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An efficient memory based differential evolution for constrained optimisation

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Abstract: Differential evolution (DE) and its diverse variants are prominently inflated by unfitting operators like mutation and crossover. Basically, DE is doesn't commit to memorise the finest effects attained in early part of the preceding peers. In this paper, a new DE (named as ^{mb}DE) based on memory mechanism is offered for constrained optimisation problems (COPs). It contained new mutation and crossover (so-called as swarm mutation and swarm crossover) created by particle swarm optimisation (PSO) circumstance. The performances ^{mb}DE has been tested over 22 CEC'2006 and 18 (10 and 30-dimensional) CEC'2010 COPs. The empirical results confirm the superiority of ^{mb}DE over many contemporary algorithms.

Keywords: differential evolution; PSO; particle swarm optimisation; mutation; crossover; elitism; COPs; constrained optimisation problems.

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1 Introduction

At the present time, constrained optimisation problems (COPs) are habitually seemed as real world problems. Solving such problems is a bigger task among investigators. From this time, a large number of evolutionary algorithms (EAs) are proposed in the literature (Slowik and Kwasnicka, 2020; Parouha and Das, 2016) to solve COPs.

Differential Evolution (designated as DE) proposed by Storn and Price (1997) is an competent and widespread ingredient in verdict of numerous EAs. It is a population-based intelligent stochastic and global search optimisation technique. Owing to its easiness and lots of benefits like easy carrying out, rationally faster, robust and comprehensive search capability (Das et al., 2009), DE has gained wide popularity over few decades. Also, it was used to solve various engineering problems for instance mechanical engineering design problem (Zhang et al., 2008), fuzzy clustering of image pixel (Das and Sil, 2010), economic load dispatch (Wang et al., 2010), to design quantisation table for JPEG baseline system (Kumar and Karpagam, 2015), etc. However, while solving multifaceted problems, DE gets stuck into local minima and causing with an untimely convergence (Das et al., 2009). Moreover, DE does not surety to extent at global optimal result in a limited stint interval.

Hence, so as to advance the presentation of elementary DE, quantities of efforts are completed in the literature (Bilal et al., 2020; Opara and Arabas, 2019; Eltaei and Mahmood, 2018; Neri and Tirronen, 2010). Accordingly, operators (mutation and crossover) selection is greatly challenged in DE. As well incorrect operator's selection of DE may lead to stagnation, untimely convergence and so on (Das and Suganthan, 2011). Over the time, to get rid of it researchers reforming DE through altered mutation schemes (Neri and Tirronen, 2010; Das and Suganthan, 2011; Gong and Cai, 2013; Das and Parouha, 2016; Dhanalakshmy et al., 2020). Likewise, researchers used mostly twofold crossover (binomial and exponential) in DE (Storn and Price, 1997). Nonetheless, it is detected that among these crossovers there is no substantial differences (Price and Storn, 2008). Additionally, hybrids of DE produce quality outcomes instead of its different improved variants (Lenin et al., 2016; Das et al., 2017). Mostly DE hybridised with particle swarm optimisation (PSO) (Kennedy and Eberhart, 1995) due to contemporary properties. The recent past well-established hybrids of DE and PSO can be found in Das and Parouha (2015), and Parouha and Das (2016). They turned out to be milestones in solving COPs.

Yet in the literature numerous alternatives and hybrids of DE have been suggested to solve COPs. But they are incompetent to deliver reasonable result, as a consequence of DE has no tool/operators to memorise the so-far best result (Wang et al., 2014), besides this in the search space it uses only the global facts. As a result, generally DE loses computing control and easily leads to premature convergence (Das et al., 2005).

In this paper, a new DE (titled as ^{mb}DE) inspired by memorised mechanism of PSO has been proposed. It contained new mutation and crossover (so-called as swarm mutation and swarm crossover) formed by p^{best} and g^{best} term of PSO. The proposed ^{mb}DE employs memory process which permits each particles to improve own search competences utilising by the knowledge of more effectual particles. The memorised knowledge as aiming at better strength may converges near to noble solutions. The practical problems are mostly structured as COPs by reason of the presence of an enormous quantity of decision variables. Henceforth, the proposed ^{mb}DE has been tested over COPs (22 CEC'2006 and 18 CEC'2010 benchmark functions) instead of unconstrained ones. As a result, proposed ^{mb}DE produces good results with better rate of convergence and exploration as expected on search space.

The paper is structured as follows. Section 2 presents overview of the conventional DE. Section 3 takes account of some constraint handling techniques. Section 4 drawn the detailed of the proposed ^{mb}DE. Analysis of results with comparison is carried out in Section 5. Lastly, the conclusion of the paper is drawn in Section 6.

2 Conventional DE

Differential evolution (DE), proposed by Storn and Price (1997) is a stochastic and population established worldwide optimisation technique. The framework of the conventional DE is set as follows.

- i *Initialisation*: at 'tth' generation population and for any d -dimensional optimisation problem np parents (target) vectors $x_i^j(t) = (x_i^1(t), x_i^2(t), \dots, x_i^d(t))$; $i = 1, 2, \dots, np$ is randomly created in given limits.
- ii *Mutation*: a mutant vector $v_i^j(t) = (v_i^1(t), v_i^2(t), \dots, v_i^d(t))$ is produced as below.

$$v_i^j(t) = x_{r_1}(t) + M \times (x_{r_2}(t) - x_{r_3}(t)) \quad (1)$$

where $r_1 \neq r_2 \neq r_3 \neq i$ and $M \in [0, 1]$ is the mutation factor.

- iii *Crossover*: a new trial vector $u_i^j(t) = (u_i^1(t), u_i^2(t), \dots, u_i^d(t))$ is made as follows.

$$u_i^j(t) = \begin{cases} v_i^j(t); & \text{if } \text{rnd}(0, 1) \leq C_r \\ x_i^j(t); & \text{if } \text{rnd}(0, 1) > C_r \end{cases} \quad (2)$$

where $C_r \in [0, 1]$ is a crossover constant.

- iv *Selection*:

$$x_i^j(t+1) = \begin{cases} u_i^j(t); & \text{if } f(u_i^j(t)) \leq f(x_i^j(t)) \\ x_i^j(t); & \text{otherwise} \end{cases} \quad (3)$$

3 Handling constraint

Constraint treatment is a puzzling task in solving COPs. To handle constraints alteration technique is widespread approaches (Deb, 1995) where COPs transfers to un-constrained one adding by penalty term into objective function as follows: F (penalty function) = f (objective function) + Ω (penalty term) $\{ R$ (penalty parameter), g (inequality constraint), h (equality constraint) $\}$. Moreover, bracket operator penalty ($\Omega = R g^2$, wherever $\alpha = \begin{cases} 0; & \text{if } \alpha \geq 0 \\ \alpha; & \text{if } \alpha < 0 \end{cases}$) among all exterior penalty is effectively used to solve COPs (Deb, 1995). Therefore, this penalty is picked in present study for solving COPs, caused by its higher efficiency.

4 Proposed scheme

After literature survey, the succeeding opinions on DE and its alternatives are noted as.

- i Kernel operators (mutation and crossover) selection is greatly challenged in DE. As well incorrect operator's selection of DE may cause to untimely convergence, stagnation and so on.

- ii Choice of proper operators puts a weighty effect in quality solution of DE.
- iii Basically DE is does not commit to memorise the finest effects attained in early part of the preceding peers.

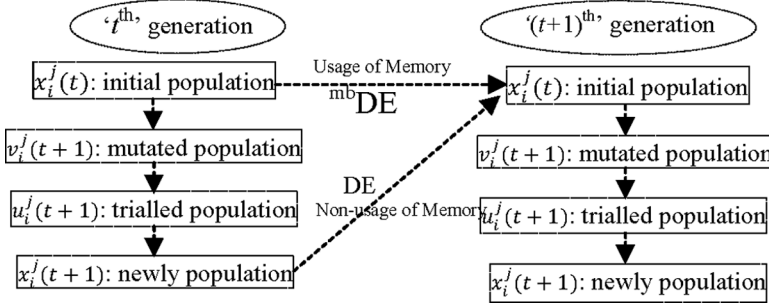
Above notes and drawbacks and benefits of DE became the main inspiration to propose an effective algorithm. The following major key points are considered for new proposed algorithm.

- i Constitution of a novel approach to remembering the finest effects/results attained in early part of the preceding peers.
- ii Necessity to improve all two-difference established mutation scheme of DE.
- iii In place of purely repetition of mutated and target vectors a slight perturbation needed to crossover operator of DE.

4.1 Usage of memory

The perfect conception of ‘usage of memory’ is lent from p^{best} and g^{best} device of PSO. Since PSO stimulated on or after social and competitive behaviour of clustered birds well-known as swarm, hence proposed operators termed as swarm mutation and swarm crossover. Moreover, suggested DE named as ^{mb}DE due to usage of memory mechanism. Pictorial demonstration of the idea of ‘usage of memory’ is clearly differentiates in Figure 1.

Figure 1 Usage of memory in ^{mb}DE vs. DE



4.2 Swarm mutation

A mutant vector $v_i^j(t) = (v_i^1(t), v_i^2(t), \dots, v_i^d(t))$ in current generation ' t ' is produced by swarm mutation using target vector $x_i^j(t) = (x_i^1(t), x_i^2(t), \dots, x_i^d(t))$ and best position vector $p_i^j(t) = (p_i^1(t), p_i^2(t), \dots, p_i^d(t))$ as follows.

$$v_i^j(t) = x_i^j(t) + \underbrace{rnd(0,1) \left| \frac{f(p_i^{best}(t))}{f(x_i^{worst}(t))} \right| \times (p_i^{best}(t) - x_i^j(t))}_{\text{cognitive component}} + \underbrace{rnd(0,1) \left| \frac{f(g^{best}(t))}{f(x_i^{worst}(t))} \right| \times (g^{best}(t) - x_i^j(t))}_{\text{social component}}$$

(4)

where $p_i^{best}(t)$ and $g^{best}(t)$: personal (individual) and global (overall) best position of the vector $p_i^j(t)$ respectively, $f(x_i^{worst}(t))$: worst (poorest) function value of vector $x_i^j(t)$. At any time, $f(x_i^{worst}(t)) = 0$, it could be changed through a large positive constant ' μ ' and $rnd(0,1)$: random number between (0, 1). Similar in PSO, the cognitive and social inheritance permits the particles to advance the search competences utilising by the knowledge of more effectual particles.

4.3 Swarm crossover

A new trial vector $u_i^j(t) = (u_i^1(t), u_i^2(t), \dots, u_i^d(t))$, is generated by swarm crossover as.

$$u_i^j(t) = \begin{cases} v_i^j(t) + rnd(0,1) \times (g^{best}(t) - p_i^{best}(t)); & \text{if } rnd(0,1) \leq C_r \\ x_i^j(t) + rnd(0,1) \times (g^{best}(t) - p_i^{best}(t)); & \text{if } rnd(0,1) > C_r \end{cases} \quad (5)$$

This adoption may to enhance convergence rate and local search ability of DE as usage of memorised concept.

4.4 Elitism

In ^{mb}DE, both proposed operator (swarm mutation and crossover) are marinating sufficient diversity. Therefore, selection operator is exchanged by elitism in ^{mb}DE cycle which works as below and retains better results.

- i Merged initial and final population getting after equation (5).
- ii Arranged (i.e., in ascending) order according to their function value.
- iii Select better halves discard the rest.

4.5 Steps of projected ^{mb}DE algorithm

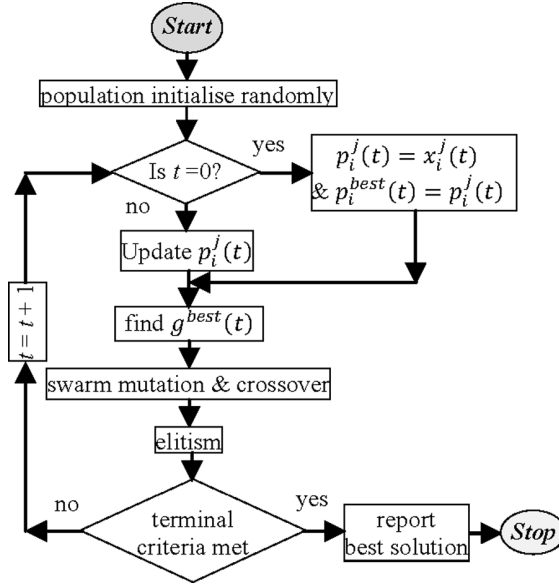
Stages of proposed ^{mb}DE are presented as:

- 1 All vectors $x_i^j(t) = (x_i^1(t), x_i^2(t), \dots, x_i^d(t))$ produces randomly in the given search range
- 2 Calculate $x_i^j(t)$
- 3 Set $t = 0$
- 4 Build $p_i^j(t) = x_i^j(t)$, $p_i^{best}(t) = p_i^j(t)$ and update $p_i^j(t) = p_i^{best}(t)$ using

$$p_i^j(t) = \begin{cases} x_i^j(t); & \text{if } f(x_i^j(t)) \leq f(p_i^j(t-1)) \\ p_i^j(t-1); & \text{otherwise} \end{cases}$$
- 5 Find $g^{best}(t)$
- 6 Apply swarm operators (mutation and crossover)
- 7 Apply elitism
- 8 Stop if the end criterion is seen, else set $t = t + 1$ and go to Step-4.

Moreover, flowchart of ^{mb}DE is given in Figure 2.

Figure 2 Flowchart of ^{mb}DE algorithm



5 Result and discussion

So as to assess the performance of ^{mb}DE, 22 CEC'2006 and 18 CEC'2010 test suit are solved. These problems are devising different taste and listed in Table 1 and 2. The more detail along with their characteristics of CEC'2006 and CEC'2010 can be found in Liang et al. (2006) and Mallipeddi and Suganthan (2010).

The ^{mb}DE is implemented in Intel (R) Core (TM) i5-2350 M CPU @ 2.30GHz 2.30GHz, RAM: 4.00 GB, Operating System: Window 10, C-free Standard 4.0. After many experiments fine tuning value of ' $C_r = 0.9$ ' and ' $R = 1000$ ' is acclaimed to use in ^{mb}DE. The overall best values in each table are highlighted with boldface italic letters and UA represents results un-availability of the corresponding algorithms.

5.1 Numerical analysis

The produced result of ^{mb}DE are equated with (i) SAMO-DE (Elsayed et al., 2011), SAMO-GA (Elsayed et al., 2011), DECV (Mezura-Montes et al., 2010), A-DDE (Mezura-Montes and Palomeque-Ortiz, 2009) and APF-GA (Tessemma and Yen, 2009) in case of 22 CEC'2006 COPs and (ii) BGR (Hsieh, 2014), SAMO-DE (Elsayed et al., 2011), SAMO-GA (Elsayed et al., 2011) and IEMA (Singh et al., 2010) in case of 18 CEC'2010 COPs. The simulations are reported on independent 25 run, 1.40×10^5 , 2×10^5 and 6×10^5 FEs (maximum no. of function evaluations) for CEC'2006, 10D CEC'2010 and 30D CEC'2010 are fixed as stopping criteria and $np = 100$ taken in both cases. These parameters are either considered equal (in case of run and population size) or minimum (in case of stop criteria) of the above listed algorithms, for fair comparison.

Table 1 CEC'2006 test suit

<i>f_{en}</i>	<i>Optimal solution</i>	<i>D</i> (dimension)	<i>Type of function</i>	$\rho = F / S $ (estimated ratio of feasible region and search space)	<i>LI</i> (No. of linear inequality constraints)	<i>NE</i> (No. of nonlinear equality constraints)	<i>NI</i> (No. of nonlinear inequality constraints)	<i>LE</i> (No. of linear equality constraints)	<i>a</i> (No. of active constraints)
g_{CEC06}^{01}	-15.0000000000	13	Quadratic	0.0111	9	0	0	0	6
g_{CEC06}^{02}	-0.8036191042	20	Nonlinear	99.9971	0	0	2	0	1
g_{CEC06}^{03}	-1.0005001000	10	Polynomial	0.0000	0	1	0	0	1
g_{CEC06}^{04}	-30665.53867178	5	Quadratic	52.1230	0	0	6	0	2
g_{CEC06}^{05}	5126.4967140071	4	Cubic	0.0000	2	3	0	0	3
g_{CEC06}^{06}	-6961.813875580	2	Cubic	0.0066	0	0	2	0	2
g_{CEC06}^{07}	24.3062090681	10	Quadratic	0.0003	3	0	5	0	6
g_{CEC06}^{08}	-0.0958250415	2	Nonlinear	0.8560	0	0	2	0	0
g_{CEC06}^{09}	680.6300573745	7	Polynomial	0.5121	0	0	4	0	2
g_{CEC06}^{10}	7049.2480205286	8	Linear	0.0010	3	0	3	0	6
g_{CEC06}^{11}	0.7499000000	2	Quadratic	0.0000	0	1	0	0	1
g_{CEC06}^{12}	-1.0000000000	3	Quadratic	4.7713	0	0	1	0	0

Table 1 CEC'2006 test suit (continued)

<i>fen.</i>	<i>Optimal solution</i>	<i>D</i> (dimension)	<i>Type of function</i>	$\rho = F / S $ (estimated ratio of feasible region and search space)	<i>LI</i> (No. of linear inequality constraints)	<i>NE</i> (No. of nonlinear equality constraints)	<i>NI</i> (No. of nonlinear inequality constraints)	<i>LE</i> (No. of linear equality constraints)	<i>a</i> (No. of active constraints)
g_{CEC06}^{13}	0.0539415140	5	Nonlinear	0.0000	0	3	0	0	3
g_{CEC06}^{14}	-47.7648884595	10	Nonlinear	0.0000	0	0	0	3	3
g_{CEC06}^{15}	961.7150222899	3	Quadratic	0.0000	0	1	0	1	2
g_{CEC06}^{16}	-1.9051552586	5	Nonlinear	0.0204	4	0	34	0	4
g_{CEC06}^{17}	8853. 539674806	6	Nonlinear	0.0000	0	4	0	0	4
g_{CEC06}^{18}	-0.8660254038	9	Quadratic	0.0000	0	0	13	0	6
g_{CEC06}^{19}	32.6555929502	15	Nonlinear	33.4761	0	0	5	0	0
g_{CEC06}^{20}	0.2049794002	24	Linear	0.0000	0	12	6	2	16
g_{CEC06}^{21}	193.7245100700	7	Linear	0.0000	0	15	1	0	6
g_{CEC06}^{22}	236.4309755040	22	Linear	0.0000	0	11	1	8	19
g_{CEC06}^{23}	— 400.0551000000	9	Linear	0.0000	0	1	2	3	6
g_{CEC06}^{24}	-5.5080132716	2	Linear	79.6556	0	0	2	0	2

Table 2 CEC'2010 test suit

<i>fen.</i>	<i>Objective type</i>	<i>Number of constraints</i>			$\rho = F / S $			<i>Search range</i>
		<i>NE</i>	<i>NI</i>		<i>10D</i>	<i>30D</i>		
c_{CEC10}^{01}	Non-separable	0	2 Non-separable		0.997689	1.0000		$[-5.12, 5.12]^D$
c_{CEC10}^{02}	Separable	1 Separable	2 Separable		0.0000	0.0000		$[-1000, 1000]^D$
c_{CEC10}^{03}	Non-separable	1 Separable	0		0.0000	0.0000		$[-50, 50]^D$
c_{CEC10}^{04}	Separable	2 Non-separable, 2 separable	0		0.0000	0.0000		$[-600, 600]^D$
c_{CEC10}^{05}	Separable	2 Separable	0		0.0000	0.0000		$[-600, 600]^D$
c_{CEC10}^{06}	Separable	2 Rotated	0		0.0000	0.0000		$[-140, 140]^D$
c_{CEC10}^{07}	Non-separable	0	1 Separable		0.505123	0.503725		$[-140, 140]^D$
c_{CEC10}^{08}	Non-Separable	0	1 Rotated		0.379512	0.375278		$[-500, 500]^D$
c_{CEC10}^{09}	Non-separable	1 Separable	0		0.0000	0.0000		$[-500, 500]^D$
c_{CEC10}^{10}	Non-separable	1 Rotated	0		0.0000	0.0000		$[-100, 100]^D$
c_{CEC10}^{11}	Rotated	1 Non-separable	0		0.0000	0.0000		$[-1000, 1000]^D$
c_{CEC10}^{12}	Separable	1 Non-separable	1 Separable		0.0000	0.0000		$[-500, 500]^D$
c_{CEC10}^{13}	Separable	0	2 Separable, 1 non-separable		0.0000	0.0000		$[-1000, 1000]^D$
c_{CEC10}^{14}	Non-separable	0	3Separable		0.003112	0.006123		$[-1000, 1000]^D$
c_{CEC10}^{15}	Non-separable	0	3 Rotated		0.003210	0.006023		$[-10, 10]^D$
c_{CEC10}^{16}	Non-separable	2 Separable	1 Separable, 1 non-separable		0.0000	0.0000		$[-10, 10]^D$
c_{CEC10}^{17}	Non-separable	1 Separable	2 Non-separable		0.0000	0.0000		$[-50, 50]^D$
c_{CEC10}^{18}	Non-separable	1 Separable	1 Separable		0.0000	0.0000		$[0, 10]^D$

Table 3 Comparative results on CEC'2006 COPs

<i>Fcn</i>	<i>Cr</i>	<i>mb</i> DE	SAMO-DE	SAMO-GA	DECV	A-DDE	APF-GA
g_{CEC06}^{01}	Best	-15.00000(1)	-15.0000(1)	-15.0000(1)	-15.0000(1)	-15.0000(1)	-15.0000(1)
	Mean	-15.00000(1)	-15.0000(1)	-15.0000(1)	-14.855(2)	-15.0000(1)	-15.0000(1)
	Std.	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	4.59E-01(2)	7.00E-06(3)	0.00e+00(1)
	Best	-0.803619(1)	-0.803619(1)	-0.80359052(4)	-0.704009(5)	-0.803605(2)	-0.803601(3)
g_{CEC06}^{02}	Mean	-0.805282(1)	-0.79873521(3)	-0.79604769(4)	-0.569458(6)	-0.771090(5)	-0.803518(2)
	Std.	1.50e-05(1)	8.8005e-03(4)	5.8025e-03(3)	9.51e-02(6)	3.66e-02(5)	1.00e-04(2)
	Best	-1.000500(1)	-1.0005(1)	-1.0005(1)	-0.461(4)	-1.0000(3)	-1.001(2)
	Mean	-1.000500(1)	-1.0005(1)	-1.0005(1)	-0.134(4)	-1.0000(3)	-1.001(2)
g_{CEC06}^{03}	Std.	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	1.17e-01(3)	9.30e-12(2)	0.00e+00(1)
	Best	-30665.5386(2)	-30665.5386(2)	-30665.5386(2)	-30665.539(1)	-30665.539(1)	-30665.5386(2)
	Mean	-30665.5386(2)	-30665.5386(2)	-30665.5386(2)	-30665.539(1)	-30665.539(1)	-30665.5386(2)
	Std.	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	1.56e-06(3)	3.20e-13(2)	0.00e+00(1)
g_{CEC06}^{05}	Best	5126.496714(1)	5126.497(2)	5126.497(2)	5126.497(2)	5126.497(2)	5126.497(2)
	Mean	5126.496714(1)	5126.497(2)	5127.976432(4)	5126.497(2)	5126.497(2)	5127.5423(3)
	Std.	0.00e+00(1)	0.00e+00(1)	1.1166e+00(3)	0.00e+00(1)	2.10e-11(2)	1.4324e+00(4)
	Best	-6961.81388(2)	-6961.81387(3)	-6961.81387(3)	-6961.814(1)	-6961.814(1)	-6961.814(1)
g_{CEC06}^{06}	Mean	-6961.81387(2)	-6961.81387(3)	-6961.81387(3)	-6961.814(1)	-6961.814(1)	-6961.814(1)
	Std.	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	2.11e-12(2)	0.00e+00(1)
	Best	24.3062091(3)	24.3062(2)	24.3062(2)	24.306(1)	24.306(1)	24.3062(2)
	Mean	24.3062092(3)	24.3096(4)	24.4113(5)	24.794(6)	24.306(1)	24.3062(2)
g_{CEC06}^{07}	Std.	1.3425e-07(2)	1.5888e-03(4)	4.5905e-02(5)	1.37e+00(6)	4.20e-05(3)	0.00e+00(1)
	Best	-0.095825(2)	-0.09582504(1)	-0.09582504(1)	-0.095825(2)	-0.095825(2)	-0.09582504(1)
	Mean	-0.095825(2)	-0.09582504(1)	-0.09582504(1)	-0.095825(2)	-0.095825(2)	-0.09582504(1)
	Std.	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	4.23e-17(2)	9.10e-10(3)	0.00e+00(1)

Table 3 Comparative results on CEC'2006 COPs (continued)

<i>Fcn</i>	<i>Cr</i>	<i>mbDE</i>	SAMO-DE	SAMO-GA	DECV	A-DDE	APF-GA
g_{CEC06}^9	Best	680.630057(2)	680.630(1)	680.630(1)	680.630(1)	680.630(1)	680.630(1)
	Mean	680.630057(2)	680.630(1)	680.634(3)	680.630(1)	680.630(1)	680.630(1)
	Std.	0.00e+00(1)	1.1567e-05(4)	1.4573e-03(5)	3.45e-07(3)	1.15e-10(2)	0.00e+00(1)
g_{CEC06}^{10}	Best	7049.248021(2)	7049.2481(3)	7049.2481(3)	7049.248(1)	7049.248(1)	7049.2481(3)
	Mean	7049.248028(2)	7059.81345(3)	7144.40311(6)	7103.548(5)	7049.248(1)	7077.6821(4)
	Std.	2.9742e-05 (1)	7.856e+00(3)	6.7860e+01(5)	1.48e+02(6)	3.23e-04(2)	51.2404(4)
g_{CEC06}^{11}	Best	0.749900(1)	0.7499(1)	0.7499(1)	0.75(2)	0.75(2)	0.7499(1)
	Mean	0.749900(1)	0.7499(1)	0.7499(1)	0.75(2)	0.75(2)	0.7499(1)
	Std.	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	1.12e-16(2)	5.35e-15(3)	0.00e+00(1)
g_{CEC06}^{12}	Best	-1.000000(1)	-1.00000000(1)	-1.00000000(1)	-1.00000000(1)	-1.00000000(1)	-1.00000000(1)
	Mean	-1.000000(1)	-1.00000000(1)	-1.00000000(1)	-1.00000000(1)	-1.00000000(1)	-1.00000000(1)
	Std.	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	4.10e-09(2)	0.00e+00(1)
g_{CEC06}^{13}	Best	0.0539415(1)	0.053942(2)	0.053942(2)	0.059798(3)	0.053942(2)	0.053942(2)
	Mean	0.0539418(1)	0.053942(2)	0.054028(3)	0.382401(5)	0.079627(4)	0.053942(2)
	Std.	5.601e-10(3)	1.7541e-08(4)	5.9414e-05(5)	2.68e-01(6)	1.000e-14(2)	0.00e+00(1)
g_{CEC06}^{14}	Best	-47.764888(2)	-47.76489(1)	-47.1883(4)	-47.764888(2)	-47.764888(2)	-47.76479(3)
	Mean	-47.764888(1)	-47.68115(5)	-46.4731793(6)	-47.722542(4)	-47.764131(3)	-47.76479(2)
	Std.	0.00e+00(1)	4.043e-02(4)	3.1590e-01(6)	1.62e-01(5)	9.0e-06(2)	1.00e-04(3)
g_{CEC06}^{15}	Best	961.715022(1)	961.71502(1)	961.71502(1)	961.715022(1)	961.715022(1)	961.71502(1)
	Mean	961.715022(1)	961.71502(1)	961.715087(2)	961.71502(1)	961.71502(1)	961.71502(1)
	Std.	0.00e+00(1)	0.00e+00(1)	5.5236e-05(3)	2.31e-13(2)	0.00e+00(1)	0.00e+00(1)
g_{CEC06}^{16}	Best	-1.905155(1)	-1.905155(1)	-1.905155(1)	-1.905155(1)	-1.905155(1)	-1.905155(1)
	Mean	-1.905155(1)	-1.905155(1)	-1.905154(2)	-1.905155(1)	-1.905155(1)	-1.905155(1)
	Std.	0.00e+00(1)	0.00e+00(1)	6.9520e-07(2)	1.10e-06(3)	0.00e+00(1)	0.00e+00(1)

Table 3 Comparative results on CEC'2006 COPs (continued)

<i>Fcn</i>	<i>Cr</i>	<i>m^b</i> DE	<i>SAMO-DE</i>	<i>SAMO-GA</i>	<i>DECV</i>	<i>A-DDE</i>	<i>APF-GA</i>
¹⁷ <i>g</i> _{CEC06}	Best	8853.539675(1)	8853.5397(2)	8853.5397(2)	8853.54128(5)	8853.54000(4)	8853.5398(3)
	Mean	8853.539698(1)	8853.5397(2)	8853.8871(3)	8919.9363(6)	8854.664(4)	8888.4876(5)
	Std.	1.56172e-08(1)	1.1500e-05(2)	1.7399e-01(3)	2.59e+01(5)	1.43e+00(4)	29.0347(6)
¹⁸ <i>g</i> _{CEC06}	Best	-0.8660254(1)	-0.866025(1)	-0.866025(1)	-0.866025(1)	-0.866025(1)	-0.866025(1)
	Mean	-0.8660251(1)	-0.866024(2)	-0.865545(5)	-0.859657(4)	-0.865925(3)	-0.865925(3)
	Std.	6.692e-07(2)	7.0436e-07(3)	4.0800e-04(4)	3.48e-02(5)	0.00e+00(1)	0.00e+00(1)
¹⁹ <i>g</i> _{CEC06}	Best	32.655592(1)	32.655593(2)	32.655592(1)	32.655593(2)	32.655593(2)	32.655593(2)
	Mean	32.655592(1)	32.757340(5)	36.427463(6)	32.660587(4)	32.658000(3)	32.655593(2)
	Std.	1.275e-08 (2)	6.145e-02(5)	1.0372e+00(6)	2.37e-02(4)	1.72e-03(3)	0.00e+00(1)
²¹ <i>g</i> _{CEC06}	Best	193.724510(1)	193.724510(1)	193.724510(1)	193.724510(1)	193.724510(1)	196.63301(2)
	Mean	193.724510(1)	193.771375(2)	246.091539(5)	198.090578(3)	193.724510(1)	199.51581(4)
	Std.	0.00e+00(1)	1.9643e-02(3)	1.4917e+01(5)	2.39e+01(6)	2.60e-04(2)	3.8656(4)
²³ <i>g</i> _{CEC06}	Best	-400.055100(1)	-396.165732(5)	-355.661000(6)	-400.055093(2)	-400.055052(3)	-399.7624(4)
	Mean	-398.156820(1)	-360.817656(5)	-194.760355(6)	-392.02961(3)	-391.41500(4)	-394.7627(2)
	Std.	1.05284e+00(1)	1.9623e+01(5)	5.3278e+01(6)	1.24e+01(4)	9.13e+00(3)	3.8656(2)
²⁴ <i>g</i> _{CEC06}	Best	-5.508013(1)	-5.508013(1)	-5.508013(1)	-5.508013(1)	-5.508013(1)	-5.508013(1)
	Mean	-5.508013(1)	-5.508013(1)	-5.508013(1)	-5.508013(1)	-5.508013(1)	-5.508013(1)
	Std.	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	2.71e-15(2)	3.12e-14(3)	0.00e+00(1)
Rank	Best	30	36	42	42	36	39
	Mean	29	49	71	66	45	44
	Std.	27	52	69	78	53	40

Table 4 Comparative results on CEC'2010 COPs

<i>fcn</i>	<i>Algorithm</i>	10-dimensional				30-dimensional			
		<i>Best</i>	<i>Mean</i>	<i>Std.</i>	<i>Std.</i>	<i>Best</i>	<i>Mean</i>	<i>Std.</i>	<i>Std.</i>
c_{CEC10}^{01}	mbDE	-7.473104e-01(1)	-7.470599e-01(1)	1.215024e-04(1)	1.215024e-04(1)	-8.218840e-01(1)	-8.209426E-01(1)	2.205013E-04(1)	2.205013E-04(1)
	BGRA	-7.473084e-01(3)	-7.470594e-01(2)	3.6753e-03(4)	3.6753e-03(4)	-8.217964e-01(4)	-7.878877e-01(5)	2.97753e-02(5)	2.97753e-02(5)
	SAMO-DE	-7.473104e-01(1)	-7.470402e-01(3)	1.350638e-03(3)	1.350638e-03(3)	-8.218838e-01(2)	-8.143674e-01(3)	4.766000e-03(2)	4.766000e-03(2)
	SAMO-GA	-7.473093e-01(2)	-7.470230e-01(4)	1.347657e-03(2)	1.347657e-03(2)	-8.217813e-01(5)	-8.115299e-01(4)	7.247940e-03(4)	7.247940e-03(4)
	IEMA	-7.4731e-01(1)	-7.431890e-01(5)	4.3301e-03(5)	4.3301e-03(5)	-8.21883e-01(3)	-8.17769e-01(2)	4.78853e-03(3)	4.78853e-03(3)
c_{CEC10}^{02}	mbDE	-2.277710e+00(1)	-2.277710e+00(1)	1.482014e-10(1)	1.482014e-10(1)	-2.280967e+00(1)	-2.277217e+00(1)	1.105830e-06(1)	1.105830e-06(1)
	BGRA	-2.277511e+00(3)	-2.272627e+00(3)	2.11504e-02(5)	2.11504e-02(5)	-2.169286e+00(5)	-2.146397e+00(4)	8.786171e-02(4)	8.786171e-02(4)
	SAMO-DE	-2.277709e+00(2)	-2.276842e+00(2)	1.154957e-03(3)	1.154957e-03(3)	-2.280962e+00(2)	-2.276111e+00(2)	3.706000e-03(2)	3.706000e-03(2)
	SAMO-GA	-2.277497e+00(4)	-2.272609e+00(4)	2.26780e-03(4)	2.26780e-03(4)	-2.265137e+00(4)	-2.252288e+00(3)	5.74730e-03(3)	5.74730e-03(3)
	IEMA	-2.7771e+00(1)	-2.7771e+00(1)	1.82278e-07(2)	1.82278e-07(2)	-2.28091e+00(3)	-1.50449e+00(5)	2.14056e+00(5)	2.14056e+00(5)
c_{CEC10}^{03}	mbDE	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	1.921468e-24(1)	1.528241e-22(1)	4.035847e-22(1)	4.035847e-22(1)
	BGRA	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	2.867279e+01(4)	2.867348e+01(4)	6.82e-05(4)	6.82e-05(4)
	SAMO-DE	0.00e+00(1)	4.1730e-23(2)	1.6405100e-22(2)	1.6405100e-22(2)	4.5996e-24(2)	4.8260e-22(2)	1.1460e-21(2)	1.1460e-21(2)
	SAMO-GA	6.492724e-22(2)	1.190663e-10(3)	2.0749471e-10(3)	2.0749471e-10(3)	5.481000e-19(3)	2.25500e-07(3)	8.15400e-07(3)	8.15400e-07(3)
	IEMA	1.46667e-16(3)	6.23456e-07(4)	1.402390e-06(4)	1.402390e-06(4)	UA(5)	UA(5)	UA(5)	UA(5)
c_{CEC10}^{04}	mbDE	-1.000000e-05(1)	-1.000000e-05(1)	0.00e+00(1)	0.00e+00(1)	-3.300462E-06(1)	-3.29196E-06(1)	2.023141E-10(1)	2.023141E-10(1)
	BGRA	3.301984e-04(3)	9.925622e-04(4)	3.83394e-04(5)	3.83394e-04(5)	5.051974E-05(4)	3.000871E-04(3)	5.891794E-03(3)	5.891794E-03(3)
	SAMO-DE	-1.000000e-05(1)	-1.000000e-05(1)	1.446083e-11(2)	1.446083e-11(2)	-3.2480E-06(2)	-2.41130E-06(2)	4.49234E-07(2)	4.49234E-07(2)
	SAMO-GA	-1.000000e-05(1)	-9.934308e-06(2)	5.1183910e-08(3)	5.1183910e-08(3)	-2.644300E-06(3)	1.470915E-03(4)	7.359431E-03(4)	7.359431E-03(4)
	IEMA	-9.9861e-06(2)	-9.91135e-06(3)	1.40239e-06(4)	1.40239e-06(4)	UA(5)	UA(5)	UA(5)	UA(5)

Table 4 Comparative results on CEC'2010 COPs (continued)

<i>fcn</i>	<i>Algorithm</i>	10-dimensional				30-dimensional			
		<i>Best</i>	<i>Mean</i>	<i>Std.</i>	<i>Best</i>	<i>Mean</i>	<i>Std.</i>	<i>Best</i>	<i>Std.</i>
c_{CEC10}^{05}	mbDE	-4.836110e+02 (1)	-4.836110e+02(1)	1.206035e-16(1)	-4.836106e+02(1)	-4.836106e+02(1)	1.102480e-10(1)		
	BGRA	-4.836106e+02(2)	-4.622261e+02(3)	1.297178e+01(3)	-4.665212e+02(3)	-4.622261e+02(3)	1.589176e+00(3)		
	SAMO-DE	-4.836106e+02(2)	-4.836106e+02(2)	4.14428e-06(2)	-4.836106e+02(1)	-4.836106e+02(1)	5.389910e-06(2)		
	SAMO-GA	-4.836106e+02 (2)	-4.016976e+02(4)	1.1126e+02(4)	-4.784754e+02(2)	-4.716434e+02(2)	3.783287e+00(4)		
	IEMA	-4.83611e+02(1)	-3.79156e+02(5)	1.79424e+02(5)	-2.86678e+02(4)	-2.7093e+02(4)	1.41169e+01(5)		
c_{CEC10}^{06}	mbDE	-5.786624e+02(2)	-5.786624e+02(2)	1.154502e-12(1)	-5.306372e+02(1)	-5.306370e+02(1)	1.20247e-05(1)		
	BGRA	-5.800043e+02(1)	-5.800043e+02(1)	3.86076e-09(2)	-5.244897e+02(5)	-5.222584e+02(3)	1.850379e+00(3)		
	SAMO-DE	-5.786624e+02(2)	-5.786562e+02(3)	9.35219e-03(3)	-5.306368e+02(2)	-5.306155e+02(2)	1.288050e-02(2)		
	SAMO-GA	-5.786619e+02 (4)	-5.777351e+02(4)	3.260495e+00(4)	-5.249595e+02(4)	-5.208121e+02(4)	3.169611e+00(4)		
	IEMA	-5.786620e+02(3)	-5.5147e+02(5)	7.35817e+01(5)	-5.29593e+02(3)	-1.32876e+02(5)	5.61042e+02(5)		
c_{CEC10}^{07}	mbDE	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)		
	BGRA	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	8.487189e-10(5)	2.481748e-09(5)	2.910792e-10(4)		
	SAMO-DE	0.00e+00(1)	7.762750e-23(2)	3.8808000e-22(3)	9.495250e-23(3)	1.782790e-13(3)	3.628040e-13(3)		
	SAMO-GA	0.00e+00(1)	7.856600e-23(3)	1.2081000e-22(2)	3.236800e-28(2)	3.236800e-28(2)	4.576600e-44(2)		
	IEMA	1.7473e-10(2)	3.25685e-09(4)	3.38717e-07(4)	4.81578e-10(4)	8.48609e-10(4)	4.84296e-10(5)		
c_{CEC10}^{08}	mbDE	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	3.456322e-28(1)	1.120451e-26(1)		
	BGRA	0.00e+00(1)	5.284814e+00(5)	3.547179e+00(4)	3.23e-08(5)	3.8574e-07(4)	1.8719e-05(4)		
	SAMO-DE	0.00e+00(1)	2.520090e-25(2)	1.2600470e-24(2)	6.425810e-21(3)	1.03293e-09(3)	2.373300e-09(3)		
	SAMO-GA	3.654300e-25(2)	3.648700e-23(3)	5.6932000e-23(3)	3.236790e-28(2)	1.59486e-24(2)	6.776570e-24(2)		
	IEMA	1.0075e-10(3)	4.0702e+00(4)	6.38287e+00(5)	1.12009e-09(4)	1.77033e+01(5)	4.08025e+01(5)		

Table 4 Comparative results on CEC'2010 COPs (continued)

<i>fcn</i>	<i>Algorithm</i>	10-dimensional				30-dimensional			
		<i>Best</i>	<i>Mean</i>	<i>Std.</i>	<i>Best</i>	<i>Mean</i>	<i>Std.</i>	<i>Best</i>	<i>Std.</i>
c_{CEC10}^{09}	mbDE	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)
	BGRA	2.7817e-20(3)	3.204581e+01(3)	5.785178e+01(3)	1.778130e+02(3)	4.10102e+02(3)	2.57192e+02(3)	1.432599e+01(2)	1.808062e+04(4)
	SAMO-DE	0.00e+00(1)	5.089752e+00(2)	2.4108277e+01(2)	1.186170e-20(2)	6.079667e+00(2)	1.432599e+01(2)	1.808062e+04(4)	4.50013e+07(5)
	SAMO-GA	1.898000e-22(2)	2.731462e+03(4)	4.62970e+03(4)	3.536309e+02(4)	1.525893e+04(4)	1.808062e+04(4)	1.808062e+04(4)	1.808062e+04(4)
	IEMA	1.20218e-09(4)	1.95109e+12(5)	5.40139e+12(5)	7.31423e+03(5)	2.98793e+07(5)	4.50013e+07(5)	4.50013e+07(5)	4.50013e+07(5)
c_{CEC10}^{10}	mbDE	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)
	BGRA	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	5.17e-08(3)	9.086640e+01(3)	1.19028e+02(3)	1.19028e+02(3)	1.19028e+02(3)
	SAMO-DE	0.00e+00(1)	4.467656E-01(2)	1.5462980E+00(2)	9.769000e-21(2)	1.960780e+01(2)	2.123749e+01(2)	2.123749e+01(2)	2.123749e+01(2)
	SAMO-GA	1.124000e-19(2)	1.732124E+02(3)	2.6369383E+02(3)	4.164404e+01(4)	6.031654e+03(4)	7.177385e+03(4)	7.177385e+03(4)	7.177385e+03(4)
	IEMA	5.401200e+09(3)	2.5613E+12(4)	3.9679E+12(4)	2.7682e+04(5)	1.58342e+07(5)	1.683630e+07(5)	1.683630e+07(5)	1.683630e+07(5)
c_{CEC10}^{11}	mbDE	-6.20500e-04(1)	-6.20500e-04(1)	1.2508278e-16(1)	-3.923439e-04(1)	-3.923432e-04(1)	2.164628e-12(1)	2.164628e-12(1)	2.164628e-12(1)
	BGRA	-1.522713e-03(2)	-1.522759e-03(3)	5.717899e-08(4)	4.8162e-06(4)	4.89179e-04(4)	3.32641e-03(4)	3.32641e-03(4)	3.32641e-03(4)
	SAMO-DE	-1.52271e-03(3)	-1.52271e-03(4)	3.6676100e-09(2)	-3.9230e-04(2)	-3.8690e-04(2)	6.14966e-06(3)	6.14966e-06(3)	6.14966e-06(3)
	SAMO-GA	-6.20500e-04(1)	-5.262000e-04(2)	4.9043000e-05(5)	-1.360000e-04(3)	-1.260000e-04(3)	4.887000e-06(2)	4.887000e-06(2)	4.887000e-06(2)
	IEMA	-1.52271e-03(3)	-1.52271e-03(4)	2.73127e-08(3)	UA(5)	UA(5)	UA(5)	UA(5)	UA(5)
c_{CEC10}^{12}	mbDE	-5.700899e+02(2)	-5.695360e+02(1)	1.0300853e+00(1)	-1.992633e-01(3)	-1.992633e-01(2)	1.104530e-10(1)	1.104530e-10(1)	1.104530e-10(1)
	BGRA	-5.701667e+02(1)	-1.218177e+02(2)	1.5347076e+02(4)	-2.589913e+00(2)	-4.897145e-02(4)	2.981746e+00(3)	2.981746e+00(3)	2.981746e+00(3)
	SAMO-DE	-5.700899e+02(2)	-1.166134e+02(3)	1.8300053e+02(5)	-1.992598e-01(4)	-1.992573e-01(3)	1.321890e-06(2)	1.321890e-06(2)	1.321890e-06(2)
	SAMO-GA	-5.697441e+02(3)	-5.588351e+01(4)	1.3518000e+02(3)	-8.535971e+01(1)	-3.493162e+00(1)	1.705561e+01(4)	1.705561e+01(4)	1.705561e+01(4)
	IEMA	-1.09735e+01(4)	-6.48172e-01(5)	2.1992800e+00(2)	UA(5)	UA(5)	UA(5)	UA(5)	UA(5)

Table 4 Comparative results on CEC'2010 COPs (continued)

<i>fcn</i>	<i>Algorithm</i>	10-dimensional				30-dimensional			
		<i>Best</i>	<i>Mean</i>	<i>Std.</i>	<i>Best</i>	<i>Mean</i>	<i>Std.</i>	<i>Best</i>	<i>Std.</i>
c_{CEC10}^{13}	mbDE	-6.842946e+01(1)	-6.842937e+01(1)	1.150708E-08(1)	-6.842946E+01(1)	-6.842937e+01(1)	1.201842e-04(1)		
	BGRA	-6.842922e+01(4)	-6.832052e+01(3)	2.186620e-01(4)	-6.642832e+01(3)	-6.432031e+01(4)	6.005242e-01(3)		
	SAMO-DE	-6.842937e+01(3)	-6.842937e+01(1)	1.5428774e-07(2)	-6.84294e+01(2)	-6.819178e+01(2)	3.891641e-01(2)		
	SAMO-GA	-6.842803e+01(5)	-6.837658e+01(2)	1.558530e-01(3)	-6.637738e+01(4)	-6.309871e+01(5)	1.286011e+00(5)		
	IEMA	-6.842940e+01(2)	-6.80182e+01(4)	1.40069e+00(5)	-6.84294e+01(2)	-6.74872e+01(3)	9.83662e-01(4)		
c_{CEC10}^{14}	mbDE	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	1.284614e-20(1)	2.640823e-12(1)		
	BGRA	2.9718e-21(3)	5.81278e-18(3)	4.217e-04(3)	3.481987e-08(4)	7.591436e-03(3)	6.955267e-02(3)		
	SAMO-DE	0.00e+00(1)	1.206380e-21(2)	2.4359280e-21(2)	1.7470e-22(2)	1.196910e-08(2)	2.569420e-08(3)		
	SAMO-GA	1.87600e-22 (2)	3.872582e+02(5)	1.2448175e+03(5)	4.145243e+00(5)	1.762866e+03(5)	5.465005e+03(5)		
	IEMA	8.0306e-10(4)	5.630810e+01(4)	1.82866e+02(4)	3.28834e-09(3)	6.15242e-02(4)	3.07356e-01(4)		
c_{CEC10}^{15}	mbDE	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	2.635720e-02(1)	1.012540e-06(1)		
	BGRA	3.74496e-17(3)	1.281766e+01(3)	5.364956e+02(2)	2.160342e+01(4)	2.160392e+01(3)	1.034748e-02(2)		
	SAMO-DE	0.00e+00(1)	7.053800e-04(2)	2.4413847e-03(4)	5.8424e-18(2)	2.112810e+00(2)	4.51067e+00(3)		
	SAMO-GA	2.825000e-20(2)	8.574500e+02(5)	1.7926000e+03(3)	8.384400e-01(3)	1.749600e+04(4)	3.411400e+04(4)		
	IEMA	9.354050e-10(4)	2.61715e+01(4)	1.57531e+08(5)	3.11876e+04(5)	2.29491e+08(5)	4.64046e+08(5)		
c_{CEC10}^{16}	mbDE	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)		
	BGRA	0.00e+00(1)	9.432e-04(2)	4.7392e-03(2)	0.00e+00(1)	0.871962e-15(2)	2.674817e-13(2)		
	SAMO-DE	0.00e+00(1)	6.469600e-03(4)	1.0870417e-02(4)	0.00e+00(1)	4.160700e-03(5)	7.677900e-03(4)		
	SAMO-GA	0.00e+00(1)	1.403400e-03(3)	4.88000e-03(3)	0.00e+00(1)	3.737000e-03(4)	6.28400e-03(3)		
	IEMA	4.44089e-16(2)	3.30299e-02(5)	2.2601300e-02(5)	6.15674e-12(2)	1.63294e-03(3)	8.1647e-03(5)		

Table 4 Comparative results on CEC'2010 COPs (continued)

<i>fcn</i>	<i>Algorithm</i>	10-dimensional			30-dimensional		
		<i>Best</i>	<i>Mean</i>	<i>Std.</i>	<i>Best</i>	<i>Mean</i>	<i>Std.</i>
c_{CEC10}^{17}	^{mb} DE	0.00e+00(1)	4.101621e-35(1)	4.851072e-35(1)	0.00e+00(1)	1.301625e-12(1)	1.060792e-12(1)
	BGRA	2.197856e-16(2)	4.398716e+01(5)	5.41752e-01(5)	1.075811e-03(3)	7.204028e-03(3)	3.43502e-03(3)
	SAMO-DE	0.00e+00(1)	1.265501e-23(2)	3.2179912e-23(2)	0.00e+00(1)	1.022630e-10(2)	1.455150e-10(2)
	SAMO-GA	0.00e+00(1)	1.270700e-02(4)	1.331029e-02(3)	0.00e+00(1)	1.343600e-02(4)	1.616200e-02(4)
	IEMA	9.47971e-15(3)	3.15093e-03(3)	1.57547e-02(4)	9.27664E-10(2)	8.83974e-02(5)	1.5109e-01(5)
c_{CEC10}^{18}	^{mb} DE	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	0.00e+00(1)	1.227620e-18(1)	1.569153e-18(1)
	BGRA	7.022090e-18(3)	3.890275e-17(3)	3.67488e-17(3)	3.250043e-04(4)	1.810866e-02(4)	1.208486e-01(4)
	SAMO-DE	1.173999e-23(2)	4.447988e-19(2)	6.6367050e-19(2)	9.192800e-18(2)	2.5681e-09(3)	6.9848e-09(3)
	SAMO-GA	4.359000e-17(4)	1.0530950e-02(5)	1.540100e-02(5)	2.818290e-01(5)	7.5357280e+00(5)	1.0517030e+01(5)
	IEMA	2.23664e-15(5)	1.617890e-14(4)	3.820340e-14(4)	1.375370e-14(3)	4.738410e-14(2)	6.57350e-14(2)
Rank	^{mb} DE	20	19	18	20	19	18
	BGRA	40	48	56	66	64	60
	SAMO-DE	27	41	47	37	43	44
	SAMO-GA	41	64	61	56	63	66
	IEMA	50	73	75	68	77	83

The comparative experimental results, i.e., best, mean, standard deviation (std.) of the objective function values are presented in Tables 3 and 4 for 22 CEC'2006 and 18 CEC'2010 respectively. From Tables 3 and 4, ^{mb}DE displays competitive or likewise results w.r.t. considered algorithms. Moreover, all algorithms are individually ranked (as '1' for the best and '2' for subsequent performer and so on) on best, mean and std. result values. It will provide in small bracket against each result in Tables 3 and 4. Also, average rank of algorithms are listed as follows: (i) on CEC'2006 COPs: ^{mb}DE (**1.24**), SAMO-GA (2.69), SAMO-DE (2.01), DECV (2.74), APF-GA (1.84) and A-DDE (2.03); (ii) on 10-dimensional CEC'2010 COPs: ^{mb}DE (**1.11**), BGRA (2.17), SAMO-GA (2.25), SAMO-DE (1.50) and IEMA (2.87); (iii) on 30-dimensional CEC'2010 COPs: ^{mb}DE (**1.05**), BGRA (3.51), SAMO-GA (3.42), SAMO-DE (2.29) and IEMA (4.22). From ranking it can be declares that the enactment of ^{mb}DE is greater than others. Eventually, ^{mb}DE produced less std. (it may 0.00E+00) for most of the both cases which designates its stability. On the whole, ^{mb}DE is produces either superior or similar results with its competitors.

5.2 Graphical and statistical analysis

Here through box plot, average FEs for all procedures under consideration for CEC'2006 and CEC'2010 are compared in Figure 3(a)–(c) respectively. From these figures it is very clear that ^{mb}DE uses less FEs as equated algorithms, which delivers a good strength to ^{mb}DE .

Figure 3 (a) Average number of FEs of ^{mb}DE with others for CEC'2006 COPs. (b) Average number of FEs of ^{mb}DE with others for 10-dimensional CEC'2010 COPs. (c) Average number of FEs of ^{mb}DE with others for 30-dimensional CEC'2010 COPs

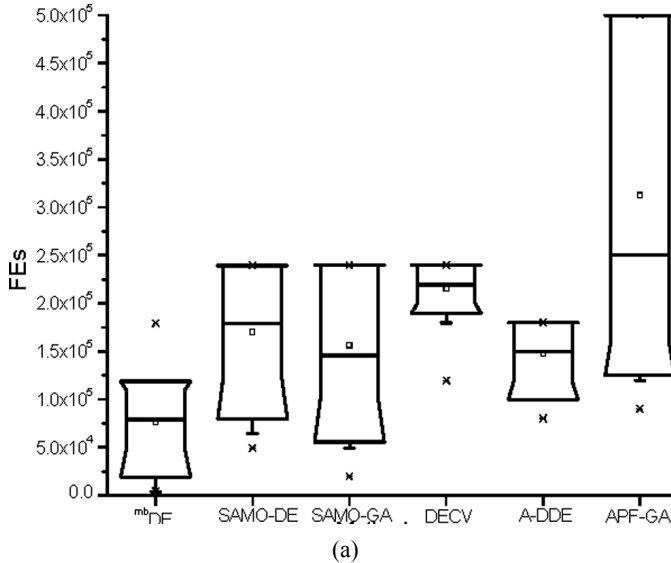


Figure 3 (a) Average number of FEs of ^{mb}DE with others for CEC'2006 COPs. (b) Average number of FEs of ^{mb}DE with others for 10-dimensional CEC'2010 COPs. (c) Average number of FEs of ^{mb}DE with others for 30-dimensional CEC'2010 COPs (continued)

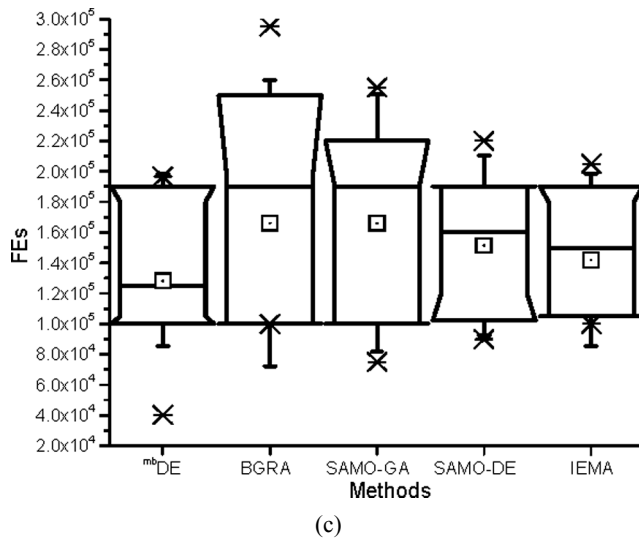
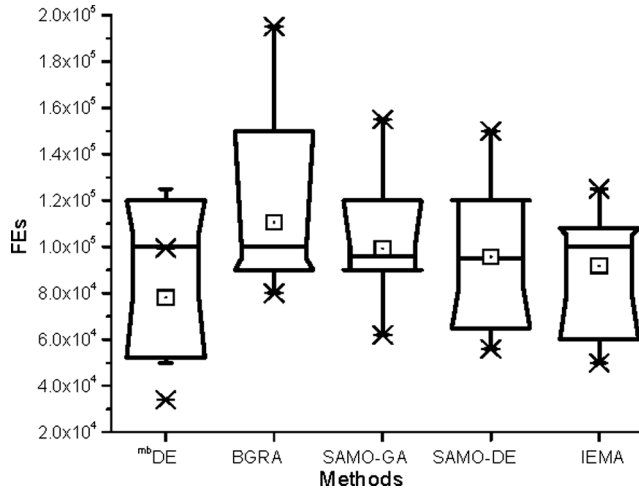


Table 5 Statistical assessment summary of ^{mb}DE along others

Comparison	Criteria	Better	Equal	Worse	WSR test	t-test
<i>For CEC'2006 COPs</i>						
^{mb} DE Vs. SAMO-DE	Best	7	11	4	≈	a
	Mean	12	8	2	≈	a
	Std.	11	11	0	≈	a
^{mb} DE Vs. SAMO-GA	Best	9	10	3	≈	a
	Mean	15	6	1	≈	a
	Std.	14	8	0	+	a+

Table 5 Statistical assessment summary of ^{mb}DE along others (continued)

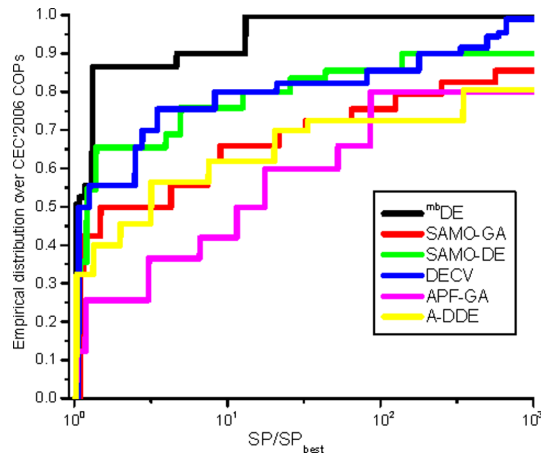
<i>Comparison</i>	<i>Criteria</i>	<i>Better</i>	<i>Equal</i>	<i>Worse</i>	<i>WSR test</i>	<i>t-test</i>
<i>For CEC'2006 COPs</i>						
^{mb} DE Vs. DECV	Best	8	10	4	≈	a
	Mean	14	6	2	≈	a
	Std.	19	3	0	+	a+
^{mb} DE Vs. A-DDE	Best	8	9	5	≈	a
	Mean	10	7	5	≈	a
	Std.	18	2	2	+	a+
^{mb} DE Vs. APF-GA	Best	10	8	4	≈	a
	Mean	11	7	4	≈	a
	Std.	7	11	4	≈	a
<i>For CEC'2010 10-dimensional COPs</i>						
^{mb} DE Vs BGRA	Best	11	5	2	≈	a
	Mean	14	3	1	≈	a
	Std.	15	3	0	+	a+
^{mb} DE Vs. SAMO-DE	Best	5	13	0	≈	a
	Mean	16	2	0	≈	a
	Std.	18	0	0	+	a+
^{mb} DE Vs. SAMO-GA	Best	13	5	0	≈	a
	Mean	18	0	0	+	a+
	Std.	18	0	0	+	a+
^{mb} DE Vs. IEMA	Best	15	3	0	≈	a
	Mean	17	1	0	≈	a
	Std.	18	0	0	+	a+
<i>For CEC'2010 30-dimensional COPs</i>						
^{mb} DE Vs BGRA	Best	16	1	1	≈	a
	Mean	18	0	0	+	a+
	Std.	18	0	0	+	a+
^{mb} DE Vs. SAMO-DE	Best	15	3	0	≈	a
	Mean	17	1	0	≈	a
	Std.	18	0	0	+	a+
^{mb} DE Vs. SAMO-GA	Best	15	2	1	≈	a
	Mean	17	0	1	≈	a
	Std.	18	0	0	+	a+
^{mb} DE Vs. IEMA	Best	14	0	0	+	a+
	Mean	14	0	0	+	a+
	Std.	14	0	0	+	a+

Furthermore, supremacy of ^{mb}DE is confirmed statistically over other algorithms through

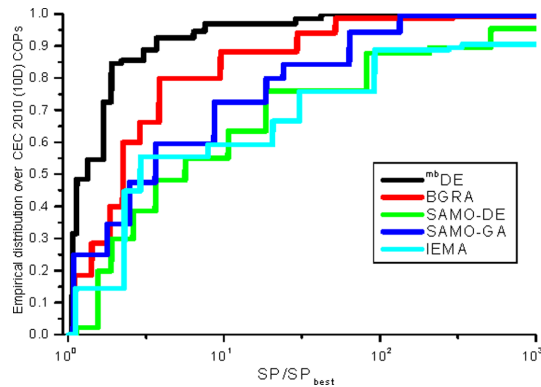
- i one-tailed t -test (with 98 degree of freedom (df) at a 5% significance level)
- ii Wilcoxon Signed Rank (WSR) test (5% significance level)
- iii ECD (empirical cumulative distribution) test
- iv performance analysis test

The details of these test can be found in (Qin et al., 2009; Sharma et al., 2012; Das and Parouha, 2015). The results of t -test and WSR test on CEC'2006 and CEC'2010 (10D and 30D) are testified in Table 5 last column. Table 5 witnessed that ^{md}DE has either 'a' or 'a+' sign (in case of t -test) and better or performs equally (in case WSR test) in most of consequence. ECD (SP/SP_{best} vs. empirical distribution) and performance analysis (pi-charts) test results on CEC'2006, 10-dimensional CEC'2010 and 30-dimensional CEC'2010 are reported in Figures 4(a)–(c) and 5(a)–(c) respectively. All these test figures favours high performance of ^{mb}DE over rest algorithms.

Figure 4 (a) ECD performance on CEC'2006 COPs. (b) ECD performance on 10-dimensional CEC'2010 COPs. (c) ECD performance on 30-dimensional CEC'2010 COPs (see online version for colours)



(a)



(b)

Figure 4 (a) ECD performance on CEC'2006 COPs. (b) ECD performance on 10-dimensional CEC'2010 COPs. (c) ECD performance on 30-dimensional CEC'2010 COPs (see online version for colours) (continued)

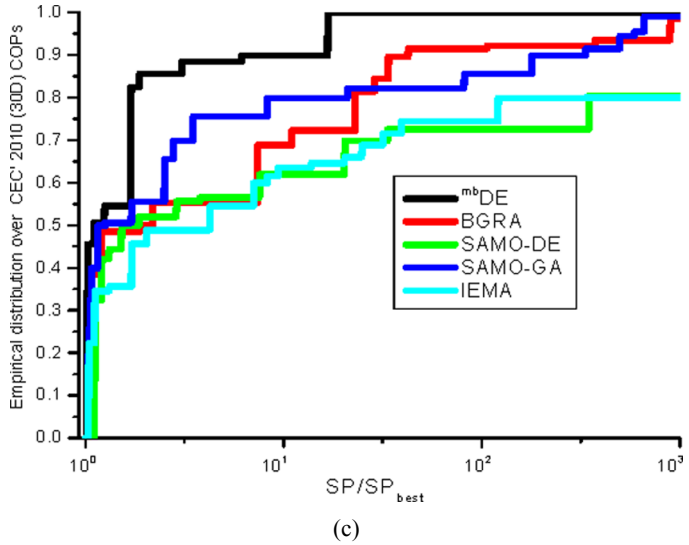
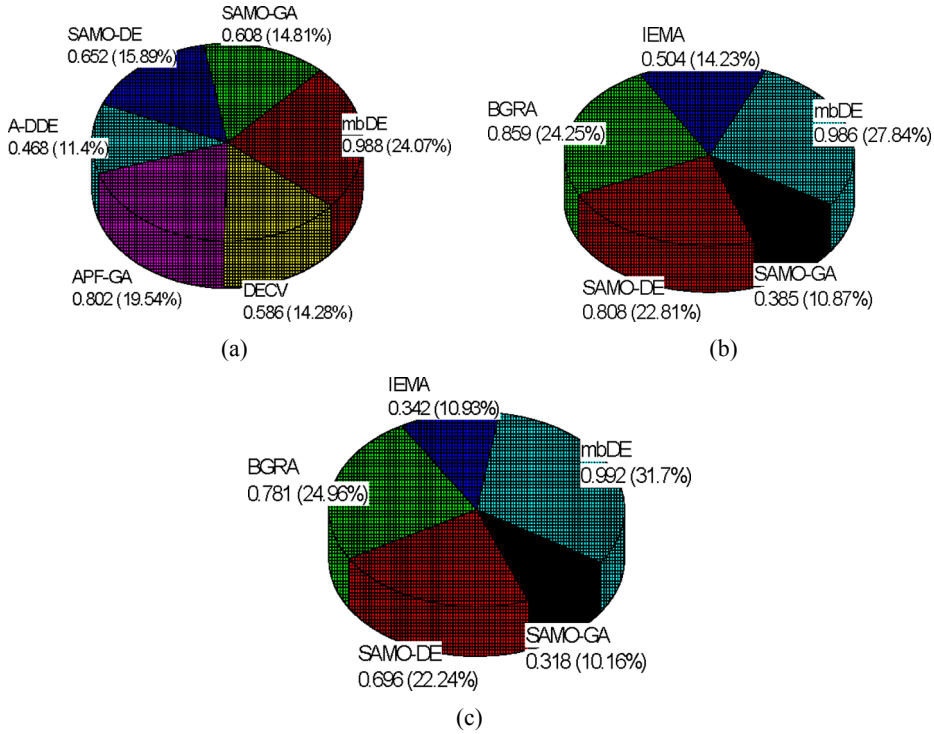


Figure 5 (a) Performance on 22 CEC'2006 COPs. (b) Performance on 10-D CEC'2010 COPs. (c) Performance on 30-D CEC'2010 COPs (see online version for colours)



5.3 Complexity analysis

According to the guidelines of CEC test suite the algorithm complexity has been investigated on 10 and 30-dimensional CEC'2010 COPs. Firstly, T_0 time (in seconds) executed through subsequent program.

```

for i= 1:1000000 do
     $x = 0.55 + (\text{double})i;$ 
     $x = x + x; x = \frac{x}{2}; x = x * x; x = \sqrt{x}, x = \log(x);$ 
     $x = e^x; x = \frac{x}{x+2};$ 
end for

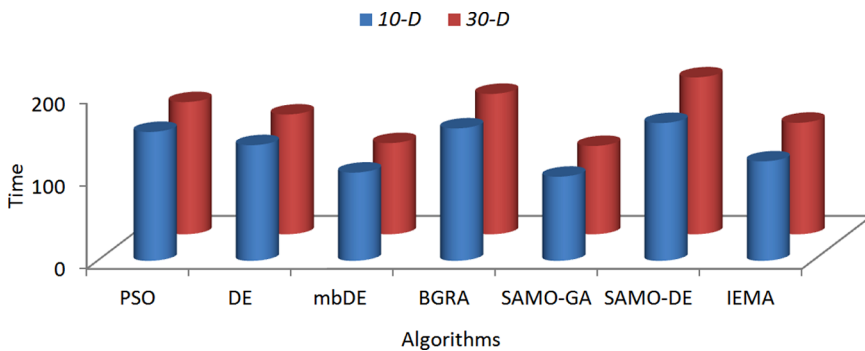
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After this computing (i) T_1 time for C16 (which include two separable equality as well as 1 separable and 1 non-separable inequality constraints) on 200,000 FEs and (ii) T_2 time (evaluated five times) for algorithms to completely execute 200,000 FEs for C16 function and store its mean value denoted as $\hat{T}_2$. Thereafter calculated $(\hat{T}_2 - T_1)/T_0$ for each algorithm and reported in Table 6. Also, complexity chart of ^{mb}DE with others are presented in Figure 6 on 10 and 30-dimensional CEC'2010 COPs. It can be observed from Table 6 and Figure 6 the time complexity of proposed ^{mb}DE is significantly less traditional DE and PSO as well as other comparative algorithms.

Table 6 Complexity results of different algorithms

Algorithm	10-dimension	30-dimension
T_0	0.3213	0.3213
T_1	0.5975	0.7854
PSO	156.23	160.25
DE	140.23	145.23
^{mb} DE	106.94	110.58
BGRA	160.52	170.25
SAMO-GA	101.94	107.07
SAMO-DE	167.01	190.25
IEMA	120.53	135.23

Figure 6 Algorithm complexity on 10 and 30-dimensional CEC'2010 COPs (see online version for colours)



6 Conclusion and future works

In this paper, a new a new DE (namely ^{mb}DE) based on memory mechanism is proposed for COPs. It memorises the finest effects attained in early part of the preceding peers through newly employed mutation and crossover (so-called as swarm mutation and swarm crossover) operators. Because the usages of the p^{best} and g^{best} concept of PSO circumstance. The performances ^{mb}DE has been tested over 22 CEC'2006 and 18 CEC'2010 (10 and 30-dimensional) COPs.

The experimental results, statistical and complexity analysis determines that ^{mb}DE carries the ultimate results and exceeds many contemporary processes in terms of convergence rate and solution excellence. The usages of the memory mechanism shape the proposed ^{mb}DE to become more effective and robust.

In future research, the proposed ^{mb}DE can be useful for solving complex un-constrained benchmark as well as real life problems. Instead of this it may applied for combinatorial optimisation problems with investigation of refined operators.

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