	0.0104	0.1860	0.3798
		MPC Only	0.2434	0.1715	13.3901	16.9528
		MPC & PSO	0.9312	0.2334	2.9279	3.0059
	X [m]	MPC & Fuzzy Logic	0.2320	0.1646	12.6286	18.4702
		MPC Only	0.2694	1.5387	1.4676	5.0723
		MPC & PSO	0.0782	0.1559	0.2743	0.3177
		MPC & Fuzzy Logic	0.2730	1.5518	1.4738	6.8141

From Table 2 we see that the values of the lateral errors increase when the longitudinal speed increases. The values of the lateral errors are minimal when using the proposed strategy MPC with constraints & PSO. Also the MPC with constraint method gives better results than the method used in Falcone et al. (2007). The second strategy of MPC with constraint and the approximation of the lateral forces are efficient and give better results than the method used also in Falcone et al. (2007). It can be that the yaw errors obtained by the proposed methods (MPC only, MPC and PSO, MPC and fuzzy logic) are always lower than those obtained by the method used in Falcone et al. (2007). The longitudinal position errors are acceptable, and its values increase as the longitudinal velocity increases. The best method used is MPC with PSO in low speeds, and MPC only in high speeds.

In Table 3, our MPC method based on PSO gives a good result when processing lateral dynamics, especially at low speeds ($v_x = 10$ m/s and 15 m/s), and obtains the same

results as the method used in Falcone et al. (2007) at high speeds ($v_x=19$ m/s and 21.5 m/s). This method is excellent for processing longitudinal dynamics as well. The second method, MPC that utilises fuzzy logic, achieves the best results at low speed and an adequate result at high speed, even if the dynamics of the lateral forces are unknown.

Figure 9 illustrates the desired trajectory and the trajectories obtained by the different methods used in this strategy. We note that the trajectories follow the desired trajectory in all cases. The difference between these methods when the longitudinal speed increases is that MPC with constraint and PSO gives the best result, the method of MPC with constraint and fuzzy logic comes in the second ranking, and the MPC method with constrained in the third ranking and better than the method used in Falcone et al. (2007) (See Table 2).

Figure 9 The trajectories with: (a) $v_x = 10$ ms; (b) $v_x = 15$ ms; (c) $v_x = 19$ ms and (d) $v_x = 21.5$ ms (see online version for colours)

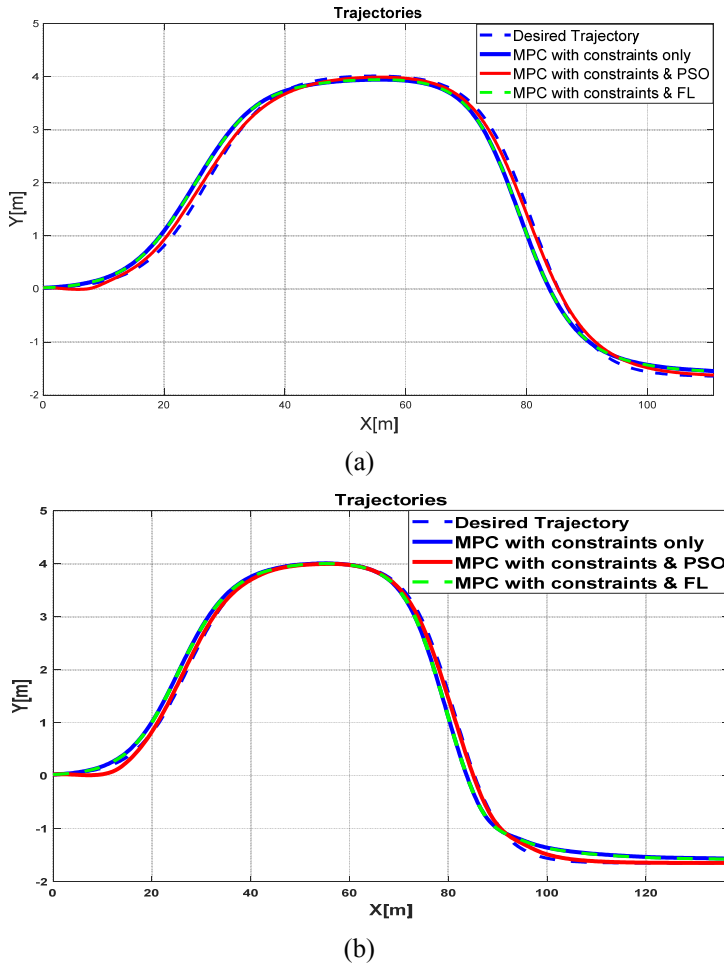
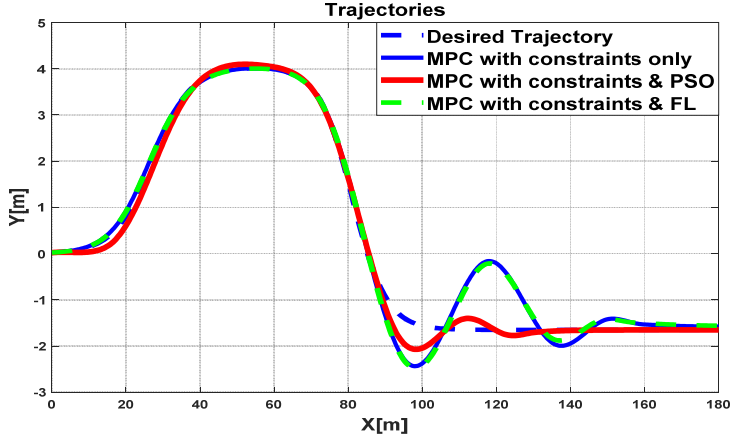
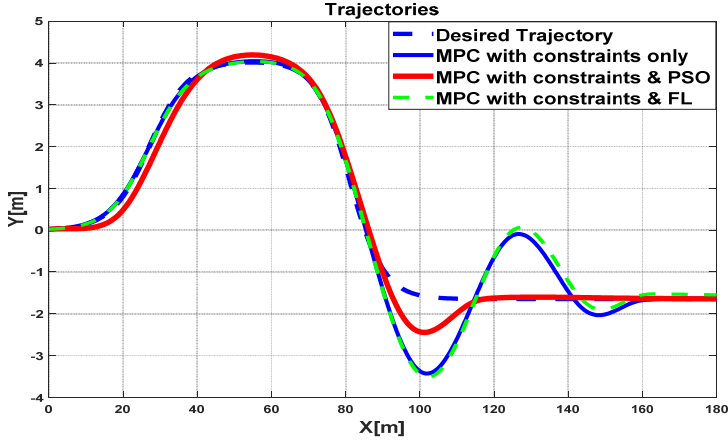


Figure 9 The trajectories with: (a) $v_x = 10$ ms; (b) $v_x = 15$ ms; (c) $v_x = 19$ ms and (d) $v_x = 21.5$ ms (see online version for colours) (continued)



(c)

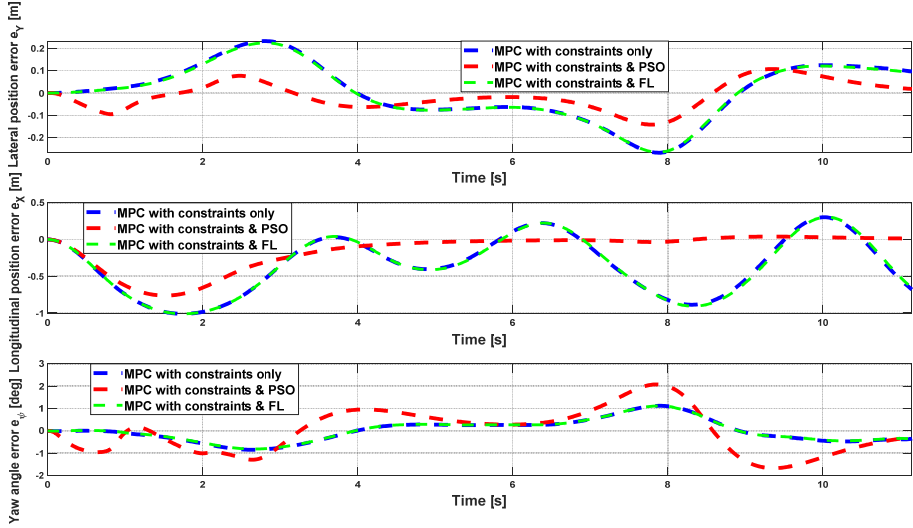


(d)

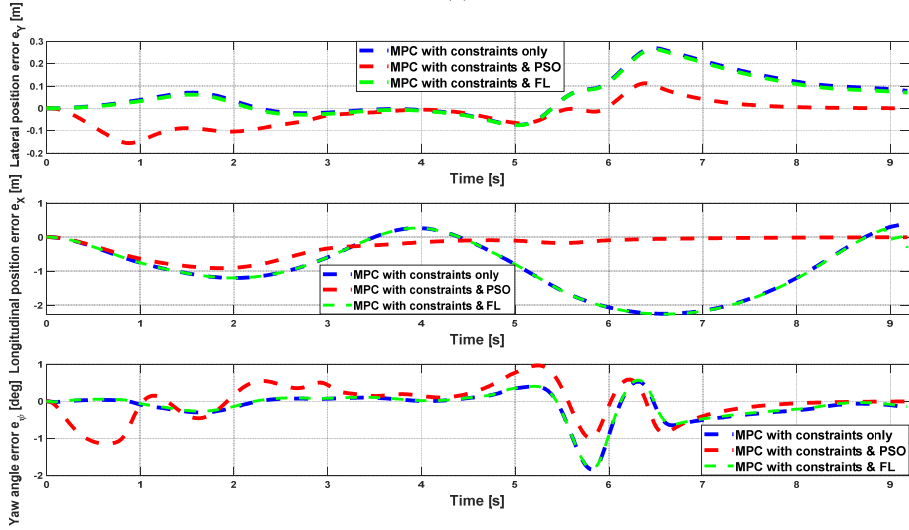
From Figure 10 we can notice that, the lateral position error varies around 0, and always less than the method used in Falcone et al. (2007) (See Table 2). When using the low longitudinal velocity $v_x = 10$ m/s and 15 m/s, the error is 6 times less than the method used in Falcone et al. (2007), for example when using $v_x = 10$ m/s the error does not exceed 0, 142 m by our method but equal to 0.96 m by using the method in Falcone et al. (2007), when using the high longitudinal velocity $v_x = 19$ m/s and 21.5 m/s, the error is almost 3 times less than the method used in Falcone et al. (2007), for example using $v_x = 19$ m/s the error does not exceed 0.56 m by our proposed method. The best method used is MPC with PSO because we find that the maximum error values found by

this method are the minimum in all the proposed methods. The MPC method with fuzzy logic is efficient and the variation of the error values found by this method are lower than the variation of the error values found by the method used in Falcone et al. (2007).

Figure 10 The errors of e_y , e_x , e_ψ with: (a) $v_x = 10$ ms; (b) $v_x = 15$ ms; (c) $v_x = 19$ ms and (d) $v_x = 21.5$ ms (see online version for colours)

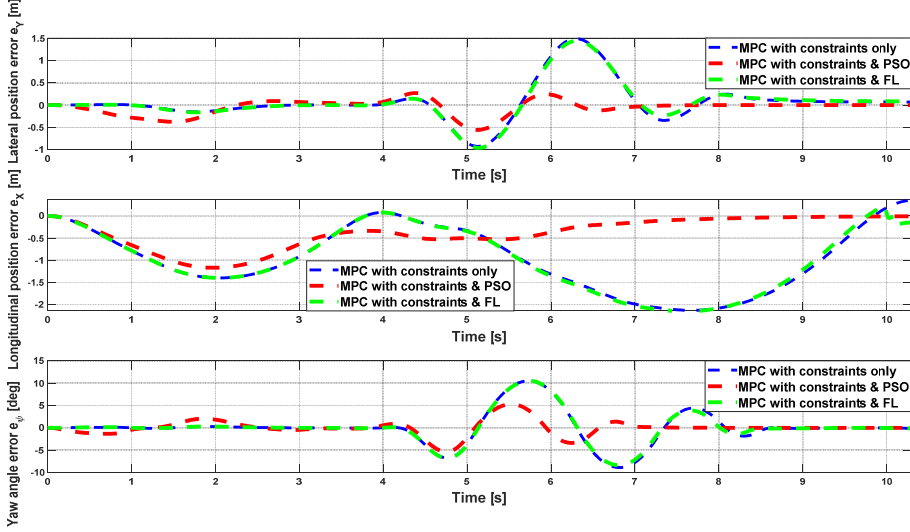


(a)

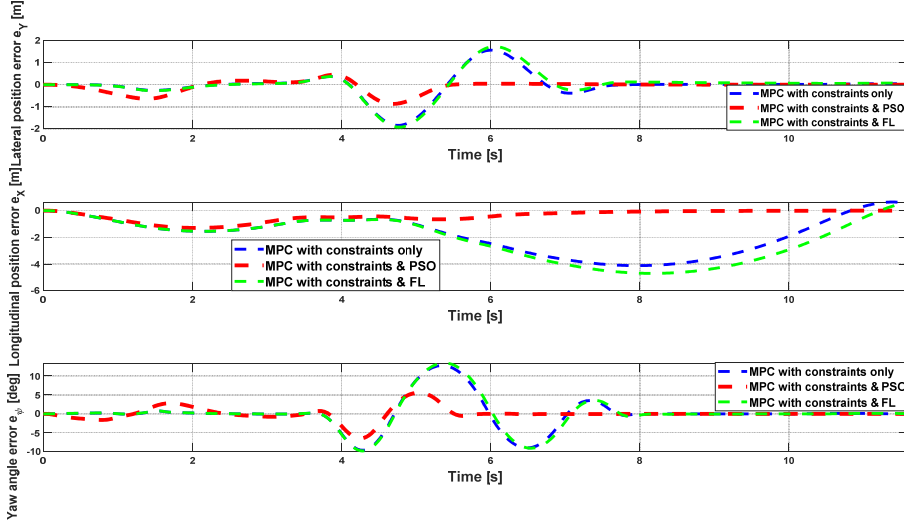


(b)

Figure 10 The errors of e_y , e_x , e_ψ with: (a) $v_x = 10$ ms; (b) $v_x = 15$ ms; (c) $v_x = 19$ ms and (d) $v_x = 21.5$ ms (see online version for colours) (continued)



(c)



(d)

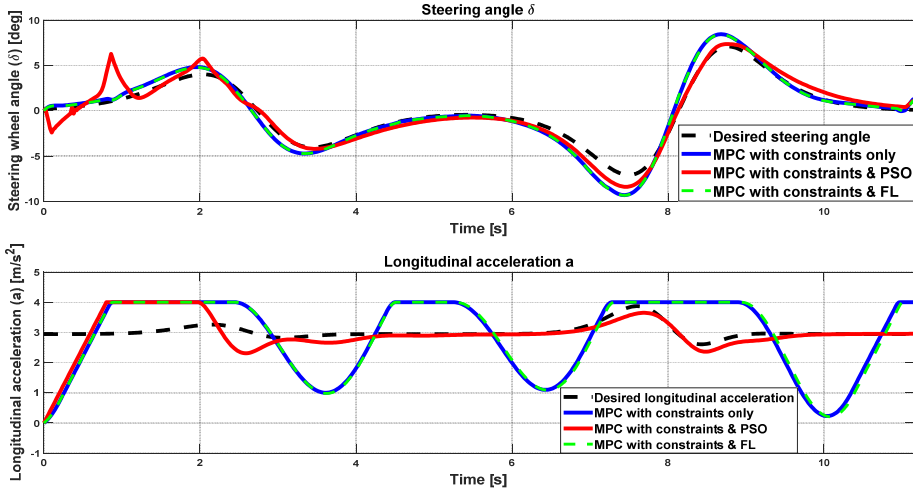
The longitudinal error decreases rapidly at the start (far from 0) then remains stable in all trajectories and the maximum value in the case of the speed $v_x = 10$ m/s is equal to 0.76 m, 0.91 m using $v_x = 15$ m/s, 1.17 m using $v_x = 19$ m/s and 1.29 m with $v_x = 21.5$ m/s.

The yaw error oscillates around 0 see Figure 10. The error obtained by the proposed methods is better and lower than that obtained by Falcone et al. (2007).

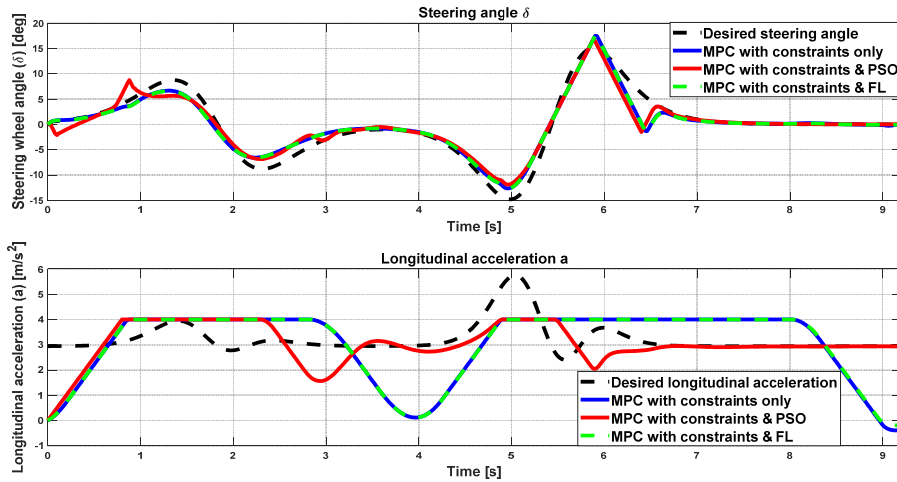
Figure 11 shows, the steering angle δ_f and longitudinal acceleration a , with the use of different longitudinal speeds.

All the curves obtained by the different methods follow the desired steering angle control curve. It is observed that the curves coincide with the desired curve at low longitudinal speed ($v_x = 10$ m/s and 15 m/s), with a small oscillation at the beginning in the case of using the MPC method with PSO. The curves are a little away from the desired curve in the high longitudinal speeds ($v_x = 19$ m/s and 21.5 m/s), and oscillate at the end of the curve in the case of MPC only and in the case of MPC use with fuzzy logic.

Figure 11 The commands steering angle δ_f and longitudinal acceleration a with: (a) $v_x = 10$ ms; (b) $v_x = 15$ ms; (c) $v_x = 19$ ms and (d) $v_x = 21.5$ ms (see online version for colours)

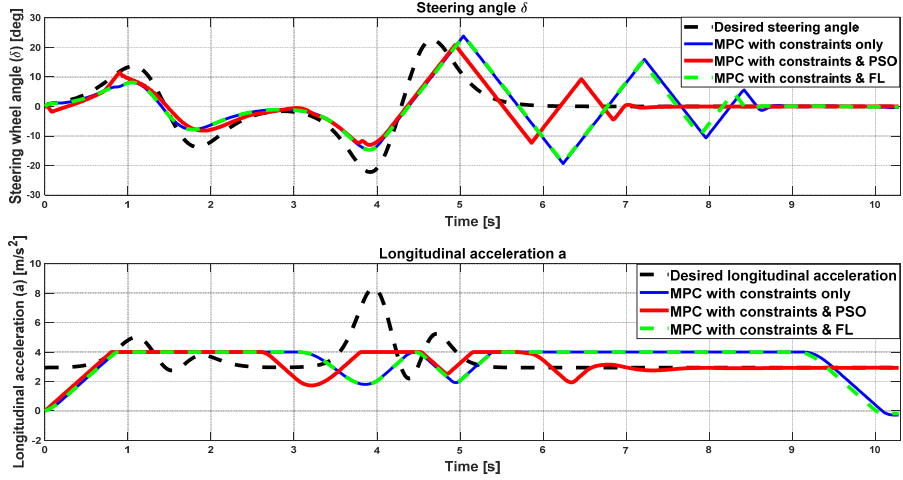


(a)

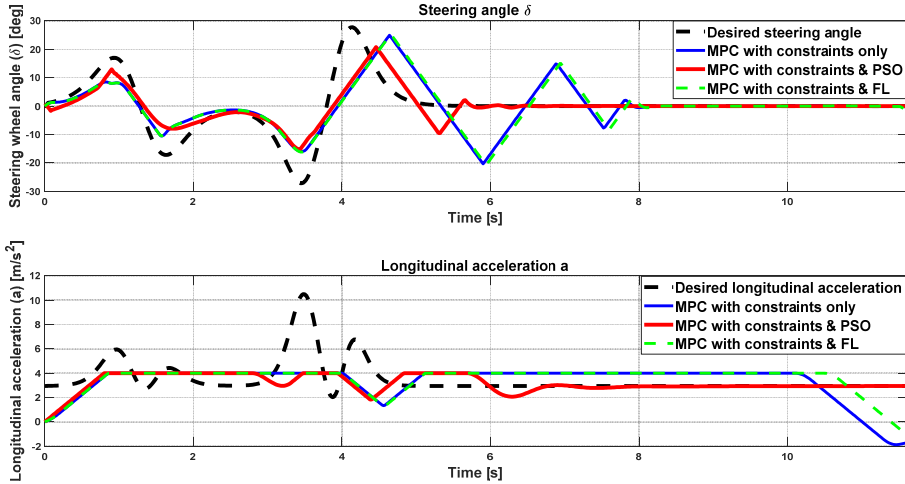


(b)

Figure 11 The commands steering angle δ_f and longitudinal acceleration a with: (a) $v_x = 10$ ms; (b) $v_x = 15$ ms; (c) $v_x = 19$ ms and (d) $v_x = 21.5$ ms (see online version for colours) (continued)



(c)



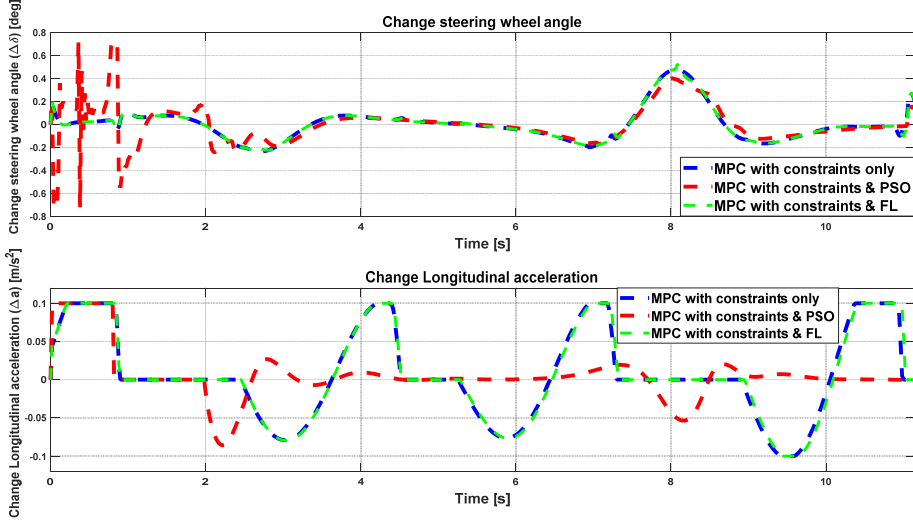
(d)

All the curves obtained by the different methods follow the desired longitudinal acceleration control curve at low longitudinal speeds with a limitation of 4 m/s^2 in the curves, especially when using the MPC method only and with fuzzy logic. The curves follow the desired curve but the constraints appear well in the case of high longitudinal velocities ($v_x = 19 \text{ m/s}$ and 21.5 m/s), and especially when the MPC method is used alone and with fuzzy logic.

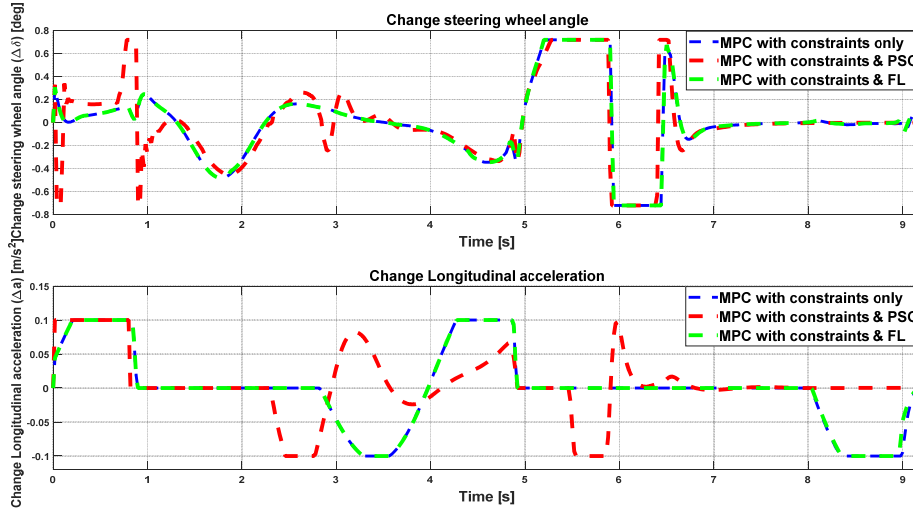
Figure 12 illustrated the change commands steering angle $\Delta\delta_f$, and longitudinal acceleration Δa , with different longitudinal speeds. The amplitude of the various

changes of the controls is weak in the low longitudinal speed, and increases during the increase of the longitudinal speed and also the oscillation appears in the use of the high longitudinal speed. It is noted that the constraints applied to the control changes are well respected ($-0.72 \text{ deg} \leq \Delta\delta_f \leq 0.72 \text{ deg}$, $-0.1 \leq \Delta a \leq 0.1$). The longitudinal acceleration change control curves are cut when they reach -0.1 or 0.1 and also the steering angle change control curves are cut on arrival -0.72 deg or 0.72 deg .

Figure 12 The change commands steering angle $\Delta\delta_f$ and longitudinal acceleration Δa with: (a) $v_x = 10 \text{ ms}$; (b) $v_x = 15 \text{ ms}$; (c) $v_x = 19 \text{ ms}$ and (d) $v_x = 21.5 \text{ ms}$ (see online version for colours)

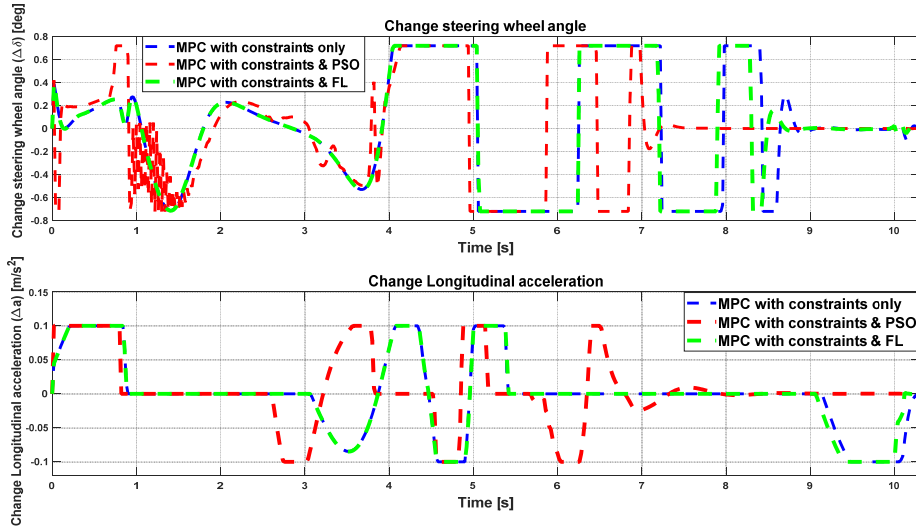


(a)

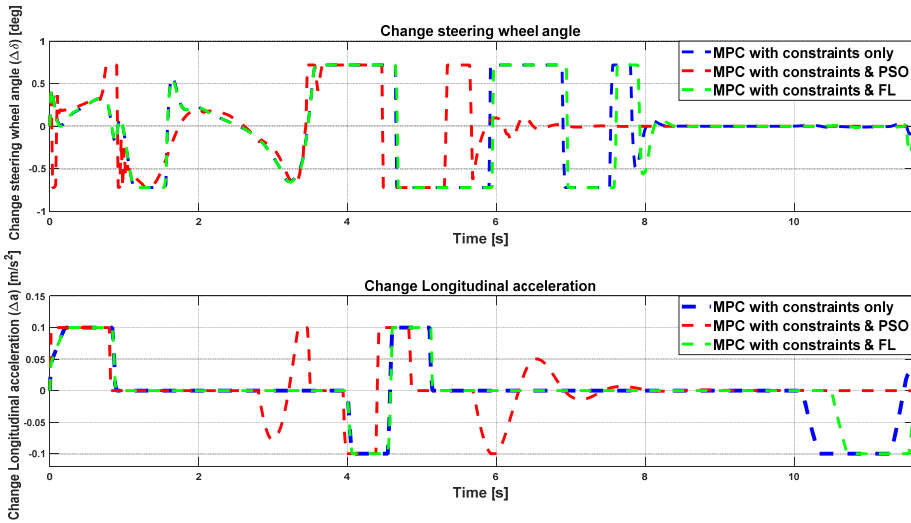


(b)

Figure 12 The change commands steering angle $\Delta \delta_f$ and longitudinal acceleration Δa with: (a) $v_x = 10$ ms; (b) $v_x = 15$ ms; (c) $v_x = 19$ ms and (d) $v_x = 21.5$ ms (see online version for colours) (continued)



(c)



(d)

From Figure 13, the velocity longitudinal used is variable as a function of time, the longitudinal speed curves obtained by the MPC methods with PSO follow the desired longitudinal speed curve. In the case of using the MPC method only and with fuzzy logic the curve follows the desired curve but with a low margin of uncertainty.

All the lateral velocity curves obtained by the different proposed methods are oscillating around 0, and are varying of the same form of the desired lateral velocity but of amplitude minus 6 times the desired velocity. And it's better for our system, to make the system more stable.

Figure 14 shows that, all of the X and Y position curves and the yaw angle curve follow their desired curves, and coincide with their desired curves at the low longitudinal speed. Weak oscillations appear at the end of the Y position curves and the yaw angle in high longitudinal speeds. The X position curve remains stable even at high longitudinal speeds.

Figure 13 The velocity longitudinal v_x and lateral v_y with: (a) $v_x = 10$ ms; (b) $v_x = 15$ ms; (c) $v_x = 19$ ms and (d) $v_x = 21.5$ ms (see online version for colours)

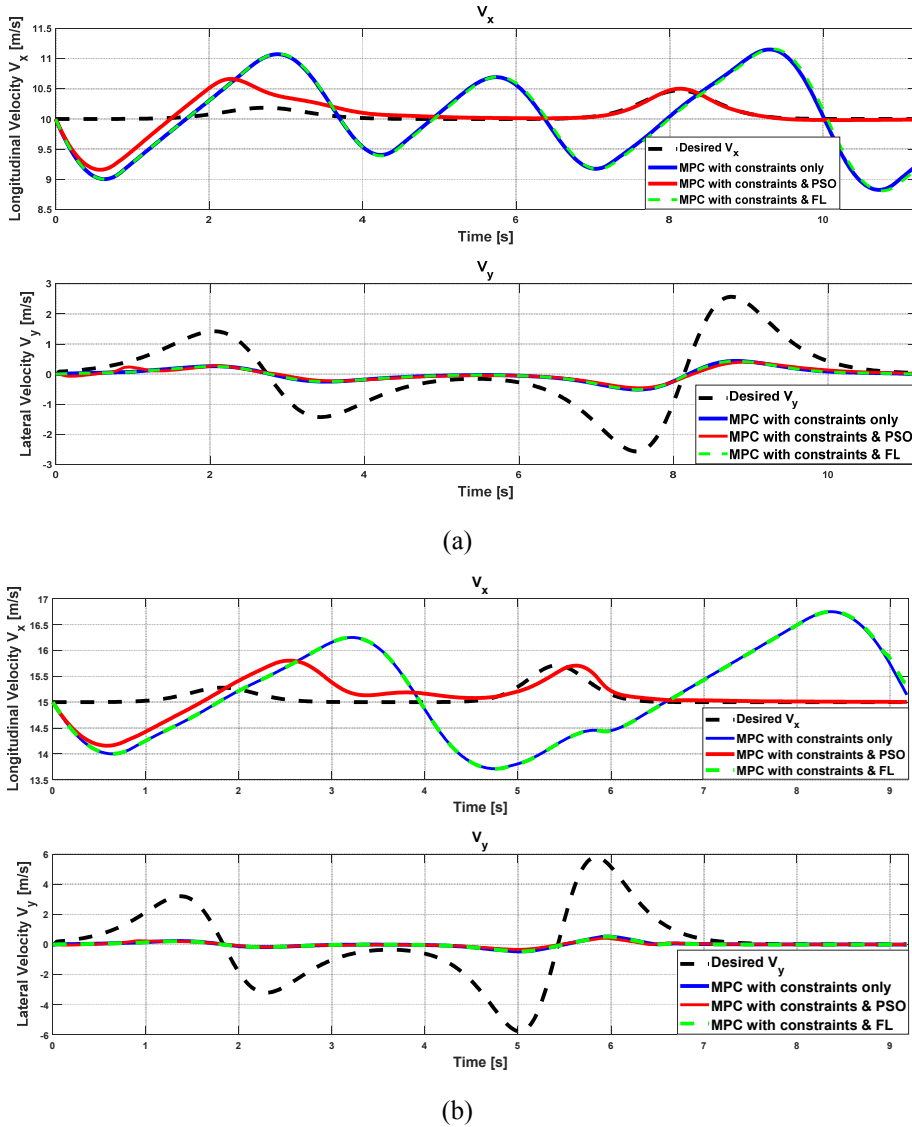
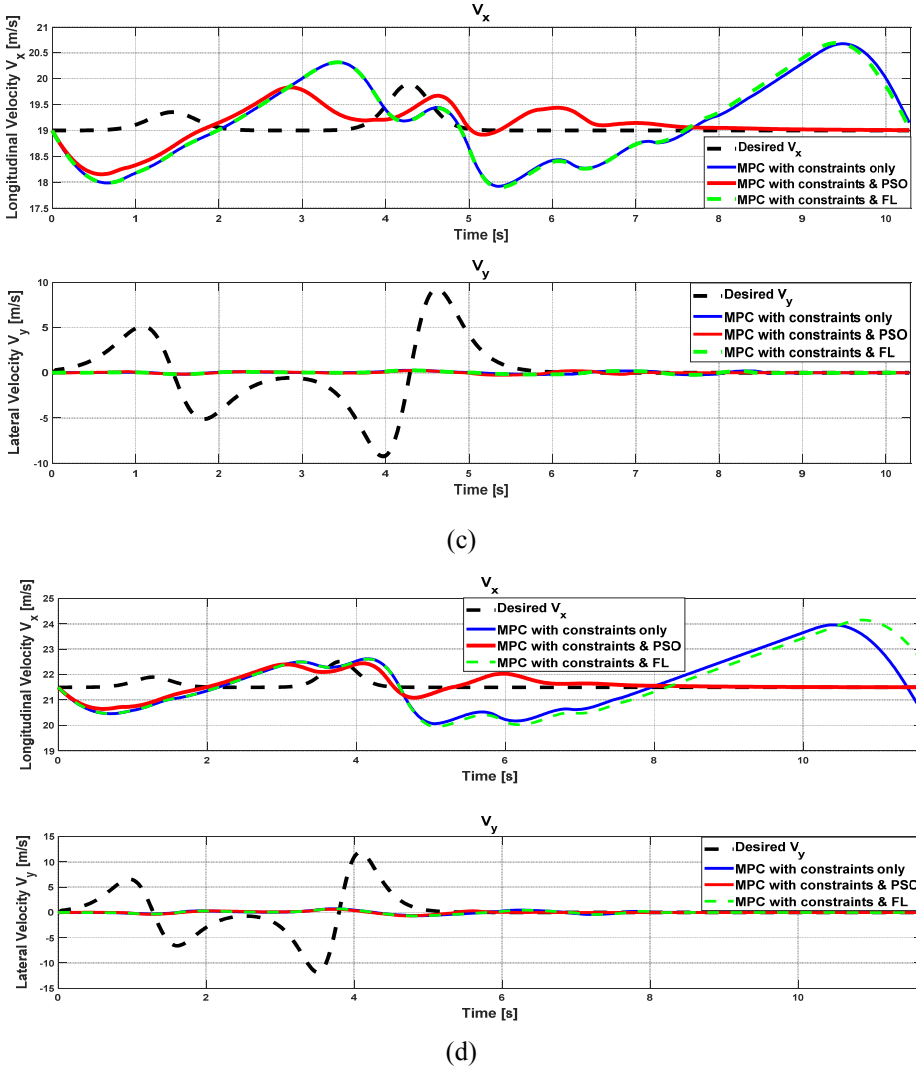


Figure 13 The velocity longitudinal v_x and lateral v_y with: (a) $v_x = 10$ ms; (b) $v_x = 15$ ms; (c) $v_x = 19$ ms and (d) $v_x = 21.5$ ms (see online version for colours) (continued)



Remarks:

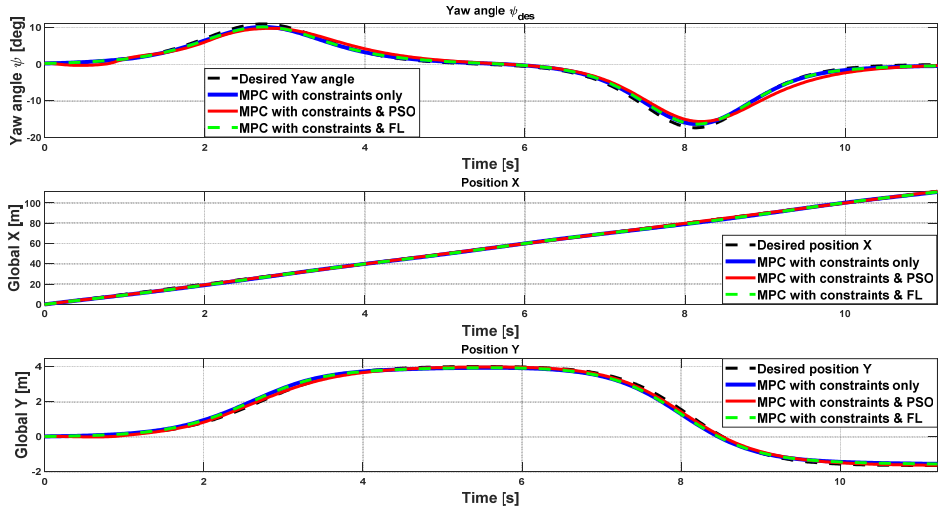
- The choice of Q , S and R weights are responsible for improving lateral and longitudinal dynamics. The choice of the weights changes the result and we can choose the weights in such a way as to improve the lateral dynamics or the longitudinal dynamics.
- The relevant choice of the values of the weights e_{v_x} , e_x and e_y determines the study objective. If we want to minimise the lateral position error, we increase the weights values of e_y and e_{v_x} , and if we want to minimise the longitudinal position error, we decrease the values weights of e_x and e_{v_x} .

Note that the MPC method with PSO is considered the best method. So in our study, the weights Q , S and R are chosen in order to improve the lateral dynamics.

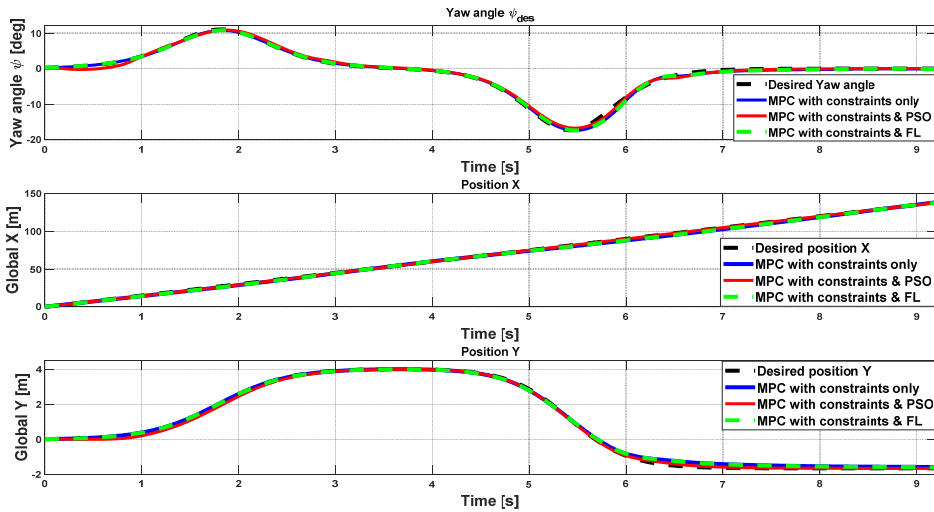
The good performances and the robustness of the MPC with constraints are demonstrated by the results of this simulation, either when applying this controller only or when combining it with another artificial intelligence technique like PSO or fuzzy logic.

The predictive control model is a robust controller, able to overcome disturbances and modelling uncertainties.

Figure 14 The position (X , Y) and the yaw angle of vehicle with: (a) $v_x = 10$ ms; (b) $v_x = 15$ ms; (c) $v_x = 19$ ms and (d) $v_x = 21.5$ ms (see online version for colours)

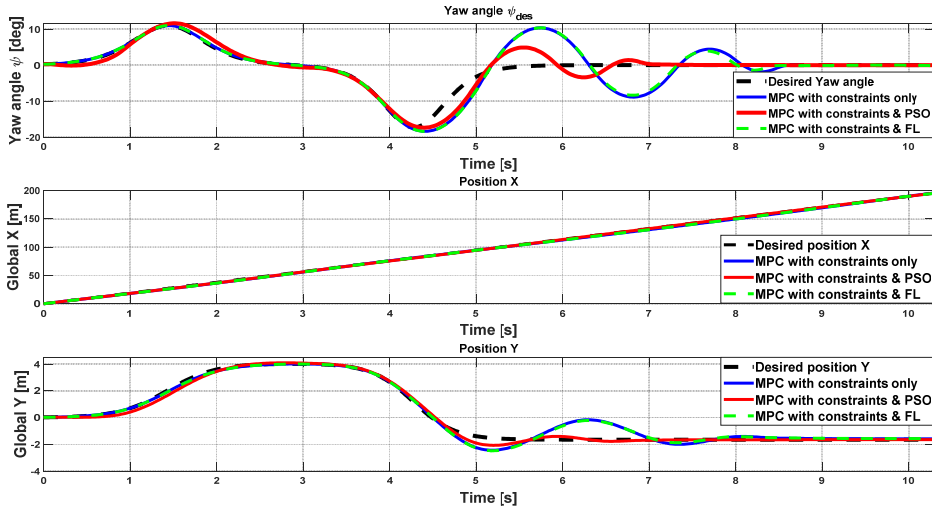


(a)

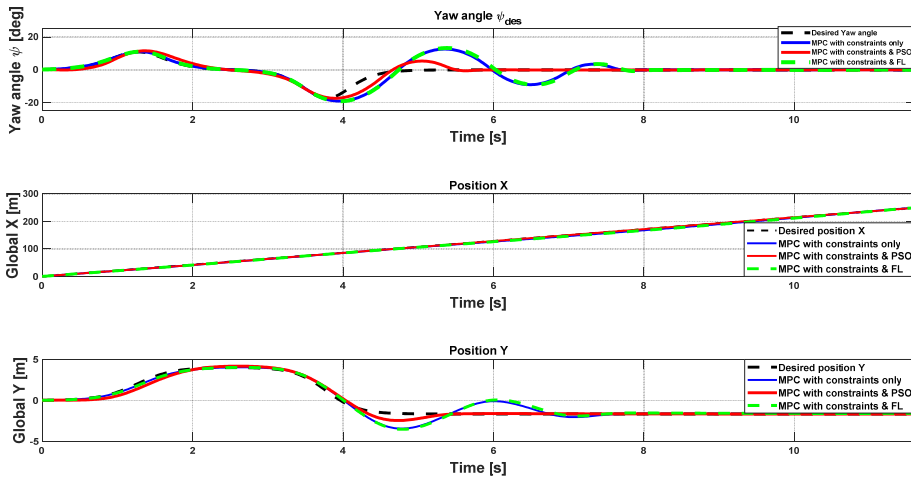


(b)

Figure 14 The position (X, Y) and the yaw angle of vehicle with: (a) $v_x = 10$ ms; (b) $v_x = 15$ ms; (c) $v_x = 19$ ms and (d) $v_x = 21.5$ ms (see online version for colours) (continued)



(c)



(d)

5 Conclusion and future works

5.1 Conclusion

In this study, the MPC controller with constraints is used to control a complex system with multi-input and multi-output (MIMO). To improve the performance, two methods of autonomous vehicle control are proposed, the first method is MPC with constraints combined with PSO, this method allows to optimise the weights of controller and allows

to obtain better results. The MPC method based on the PSO makes possible to obtain results up to 6 times better than the results available in the literature. The second method is the use of MPC with constraints combined with fuzzy logic. The effectiveness and good results of this method can be achieved without knowledge of the lateral dynamics of the system. Fuzzy logic is used to approximate these dynamics. This method remains more efficient than MPC with constraints alone and less efficient than MPC with constraints combined with PSO, but the advantage of this method is to control the system without knowing their dynamics. The choice of the weights Q , S and R , allows us to favour either the improvement of the lateral dynamics, or the longitudinal dynamics.

The MPC with Constraints is combined with the artificial intelligence techniques to minimise the lateral and longitudinal dynamic error at variable longitudinal velocities. It allows the production of complex control laws but it provides better results. The simulation results demonstrate the improvement of the results.

5.2 Future works

Further research is necessary to resolve a number of open issues. Below are some of them:

- the use of MPC combined with the neural network to approximate lateral forces
- the use of MPC combined with neural network or fuzzy logic, or adaptive neuro-fuzzy inference system (ANFIS) to approximate longitudinal forces
- study of a complete autonomous vehicle system taking into account vertical forces
- adds other constraints (sub-objective) to the cost function.

An in-depth study of artificial intelligence combined with MPC will be developed to ameliorate the performance of the MPC and the system in future works.

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