Bubbles in the virtual finance: an application of the Phillips-Wu-Yu (2011) methodology on the bitcoin price

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Abstract: This paper aims to detect the existence of speculative bubbles in the bitcoin US price by using a year by year ADF test, initiated by Dickey and Fuller (1981), and SADF test, initiated by Phillips et al. (2011). Over the period 2011–2020, we detect several episodes of bubbles during specific times of our study period but most importantly we detect a huge bubble in 2017 and 2019.

Keywords: speculative bubbles; crypto-currency; bitcoin; augmented Dickey-Fuller ADF; SADF; virtual finance; virtual bubbles.

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1 Introduction

The financial market is inefficient. At least, that is what theories tend to prove by highlighting the speculative bubbles that have paraded over the years. Shiller (2014) proves that the deviation of the price from its real value is a phenomenon which is spreading more and more as the euphoria of the investors grows according to what is considered as 'fashion and fads'. The pursuit of a particular asset is done less and less on

the basis of the fundamentals of finance and more through a 'herding behaviour' adopted by the least advantageous ones when it comes to information's. The behaviour of investor in the market starts being driven by overconfidence and whims which causes hysteria during the acquisition of a particular asset. The unidirectional movement of the crowd causes a significant confrontation between supply and demand which ends up leaving room for the aberrant increase of the prices. The aim of this kind of behaviour is not based on the rate of return on investment but on the anticipations that the asset can be sold to someone else at a higher price; which leads to an irrational rise in the price. This irrational increase is called in financial jargon 'speculative bubble' which is a deviation of the asset price from its fundamental value. Generally, this phenomenon turns out to be unobservable in the market as fundamentals are not always estimated. It even happens that the fundamental value reflects a consensus that assigns it a value other than its production cost (the currency case). We end up catching the phenomenon when the owners of the assets start selling at the same time making the bubble burst. Investor then have no choices than watching their money fly away as heavy repercussions are caused to their portfolios The great recession of 2008 is a perfect example of the virtual wealth of investors. The subprime crisis was not unanimous on the recording of losses which led some investors to lose hope in the performance of the financial market and consequently, in the operating of the states which are supposed to protect them and protect their wealth.

As if he had recorded the cry of distress of market participants, Nakamoto (2008) introduces a virtual currency in perfect decorrelation with the states and responding only to the law of supply and demand, all linked to a system called blockchain. This cryptocurrency has got the name of bitcoin. Introduced through Nakamoto's (2008) article, bitcoin is a crypto-currency or virtual currency that derives its logic from cryptography. It has become so popular that its price rose from USD0.1 in 2009 to almost USD20,000 in 2017, attracting various speculators looking to make short-term profits. It also attracted investors who are looking for a long-term investment since the bitcoin will soon become as scarce as gold and scarcity is expensive. Quite bold when we know that no consensus has been created when it comes to the real value of this crypto-currency. As Cheah and Fry (2015) estimates the real value of bitcoin to be equal to zero, Hayes (2016) stipulates that it's equal to the blow in electricity for minors. Concretely, these thematic remains a mystery but one thing is certain, the real finance dogma is shaken and speculative bubbles, found in financial markets, are appearing in the virtual. We then witness the transaction from real finance to virtual finance.

The aim of our paper is to study the price fluctuation of the bitcoin and spotlight if there is any sign of a bubble by detecting and datestamping them. We use the PWY (Phillips et al., 2011) methodology that is based on specifications of the augmented Dickey-Fuller (ADF) test.

This paper is structured as follow: Section 2 describes the bitcoin and its technology. Section 3 describes the methodology that we are going to use. Section 4 presents our main results. Finally, Section 5 represents the conclusion.

2 Literature review

According to Ahamad et al. (2013), fiat money has been categorised as old, old-fashioned, not enough, mismanaged by its governments, and is beginning to cause social unrest. For this reason and in response to the central bank's failure to manage

financial crises, a new kind of currency has emerged under the name of crypto-currency. It is indeed a means of exchange but is not yet accepted by all since it was not integrated through the decision of a particular state and represents no source of tangible wealth. Bariviera (2017) stipulates that this new type of currency is a libertarian response to the failure of the financial system which has shown its flaws through the various financial crises in recent years. It is extremely popular with people who hate government involvement in the regulation and creation of money. It also offers an alternative to those who fear a runaway inflation due the 'quantitative easing' policies (Moore and Christin, 2013). Thanks to the popularity of the system behind this virtual money, hundreds of different crypto-currencies have emerged. However, the first, best known and most widely used remains the bitcoin with a market capitalisation approximately 160 billion US dollars (USD) in 2019 according to Da Cunha and Silva (2019).

First introduced by a person or a group of person hidden under the pseudonym of Satoshi Nakamoto in 2008, the first stable release of bitcoin officially went into effect in January 2009. Su et al. (2018) argues that the fact that there is no coins or banknotes of bitcoin and that it is relatively poor as a unit of account and storage of value (properties of a currency) are the reasons that crypto-currencies are qualified as virtual. Since Nakamoto (2008) has last been heard of in 2011, it is the miners who exist all over the world and who offers their computing power to the service of the system, that deals with the transactions and the security of the dematerialised system. Indeed, what make the bitcoin more and more popular is the technology that it is related to. Using the block chain, which is similar to a 'book account' that records all transactions between the markets participants, miners carry out their computing power at the network layout. They validate the transactions circulating on the network while gathering them in blocks. They then add them to the register by forming a chain of blocks; knowing that a block consists of 300 transactions. Hughes et al. (2019) explains that a block chain is a ledger of transactions or blocks that make a linear chain of all transactions that has been made since the first one. Each block contains the last group of transactions and is similar to a public registry that details the history of bitcoin. If bitcoin changes owners, this movement appears in the block chain but it does not list the names of the owners. Traceability of transactions is therefore ensured with this system hence the indifference to the need for an entity that would play the role of a central bank. This also means that the transaction fees are absent since intermediaries fees are eliminated (Lo and Wang, 2014). All those elements contributed to raise the popularity of the bitcoin that attracted different types of investors all over the world.

Since the exchange with other currency is carried through a variable rate which responds only to the law of supply and demand, the number of transactions increased highly and bitcoin became quickly a highly speculative asset with excessive price volatility. Williams (2014) explains that the price volatility of the bitcoin (BTC) is 7 times higher than gold, 8 times higher than S&P 500 and 18 times higher than the USD. Blau (2017), meanwhile, found that this price volatility doubled the average volatility of 51 ordinary currencies from July 2010 to June 2014. While the real reasons for this price spike are still open to debate, a common explanation known as the 'Satoshi cycle' suggests that there is a strong correlation between Google searches for 'bitcoin' and the current price of bitcoin according to Chan et al. (2018).

As this crypto-currency is settled to be 'something' between gold and USD (Baur et al., 2018), the number of bitcoins will never exceed 21 million. Miners are paid by bitcoins [25 bitcoins in 2016 according to Gobel et al. (2016)] to ensure the security of

the system. This remuneration is made in fraction of newly created bitcoins and constitutes the only possible way to give rise to new bitcoins (Loi, 2017). The completion of the maximal programmed units is estimated for 2,140 according to Hendrickson et al. (2016). Halaburda and Sarvary (2016) believe that this limitation in number is motivated by the desire to ensure the scarcity of bitcoins in order to make it similar to gold. Enough to attract many investors.

3 Data and estimations

3.1 Data

The data used in this paper is sourced from ABC Bourse and includes daily closing prices for the bitcoin going from 10 February 2011 to 2 April 2020, yielding a total of 3,340 observations. Even though the bitcoin officially got introduced in 2009, we could not get a hold of the pricing data before 2011 and that is due to the fact that the price did not increase above \$1 until the beginning of 2011. Since then, the price of bitcoin started climbing hitting \$100 in 2013. Within the latest three months of that year, the price got multiplied by 10 and hit his first \$1,000 per bitcoin on November 2013.

Even though that rose in the price did not last long within the three years that follows, dropping under \$300, it started rising again in 2017 hitting an historical price of \$19,872.62 on the last month of the year. Starting from then, the price went under \$5,000 on the last month of the next year but that did not last long since the bitcoin price took a value of \$13,879 on June 2019 and \$10,598 on February 2020.

3.2 Methodology

3.2.1 The ADF test for a bubble

The best suitable approaches that can be used to detect bubbles in our case are the models of PWY (Phillips et al., 2011) and PSY (Phillips et al., 2015). It is capable of detecting bubbles without going through fundamentals, specifically: the fundamental value.

As a starting point, we follow the equation of Phillips et al. (2015) which provides other bubble generating mechanisms like intrinsic bubbles and time-varying discount factor fundamentals. We then use the asset pricing equation that follows:

$$P_t = \sum_{I=0}^{\infty} \left(\frac{1}{1+r_f}\right)^i E_t\left(U_{t+i}\right) + B$$

where P_t is the price of the crypto-currency used, r_f the risk-free interest rate, E_t is the expectation, U_t the unobservable fundamentals and B_t is the bubble component.

$$E_t\left(B_{t+i}\right) = \left(1 + r_f\right)B_t$$

If $B_t = 0$ there is no bubble in our sample and the degree of non-stationarity of the price is controlled by unobservable fundamentals. If a bubble is observed, the unobservable fundamentals are in the origin of their creation. The explosive or mildly explosive behaviour in asset price is then considered as an indicator of market exuberance during the inflationary phase of a bubble.

The general form of the Dickey-Fuller (Dickey and Fuller, 1981) test is based on the following regression:

$$\Delta P_t = \hat{\alpha}_{\eta, r_2}, \, \hat{\beta}_{\eta, r_2} P_{t-1} + \sum_{i=1}^k \hat{\psi}^i_{\eta, r_2} \Delta P_{t-i} + \varepsilon_t$$

where P_t is the crypto-currency price with the fundamental value, ε_t is the error term, *K* is the order of lags and $\varepsilon_t(0, \sigma_{f_1, f_2}^2)$. *K* is the (transient) lag order that controls the autocorrelation, and is selected by minimising the Akaike information criterion.

The ADF statistic based on this regression is denoted by ADF_n^r while the number of observation in the regression is $T_w = Tr_w$. The window of the regression moves through the entire sample starting from the fraction r_1^{TH} and ending to the r_2^{TH} , where $r_2 = r_1 + r_w$ and r_w is the partial size of the regression's window.

Phillips et al. (2011) look directly at the non-linear explosive behaviour which means that the null hypothesis in their test is H_0 : $\beta = 1$ (there is a unit root behaviour) while the alternative hypothesis is H_1 : $\beta > 1$ (there is an explosive behaviour).

The method of Phillips et al. (2011) starts the calculation of the long-term Dickey-Fuller statistics from the right-hand side in the recursive regression, which means that the initial observation of each regression is fixated to be the first observation of the full sample while the number of observations continues to increase until the full sample is used.

We employ then the sup augmented Dickey-Fuller (SADF) statistics, developed by PWY (Phillips et al., 2011), and the generalised SADF, developed by PSY (Phillips et al., 2015) that is useful to detect the presence of multiple bubbles.

3.2.2 The PWY test for multiple bubbles

To test for an explosive behaviour in the asset price, PWY (Phillips et al., 2011) proposed a new mechanism that can detect the origin of the explosive collapse of economic exuberance as well as the dates.

This new mechanism is based on a repeated estimation of the ADF model on a continuously expanding sample sequence and is obtained as the maximum value of the statistical sequence corresponding to the ADF test. The window size (fraction) r_w , in this case, extends from r_0 to 1; while r_0 is the smallest fraction of the width of the sample and 1 is the largest fraction of the window (the overall size of the sample).

The starting point r_1 of the sampling sequence is set to 0 so that the end point of each sample r_2 is equal to r_w and goes from r_0 to 1. The ADF statistic for a sample between 0 and r_2 is denoted by $ADF_0^{r_2}$. The PWY test is then a sup statistic based on direct recursive regression and is simply defined as:

 $SADF(r_0) = \sup ADF_0^{r_2}$ $r_2 \in [r_0, 1]$

4 Results and interpretations

Our empirical test is divided into three parts: The first part consists in identifying the normality of the series through the analysis of descriptive statistics. The second part represents the ADF unit root test which consists in detecting the stationarity of the series. The third and final part is to detect if there is speculative bubbles in the price of bitcoin through the SADF test. All the tests were carried out on the E-Views 9 software.

Table 1 presents the results of the descriptive statistics of our series where we find that the Kurtosis coefficient is less than 3 for the year 2012, 2014, 2016, 2019 and 2020 reflecting the degree of flattening of the distribution which is more flattened than the normal distribution (platykurtic). The rest of the years shows a Kurtosis greater than 3 which means that the distribution is leptokurtic. The Skewness coefficient, representing the degree of asymmetry in the series, is positive and greater than 0 (except for 2019 and 2020) indicating asymmetry to the right (left for 2019 and 2020). On the other hand, the Jarque-Bera test has a high value compared to the probability which allows us to reject the null hypothesis of normality of the distribution for all our samples. The series are therefore not normal and does not follow a normal distribution.

Table 2 presents the results of the unit root test which shows a probability p-value greater than the critical values (1%, 5% and 10%) for the price of all the studied years except for 2018. This result drives us to accept the null hypothesis of the ADF test that stipulates that our series present a unit root and are not stationary.

Our third part is shown in Table 3 where we present the results of the SADF tests. The statistics are compared with the critical values obtained thanks to the Monte Carlo simulation with 1,000 replications for each observation.

The p-value related to the SADF test (0.000) is lower than the critical values (1%, 5% and 10%) for all the distributions except for 2014, 2015, 2018 and 2020 distributions. This result drives us to reject the null hypothesis of the absence of bubbles which means that our studied period contains some explosive behaviour. Remember that the assumptions of the SADF test are the opposite of the ADF test.

Our results are simulated on Figures 1 and 2.

The two figures shows three lines, one in green (the future prices), one in red (the critical values) and finally a blue line (the calculated sequences). When the calculated sequence goes above the red critical values, we then talk about the presence of a bubble.

As it is shown on both the figures, the price of bitcoin was the victim of at least one speculative attack through all the studied years except for 2014, 2015, 2018 and 2020. According to the SADF test, we obtain ten distinct periods where some bubbles occurred. Those periods are illustrated in Table 4 where we summarise the bubbles periods and timestamp them.

	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
	6.800992	8.425	198.343	540.3847	278.8633	576.7104	4,123.728	7,804.635	7,531.166	8,276.771
Aedian	4.339900	7.000	115.64	517.1544	255.1148	595.1708	2,635.196	7,117.802	8,055.000	8,697.500
Aaximum	31.90990	16.41	1,163.00	995.000	497.4528	984.4308	19,872.62	17,170.36	13,929.8	10,333.00
linimum	0.747800	4.38	13.15	316.6076	211.8775	373.000	819.8256	3,385.851	3,421.800	4,826.00
std. deviation	6.146306	3.193	258.816	148.6232	61.31153	139.9154	4,187.872	2,543.979	2,758.083	1,436.139
skewness	1.306961	0.285	2.258	0.538418	1.708588	0.378053	2.035598	1.51585	-0.04006	-0.62854
Curtosis	4.498761	1.455	6.994	2.655723	5.155873	2.373202	6.673598	5.167132	1.835706	2.388997
arque-Berra	122.9431	41.354	552.891	19.43781	248.2743	14.70970	457.3139	152.0993	20.71378	7.570136
robability	0.00000	0.0000	0.0000	0.00000	0.00000	0.0006369	0.00000	0.0000	0.000032	0.019198
bservations	325	365	365	365	365	365	365	365	365	93

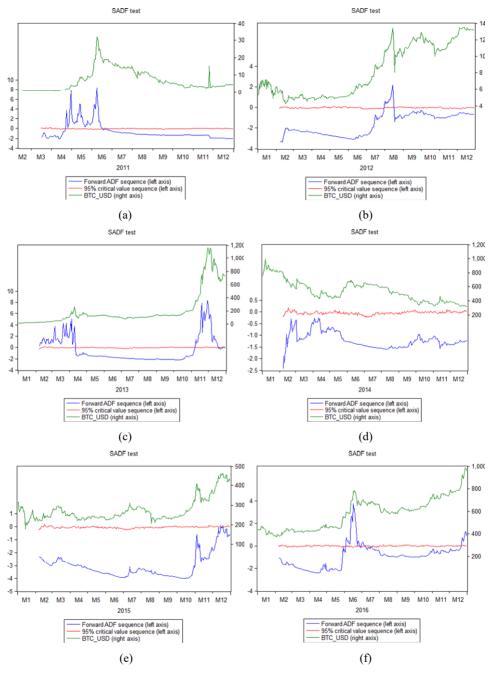
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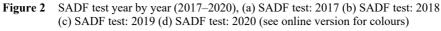
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Table 2ADF, results of the unit root tests

		Akaike (intercept)		Akai	Akaike (trend and intercept)	(tdəz.		Akaike (none)	
I	Lag	T_stat	Prob	Lag	T_stat	Prob	Lag	$T_{_stat}$	Prob
BTC/USD 2011	5	-1.9944	0.2893	5	-1.96468	0.6178	5	-1.197241	0.2116
BTC/USD 2012	5	-0.295665	0.9226	5	-2.159804	0.5101	5	1.078835	0.927
BTC/USD 2013	9	-0.99457	0.7562	9	-2.002722	0.5974	9	-0.269489	0.5885
BTC/USD 2014	1	-1.547024	0.5087	1	-2.418882	0.3691	1	-1.551615	0.1134
BTC/USD 2015	8	-0.377284	0.9099	8	-1.896789	0.6541	8	0.649033	0.8559
BTC/USD 2016	2	0.470051	0.9855	2	-1.743646	0.7298	2	1.824412	0.9839
BTC/USD 2017	3	0.557617	0.9884	С	-1.247625	0.8983	3	1.552631	0.9706
BTC/USD 2018	1	-2.67252	0.0798	1	-3.432604	0.0487	1	-2.557815	0.0104
BTC/USD 2019	2	-1.492873	0.5363	2	-0.975857	0.9447	2	0.034048	0.693
BTC/USD 2020	1	-1.159086	0.6891	1	-2.026821	0.5787	1	-0.260296	0.5897
				Critical	Critical values				
1%		-3.508326			-4.06829			-2.592129	
5%		-2.895512			-3.462912			-1.944619	
10%		-2.584952			-3.157836			-1.614288	

Figure 1 SADF test year by year (2011–2016), (a) SADF test: 2011 (b) SADF test: 2012 (c) SADF test: 2013 (d) SADF test: 2014 (e) SADF test: 2015 (f) SADF test: 2016 (see online version for colours)





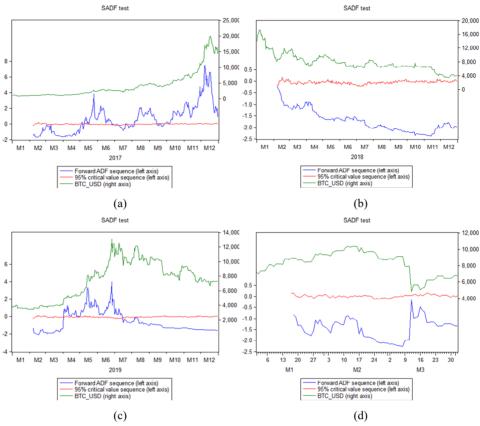


Table 3SADF test statistics

SADF	Test stat. —	Fin	nite sample critical val	lues
SADF	Test stat.	90%	95%	99%
Window	size: 38			
2011	8.340475 (0.0000)	1.196545	1.42931	2.021326
2012	2.167474 (0.0040)	1.254185	1.491772	1.934078
2013	8.314494 (0.0000)	1.225993	1.580124	2.03928
2014	-0.263470 (0.7240)	1.225993	1.580124	2.039288
2015	0.117949 (0.5270)	1.225993	1.580124	2.039288
2016	3.737337 (0.0000)	1.243265	1.541343	2.000392
2017	7.478917 (0.0000)	1.225993	1.580124	2.039288
2018	-0.238895 (0.7080)	1.225993	1.580124	2.039288
2019	4.065988 (0.0000)	1.225993	1.580124	2.039288
2020	-0.155663 (0.5540)	1.063077	1.371218	1.986935

Note: Critical values are based on Monte Carlo simulation.

Number	Starting day	Ending day	Peak day	Peak price (\$)	Crash end price (\$)	Crash size
1	04/21/2011	06/17/2011	04/23/2011	31,9099	18,34	43%
2	07/30/2012	08/18/2012	08/14/2012	16,41	11,86	28%
3	02/07/2013	04/12/2013	04/10/2013	259,34	112	57%
4	11/07/2013	12/18/2013	12/04/2013	1153,27	680,7	41%
5	06/03/2016	07/10/2016	06/16/2016	774,7674	651,0037	16%
6	12/20/2016	31/12/2016	07/03/2016	984,4308	956,3568	3%
7	05/08/2017	07/07/2017	05/25/2017	2697,207	2599,836	4%
8	08/03/2017	09/14/2017	09/02/2017	4977,837	3937,341	21%
9	09/29/2017	31/12/2017	12/17/2017	19872,62	14211,04	28%
10	04/02/2019	07/16/2019	06/26/2019	13929,8	9424,8	32%

Table 4Summary of the bitcoin bubbles periods

The first bubble is shown in 2011 [Figure 1(a)] where the bitcoin prices reached \$31, knowing that at the beginning of the year it did not even hit \$1. This bubble crushes down only two days after to appear in 2012 [Figure 1(b)]. This sudden rise and drop is due to the popularity of the bitcoin phenomenon that started attracting a limited attention on that period. After the burst of the second bubble in August 2012, the price did not climb above \$16 until the next year. Indeed, in 2013 the popularity of the bitcoin suddenly climbed to take on an international scale. Prices became very significant reaching \$200 in April 2013 and \$1,000 during the last month of the year. We can explain the two bubbles existing on that year [Figure 1(c)] by the excessive media coverage of the phenomenon which has aroused the interest of several types of investors. This media coverage led to some speculative attacks which raised the price of the bitcoin from \$13.24 at the start of the year to \$259.34 in April 2013. The price finally registered a flagrant drop of 68% within a week reaching \$82 due to the suspension of the trading at Mt. Gox for a 'market cool-down' (Cheung et al., 2015). The second bubble in 2013 came after another climb of the bitcoin popularity as the US federal judge and the German tax authorities declared that bitcoin shares some characteristics with functional currencies. The prices then ended up dropping after the Chinese Government decides to ban the use of bitcoin by financial institutions and business. On February 2014, the price crushes due to the bankruptcy of Mt. Gox, the largest exchange platform that was robbed of 850,000 bitcoin through a hacking. Since bubbles continue to occur on 2016 [Figure 1(f)], the major ones happened in 2017 where bitcoin prices took another level. As the number of businesses accepting bitcoin increases, the Japanese government recognises this crypto-currency as legal means of payment leading the bitcoin price above \$2,000. This price is multiplied by two on the period between August 2017 and September 2017 thanks to the launch of bitcoin Futures which ended up attracting more and more investor. On 17 December, the bitcoin price hits a historical peak of \$19,872 against \$278.86 in 2015; an increase of 7,026% in just two years. The price then drops after the announcement of the increasing supervision of exchanges in South Korea. Since then, the bitcoin price resumed a downward trend to reach \$3,841.9 in December of the ensuing year but this drop is quickly caught up and bitcoin registers an average a price of \$7,531,166 in 2019. As we can see on Table 4, the price fluctuations of bitcoin continue to rise and drop recording some major crash size in a retrained period (up to 57% in two days). That fluctuation results a huge bubble in 2019 [Figure 2(c)] that went from 4 February to 16 July. Such results show that the price of bitcoin is completely disconnected from its fundamentals.

In 2020, the price of this crypto-currency is halved in a space of a month, going from \$10,482.6 in February to \$4,857.1 in March, losing up to 53% of its value due to the COVID-19 crisis.

5 Conclusions

In this paper, we studied the bitcoin price by using the right tailed tests introduced by PWY (Phillips et al., 2011) to see if there is an emergence of speculative bubbles that can affect the price of the bitcoin. Through the ADF test, we find evidence of the non-stationarity of our series which confirms that the bitcoin price is detached from its fundamentals. We detect ten periods of explosive behaviour for the period going from 2011 to 2020 thanks to the SADF test and we find out that those bubbles are most likely driven by the high medialisation of the bitcoin and the fascination of the technology that is behind this crypto-currency. Indeed, despite the excessive rise in the price and the mystery linking its intrinsic value, investors have not stopped taking long position even when its price exceeded \$10,000 on 2017. We are in a case where the initial increase in price generates expectations of future increases that attract investors wishing to make capital gains. Even though the SADF test detected multiple bubbles, nothing can prove that the bitcoin price is right now at a correct phase neither does it mean that the bitcoin is not the biggest bubble that ever happened in the financial market.

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