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Auction game of agro-product under cobweb phenomenon of supply and demand

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Abstract: Auction of agro-product between buyers and a farmer deals with first-price auctions in the situation of the cobweb model of supply and demand. The first price bidding auction lies in the neighbourhood of a balanced price of the indivisible agro-product. The selection of bid price holds the Weibull probability density function. We measure the available agro-product at the balance point of demand and supply which is associated with the balanced price. A new computational approach has been employed to obtain the optimum expected bidding value to get the win as well as the expected revenue earned by the farmer in the first price auction of agro-products under cobweb nature price fluctuations.

Keywords: agro-product; cobweb phenomenon; first-price auction; Weibull distribution; inventory level.

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1 Introduction

A market is a set of arrangements by which buyers and sellers come into contact and exchange goods or services and in many cases, this exchange is carried out via auction. Hence, the analysis of the auction becomes very important, especially for the wholesale trades of agro-products. The major challenge in the auction of agro-product is the uncontrolled influence of the wholesale network, especially in the import of fruits and vegetables. Wholesale market institutions are essential to market in the trade of agro-products in developed countries. Kuzman (2019) observes that the wholesale market is a legal entity registered for carrying out activities of the wholesale organisation in accordance with regulations on the registration of economic entities. They can trade through auctions for fast sales and also have an informative role at the regional and national market level, especially for buyers and sellers. The electronic and printed media as well as media from the field of agro-industry support in aspect of reporting about the auction. Krishna (2002) focused on the expected selling price in a symmetric first price auction which is the same as in the second price auction game but the risk-neutral seller is indifferent between the two formats and in particular instances, the price in one or the other auction may be higher. Zhang and Sogn-Grundvåg (2023) investigate the impact of reserve prices on bidder participation in the main Norwegian auction market for frozen Atlantic cod. They found that the reserve price actually has no negative influence on bidder participation. Das (2023) gives a model of collusion formation and a method of measuring the degree of it among the traders/bidders in the agricultural commodity markets in India. The important assumption is that the bidding is repetitive with a set of common bidders.

Chiu et al. (2022) estimate the pricing efficiency of a livestock auction market using a two-tier stochastic frontier model. This model captures the pricing inefficiency of a hedonic pricing model for livestock via exponential error terms above and below the hedonic model specification. GeirSoqn-Grundvag and Zhang (2023) examine the comparison between selling agricultural commodities by auction or directly. Hedonic price analysis using transaction data indicates that direct sales are more informationally efficient than auctions. Meanwhile, auctions gain a price premium over direct sales holding other variables constant. However, a substantial increase in the use of direct sales indicates that their information efficiency is more important to sellers than the small price premium provided by auctions. Milgrom (1989) analysed that an experienced estimator working in familiar terrain can lose money if he does not understand the subtleties of competitive bidding. Myerson (1981) made an auction design of a seller whose single object, is to be sold to one of several possible buyers. The seller has imperfect information about the bidding price. The theoretical and empirical research tend to achieve the effect of efficiency and maximum revenue.

In this paper, we develop a model on the first price sealed auction game of agro-product under the cobweb phenomenon that arises in agricultural markets. From this model, we are able to get the optimum bidding price of bidders and marginal revenue gained by auctioneers (or farmers). This paper also aims to provide the application of auction games to the wholesale of agro-products under the cobweb nature of supply and demand. Due to a lack of mathematical intuition in agro-products auctions, the farmers are not able to get suitable revenue and bidders are also not able to bid their optimum bidding value. We provide a mathematical model to solve the problem of the agro-product auction in terms of optimum expected bidding value and also compute the inventory level at the auction time with its associated actual value. However, auctions are a classical example of trade with mutually expanded valuations, so theoretical and empirical researches tend to achieve the effect of efficiency and maximum revenue. Nowadays, the online auction system is very popular for buying and selling products. It facilitates easier, faster, and more reliable auction methods. Nalinipriya et al. (2019) introduced a dynamic system to sell and buy agricultural products based on auction by developing a software environment for farmers to gain maximum profit. An online auction administrator sells the product to the desired person. Crespi and Xia (2015) presented an analytical model of a first-price sealed-bid cattle auction a spot and coordinated markets are interconnected. The model reveals that the conventional wisdom that market coordination negatively affects the bid price in the spot market is an oversimplification. Zossou et al. (2022) combine ‘exogenous’ information treatments on the benefits of a locally produced improved rice processing technology with an ‘endogenous’ information treatment. They assess the effects of these information treatments on consumers’ choice and valuation of parboiled rice in Benin. Agricultural products are most commonly produced depending on the demand of the past period, which is also called as cobweb cycle. Here farmers do not have actual knowledge about the future demand of their products. So it is challenging to come up with a model to predict the future demands so that the products are adequately produced.

Gloser-Chahoud et al. (2017) use a classical cobweb theorem and link the dynamics of row material markets and commodity price fluctuation to a delayed adjustment of supply. Poitras (2023) examines the first wave of linear cobweb theory initiated by Tinbergen (1930), Schultz (1930) and Ricci (1930) and proceeds to consider the evolution of price expectations in the second wave of cobweb models associated with endogenous cycles in commodity markets. The role of modern cobweb theory in discussions about the stability of market equilibrium and the connection to processes with rational expectations is assessed. Dieci et al. (2022) develop a cobweb model in which firms, facing a two-period production delay, have access to a flexible and inflexible production technology. Cavalli et al. (2021) study a class of agricultural and energy markets. They can be characterised by seasonality with a precise in periodicity of demand, and supply. They analytically prove the emergence of dynamical scenarios that are missing in the classic cobweb model for non-seasonal markets. Mustafa et al. (2023) give a review based on the PRISMA framework to systematically identify and analyse literature from two important databases. This work has been grouped by volatility drivers, governance levels, theoretical approaches, and background data types. It also highlights the importance of a paradigm shift in researching agriculture price volatility to addressing food crises, considering principal objectives and perspectives such as food security, poverty alleviation, sustainability in food value chains, and resilience of food systems across the globe. Farhadi et al. (2023) consider an outsourcing problem where

a software agent procures multiple services from providers with uncertain reliabilities to complete a computational task before a strict deadline. The service consumer's goal is to design an outsourcing strategy to optimise a specific objective function. This objective function can be different based on the consumer's nature; a socially focused consumer often aims to maximise social welfare, while a self-interested consumer often aims to maximise its utility. However, in both cases, the objective function depends on the providers' execution costs, which are privately held by the self-interested providers and hence may be misreported to influence the consumer's decisions.

The price of agro-products also depends on demand and vice-versa, so it is also important to come up with a method to find the best price for farmers as well as for consumers. Axell (1988) presented the analyses of the outcome of a price formation process with information costs. They examine the equilibria in the markets under two situations:

- 1 consumers are having no information about prices but have information about the distribution of prices
- 2 when consumers are not informed about the distribution of prices and conclude that the equilibrium might differ considerably.

They concluded that if firms have to make experiments to find out the profit function, the market will approach the price dispersion configuration. Mishra and Mishra (2008) determine the price of EOQ under the perfect competition of the marketing structure under a dynamic situation with marginal revenue and marginal cost approach of optimisation technique. Mishra and Mishra (2011) have computed the price at any time t under the cobweb phenomenon. This chapter is focused on the imperfect marketing structure problems, where the price is not fixed and the elasticity of demand is more. Hey (1974) thought that if each firm knows that it faces a downward-sloping demand curve. The demand has to be estimated and positioned. It depends on the relation of that firm's price to the prices charged by other firms and given the distribution function $g(p|k) = kf(p)[1 - F(p)]^{(k-1)}$, where customers sample k firms and buy from the cheapest firm. If $f(p)$ is the density function of the distribution of prices let $F(p)$ be the distribution function. Sen et al. (2018) discussed the price of agro-products in the convergent, divergent, and enclosed nature of cobweb models of prices. Taleizadeh et al. (2015), in their model, assume the production process both healthy and defective items are generated. They also assume that defective items can be sold at a lower price than the selling price. Their goal was to maximise the total profit of each member. To solve this model the Stackelberg approach is employed between the members. Tiwari et al. (2018) explicitly analysed the emerging retail scenario the impact of price, inventory level, and the integration among retailers and suppliers with limited shelf space.

Various auction methods have been employed to sell agro-products in order that both farmers and consumers are satisfied with the price. Zou et al. (2018) formulated double-sided auction games as a pair of single-sided auction games, which are coupled through potential resources. They designed a decentralised iteration procedure to achieve an efficient solution. Gan et al. (2005) use a single-period auction game model for analysing strategic behaviour in the pool-based electricity market. Kim and Koh (2022) consider independent private value auctions in which prior to bidding, each bidder engages in fully flexible information acquisition, choosing not only how much information but also what information to acquire about his value. They also analyse

how the reserve price influences bidders' information-acquisition behaviour and study its implications for the seller's expected revenue. Byde (2003) describes an evolution-based method for determining auction mechanisms and applies it to first-price and second-price sealed auctions. He replicates known results and extends the literature by establishing better mechanisms over standard mechanisms. Laffont (1997) surveyed auction data available and conclude that predictions are highly dependent on unobservable restrictions and suggested that to develop structural econometrics of auction data with only a few restrictions. Wilson (1992) discusses various forms of auctions and presents the formulations used in the main models and finds that auctions remain to be the simplest and most familiar means of price determination for many tradings without a middleman. Other motivational works in this direction are Sood et al. (2011) and Edalat et al. (2012). Current related work is done by Kawai and Nakabayashi (2022) and Zhang et al. (2021).

This paper consists of a mathematical model in Section 3, this section covers the cobweb phenomenon of agro product, inventory level, the total cost function of the agro product, and computation of the actual price of agro product at an equilibrium position under the cobweb phenomenon. Section 4 focuses on the assumption of agro-product first price bidding auction and computation of optimum value. Section 5 contains the revenue equivalence theorem and the problem of bidding, Section 6 shows the proposition based on obtained results. Section 7 contains sensitivity analysis which consists of figures and tables and their observations. And Section 8 shows the conclusion of the paper.

2 Notations

Notations in Table 1 are used in the paper.

Table 1 Notation and description

$\mathbb{R}$	Set of real numbers
S_t	Quantity supplied at time t
D_t	Quantity demanded at the time t
I_t	Inventory level at time t
P_{it}	Actual price of the item to i^{th} agent at time t in an equilibrium situation
θ	Constant rate of deterioration
Θ	Total deteriorated quantity in one cycle
$\overline{P_{it}}$	The upper value of the item price at time t to the i^{th} agent
$\underline{P_{it}}$	The lower value of the item price at time t to the i^{th} agent
n	Number of bidders
C_{Au}	Auction set up cost per auction
C_d	Deterioration cost per unit item

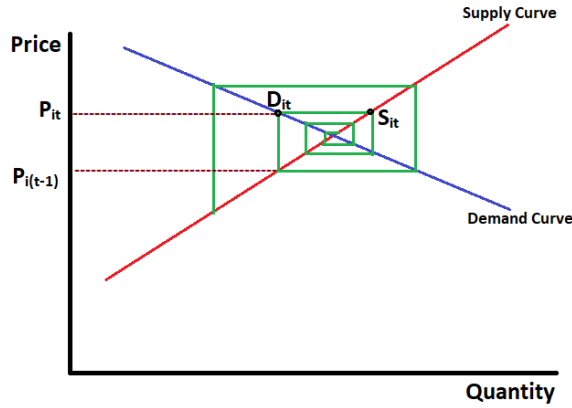
3 Mathematical model

3.1 Cobweb phenomenon of agro-product

Here, we focus on the agro-product market which deals with incomplete information about the price. The farmer faces difficulty in taking the decision of the actual amount

of production of agro-product to fulfil the current demand. The farmers cultivate the agro-product based on the price at that time. But due to unpredicted weather conditions and the time lag between the harvesting and cultivation price of the agro-product fluctuates. So, the current supply of agro-product by the i^{th} agent is based on the preceding price but the current demand is based on the current price. This is the main cause of the cobweb nature of demand and supply as shown in Figure 1. Consequently, we get the following two relations. $D_{it} = b_i P_{it} + a_i$ and $S_{it} = g_i P_{i(t-1)} + h_i$, $i \in N$, $i = 1, 2, 3, \dots, n$ where, P_{it} = actual price of the item to i^{th} agent at time t , D_{it} = quantity demanded by the i^{th} agent at time t , S_{it} = quantity supplied to the i^{th} agent at time t .

Figure 1 Demand vs. price and supply vs. price graph for a product (see online version for colours)



3.2 Inventory level and total cost of agro-product

Consider the demand-price graph and the supply-price graph of the i^{th} agent at time t given by

$$D_{it} = b_i P_{it} + a_i \text{ and } S_{it} = g_i P_{i(t-1)} + h_i, \quad i \in N, \quad i = 1, 2, 3, \dots, n$$

For equilibrium situation,

$$S_{it} + D_{it} = 0.$$

Differentiating w.r.t. time t , we get

$$\begin{aligned} \frac{dS_{it}}{dt} &= \frac{dI_t}{dt} + \theta I_t = -\frac{dD_{it}}{dt} \\ \frac{dI_t}{dt} + \theta I_t &= -b_i \frac{dP_{it}}{dt} \end{aligned} \tag{1}$$

$$\therefore \frac{dI_t}{dt} + \theta I_t = g_i \frac{dP_{i(t-1)}}{dt} = -\frac{g_i}{b_i} \left(\frac{dI_{t-1}}{dt} + \theta I_{t-1} \right) \tag{2}$$

$$\Rightarrow \frac{d(I_t + \nu_i I_{t-1})}{dt} + \theta (I_t + \nu_i I_{t-1}) = 0$$

where $\frac{g_i}{b_i} = \nu_i$.

Let

$$(I_t + \nu_i I_{t-1}) = U_{it} \Rightarrow \frac{dU_{it}}{dt} = -\theta U_{it} \Rightarrow U_{it} = Ae^{-\theta t}.$$

Initial condition,

$$\begin{aligned} U_{i1} &= I_1 = Ae^{-\theta} \Rightarrow A = I_1 e^{\theta} \Rightarrow U_{it} = I_1 e^{-\theta(t-1)} \\ \therefore I_t + \nu_i I_{t-1} &= I_1 e^{-\theta(t-1)} \\ \therefore I_2 + \nu_i I_1 &= I_1 e^{-\theta} \\ I_3 &= I_1 e^{-2\theta} - \nu_i I_2 \Rightarrow I_3 = I_1 e^{-2\theta} - \nu_i I_1 e^{-\theta} + \nu_i^2 I_1 \\ I_4 &= I_1 e^{-3\theta} - \nu_i I_3 \Rightarrow I_4 = I_1 e^{-3\theta} - \nu_i I_1 e^{-2\theta} + \nu_i^2 I_1 e^{-\theta} - \nu_i^3 I_1 \end{aligned} \quad (3)$$

Similarly, by mathematical induction, we can write

$$\begin{aligned} I_n &= I_1 e^{-(n-1)\theta} - \nu_i I_1 e^{-(n-2)\theta} + \nu_i^2 I_1 e^{-(n-3)\theta} - \dots + (-1)^{n-2} \nu_i^{n-2} I_1 e^{-\theta} \\ &\quad + (-1)^{n-1} \nu_i^{n-1} I_1 \\ &= I_1 e^{-\theta(n-1)} \left[\frac{1 - (-1)^n \nu_i^n e^{n\theta}}{1 + \nu_i e^{\theta}} \right] \end{aligned}$$

Hence, the level of inventory at time t is defined as

$$I_t = I_1 e^{-\theta(t-1)} \left[\frac{1 + (-1)^{t+1} \left(\frac{g_i}{b_i} \right)^t e^{t\theta}}{1 + \frac{g_i}{b_i} e^{\theta}} \right]$$

Since,

$$\begin{aligned} \frac{d(I_t + \nu_i I_{t-1})}{dt} + \theta(I_t + \nu_i I_{t-1}) &= 0 \\ \therefore \int_1^T \theta(t) dt &= - \int_1^T \frac{d(I_t + \nu_i I_{t-1})}{I_t + \nu_i I_{t-1}} \\ \therefore \Theta &= - |\log(I_t + \nu_i I_{t-1})|_1^T = \log \left(\frac{I_1 + \nu_i I_0}{I_T + \nu_i I_{T-1}} \right) \end{aligned}$$

Since, $I_0 = 0$, therefore, $\Theta = \log \left(\frac{I_1}{I_T + \nu_i I_{T-1}} \right)$, where I_1 is the initial inventory at unit time ($t = 1$).

Holding cost of inventory till delivery to the bidder,

$$HC = C_h \int_0^T I_1 e^{-\theta(t-1)} \left[\frac{1 + (-1)^{t+1} \left(\frac{g_i}{b_i} \right)^t e^{t\theta}}{1 + \frac{g_i}{b_i} e^{\theta}} \right] dt.$$

Also, the auction set up cost = $C_{Au} \left(\frac{D}{Q} \right)$ and deterioration cost,

$$C_d \Theta = C_d \log \left(\frac{I_1}{I_T + \nu_i I_{T-1}} \right).$$

$$\therefore \text{Total cost} = C_h \int_0^T I_1 e^{-\theta(t-1)} \left[\frac{1 + (-1)^{t+1} \left(\frac{g_i}{b_i} \right)^t e^{t\theta}}{1 + \frac{g_i}{b_i} e^\theta} \right] dt$$

$$+ C_{Au} \left(\frac{D}{Q} \right) + C_d \log \left(\frac{I_1}{I_T + \nu_i I_{T-1}} \right).$$

3.3 Computation of actual price of agro-product at equilibrium position under cobweb phenomenon

Since, $D_{it} = b_i P_{it} + a_i$ and $S_{it} = g_i P_{it-1} + h_i$, $i \in N_{CL_l}$, $l = 1, 2, 3, \dots, K$ and for equilibrium $b_i P_{it} + a_i = g_i P_{it-1} + h_i \Rightarrow b_i P_{it} - g_i P_{it-1} = h_i - a_i$.

- For $t = 1$,

$$P_{i1} = \frac{h_i - a_i}{b_i} + \frac{g_i}{b_i} P_{i0}$$

- For $t = 2$

$$P_{i2} = \frac{h_i - a_i}{b_i} + \frac{g_i}{b_i} \left\{ \frac{h_i - a_i}{b_i} + \frac{g_i}{b_i} P_{i0} \right\}.$$

By using the mathematical induction method for any value of t we can write

$$P_{it} = \left(\frac{h_i - a_i}{b_i} \right) \left\{ 1 + \frac{g_i}{b_i} + \left(\frac{g_i}{b_i} \right)^2 + \left(\frac{g_i}{b_i} \right)^3 + \dots + \left(\frac{g_i}{b_i} \right)^t \right\} + \left(\frac{g_i}{b_i} \right)^t P_{i0}$$

$$P_{it} = \left(\frac{h_i - a_i}{b_i} \right) \frac{\left\{ 1 - \left(\frac{g_i}{b_i} \right)^t \right\}}{\left(1 - \frac{g_i}{b_i} \right)} + \left(\frac{g_i}{b_i} \right)^t P_{i0}$$

$$P_{it} = \left(\frac{h_i - a_i}{b_i - g_i} \right) \left\{ 1 - \left(\frac{g_i}{b_i} \right)^t \right\} + \left(\frac{g_i}{b_i} \right)^t P_{i0}$$

$$= \left(\frac{h_i - a_i}{g_i - b_i} \right) \left\{ \left(\frac{g_i}{b_i} \right)^t - 1 \right\} + \left(\frac{g_i}{b_i} \right)^t P_{i0}.$$

Hence, the balance price value of an agro-product is defined below in terms of time t given by

$$P_{it} = \left(\frac{g_i}{b_i} \right)^t \left\{ P_{i0} + \left(\frac{h_i - a_i}{g_i - b_i} \right) \right\} - \left(\frac{h_i - a_i}{g_i - b_i} \right) \quad (4)$$

Equation (4) is convergent cobweb price, since $g_i < b_i$ and it will under divergent cobweb price $g_i > b_i$. At $g_i = b_i$ is enclosed cobweb price.

4 Assumption of agro-product first price bidding auction

We consider this balanced price value as the actual value of i bidder to make a bid. So, P_{it} denotes the willingness of agents i to buy the product. Assume that $f(P_{it})$ is the probability of agent i getting the object when the vector of announced types is $P_t = (P_{1t}, P_{2t}, \dots, P_{nt})$. Consider an auction scenario with the following assumptions

- 1 n risk-neutral bidders (buyers) $1, 2, 3, \dots, n$.
- 2 The valuation of bidder i ($i = 1, 2, 3, \dots, n$) is a real interval $[P_{it}, \overline{P_{it}}] \subset \mathbb{R}$ with $P_{it} < \overline{P_{it}}$.
- 3 The valuation of bidder i ($i = 1, 2, 3, \dots, n$) is drawn from $[P_{it}, \overline{P_{it}}]$ with a strictly positive density $f(P_{it}) > 0$. Let $F(P_{it})$ be the cumulative distribution function.
- 4 The types of bidders signal $P_{1t}, P_{2t}, \dots, P_{nt}$ are statistically independent.
- 5 If $f(P_{it})$ is the density function of bidding drawn from $[P_{it}, \overline{P_{it}}]$ and the cumulative distribution is $F(p_{it})$. The Weibull density function is a good model to look for the lowest bidding price. Hence

$$f(P_{it}) = \begin{cases} \alpha \beta P_{it}^{\beta-1} e^{-\alpha P_{it}^\beta}, & \text{if } P_{it} > 0, \alpha > 0, \beta > 0 \\ 0, & \text{otherwise.} \end{cases}$$

- 6 And cumulative distribution function $F(P_{it}) = 1 - e^{-\alpha P_{it}^\beta}$.

We consider the first price auction for an agro-product in which demand, supply, and actual price hold the cobweb theorem. In this paper, we compute the appropriate bidding price which maximises the expected utility of the bidder. Let $B(P_{it}) = B_i$ be the bidding value of a product when its actual value is P_{it} . The i^{th} bidder will win the bidding when all other bidders will bid less than B_i value for the same agro-product. Therefore, the expected utility of bidding B_i value is given by

$$\begin{aligned} U(P_{it}, B_i) &= (P_{it} - B_i) \\ &\cdot \text{Prob}(B(P_{it}) = B_j \leq B_i : i \neq j, \forall j = 1, 2, 3, \dots, n-1) \\ &= (P_{it} - B_i) \cdot [F(B^{-1}(B_i))]^{n-1} \\ \therefore \max_{B_i} U(P_{it}, B_i) &= \max_{B_i} (P_{it} - B_i) \cdot [F(B^{-1}(B_i))]^{n-1} \end{aligned}$$

Differentiating $(P_{it} - B_i) [F(B^{-1}(B_i))]^{n-1}$ w.r.t. B_i and equating to zero, we get

$$(P_{it} - B_i)(n-1) [F(B^{-1}(B_i))]^{n-2} f(B^{-1}(B_i)) \frac{1}{B'} - [F(B^{-1}(B_i))]^{n-1} = 0$$

$$\begin{aligned} \therefore \frac{dB_i}{dP_{it}} &= B' = \frac{(P_{it} - B_i)(n-1) [F(B^{-1}(B_i))]^{n-2} f(B^{-1}(B_i))}{[F(B^{-1}(B_i))]^{n-1}} \\ &= \frac{(P_{it} - B_i)(n-1)f(P_{it})}{F(P_{it})} \end{aligned}$$

or

$$\begin{aligned} \frac{dB}{dP_{it}} &= \frac{(P_{it} - B_i)(n-1)f(P_{it})}{F(P_{it})} = \frac{\alpha\beta(P_{it} - B_i)(n-1)P_{it}^{\beta-1}e^{-\alpha P_{it}^\beta}}{1 - e^{-\alpha P_{it}^\beta}} \\ \Rightarrow \frac{dB_i}{dP_{it}} + \frac{\alpha\beta(n-1)P_{it}^{\beta-1}e^{-\alpha P_{it}^\beta}}{1 - e^{-\alpha P_{it}^\beta}} B_i &= \frac{\alpha\beta(n-1)P_{it}^\beta e^{-\alpha P_{it}^\beta}}{1 - e^{-\alpha P_{it}^\beta}} \end{aligned}$$

Solving the above linear ODE, we get

$$\begin{aligned} B_i e^{\int \frac{\alpha\beta(n-1)P_{it}^{\beta-1}e^{-\alpha P_{it}^\beta}}{1 - e^{-\alpha P_{it}^\beta}} dP_{it}} &= \int \frac{\alpha\beta(n-1)P_{it}^{\beta-1}e^{-\alpha P_{it}^\beta}}{1 - e^{-\alpha P_{it}^\beta}} e^{\int \frac{\alpha\beta(n-1)P_{it}^{\beta-1}e^{-\alpha P_{it}^\beta}}{1 - e^{-\alpha P_{it}^\beta}} dP_{it}} dP_{it} \\ &= \int \frac{\alpha\beta(n-1)P_{it}^{\beta-1}e^{-\alpha P_{it}^\beta}}{1 - e^{-\alpha P_{it}^\beta}} e^{\int \frac{\alpha\beta(n-1)P_{it}^{\beta-1}e^{-\alpha P_{it}^\beta}}{1 - e^{-\alpha P_{it}^\beta}} dP_{it}} dP_{it} \end{aligned}$$

Putting $u = 1 - e^{-\alpha P_{it}^\beta}$, we get

$$\begin{aligned} B_i e^{(n-1) \int \frac{du}{u}} &= \int (n-1) e^{(n-1) \int \frac{du}{u}} P_{it} \frac{du}{u} \\ B_i u^{n-1} &= \int (n-1) u^{(n-2)} P_{it} du \\ &= (n-1) \left[P_{it} \int u^{n-2} du - \int \frac{dP_{it}}{du} \left(\int u^{n-2} du \right) du \right] \\ &= (n-1) \left[P_{it} \frac{u^{n-1}}{n-1} - \int \frac{u^{n-1}}{n-1} \frac{dP_{it}}{du} du \right] \\ B_i &= \left[P_{it} - \frac{\int u^{(n-1)} dP_{it}}{u^{n-1}} \right] + C, \quad C = \text{constant.} \end{aligned}$$

But, since $P_{it} \in [\underline{P_{it}}, \overline{P_{it}}]$, hence

$$B^*(P_{it}) = P_{it} - \frac{\int_{\underline{P_{it}}}^{\overline{P_{it}}} [1 - e^{-\alpha P_{it}^\beta}]^{n-1} dP_{it}}{[1 - e^{-\alpha P_{it}^\beta}]^{n-1}} \quad (5)$$

5 Revenue equivalence theorem

Revenue equivalence theorem: suppose the bidder has independent and identically distributed valuations and is risk-neutral any symmetric and increasing equilibrium of a direct revelation auction A that assigns the item to the highest bidder such that the expected payment of bidder with value 0 is 0, yields the same expected revenue.

5.1 Problem

Let $B : [0, P_{it}] \rightarrow \mathbb{R}$, What is the revenue from the first price auction? It is the expected winning bid or the expected bid of the bidder with the highest price.

5.2 Solution

Let, $B : [0, P_{it}] \rightarrow \mathbb{R}$ be a symmetric equilibrium strategy for auction A . We will characterise $m^A(P_{it})$, the expected payment of a fixed bidder i if he bids $B(P_{it})$. Let $u^A(B_i, P_{it})$ be expected utility of bidder i if he bids $B(P_{it})$

$$\begin{aligned} u^A(B_i, P_{it}) &= P_{it} \cdot Pr[i \text{ wins with bid } B(P_{it})] - m^A(P_{it}) \\ u^A(B_i, P_{it}) &= P_{it} \cdot G(B_i | B_{i-1} \leq B_i) - m^A(P_{it}) \end{aligned} \quad (6)$$

Since bidder i chooses B_i to maximise his expected utility, it must hold

$$\frac{\partial u^A(B_i, P_{it})}{\partial B_i} = P_{it} \cdot g(B_i | B_{i-1} \leq B_i) - \frac{d}{dB_i} m^A(P_{it}) = 0 \quad (7)$$

Since, furthermore it is optimal to report $B_i = P_{it}$, we obtain

$$P_{it} \cdot g(P_{it} | P_{(i-1)t} \leq P_{it}) - \frac{d}{dP_{it}} m^A(P_{it}) = 0 \quad (8)$$

Finally, by solving the differential equation

$$m^A(\overline{P_{it}}) = m^A(\underline{P_{it}}) + \int_{\underline{P_{it}}}^{\overline{P_{it}}} P_{it} \cdot g(P_{it} | P_{(i-1)t} \leq P_{it}) dP_{it} \quad (9)$$

$$m^A(\overline{P_{it}}) = 0 + \int_{\underline{P_{it}}}^{\overline{P_{it}}} yg(y) dy \quad (10)$$

Note that $m^A(p_{it})$ does not depend on A , hence the theorem holds. The expected optimal revenue gained by the auctioneer is

$$\begin{aligned} &= n \int_{\underline{P_{it}}}^{\overline{P_{it}}} m^A(B^*(P_{it}) f(P_{it})) dP_{it} \\ &= \int_{\underline{P_{it}}}^{\overline{P_{it}}} \left[\int_0^{B^*(P_{it})} n B_i g(B_i | B_1, B_2, \dots, B_n) \right] dB_i f(P_{it}) dP_{it} \\ &= \int_{\underline{P_{it}}}^{\overline{P_{it}}} \left[\int_0^{B^*(P_{it})} B_i n f(B_i) \left[e^{-\alpha B_i^\beta} \right]^{n-1} \right] dB_i \alpha \beta P_{it}^{\beta-1} e^{-\alpha P_{it}^\beta} dP_{it} \\ &= \int_{\underline{P_{it}}}^{\overline{P_{it}}} \left[\int_0^{B^*(P_{it})} B_i n \alpha \beta B_i^{\beta-1} \left[e^{-\alpha B_i^\beta} \right]^n \right] dB_i \alpha \beta P_{it}^{\beta-1} e^{-\alpha P_{it}^\beta} dP_{it} \\ &= \int_{\underline{P_{it}}}^{\overline{P_{it}}} \alpha \beta P_{it}^{\beta-1} e^{-\alpha P_{it}^\beta} \left[\int_0^{B^*(P_{it})} B_i n \alpha \beta B_i^{\beta-1} \left[e^{-\alpha B_i^\beta} \right]^n dB_i \right] dP_{it} \end{aligned} \quad (11)$$

6 Propositions

Proposition 1: Actual price of the item to i^{th} agent at time t in an equilibrium situation of linear supply $S_{it} = g_i P_{it} + h_i$ and linear demand $D_{it} = b_i P_{it} + a_i$ under cobweb phenomenon is

$$P_{it} = \left(\frac{g_i}{b_i}\right)^t \left\{ P_{i0} + \left(\frac{h_i - a_i}{g_i - b_i}\right) \right\} - \left(\frac{h_i - a_i}{g_i - b_i}\right).$$

Proposition 2: If the demand rate and supply rate of deteriorated items is equal at any time t , then the Inventory level in the market is given by

$$I_t = I_1 e^{-\theta(t-1)} \left[\frac{1 + (-1)^{t+1} \left(\frac{g_i}{b_i}\right)^t e^{t\theta}}{1 + \frac{g_i}{b_i} e^\theta} \right].$$

Proposition 3: If the agro-product auctioneer has demand and supply as a linear function of price and θ is the constant rate of deterioration of an item with C_d per unit cost of the deteriorated item. Then the total cost of the auction is given below

$$\begin{aligned} \text{Total cost} &= C_h \int_0^T I_1 e^{-\theta(t-1)} \left[\frac{1 + (-1)^{t+1} \left(\frac{g_i}{b_i}\right)^t e^{t\theta}}{1 + \frac{g_i}{b_i} e^\theta} \right] dt + C_{Au} \left(\frac{D}{Q}\right) \\ &\quad + C_d \log \left(\frac{I_1}{I_T + \nu_i I_{T-1}} \right). \end{aligned}$$

Proposition 4: Under the first price auction of an agro-product where demand and supply are the linear functions of price and hold the cobweb theorem and auction bidding holds the Weibull density function

$$f(P_{it}) = \begin{cases} \alpha \beta P_{it}^{\beta-1} e^{-\alpha P_{it}^\beta}, & \text{if } P_{it} > 0, \alpha > 0, \beta > 0 \\ 0, & \text{otherwise.} \end{cases}$$

The bidding price drawn from $[P_{it}, \overline{P_{it}}]$ with CDF $F(P_{it}) = 1 - e^{-\alpha P_{it}^\beta}$, then the optimum bidding value for the bidder is

$$B^*(P_{it}) = P_{it} - \frac{\int_{P_{it}}^{\overline{P_{it}}} [1 - e^{-\alpha P_{it}^\beta}]^{n-1} dP_{it}}{[1 - e^{-\alpha P_{it}^\beta}]^{n-1}}.$$

Proposition 5: Let $B : [0, P_{it}] \rightarrow \mathbb{R}$ be a symmetric equilibrium strategy for auction A and $m^A(P_{it})$, the expected payment of a fixed bidder i if he bids $B(P_{it})$. Here m^A does not depend on auction A . Then the expected optimal revenue gained by the auctioneer (or farmer)

$$= \int_{\underline{P_{it}}}^{\overline{P_{it}}} \alpha \beta P_{it}^{\beta-1} e^{-\alpha P_{it}^\beta} \left[\int_0^{B^*(P_{it})} n \alpha \beta B_i^\beta [e^{-\alpha B_i^\beta}]^n dB_i \right] dP_{it}.$$

7 Sensitivity analysis

- 1 Suppose, k is a particular bidder and his supply and demand curve is given by

$$D_{kt} = b_k P_{kt} + a_k = 0.9P_{kt} + 100 \text{ and}$$

$$S_{kt} = g_k P_{k(t-1)} + h_k = -0.6P_{k(t-1)} + 700$$

Let the initial price of the agro product is $P_{k0} = \text{Rs. } 150$, therefore

$P_{kt} = (-2/3)^t(-250) + 400$. It is assumed that the agro-product has a constant rate of deterioration $\theta = 25\%$ per day and $I_1 = 500$ units. Let, $n = 3$, $\alpha = 0.2$, $\beta = 0.5$, $\underline{P}_{kt} = P_{kt} - 20$, $\overline{P}_{kt} = P_{kt} + 20$.

- 2 Suppose, $b_k = 0.9$, $a_k = 100$, $h_k = 700$, $P_{k0} = \text{Rs. } 150$, $\theta = 25\%$ per day, $I_1 = 500$ units. Then a table of values of P_{kt} for different values of g_k is shown in Table 3.
- 3 Suppose, $g_k = 0.9$, $a_k = 100$, $h_k = 700$, $P_{k0} = 150$, $\theta = 25\%$ per day, $I_1 = 500$. Then table of values of P_{kt} for different values of b_k is shown in Table 4.

Table 2 Time vs. price, inventory level and optimum bidding, expected revenue

t	P_{kt}	I_t	$B^*(P_{kt})$	$Ex. \text{ rev.}$	t	P_{kt}	I_t	$B^*(P_{kt})$	$Ex. \text{ rev.}$
1	566.67	500	526.67	0.0080	11	402.89	233.50	362.89	0.0200
2	288.89	722.73	248.90	0.0437	12	398.07	187.63	358.07	0.0206
3	474.07	785.09	434.07	0.0131	13	401.28	149.98	361.28	0.0202
4	350.62	759.58	310.62	0.0281	14	399.14	119.37	359.14	0.0205
5	432.92	690.32	392.92	0.0167	15	400.57	94.68	360.57	0.0203
6	378.05	603.47	338.05	0.0235	16	399.62	74.88	359.62	0.0204
7	414.63	513.88	374.63	0.0186	17	400.25	59.08	360.25	0.0204
8	390.25	429.47	350.25	0.0217	18	399.83	46.52	359.83	0.0204
9	406.50	353.98	366.50	0.0115	19	400.11	36.57	360.11	0.0204
10	395.66	288.69	355.66	0.0210	20	399.92	28.70	359.92	0.0204

Figure 2 Price (P_{kt}) vs. time (t) graph (see online version for colours)

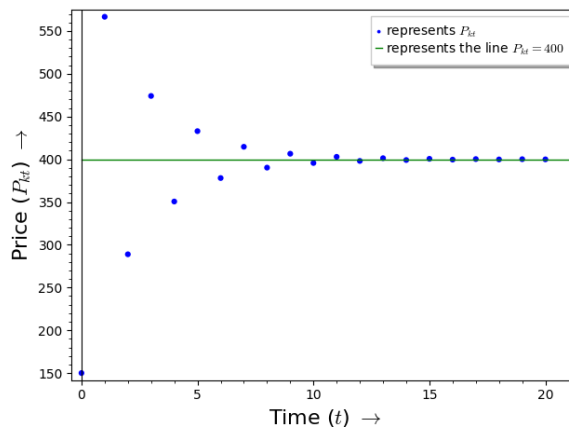


Figure 3 Inventory level (I_t) vs. time (t) graph (see online version for colours)

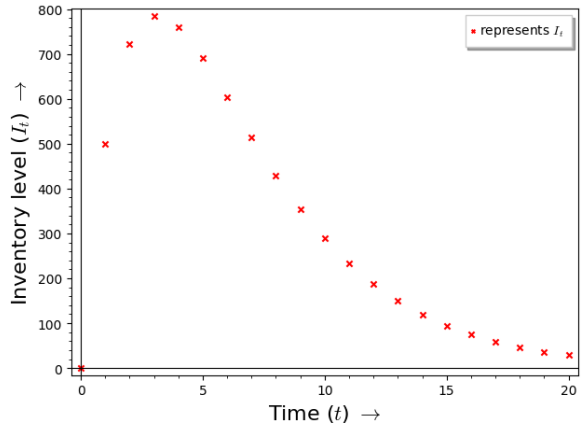


Table 3 Supply elasticities (g_k) vs. price at various times

t	P_{kt}			t	P_{kt}		
	$g_k = -0.2$	$g_k = -0.5$	$g_k = -0.8$		$g_k = -0.2$	$g_k = -0.5$	$g_k = -0.8$
1	633.33	583.33	533.33	11	545.45	429	408.49
2	525.93	342.59	192.59	12	545.45	428.33	303.56
3	549.79	476.34	495.47	13	545.45	428.70	396.83
4	544.49	402.03	226.25	14	545.45	428.50	313.93
5	545.67	443.31	465.56	15	545.45	428.61	387.62
6	545.41	420.38	252.84	16	545.45	428.55	322.11
7	545.47	433.12	441.92	17	545.45	428.58	380.34
8	545.45	426.04	273.85	18	545.45	428.56	328.58
9	545.45	429.98	423.25	19	545.45	428.58	374.59
10	545.45	427.79	290.45	20	545.45	428.57	333.70

Figure 4 Price (P_{kt}) vs. time (t) graph for different values of g_k (see online version for colours)

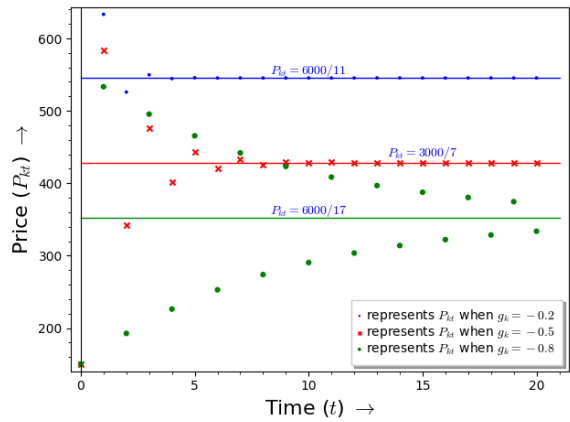


Table 4 Supply elasticities g_k vs. level of inventory at various times

t	I_t			t	I_t		
	$g_k = -0.2$	$g_k = -0.5$	$g_k = -0.8$		$g_k = -0.2$	$g_k = -0.5$	$g_k = -0.8$
1	500.00	500.00	500.00	11	57.43	139.69	952.88
2	500.51	667.18	833.84	12	44.73	109.57	878.97
3	414.49	673.92	1,044.46	13	34.83	85.77	806.20
4	328.29	610.58	1,164.59	14	27.13	67.04	736.01
5	256.89	523.15	1,219.13	15	21.13	52.34	669.33
6	200.34	433.89	1,226.93	16	16.45	40.84	606.72
7	156.09	352.62	1,202.17	17	12.81	31.85	548.46
8	121.57	282.76	1,155.48	18	9.98	24.82	494.66
9	94.68	224.77	1,094.76	19	7.77	19.35	445.25
10	73.74	117.57	1,025.82	20	6.05	15.07	400.10

Figure 5 Time (t) vs. inventory level (I_t) at various elasticity of supply values (see online version for colours)

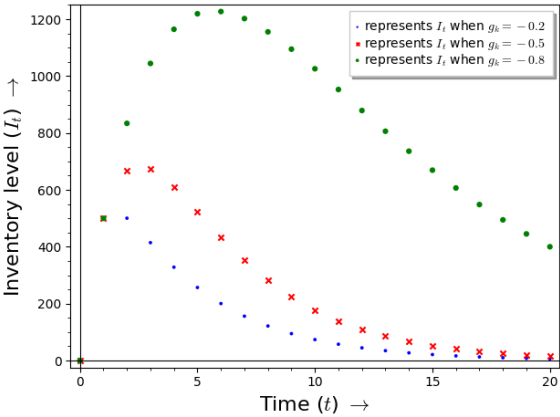


Table 5 Price (P_{kt}) vs. elasticity of demand at different values of time (t)

t	P_{kt}			t	P_{kt}		
	$b_k = 0.4$	$b_k = 0.6$	$b_k = 0.8$		$b_k = 0.4$	$b_k = 0.6$	$b_k = 0.8$
1	1,387.50	925.00	693.75	11	887.01	666.92	545.46
2	459.38	537.50	489.84	12	834.74	666.54	545.45
3	1,155.47	731.25	566.31	13	873.94	666.73	545.46
4	633.40	634.38	537.63	14	844.54	666.64	545.45
5	1,024.95	682.81	548.39	15	866.59	666.68	545.45
6	731.29	658.59	544.35	16	850.06	666.66	545.45
7	951.54	670.70	545.37	17	862.46	666.67	545.45
8	786.35	664.65	545.30	18	853.16	666.66	545.45
9	910.24	667.68	545.51	19	860.13	666.67	545.45
10	817.32	666.16	545.43	20	854.90	666.67	545.45

Figure 6 Price (P_{kt}) vs. time (t) graph for different values of b_k (see online version for colours)

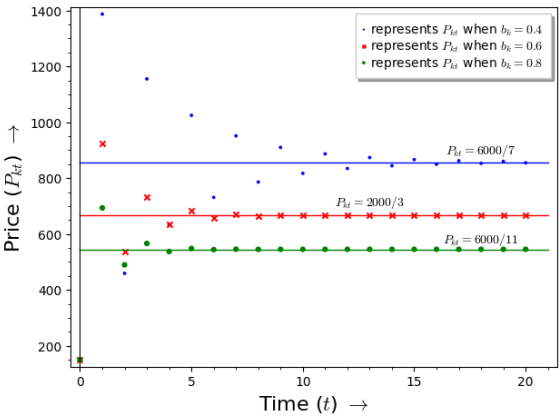
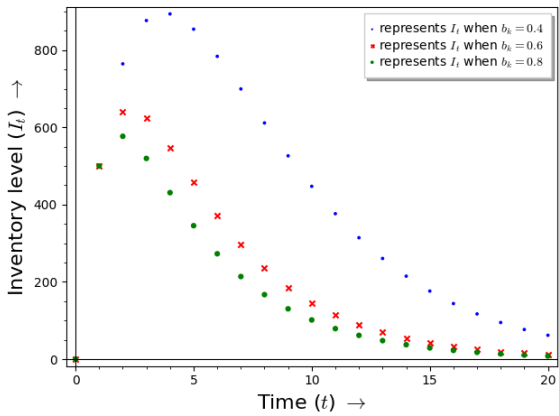


Table 6 Supply elasticities g_k vs. level of inventory at various times

t	I_t			t	I_t		
	$b_k = 0.4$	$b_k = 0.6$	$b_k = 0.8$		$b_k = 0.4$	$b_k = 0.6$	$b_k = 0.8$
1	500.00	500.00	500.00	11	376.60	113.77	79.13
2	764.40	639.40	576.90	12	314.41	88.85	61.64
3	876.57	622.97	519.60	13	260.70	69.32	48.01
4	893.61	547.57	431.03	14	214.91	54.05	37.39
5	854.15	457.77	345.58	15	176.28	42.12	29.12
6	783.86	372.14	272.84	16	143.97	32.82	22.68
7	699.46	297.63	213.88	17	117.14	25.57	17.66
8	611.48	235.70	167.09	18	94.99	19.92	13.76
9	526.28	185.52	130.33	19	76.79	15.51	10.71
10	447.41	145.46	101.57	20	61.92	12.08	8.34

Figure 7 Inventory level (I_t) vs. elasticity of demand with different values of time (t) (see online version for colours)



7.1 Sensitivity analysis of the shape parameter (α)

Suppose, k is a particular bidder and his supply and demand curve is given by

$$D_{kt} = b_k P_{kt} + a_k = 0.9P_{kt} + 100 \text{ and} \\ S_{kt} = g_k P_{k(t-1)} + h_k = -0.6P_{k(t-1)} + 700$$

Let the initial price of the agro product is $P_{k0} = \text{Rs.}150$, $b_k = 0.9$, $g_k = -0.6$ and $a_k = 100$, $h_k = 700$, $P_{k0} = \text{Rs.}150$. It is assumed that the agro-product has a constant rate of deterioration $\theta = 25\%$ per day and $I_1 = 500$ units. Let, $n = 3$, $\beta = 0.5$, $\underline{P}_{kt} = P_{kt} - 20$, $\overline{P}_{kt} = P_{kt} + 20$, we take $\beta = 0.5$.

To calculate the value of $B^*(P_{kt})$ for different values of α at different points of time we have the results mentioned in Table 7.

Table 7 Optimal bidding ($B^*(P_{kt})$) vs. α at various times

t	$B^*(P_{kt})$				t	$B^*(P_{kt})$			
	$\alpha = 0.01$	$\alpha = 0.1$	$\alpha = 1$	$\alpha = 10$		$\alpha = 0.01$	$\alpha = 0.1$	$\alpha = 1$	$\alpha = 10$
1	526.67	526.67	526.67	526.67	11	362.89	362.89	362.89	362.89
2	248.90	248.90	248.90	248.90	12	358.07	358.07	358.07	358.07
3	434.07	434.07	434.07	434.07	13	361.28	361.28	361.28	361.28
4	310.62	310.62	310.62	310.62	14	359.14	359.14	359.14	359.14
5	392.92	392.92	392.92	392.92	15	360.57	360.57	360.57	360.57
6	338.05	338.05	338.05	338.05	16	359.62	359.62	359.62	359.62
7	374.63	374.63	374.63	374.63	17	360.25	360.25	360.25	360.25
8	350.25	350.25	350.25	350.25	18	359.83	359.83	359.83	359.83
9	366.50	366.50	366.50	366.50	19	360.11	360.11	360.11	360.11
10	355.66	355.66	355.66	355.66	20	359.92	359.92	359.92	359.92

We also make a table for expected revenue for different values of α at different points in time as shown in Table 8.

Table 8 Expected revenue vs. α at various times

t	<i>Expected revenue</i>				t	<i>Expected revenue</i>			
	$\alpha = 0.01$	$\alpha = 0.1$	$\alpha = 1$	$\alpha = 10$		$\alpha = 0.01$	$\alpha = 0.1$	$\alpha = 1$	$\alpha = 10$
2	0.2747	0.4075	0	0	12	0.3659	0.2797	0	0
4	0.3281	0.3281	0	0	14	0.3667	0.2787	0	0
6	0.3502	0.2989	0	0	16	0.3671	0.2783	0	0
8	0.3598	0.2870	0	0	18	0.3672	0.2781	0	0
10	0.3640	0.2819	0	0	20	0.3673	0.2780	0	0

7.2 Sensitivity analysis of the size parameter (β)

We take $\alpha = 0.2$, keep all other data same as considered for Table 7 and calculate the value of $B^*(P_{kt})$ for different values of β at different points in time.

Table 9 Optimal bidding ($B^*(P_{kt})$) vs. β at various times

t	$B^*(P_{kt})$				t	$B^*(P_{kt})$			
	$\beta = 0.01$	$\beta = 0.1$	$\beta = 1$	$\beta = 10$		$\beta = 0.01$	$\beta = 0.1$	$\beta = 1$	$\beta = 10$
1	526.67	526.67	526.67	526.67	11	362.89	362.89	362.89	362.89
2	248.90	248.90	248.90	248.90	12	358.07	358.07	358.07	358.07
3	434.07	434.07	434.07	434.07	13	361.28	361.28	361.28	361.28
4	310.62	310.62	310.62	310.62	14	359.14	359.14	359.14	359.14
5	392.92	392.92	392.92	392.92	15	360.57	360.57	360.57	360.57
6	338.05	338.05	338.05	338.05	16	359.62	359.62	359.62	359.62
7	374.63	374.63	374.63	374.63	17	360.25	360.25	360.25	360.25
8	350.25	350.25	350.25	350.25	18	359.83	359.83	359.83	359.83
9	366.50	366.50	366.50	366.50	19	360.11	360.11	360.11	360.11
10	355.66	355.66	355.66	355.66	20	359.92	359.92	359.92	359.92

We also make a table for expected revenue for different values of β at different points in time as shown in Table 10.

Table 10 Expected revenue vs. β at various times

t	<i>Expected revenue</i>				t	<i>Expected revenue</i>			
	$\beta = 0.01$	$\beta = 0.1$	$\beta = 1$	$\beta = 10$		$\beta = 0.01$	$\beta = 0.1$	$\beta = 1$	$\beta = 10$
2	0.0002	0.0313	0	0	12	0.0002	0.0333	0	0
4	0.0002	0.0326	0	0	14	0.0002	0.0334	0	0
6	0.0002	0.0330	0	0	16	0.0002	0.0334	0	0
8	0.0002	0.0332	0	0	18	0.0002	0.0334	0	0
10	0.0002	0.0333	0	0	20	0.0002	0.0334	0	0

7.3 Observations

- 1 From Table 2 and Figure 2, we find that the fluctuation of the price of seasonal agro-product converges at a particular price after some time and the optimal bidding value is always less than the actual value of agro-product.
- 2 Table 2 and Figure 3 indicate that the inventory level of agro-products increases with an increase in time under a constant rate of deterioration till a particular time after that it starts to fall with the increase of time. We also observe that the expected revenue gains by the auctioneer tend to be zero when the bidder bids optimum bidding.
- 3 From Table 2, we get a very interesting result that the optimum value of the bidding is approximately equal to the actual value of the agro-products for the bidder minus the length of the range of guessed interval of bidding.
- 4 Table 3 and Figure 4 provide the information regarding the convergence of price if the slope of the supply increases then the time of convergence of price also increases. And we also observe that if the slope of the supply decreases then the convergence price value becomes high.

- 5 From Table 4 and Figure 5, we observe that at the initial time, the inventory level of agro-product is the same but the peak of the inventory level of agro-product is proportional to the elasticity of supply.
- 6 Table 5 and Figure 6 show that the price converges fast for the high value of elasticity of the demand and if the elasticity of demand decreases then the value of the convergent price increases and the time of convergence of price also increases.
- 7 Table 6 and Figure 7 conclude that the peak of inventory level is inversely proportional to the elasticity of demand. The initial inventory level is the same for all values of elasticity of demand.
- 8 From Table 7, we observe that the optimal bid converges with an increase in time but it is unaltered with a change in shape parameter of the probability distribution curve of selection of bidding price. But from Table 8, expected revenue is oscillatory in nature between the decrease-increase trend with the increase in the value of shape parameter (α).
- 9 From Table 7, we also observe that for any fixed value of the shape parameter, (α), the expected revenue gained by the farmer converges at a finite value with an increase of time.
- 10 For different values of size parameter (β) at a particular time the optimal bidding price remains unaltered but for variation in time it converges at a particular value of price. The expected revenue also converges to zero with the variation of size parameters and time.

8 Conclusions

This paper gives the mathematical intuition about the optimal bidding price of agro-product to a bidder as well as the expected revenue gained by the farmers with the optimum bidding price. The inventory level of agro-products at any time t has been formulated in terms of deterioration rate and the ratio of the slope of supply. The sensitivity analysis of parameters provides very useful results about the level of the available inventory at a time t to get a guess about the optimum bid. It is approximately equal to the actual value of the agro-products to the bidder minus the length of the guessed interval of bidding. The peak of the inventory level of agro-product is proportional to the elasticity of supply. The peak of the inventory level is inversely proportional to the elasticity of demand. The initial inventory level is the same for all values of elasticity of demand.

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