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# Reliability assignment of a heavy-duty CNC machine tool spindle system based on fault tree analysis

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**Abstract:** Research on the distribution of machine tool reliability indexes is of great significance to improve the reliability of machine tools in the design and manufacture stages. Owing to the complex structure of heavy-duty CNC machine tools, multi-source and heterogeneous failure causes, etc., the traditional single method cannot get accurate results. In this paper, based on the FTA method, a three-level machine tool fault tree model is established to carry out research on reliability index assignment. Then, as the levels progress, the distribution of reliability indicators is based on probability importance analysis, reliability redistribution method, and AHP respectively. Finally, the top-down reliability assignment results are obtained. After analysis, the structure obtained by combining multiple reliability allocation methods is more in line with the actual situation. According to the results, the weak parts of the machine tool are designed and improved, thereby improving the reliability of the machine tool.

**Keywords:** heavy-duty CNC machine tool spindle system; FTA; reliability assignment; reliability index.

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### 1 Introduction

As an 'industrial mother machine', heavy-duty CNC machine tools are of great significance to the development of the manufacturing industry. Improving the reliability of this type of machine tool is particularly important for promoting the development of aerospace, military, and navigation industries (Catelani et al., 2016, 2017; Wu et al., 2018). Spindle system is one of the subsystems of heavy CNC machine tools with relatively high frequency of failures. Therefore, it is necessary to analyse the reliability of the spindle system of machine tools (Kim and Kim, 2021; Liu et al., 2021; Yalaoui et al., 2005). In the field of reliability research, the reliability distribution is directly related to the reliability of the machine tool design stage. The research on reliability distribution based on the fault data of the machine tool in the use stage has important reference significance for its design and remanufacturing to improve the reliability of the machine tool (Eldosouky et al., 2021; Gao et al., 2020; Liu et al., 2020). Owing to the complex structure of heavy-duty CNC machine tools, few fault samples, difficulty in fault tracing and fuzzy causes of failures, it is impossible to use a single method to ensure effective reliability assignment for heavy-duty CNC machine tools. The assignment of reliability indicators is a step-by-step iterative process, so it is necessary to select different methods for reliability assignment at different stages according to the actual situation.

Reliability assignment refers to assigning weights to each system and component according to a certain algorithm based on the reliability index specified by the product. A large number of scholars have made corresponding contributions to the research on reliability distribution theory. At present, the widely used methods include AGREE distribution method, score distribution method, proportional combination method and so on. Among them, although the AGREE assignment method considers the complexity and importance of each component system, this method is often used in the analysis of the

reliability of electronic systems, and it is not considered enough when analysing other situations (Hongbin et al., 2009; Li et al., 2018). The scoring distribution method makes up for the information defects caused by insufficient fault data by introducing subjective information, but the distribution results are greatly affected by the personal judgment and cognitive level of experts, and the distribution efficiency is low. The proportional distribution method needs to find similar systems for reference and is not universal. In order to solve the incompleteness of a single method, different methods are used comprehensively for the reliability distribution. The distribution hierarchy model is established based on the fault tree, and the Bayesian network is used to quantify the distribution weight. Based on Edgeworth series method and data envelopment analysis method, the assignment results can be made more accurate. When assigning reliability to systems with fuzzy data, the introduction of AHP based on fault tree can be supplemented by subjective information. In the face of complex heavy-duty CNC machine tools, it is necessary to integrate a variety of methods to allocate reliability for different situations (Wei et al., 2012; Zhang et al., 2019).

In this paper, based on fault tree analysis and synthesising a variety of reliability distribution methods, the reliability distribution of machine tools is carried out (Ding et al., 2021; Nair et al., 2020; Xu et al., 2020). First, analyse the fault tree of the spindle system of the heavy CNC machine tool, establish three-layer structure events, calculate the probability importance of the events and then obtain the weight of the events to be assigned, and obtain the reliability assignment result of the certain events, that is, the reliability of the intermediate events index. Secondly, use the reliability redistribution method to distribute the reliability of the intermediate events of the fault tree. Finally, the reliability index is further assigned to the bottom event by using the AHP, and the assignment result is further refined. The reliability assignment index can be mapped to the parts of the comprehensive reliability assignment method based on the fault tree are more in line with the actual situation, which is conducive to improving the maintenance strategy of the machine tool, providing a reference for the reliability design of the machine tool.

### 2 The fault tree-based reliability distribution model for spindle systems

#### 2.1 Determine system reliability assignment metrics

When assigning reliability indexes, the Mean Time Between Failures (MTBF) of the system is usually selected as the reliability index of reliability assignment and this method can visually show the advantages and disadvantages of the assigned system reliability (Wu and Sun, 2021). In this paper, when assigning reliability to the spindle system of a heavy-duty CNC machine tool, a fault tree model needs to be established first, and a quantitative analysis is performed based on the probability importance to obtain the distribution weight of the reliability index. When calculating the probability importance of a fault event, the failure probability needs to be used (Kalantari et al., 2020; Parihar and Chakraborty, 2020; Ryu and Jae, 2021). So the collected fault data is processed to obtain the fault probability of the fault event, which is used as the reliability assignment index in this paper, and is transformed by:

$$\lambda = \frac{1}{\text{MTBF}} = \frac{c}{N \cdot \Delta t} \tag{1}$$

Among them, c is the number of faulty components within the considered time range  $\Delta t$ ; N is the number of all components;  $\Delta t$  is the considered time range.

$$q_i = 1 - e^{-\lambda_i t}, (i = 1, 2, 3, ..., n)$$
<sup>(2)</sup>

The obtained failure probability is replaced by the reliability assignment index, which is expressed as:

$$P_{S}^{*}(P_{1},P_{2},L,P_{i},L,P_{n}) \leq P_{S}$$

$$\tag{3}$$

In the formula,  $P_s$  is the fault probability of the fault tree top event before assignment;  $P_s^*$  is the fault probability of the fault tree top event after assignment;  $P_i$  is the *i*-th event fault probability.

The fault probability of the fault tree top event after assignment is lower than the fault probability before assignment, otherwise recalculate until it meets the requirements.

#### 2.2 Reliability assignment method of top event based on probability importance

Reliability assignment of fault tree top events based on probability importance analysis method. Probabilistic importance is often used in the quantitative analysis of fault trees, and the value of this value can reflect the impact of fault tree events on the overall fault probability. Therefore, the probability importance is introduced into the reliability assignment method, and the reliability of the fault tree top event is allocated according to the probability importance.

The process of calculating the probability importance of each event is as follows:

1) Assuming that there are n fault events in the system fault tree, the probability importance of the *j*-th event is as follows:

$$I_{j} = \frac{\partial P_{s}}{\partial P_{j}}, 1 \le i \le j \le n$$

$$\tag{4}$$

When the logic gate is an 'OR' gate, the probability importance formula can be simplified as:

$$I_{j} = \frac{\partial P_{s}}{\partial P_{j}} = \prod_{j=1, i \neq j}^{n} (1 - P_{i})$$
(5)

When the logic gate is an 'AND' gate, the probability importance formula can be simplified as:

$$I_{j} = \frac{\partial P_{s}}{\partial P_{j}} = \prod_{j=1, i \neq j}^{n} P_{i}$$
(6)

The calculation method of the failure probability  $P_s$  of the fault tree top event is as:

If the logic gate is an 'OR' gate:

$$P_{S} = \bigcup_{i=1}^{n} P_{i} = 1 - \prod_{i=1}^{r} (1 - P_{i})$$
(7)

If the logic gate is an AND gate:

$$P_S = \bigcap_{i=1}^n P_i = \prod_{i=1}^n P_i \tag{8}$$

2) Reliability assignment of top events based on probability importance (Burdyshev and Tyurin, 2021; Umma et al., 2021)

Substitute the calculated fault tree event probability importance into the following formula (9) to obtain the difference  $\Delta q$  that needs to be adjusted after assignment, and then the specific fault probability after assignment can be obtained, as shown in formula (10).

$$\Delta q_1 : \Delta q_2 : \dots : \Delta q_i = I_1 : I_2 : \dots : I_i \tag{9}$$

Reliability assignment to event i:

$$P_i^* = P_i + \Delta q_i \tag{10}$$

where  $P_i$  is the probability of failure of event *i* before assignment;  $I_i$  is the probability importance of event *i*;  $\Delta q$  assigns the difference between the probability of failure before and after event i that needs to be adjusted.

The total fault probability  $P_s^*$  after distribution is as:

$$P_{S}^{*} = 1 - \prod_{i=1}^{n} \left( 1 - \left( P_{i} + \Delta q_{i} \right) \right)$$
(11)

Complete the reliability assignment of fault tree top events through the above content. And get the fault probability value after the intermediate events of the fault tree are assigned.

### 2.3 Reliability assignment of fault tree intermediate events based on reliability reassignment method

The reliability redistribution method is often used in the series system. The intermediate event of the fault tree is the bridge connecting the previous and the next, and it is also the key of the series system in the fault tree. Therefore, the reliability redistribution method is adopted here. First, determine the relationship between each component unit and the system on the reliability index, and then sort the reliability index of each component unit in descending order. Units with high reliability indexes will improve the reliability level, while those with low reliability indexes. The unit of the index will keep the reliability level unchanged, so that the reliability of the system can meet the expected requirements. Using reliability redistribution method for intermediate events can not only ensure accurate distribution but also improve distribution efficiency.

The specific steps of the reliability reassignment method are as follows:

1) Sort events in descending order of failure probability as:

$$P_1 > P_2 > \dots > P_{k_0} > P_{k_{n+1}} > \dots > P_n \tag{12}$$

2) Principles of Reassignment; According to Reliability, Reduce the relatively high failure probabilities  $P_1, P_2, ..., P_k$  in the event to a fixed value of  $P_0$ , the original lower failure probability,  $P_{k0}, P_{k0+1}, ..., P_n$  will remain unchanged.  $P_s$  can be obtained by equation (13).

$$P_{S} = \phi(P_{1}, P_{2}, ..., P_{n}) = 1 - (1 - P_{0})^{k_{0}} / \prod_{i=k_{0}+1}^{n} (1 - P_{i})$$
(13)

The total failure probability  $P_s$  should be greater than the expected failure probability index  $P_s^*$ , then equation (1) can be rewritten as:

$$P_{S}^{*} \leq P_{S} = 1 - \left(1 - P_{0}\right)^{k_{0}} / \prod_{i=k_{0}+1}^{n} \left(1 - P_{i}\right)$$
(14)

Determine  $k_0$  and fixed value  $P_0$ , and make it satisfy the equation (15).

$$P_0 = 1 - \left(1 - P_S^* / \prod_{i=j+1}^{n+1} \left(1 - P_j\right)\right)^{1/k_0}$$
(15)

### 2.4 Reliability assignment results of bottom events based on AHP

Since basic events are the most fundamental cause of failures and involve various influencing factors in the spindle system of heavy-duty CNC machine tools, it is difficult to obtain reasonable assignment results using traditional assignment methods. Therefore, this layer adopts the reliability assignment method combined with the analytic hierarchy process, and its purpose is to divide the target layer, the criterion layer, and the scheme layer into the relevant factors of the main shaft system through the hierarchical structure. The reliability index is the target level, the influencing factor is the criterion level and the assignment object is the scheme level. Then, the experts judge the degree of influence of the criterion level on the scheme level, and obtain the assignment relationship of the assignment object reliability index and finally obtain the reliability of the assignment object Sexual index (Li et al., 2021a).

The steps of the reliability assignment method based on the AHP are as follows:

1) Establish the hierarchy required for analytic hierarchy process: First divide the hierarchy, as shown in Figure 1.

In Figure 1, the connection between each unit of the target layer, the criterion layer and the solution layer represents the connection between the units, and the factors or indicators between the same layer are independent of each other.

2) Construct the judgment matrix of the criterion layer: According to Figure 1, the affiliation between each unit can be clarified, and the influence factor 1 of the criterion layer is taken as an example to illustrate. The influence factor 1 is the upper layer of the indicators of each subsystem in the scheme layer. Weight assignment, when the weight can be obtained by quantitative calculation, the corresponding

weight value can be directly determined. Pairwise comparisons of risk factors representing the relative importance among risk factors are performed using Saaty's method that is designed to evaluate the degree to which a factor is important than comparing one, see Table 1(Li et al., 2021b).





 Table 1
 Pairwise comparisons of Saaty's method

$u_i'/u_j'$	Equally	Weakly	Fairly	Strongly	Absolutely
	important	important	important	important	important
$a_{ij}$	1	3	5	7	9
$u_i'/u_j'$	Equally	Weakly	Fairly	Strongly	Absolutely
	unimportant	unimportant	unimportant	unimportant	unimportant
a <sub>ij</sub>	1	1/3	1/5	1/7	1/9

From the same to the absolute strong, every two levels can be quantified by 2, 4, 6 and 8 in turn.

3) Calculate the weight vector of the criterion layer and check the consistency: The weight of each factor to the criterion layer can be obtained through the judgment matrix. Consistency test is to ensure the rationality of constructing judgment matrix. The specific method is as follows:

The consistency indicator is used to quantify the degree of inconsistency of the judgment matrix, denoted as *C*, which can arranged by:

$$C = \left(\lambda_{max} - n\right) / \left(n - 1\right) \tag{16}$$

When C = 0, the judgment matrix is consistent, the larger the value of C, the more serious the inconsistency of the judgment matrix.

Whether the weight of the random consistency test is reasonable, expressed by  $C_k$ , show as:

$$C_R = C/R \tag{17}$$

Among them, *R* is obtained through a reliable check table, as shown in Table 2.

Table 2Random consistency index value

п	1	2	3	4	5	6	7
R	0	0	0.58	0.90	1.12	1.24	1.32

4) *Calculate the weight vector*: If the relative weight of the target layer relative to the criterion layer is  $w = (w_1, w_2, ..., w_n)$ . The relative weight of a certain influencing factor at the criterion level to the scheme level is  $v_i = (v_1^i, v_2^i, ..., v_m^i)^T$ . Then, the combined weight vector of the scheme layer to the target layer is  $w_i$ , and that can be rewritten by equation (18).

$$w_i = w v_i^T \tag{18}$$

5) Allocate reliability index: The importance of system failure events to the system is the combination weight vector, and the assignment of indicators is completed based on the importance of the assignment. The fault probability  $P_i$  of the *i*-th unit consisting of *m* units in the scheme layer is as follows:

$$P_{i} = P \frac{1}{w_{i}} / \sum_{i=1}^{m} \frac{1}{w_{i}}$$
(19)

where *P* is the failure probability allowed by the top event.

### 3 Case study

Heavy-duty CNC machine tool is one of the most typical representatives of complex electromechanical products. It is especially important in the manufacturing and processing fields, such as the aviation industry and national defence and military industries. Especially in the development stage, it is very important to reasonably allocate the reliability indicators of the whole machine or system to each system or part. Reasonable reliability assignment can improve the reliability of the whole machine or system, and at the same time improve the quality of processed products (Baladeh and Zio, 2021). Given that the spindle system is one of the core functional systems of heavy-duty CNC machine tools (Zhou and Li, 2011; Zhang and Wu, 2012). Therefore, this research uses the spindle system of the THP6513 heavy-duty CNC horizontal milling and boring machine as a practical application to verify the feasibility and effectiveness of the proposed method. Figure 2 is a schematic diagram of the structure of the spindle system. The fault tree of the spindle system is established through the faults that have occurred, as shown in Figure 3, and the intermediate events and basic event descriptions are shown in Tables 3 and 4.

Figure 2 Spindle structure diagram. 1-Cone clamping device, 2-Link, 3-The front cover, 4-Frontend bearing, 5-Case, 6-Spindle, 7-Disc spring, 8-Rear bearing, 9-Piston, 10-Broachcylinder



Figure 3 Spindle system fault tree



### 3.1 Reliability assignment of top events in spindle system based on fault tree

For the fault tree of the spindle system, the minimum cut set of the fault tree is obtained by the descending method. Using the fault maintenance data of an enterprise in the past 3 years, the fault interval time data is obtained through data processing, and it is substituted into equations (1) and (2) to obtain after processing. Event failure probability in the fault tree. Then, the probability importance of each basic event can be obtained by combining equations (7) and (8). The failure probability and probability importance of basic events are obtained as shown in Table 5.

Mode number	Failure mode	Mode number	Failure mode
A1	Clamping device failure	В5	Solenoid valve failure
A2	Hydraulic system failure	B6	The oil pump cannot supply oil
A3	Abnormal noise from the spindle	B7	Vibration is big
A4	Spindle system alarm	B8	Encoder overcurrent
A5	Spindle speed is failure	В9	overcurrent
B1	Fixture component failure	B10	Temperature is too high
B2	Fixture operation failure	B11	Spindle does not rotate
B3	Piston is unstable	B12	Spindle speed is unstable
B4	Overflow valve blocked		

 Table 3
 Intermediate event description table of fault tree

**Table 4**Fault tree basic event description table

Mode number	Failure mode	Mode number	Failure mode
X1	The main motor coupling is loose	X21	Bearing sleeve wear
X2	Temperature switch circuit virtual connection	X22	Preload is too large
X3	Broken claw	X23	Insufficient of lubricating oil
X4	Butterfly spring crack	X24	Circuit current is too large
X5	Loose nut of the pull claw	X25	The fuse is not disconnected
X6	The connecting rod thread is defective	X26	Motor short circuit
X7	Piston ring damaged	X27	Poor power connection
X8	hydro-cylinder Internal leakage	X28	Coding line is disconnected
X9	hydro-cylinder crawl	X29	Inverter acceleration and deceleration time is too short
X10	hydro-cylinder External leakage	X30	Insufficient coolant
X11	No pressure in upper chamber of main valve	X31	Front bearing heating
X12	Spring force is too small	X32	Large cutting volume
X13	Burnt coil	X33	Spindle stuck
X14	Slow response	X34	Poor connection between control unit and motor
X15	Oil viscosity is too high	X35	The printed circuit board is dirty
X16	Blocked suction pipe	X36	Broach not in place
X17	Insufficient oil	X37	Override switch circuit is loose
X18	The adjustment screw is loose	X38	The grating is too dirty
X19	Oil pump speed is too low	X39	Aging of electronic components
X20	Pre-tightening force is too small		

Mode number	Failure probability P <sub>i</sub>	Probability importance $I_i$	Mode number	Failure probability P <sub>i</sub>	Probability importance $I_i$
X1	$2.3335 \times 10^{-5}$	0.9990422	X21	$3.0662 \times 10^{-5}$	0.9990495
X2	$3.4775 \times 10^{-5}$	0.9990536	X22	$2.3335 \times 10^{-5}$	0.9990422
X3	$3.4856 \times 10^{-5}$	0.9990537	X23	$3.4775 \times 10^{-5}$	0.9990536
X4	$2.3335 \times 10^{-5}$	0.9990422	X24	$3.0662 \times 10^{-5}$	0.9989956
X5	$3.0662 \times 10^{-5}$	0.9990495	X25	$2.3335 \times 10^{-5}$	0.9989882
X6	$2.3335 \times 10^{-5}$	0.9990422	X26	$3.4775 \times 10^{-5}$	0.9990536
X7	$2.3335 \times 10^{-5}$	0.9990422	X27	$3.0662 \times 10^{-5}$	0.9990495
X8	$3.4775 \times 10^{-5}$	0.9990536	X28	$2.3335 \times 10^{-5}$	0.9990422
X9	$3.4775 \times 10^{-5}$	0.9990536	X29	$2.3335 \times 10^{-5}$	0.9990422
X10	$3.4855 \times 10^{-5}$	0.9990537	X30	$3.4775 \times 10^{-5}$	0.9990536
X11	$2.3335 \times 10^{-5}$	0.9989956	X31	$3.4855 \times 10^{-5}$	0.9990537
X12	$2.3335 \times 10^{-5}$	0.9989956	X32	$3.0662 \times 10^{-5}$	0.9990495
X13	$3.4775 \times 10^{-5}$	0.9990536	X33	$2.3335 \times 10^{-5}$	0.9990422
X14	$3.0662 \times 10^{-5}$	0.9990495	X34	$3.0662 \times 10^{-5}$	0.9990495
X15	$2.3335 \times 10^{-5}$	0.9990422	X35	$2.3335 \times 10^{-5}$	0.9990422
X16	$3.4775 \times 10^{-5}$	0.9990536	X36	$2.3335 \times 10^{-5}$	0.9990422
X17	$2.3335 \times 10^{-5}$	0.9990422	X37	$2.3335 \times 10^{-5}$	0.9990422
X18	$2.3335 \times 10^{-5}$	0.9990422	X38	$2.3335 \times 10^{-5}$	0.9990422
X19	$2.3335 \times 10^{-5}$	0.9990422	X39	$2.3335 \times 10^{-5}$	0.9990422
X20	$3.4775 \times 10^{-5}$	0.9990536			

 Table 5
 Basic event failure probability and probability importance table

Taking the fault probability  $P_i$  of the basic event of the fault tree in Table 1, the fault probability of the intermediate event can be obtained according to equations (4) and (5), and substituting it into equation (7) to obtain the probability importance of the intermediate event in the second layer of the fault tree. The calculation results are shown in Table 6.

Mode number	Failure probability	Probability importance $I_i$
A1	$1.1218 \times 10^{-4}$	0.999130991
A2	$3.2122 \times 10^{-4}$	0.999327484
A3	$1.4686 \times 10^{-4}$	0.999161508
A4	$2.4713 \times 10^{-4}$	0.999265857
A5	$1.7066 \times 10^{-4}$	0.999189432

**Table 6** Probabilistic importance and failure probability of second-level events

According to equations (5) and (6), the probability of occurrence of the fault tree top event is  $9.8109 \times 10^{-4}$ . It is now required that the fault probability can be reduced to  $5.0 \times 10^{-4}$  in the newly designed spindle system. Redistribute the fault probabilities of events A1, A2, A3, A4 and A5, and establish a system of equations from equations (9) to (11).

$$\begin{cases} \Delta q_1 : \Delta q_2 : \dots : \Delta q_i = I_1 : I_2 : \dots : I_i \\ P_i^* = P_i + \Delta q_i \\ P_S^* = 1 - \prod_{i=1}^n (1 - (P_i + \Delta q_i)) \end{cases}$$
(20)

The MATLAB software can be used to calculate the result of the top event assignment, as shown in Table 7.

Mode number	Failure probability before assignment	Failure probability after assignment
A1	$1.1218 \times 10^{-4}$	$1.2596 \times 10^{-5}$
A2	$3.2122 \times 10^{-4}$	$2.2162 \times 10^{-4}$
A3	$1.4686 \times 10^{-4}$	$4.7273 \times 10^{-5}$
A4	$2.4713 \times 10^{-4}$	$1.4753 \times 10^{-4}$
A5	$1.7066 \times 10^{-4}$	$7.1070 \times 10^{-5}$

**Table 7**Top event assignment results comparison

In order to have a clearer and more intuitive understanding of the comparison results after top event assignment, as shown in Figure 4.

Figure 4	Top event assignment result comparison chart (a) top event assignment results graph
	(b) Analysis chart of the proportion of top event assignment results



Failure probability allocation Failure probability after allocation

Figure 4 Top event assignment result comparison chart (a) top event assignment results graph (b) Analysis chart of the proportion of top event assignment results (continued)



(b)

It can be seen from Table 7 and Figure 4 that the failure probability of events A2, A4 and A5 is still very high. According to the principle of reliability assignment, it is known that for high failure probability events, the parts with higher allocated reliability index are more difficult to be improved. Therefore, when the conditions are met, the events with high failure probability should be allocated to a lower reliability index. The comparison of the assignment results proves that the reliability index allocated to each event is reasonable.

### 3.2 Reliability assignment of intermediate events in spindle system based on fault tree

After the assignment of the reliability index of the top event is completed, the failure probability of the first-level intermediate event has changed. The reliability redistribution method needs to be used for the first-level intermediate events. The event 'A1 fixture loosening and clamping device failure' is taken as an example for illustration. The specific steps are as follows:

1) *First, sort the events in descending order of failure probability*: The failure probability of events B1 and B2 at the next level of the A1 event in the fault tree is:

$$P_{B1} = 8.8844 \times 10^{-5}, P_{B2} = 5.3996 \times 10^{-5} \text{ so } P_{B2} < P_{B1}$$
 (21)

Reduce the failure probability of events with a higher failure probability  $P_{B1}$  to  $P_0$  and keep the failure probability of  $P_{B2}$  unchanged. From Table 5, it can be seen that the failure probability of A1 after assignment is  $P_s^* = 1.2596 \times 10^{-5}$ , and the relationship between the failure probability of A1 and the failure probability of events B1 and B2 is expressed by equation (13).

Use trial and error method to determine  $k_o$  and  $P_o$ . When  $k_o = 1$ ,

$$P_{B2} = 5.3996 \times 10^{-5}$$

$$P_0 = 1 - \frac{\left(1 - P_s^*\right)}{\left(1 - P_{B2}\right)} = -4.1402 \times 10^{-5} < 0$$
(22)

When  $k_o = 2$ ,

$$P_{S}^{*} = 1 - (1 - P_{0})^{2}$$

$$P_{0} = 1 - (1 - P_{S}^{*})^{\frac{1}{2}} = 6.2980 \times 10^{-6}$$
(23)

Since there are B1 and B2 in the next layer of event A1, it can be known from equation (13) that when the failure probability of events B1 and B2 are adjusted, the failure probability after the assignment of event A1 can be met. At this time, the failure probability of events B1 and B2 is  $6.2980 \times 10^{-6}$ .

Similarly, events A2, A3, A4 and A5 can be filtered through equation (13) to screen out the secondary events that need to adjust the failure probability. The results are shown in Table 8 and Figure 5.

	events				
Mode number	Failure probability before assignment	Failure probability after assignment	Mode number	Failure probability before assignment	Failure probability after assignment
B1	$8.8844 \times 10^{-5}$	$6.2980 \times 10^{-6}$	B8	$7.1550 \times 10^{-10}$	$7.1550 \times 10^{-10}$
B2	5.3996×10 <sup>-5</sup>	$6.2980 \times 10^{-6}$	X1	2.3335×10 <sup>-5</sup>	2.3335×10 <sup>-5</sup>
В3	$1.2772 \times 10^{-4}$	$1.2772 \times 10^{-4}$	X2	$3.4770 \times 10^{-5}$	$3.4770 \times 10^{-5}$
B4	$5.4452 \times 10^{-10}$	$5.4452 \times 10^{-10}$	В9	$1.1210 \times 10^{-4}$	$5.6384 \times 10^{-5}$
В5	6.5431×10 <sup>-5</sup>	6.5431×10 <sup>-5</sup>	B10	$1.0028 \times 10^{-4}$	$5.6384 \times 10^{-5}$
B6	$1.2810 \times 10^{-4}$	$2.8482 \times 10^{-5}$	B11	$1.0066 \times 10^{-4}$	$1.0671 \times 10^{-6}$
B7	$1.2353 \times 10^{-4}$	$7.0606 \times 10^{-5}$	B12	$7.0003 \times 10^{-5}$	$7.0003 \times 10^{-5}$

 Table 8
 Comparison of reliability assignment results of fault tree secondary intermediate events

Figure 5 Comparison chart of first-level event assignment results (a) Intermediate event assignment result graph (b) Analysis chart of the proportion of intermediate event assignment results



From Table 8 and Figure 5, it can be seen that after the assignment of the first-level intermediate events is completed, the new failure probability of the second-level event is obtained. Among them, B1, B2, B6, B7, B9, B10, B11 events need to be adjusted to the next layer of events to meet the new failure probability requirements. Secondary events that require a change in fault probability are shown in Table 9.

Mode number	Failure probability before assignment	Failure probability after assignment	Mode number	Failure probability before assignment	Failure probability after assignment
B1	$8.8844 \times 10^{-5}$	6.2980×10 <sup>-6</sup>	В9	$1.1210 \times 10^{-4}$	5.6384×10 <sup>-5</sup>
B2	5.3996×10 <sup>-5</sup>	$6.2980 \times 10^{-6}$	B10	$1.0028 \times 10^{-4}$	$5.6384 \times 10^{-5}$
B6	$1.2810 \times 10^{-4}$	$2.8482 \times 10^{-5}$	B11	$1.0066 \times 10^{-4}$	$1.0671 \times 10^{-6}$
B7	$1.2353 \times 10^{-4}$	$7.0606 \times 10^{-5}$			

 Table 9
 Secondary intermediate events that need to change the probability of failure

## 3.3 Calculation of reliability assignment for secondary intermediate events in fault trees

After the reliability assignment of the first-level intermediate events is completed, this paper assigns the reliability indicators to the second-level intermediate events that need to be adjusted based on the AHP. The specific steps are as follows:

1) *Establish a hierarchy of basic events*: Establishing the hierarchy of basic events requires consideration of influencing factors such as event frequency M1, event importance M2, maintenance difficulty M3 and cost M4. Therefore, the basic event hierarchy is established with M1, M2, M3 and M4 as the criterion level, as shown in Figure 6.





 Table 10
 Criterion-level judgment matrix

Ε	M1	M2	<i>M3</i>	<i>M</i> 4
M1	1	3	2	3
M2	1/3	1	2	1
M3	1/2	1/2	1	1/2
M4	1/3	1	2	1

2) The calculation scheme layer checks the weight vector of the criterion layer and conducts the consistency check. Input the matrix into the MATLAB software to obtain the maximum eigenvalue  $\lambda_{max} = 4.1545$ , and the consistency index can be obtained by equation (14).

$$C = (\lambda_{\max} - n) / (n - 1) = (4.1545 - 4) / 3 = 0.0515$$
(24)

According to Table 2 and equation (17), we can get the value of  $C_R$ .

$$C_{R} = C / R = 0.0572 < 0.1 \tag{25}$$

Using the eigenvalue method, the weight of the scheme layer to the criterion layer can be obtained as: (0.6326, 0.1343, 0.0988, 0.1343).

3) *Calculate the weight of the scheme layer to the criterion layer*: The M1 and M2 of the basic event are calculated based on the failure probability and probability importance of each basic event, so the M1 and M2 judgment matrix of each basic event can be directly calculated. The judgment matrix of M3 and M4 is judged by experts according to Table 1.

This article takes the event B1 as an example for the calculation of the tool claw failure. The calculation process of other events is the same as that of the event B1, so the calculation results are directly given.

The judgment matrix of event B1 under 'event frequency M1' is shown in Table 11.

Table 11	Judgment	matrix	for	frequent	events
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M1	X3	X4	X5
X3	1	1.4935	1.1366
X4	0.6696	1	0.7610
X5	0.8798	1.3140	1

Use MATLAB software to find the maximum eigenvalue of 3, test consistency index  $C_R = 0 < 0.1$  and pass the test, the judgment matrix is reasonable. The weight is (0.2967, 0.3670, 0.3363).

The judgment matrix of event B1 under 'event frequency M2' is shown in Table 12.

M1	X3	X4	X5
X3	1	1.000011511	1.000004204
X4	0.999988489	1	0.999992693
X5	0.999995796	1.000007307	1

Table 12Judgment matrix of event importance.

Use MATLAB software to find the maximum eigenvalue of 3, test consistency index  $C_R = 0 < 0.1$  and pass the test, the judgment matrix is reasonable. The weight is (0.3561, 0.3386, 0.3053).

The judgment matrix of event B1 under 'difficulty of maintenance M3' is shown in Table 13.

M3	X3	X4	X5
X3	1	1	2
X4	1	1	2
X5	1/2	1/2	1

 Table 13
 Judgement matrix of maintenance difficulty

Use MATLAB software to find the maximum eigenvalue of 3, test consistency index  $C_R = 0 < 0.1$  and pass the test, the judgment matrix is reasonable. The weight is (0.3238, 0.4508, 0.2254).

The judgment matrix of event B1 under 'cost M4' is shown in Table 14.

Table 14Cost judgment matrix

M4	Х3	X4	Х5
X3	1	1	3
X4	1	1	3
X5	1/3	1/3	1

Use MATLAB software to find the maximum eigenvalue of 3, test consistency index  $C_R = 0 < 0.1$  and pass the test, the judgment matrix is reasonable. The weight is (0.3892, 0.4466, 0.1643).

- 4) According to equation (16), the weights of events X3, X4 and X5 relative to the target layer can be expressed by the following equation:
  - $w_{X3} = (0.6326, 0.1343, 0.0988, 0.1343)(0.2967, 0.3561, 0.3238, 0.3892)^{T} = 0.3198$   $w_{X3} = (0.6326, 0.1343, 0.0988, 0.1343)(0.3670, 0.3386, 0.4508, 0.4466)^{T} = 0.3822 \quad (26)$  $w_{X3} = (0.6326, 0.1343, 0.0988, 0.1343)(0.3363, 0.3053, 0.2254, 0.1643)^{T} = 0.2981$

Therefore, the weight of event B1 is (0.3198, 0.3822, 0.2981).

The same can be obtained, the weights of events B2, B6, B7, B9, B10 and B11 relative to the target layer can be expressed by the following equation:

$$w_{B2} = (0.4629, 0.5017)$$

$$w_{B6} = (0.2081, 0.1526, 0.1869, 0.1984, 0.2369)$$

$$w_{B7} = (0.2106, 0.3932, 0.2386, 0.1576)$$

$$w_{B9} = (0.2799, 0.2330, 0.22960.2575)$$

$$w_{B10} = (0.2767, 0.3235, 0.3998)$$

$$w_{B11} = (0.2333, 0.1864, 0.3189, 0.2614)$$
(27)

5) The reliability index is allocated according to the vector. Use equation (19). Obtain the failure probability of each basic event. Take event B1 as an example.

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$$P_{\chi_{3}} = \left(6.2980 \times 10^{-6}\right) \times \frac{\left(\frac{1}{0.3198}\right)}{\left(\frac{1}{0.3198} + \frac{1}{0.3822} + \frac{1}{0.2981}\right)} = 2.16 \times 10^{-6}$$

$$P_{\chi_{4}} = \left(6.2980 \times 10^{-6}\right) \times \frac{\left(\frac{1}{0.3822}\right)}{\left(\frac{1}{0.3198} + \frac{1}{0.3822} + \frac{1}{0.2981}\right)} = 1.81 \times 10^{-6}$$

$$P_{\chi_{5}} = \left(6.2980 \times 10^{-6}\right) \times \frac{\left(\frac{1}{0.2981}\right)}{\left(\frac{1}{0.3198} + \frac{1}{0.3822} + \frac{1}{0.2981}\right)} = 2.32 \times 10^{-6}$$

$$(28)$$

In the same way, the failure probability of the basic events of events B2, B6, B7, B9, B10 and B11 can be obtained. Since there are repeated basic events in the event, in order to improve the reliability level, the failure probability value of the basic event with a lower failure probability is selected and the result is as follows Table 15, as shown in Figure 7.

Mode number	Failure probability before assignment	Failure probability after assignment	Mode number	Failure probability before assignment	Failure probability after assignment
X3	$3.4775 \times 10^{-5}$	$3.56 \times 10^{-7}$	X23	$3.4775 \times 10^{-5}$	2.51×10 <sup>-5</sup>
X4	$2.3335 \times 10^{-5}$	$3.32 \times 10^{-7}$	X26	$3.4775 \times 10^{-5}$	$1.25 \times 10^{-5}$
X5	$3.0662 \times 10^{-5}$	$3.45 \times 10^{-7}$	X27	$3.0662 \times 10^{-5}$	$1.50 \times 10^{-5}$
X6	$2.3335 \times 10^{-5}$	$3.10 \times 10^{-6}$	X28	$2.3335 \times 10^{-5}$	$1.52 \times 10^{-5}$
X15	$2.3335 \times 10^{-5}$	$5.27 \times 10^{-6}$	X29	$2.3335 \times 10^{-5}$	$1.36 \times 10^{-5}$
X16	$3.4775 \times 10^{-5}$	$7.19 \times 10^{-6}$	X30	$3.4775 \times 10^{-5}$	$2.21 \times 10^{-5}$
X17	$2.3335 \times 10^{-5}$	$5.87 \times 10^{-6}$	X31	$3.4855 \times 10^{-5}$	$1.89 \times 10^{-5}$
X18	$2.3335 \times 10^{-5}$	$5.53 \times 10^{-6}$	X32	$3.0662 \times 10^{-5}$	$1.53 \times 10^{-5}$
X19	$2.3335 \times 10^{-5}$	$4.63 \times 10^{-6}$	X33	$2.3335 \times 10^{-5}$	$2.75 \times 10^{-7}$
X20	$3.4775 \times 10^{-5}$	$1.88 \times 10^{-5}$	X34	$3.0662 \times 10^{-5}$	$3.45 \times 10^{-7}$
X21	$3.0662 \times 10^{-5}$	$1.01 \times 10^{-5}$	X35	$2.3335 \times 10^{-5}$	$2.01 \times 10^{-7}$
X22	$2.3335 \times 10^{-5}$	1.66×10 <sup>-5</sup>	X36	$2.3335 \times 10^{-5}$	$2.46 \times 10^{-7}$

Comparison of failure probability assignment results for secondary events Table 15





Through calculation, the reliability assignment of the CNC machine tool spindle system is finally completed. From Table 15 and Figure 7, it can be seen that the tool claw is damaged X3, the butterfly spring is cracked X4, and the connecting rod and the tool claw nut are loosened X5, Defects in connecting rod thread processing X6, excessive oil viscosity X15, impurity blockage in the suction pipe X16, insufficient oil volume X17, loose pressure adjustment screw X18, oil pump speed too low X19, small preload X20,

bearing sleeve wear X21, preload large force X22, insufficient lubricating oil X23, partial motor short circuit X26, poor power connection X27, spindle code line disconnection X28, inverter acceleration and deceleration time is too short X29, insufficient coolant X30, large cutting amount X31, front bearing heating X32, Spindle stuck X33, poor connection between spindle control unit and motor X34, printed circuit board dirty X35, broach not in place X36 are all key considerations for redesigning the spindle system.

### 4 Conclusion

Use the fault tree structure to carry out reliability assignment, and the fault events to indicate the failure of machine tool subsystems and parts, so that the assignment task of reliability assignment is clear and the reliability index can be distributed down layer by layer, and finally reliability indexes are allocated to the parts of the system. Owing to the different content of events between different layers in the fault tree, the influencing factors and the number of events included, the methods suitable for each layer are used to allocate methods: use probability importance for top events, reliability reassignment method for intermediate events and analytic hierarchy process for bottom event. This method makes the assignment process clearer and makes the assignment result more accurate. At the same time, this method effectively solves the problems of incomplete considerations and low assignment results are useful for system optimisation and design. All have a certain reference value.

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