



European J. of Industrial Engineering

ISSN online: 1751-5262 - ISSN print: 1751-5254

<https://www.inderscience.com/ejie>

Optimising logistics packaging and replenishing policies for deteriorating items in e-commerce environments

Tien-Yu Lin

DOI: [10.1504/EJIE.2025.10067748](https://doi.org/10.1504/EJIE.2025.10067748)

Article History:

Received:	06 June 2023
Last revised:	27 October 2023
Accepted:	15 February 2024
Published online:	25 August 2025

Optimising logistics packaging and replenishing policies for deteriorating items in e-commerce environments

Tien-Yu Lin

Qiaoxing College of Economics and Management,
Fujian Polytechnic Normal University,
Fuzhou, Fujian, China
Email: 3146375768@qq.com

Abstract: This paper develops an instantaneously deteriorating inventory model for an e-commerce environment wherein the retailer employs an improved logistics packaging policy to reduce the rate of damage to goods in transit. Because of the difficulty in finding a closed-form solution, an efficient algorithm has been developed to find the optimal solution for the proposed model. The theoretical results show that the deteriorating rate could impact the optimal solution and the average total profit. The replenishment time for the proposed model with return rate is shorter than that for the traditional EOQ model. Numerical examples illustrate the proposed model and algorithm. Moreover, sensitivity analysis illustrates the effects of four parameters of importance (i.e., set-up cost, demand rate, holding cost, and deteriorating rate) on the optimal strategy. The discussion of numerical and sensitivity analysis results provide some managerial insights. [Submitted: 6 June 2023; Accepted: 15 February 2024]

Keywords: inventory; logistics packing policy; deterioration; e-commerce; replenishing policy.

Reference to this paper should be made as follows: Lin, T-Y. (2025) 'Optimising logistics packaging and replenishing policies for deteriorating items in e-commerce environments', *European J. Industrial Engineering*, Vol. 20, No. 3, pp.317–334.

Biographical notes: Tien-Yu Lin is a Professor at Qiaoxing College of Economics and Management, Fujian Polytechnic Normal University, located in Fujian, China. He earned his PhD in Business Administration from National Chung Cheng University in Min-Xiong, Taiwan (China). His research has been published in various journals, including the *Journal of the Operational Research Society*, *Applied Mathematical Modelling*, *Computers & Industrial Engineering*, *International Journal of Systems Science*, *TOP*, and *Journal of Industrial and Management Optimization*. His current research interests focus on supply chain management, inventory control, decision analysis, and consumer behaviour.

1 Introduction

Traditional inventory models assume that inventory decreases solely due to supply, demand, and sales. However, in fact, most inventories decrease in value and quantity

over time owing to deterioration, volatility, and spoilage, among other factors. The best examples are fresh fruits and vegetables, meat, and gasoline. This type of inventory is collectively referred to as ‘deteriorating inventory’. Because deteriorating inventories exist in daily life and incur additional costs, failure to include deteriorating factors in inventory models’ exploration renders the inventory model structure incomplete besides affecting decision-making, which means that a deteriorating item impacts the inventory level (Maihami et al., 2019; Pando et al., 2018; Jaber et al., 2022). The recent boom in fresh supermarkets, semiconductors, and high-tech products has made managing deteriorating inventory even more crucial.

Ghare and Schrader (1963) were pioneers focused on the deterioration issue; they discussed the inventory model by assuming that an item’s deteriorating rate is subject to an exponential distribution. They also developed an economic order quantity (EOQ) model with a fixed constant value for the deteriorating rate. Many researchers have since focused on this topic. Research on deteriorating commodities is divided into two main categories: instantaneous deterioration and non-instantaneous deterioration. For instantaneous deterioration items, researchers believe that deterioration occurs immediately after an item is purchased. For example, Covert and Philip (1973) studied an EOQ inventory model for deteriorating items with a two-parameter Weibull distribution. Dave and Patel (1981) developed a (T, S_i) policy inventory model for exponentially deteriorating items that considered proportional demand and no shortages. Meanwhile, Hariga (1996) developed optimal EOQ models for deteriorating items with time-varying demand, in which the deterioration rate is treated as a constant fraction of the on-hand inventory. Several survey papers, including Teng et al. (2002), Bakker et al. (2012) and Teng et al. (2016), have summarised additional research in this field. Moreover, Goyal and Giri (2001) conducted a comprehensive literature review on deteriorating inventory. Recently, interesting research efforts for deteriorating items have been in extending this topic to supply chain systems, in which some researchers (Tiwari et al., 2018; Lowalekar and Basu, 2020; Zhang et al., 2022) considered an integrated two- or three-level supply chain for a deteriorating product. Specifically, Daryanto et al. (2019) incorporated carbon emissions and item deterioration into the supply chain model to optimise delivery schedules from the supplier to the third-party logistics service provider and then to the buyer. Although this study considered the shipment schedule, it did not address the logistical packaging issue. Other recent works of interesting extensions to the instantaneous deterioration items are those of Das et al. (2022), Jain and Singh (2022), Tiwari et al. (2022) and Sebatjane (2022).

Researchers believe that non-instantaneous deterioration items would maintain quality or the original condition for some time (Maihami and Abadi, 2012; Li et al., 2019). Various inventory models for non-deteriorating items have been studied recently. For example, some researchers (Tiwari et al., 2018; Mashud et al., 2021; Almathkour and Benkherouf, 2023) have incorporated the concept of back-ordering, which refers to a delay in the delivery of goods to consumers, into non-instantaneous deterioration inventory models. Meanwhile, others (Soni, 2013; Tiwari et al., 2019; Tripathy et al., 2022) incorporated a vendor’s permissible delay in payment, which allows the buyer’s purchase cost to be paid at a later date without incurring interest charges, into non-instantaneous deterioration inventory models. Some researchers (Khakzad and Gholamian, 2020; Liao et al., 2020; Tavassoli et al., 2022) extended the non-instantaneous deterioration inventory models to the field of advance payment, in which a buyer pays the entire or partial purchase price before receiving the order. Other

relevant papers on this topic are those of Mishra et al. (2020), Maihami et al. (2021) and Meena et al. (2022), among others. No studies have discussed the packaging logistics issues for non-instantaneous deterioration items.

It is clear from the preceding discussion that although the extended models of deteriorating and non-deteriorating items have been studied in various aspects, no previous article has studied the logistics packaging strategy in conjunction with the above models, especially important in the development of e-commerce today. Evidence suggests that enhanced logistics packaging reduces damage to goods during delivery, thereby effectively lowering the return rate and compensation for damage to goods in the e-commerce business. The e-tailing of 3C (i.e., computer, communication, and consumer electronics) commodities in China is an excellent example of a deteriorating item inventory model.

Because of the spread of COVID-19, e-commerce has altered customers' traditional purchasing behaviour and significantly impacted human economic life. This phenomenon has led to retailers increasingly selling goods to customers online via the internet. In general, e-tailers entrust third-party logistics with delivering customers' orders. However, to complete their deliveries as quickly as possible, employees of third-party logistics handle customers' deliveries arbitrarily, resulting in the return of goods and compensation for damage caused during the transportation process. This caused the retailer to bear additional costs and damaged the retailer's goodwill. According to a report released by the National Retail Federation and Appriss Retail Retailers (National Retail Federation, 2022), goods returned by consumers are expected to run up to \$761 billion in merchandise sold last year. This accounts for an average of 16.6% of total US retail sales, which soared to \$4.583 trillion in 2021. E-tailers are actively looking for viable solutions to avoid the huge losses caused by the high return rate.

Strengthening packaging strategies, such as using new packaging materials, packaging technology, and packaging methods, is one of the most important ways of avoiding damage caused by logistics transportation. Most previous researchers on packaging issues focused on the physical or marketing properties of packaging materials. According to some researchers (Singh, 1992; Jarimopas et al., 2005; Idah et al., 2012; Sebatjane, 2022), enhancing the packaging material and technique can minimise mechanical damage to packaged products and improve the quality of fresh fruits/vegetables. Chonhenchob et al. (2009) compared the quality of fresh fruits/vegetables after being shipped in trucking transport by quantifying the level of bruises and cuts on fruits/vegetables, concluding that proper packaging reduced damage during truck transportation of fresh produce. Some researchers have shown package design to have a significant effect on the cooling performance of ventilated package designs used in fresh fruit handling, including energy use and efficiency (Zou et al., 2006; Delele et al., 2013; Defraeye et al., 2014; Han et al., 2015). Recently, Vladić et al. (2015) explored the impact of packaging shape design on consumer perception. Their research on consumers of fast-moving consumer goods provided clear insight into the impact of packaging shape on the perception of packaging characteristics. Meanwhile, Fadji et al. (2016) simulated shipping damage on ventilated cardboard packages and assessed the sensitivity of apples inside the package to vibration bruising damage. They found improper packaging to result in fruit bruise damage and the type of packaging used during fruit handling to affect the physical quality of the fruit. Resimović et al. (2022) demonstrated that the material influences the perception of the product's naturalness and

environmental friendliness; they examined consumers' ecological awareness during shopping activities based on consumers' choice of product packaging. Although packaging policy importance evoked mention in these studies, no article focused on returned items resulting from poor packaging logistics, which is critical in e-commerce.

Integrating the preceding discussions, this paper, therefore, focused on the optimal replenishing and logistics packaging policies of instantaneously deteriorating items in which retailers increase additional costs to improve logistics packaging techniques or materials and reduce the rate of damage to goods in transit. The contributions of this study are threefold. First, to the best of our knowledge, this study is the first to incorporate replenishing and logistics packaging policies into the deteriorating inventory issue. Second, it inspires a new and vital focus on logistics packaging policies, which can significantly reduce the return rate resulting from improper packaging materials and techniques in the e-commerce business. Third, the theoretical and sensitivity analysis validates the conditions for achieving the lowest cost at various combinations of cycle replenishing and packaging policies, which is useful for retailers in the e-commerce space.

The remainder of the paper is structured as follows. Section 2 illustrates all the notations and assumptions covered in this paper. Section 3 summarises the problem and constructs the mathematical model. Section 4 explores and derives both theoretical results and optimal solutions. Section 5 provides numerical examples for the proposed model and presents the sensitivity analysis to gain managerial insights. Finally, Section 6 concludes the paper.

2 Notations and assumptions

To develop the mathematical model for optimal replenishing and logistics packaging strategies in e-commerce business, we introduce the following necessary notations and assumptions.

2.1 Notations

2.1.1 Parameters

- A The set up cost per order.
- L The order lot size.
- h The unit holding cost per cycle.
- h_d The unit holding cost for defective per cycle.
- P_g The unit price of perfect items sold.
- P_d The unit price of defective items sold.
- x The returned rate representing that customers return defective goods, which have been damaged in transit after employing enhanced packaging material.
- π The returned rate representing that customers return defective goods, which have been damaged in transit before employing enhanced packaging material.

- θ The instantaneous deteriorating rate.
- Ω The demand rate.
- i_0 The initial unit packing cost.
- u The unit-purchasing cost.
- f The cost per unit of returned goods (including handling, shipping, testing, etc.).

2.1.2 Decision variables

- T The replenishment time per cycle.
- i The new unit-packing cost in which the retailer enhancing the logistics packaging strategy (including different materials, techniques, or methods), $i > i_0$.

2.2 Assumptions

The following assumptions are needed to simplify and develop the mathematical model:

- 1 The defect items received by the customer are mainly caused by the delivery process.
- 2 The retailer stores the customer returns in another warehouse and sells these returns in whole lots on the secondary market at a low price. Therefore, we assume the deterioration characteristics can be ignored after the product is sold.
- 3 The e-tailer improves their packaging policy (including new materials, techniques, or methods) to reduce product damage during the delivery process, which means that the new unit-packaging cost (i) is greater than the original one (i_0). The relationship between product damage returned rate and a new packaging material investment for each cycle time can be treated as an exponential investment cost function and can be expressed as $x = \pi e^{\frac{1-i}{i_0}}$, for $i_0 \leq i$ and π is the original returned rate caused by the unenhanced packing behaviour (i.e., $x \leq \pi$).
- 4 The demand rate is known, and no shortages occur.

3 The model

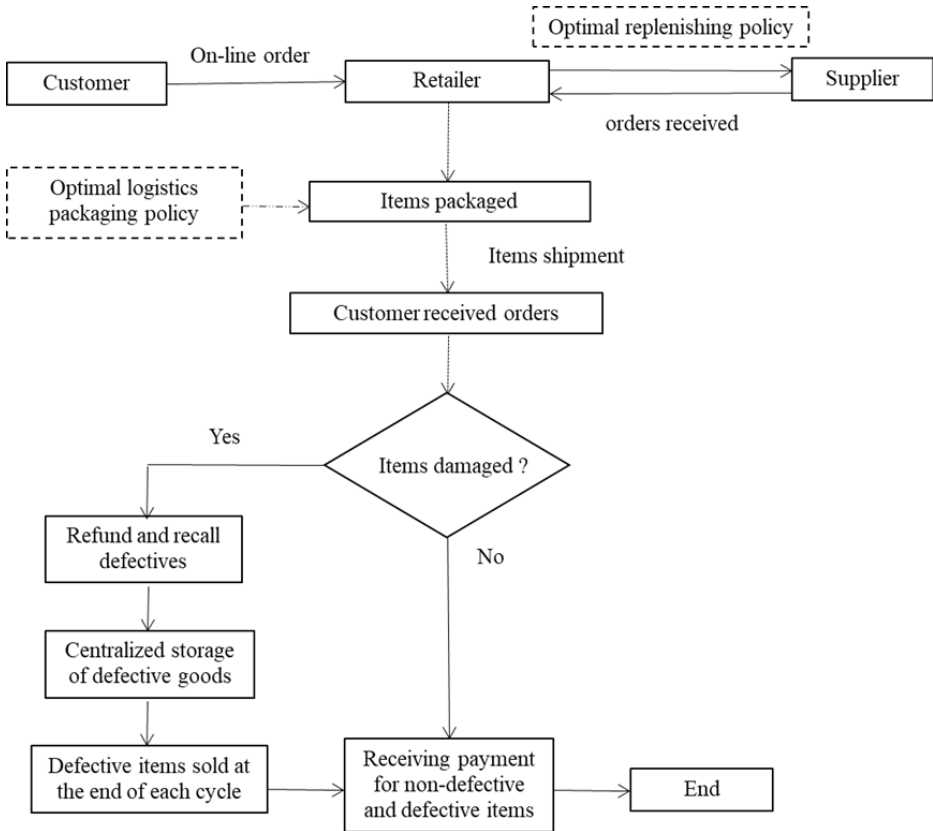
3.1 Problem description

Faced with a competitive business environment, e-tailers are attempting to optimise their logistics processes to reduce logistics risks and costs. A major factor affecting the modern logistics industry is logistics packaging technology. In the transport packaging design of commodities, stakeholders must consider not only the safety of commodities during transport but also the efficiency of transport loading and unloading operations. With the rapid development of e-commerce, online shopping has grown and become a new trend in people's lives. Commodities in the e-commerce business are often entrusted to third-party logistics for transportation and distribution. Employees of third-party logistics are rough and casual in handling goods to expedite the operation, which may result in damage to poorly packaged items during transportation. Customers frequently return

damaged items, resulting in additional costs and a loss of goodwill. Thus, this paper proposes that e-tailers improve their packaging policies to reduce the damage to items during the delivery process, which raises unit-packaging costs but lowers the damage rate during the delivery process. China's 3C e-tailers are the best example. The scenario for this study is as follows.

E-tailers typically receive the customer's order from the online platform for rapidly deteriorating items. When orders are received, they wrap the product in logistics packaging and notify the third-party logistics company to deliver the product to the customer. However, third-party logistics may deliver items damaged due to improper packaging types during merchandise delivery to customers, which may be mitigated by the retailer's packaging policy (including different materials, techniques, and methods). Customers who receive defective items request that the retailer refund their money and then return the defective items. The retailer stores the returned merchandise in a different warehouse where the unit holding cost is less than that of perfect merchandise. At the end of each cycle, they sell these defective items in whole lots on the secondary market at a lower price. Because damaged items often incur significant additional costs for retailers, retailers actively adopt new packaging policies to reduce the rate of damage to goods in transit; however, such new packaging policies raise unit-packaging costs. This study simultaneously determines the optimal replenishing and packaging policies that maximise the retailer's profit. Figure 1 depicts the flowchart of the problem description.

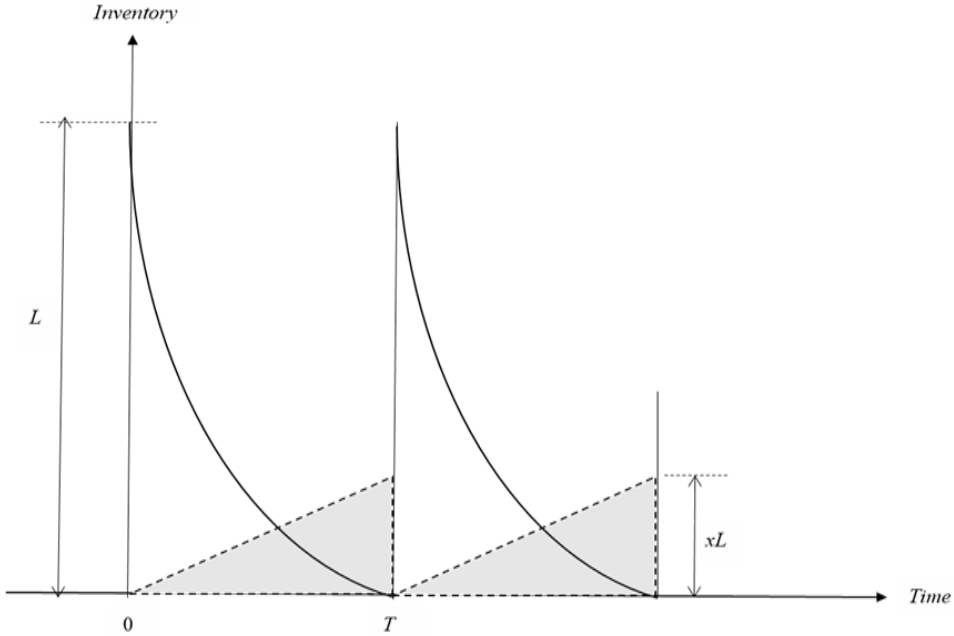
Figure 1 The flowchart of the problem description



3.2 The mathematical model

This section constructs an instantaneously deteriorating inventory model in an e-commerce environment where e-tailers use a new logistics packaging policy (including different materials, techniques, or methods) to reduce damage caused by collisions or crushing of goods during transportation. Figure 2 depicts the behaviour of a rapidly deteriorating inventory system with returned defective items.

Figure 2 The behaviour of a deteriorating inventory model with defective items returned



As shown in Figure 2, the inventory level is depleted by demand and deterioration during a replenishment period, which is governed by the following differential equation:

$$\frac{dI(t)}{dt} + \theta I(t) = -\Omega, \quad 0 \leq t \leq T, \quad (1)$$

with the boundary condition $I(T) = 0$.

Solving equation (1), we have

$$I(t) = \frac{\Omega}{\theta} [e^{\theta(T-t)} - 1], \quad 0 \leq t \leq T$$

Consequently, the retailer's order quantity is

$$L = I(0) = \frac{\Omega}{\theta} [e^{\theta T} - 1]$$

The retailer sells perfect items on the market at a price per unit of P_g . Alternatively, the product return rate due to a collision during delivery to the consumer is x , which is

caused by insufficient packaging strength and, according to Assumption 3, can be expressed as $x = \pi e^{\frac{1-i}{i_0}}$. When customers receive the defective items, they immediately return them to the retailer, who stores them in another warehouse. At the end of each cycle, the retailer sells the defective items on the secondary market for P_d ($P_g \gg P_d$) per unit. We also know that the instantaneous deterioration rate is θ . This means that these deterioration items cannot be sold to customers. Therefore, the total revenue in each cycle is

$$\begin{aligned} TR(T, i) = & P_g \left[\frac{\Omega}{\theta} (e^{\theta T} - 1) \right] (1 - \theta) \left[1 - \pi e^{\left(1 - \frac{i}{i_0}\right)} \right] \\ & + P_d \left[\frac{\Omega}{\theta} (e^{\theta T} - 1) \right] (1 - \theta) \left[\pi e^{\left(1 - \frac{i}{i_0}\right)} \right] \end{aligned} \quad (2)$$

The total purchase cost in full is $u \left[\frac{\Omega}{\theta} (e^{\theta T} - 1) \right]$ each cycle, whereas the set-up cost is A . The cost of packaging for a new logistics packaging policy (materials, techniques, and methods) is $i \left[\frac{\Omega}{\theta} (e^{\theta T} - 1) \right] (1 - \theta)$ for each cycle. The retailer's holding cost for all items per cycle is as follows:

$$\begin{aligned} h \cdot \int_0^T I(t) dt &= h \cdot \int_0^T \frac{\Omega}{\theta} [e^{\theta(T-t)} - 1] dt, \\ &= \frac{\Omega h}{\theta} \left\{ \frac{1}{\theta} [e^{\theta T} - 1] - T \right\} \end{aligned}$$

Figure 2 depicts the number of returned items as a shaded area, which is a function of the return rate, a new unit-packaging cost, and the number of orders. Therefore, the cost of storing returned items is as follows:

$$HC_d = \frac{\Omega h_d}{2\theta} [e^{\theta T} - 1] (1 - \theta) T \pi e^{\left(1 - \frac{i}{i_0}\right)}.$$

The return cost is also affected by the new unit-packaging cost, refund rate, and order quantity. Thus, we obtained the following return cost:

$$RC = f\pi \left[\frac{\Omega}{\theta} (e^{\theta T} - 1) \right] (1 - \theta) \left[e^{\left(1 - \frac{i}{i_0}\right)} \right].$$

Integrating the above costs, we obtained the total cost per cycle as follows:

$$\begin{aligned} TC(T, i) = & A + u \left[\frac{\Omega}{\theta} (e^{\theta T} - 1) \right] + \frac{\Omega h}{\theta} \left\{ \frac{1}{\theta} [e^{\theta T} - 1] - T \right\} + i \left[\frac{\Omega}{\theta} (e^{\theta T} - 1) \right] (1 - \theta) \\ & + f\pi \left[\frac{\Omega}{\theta} (e^{\theta T} - 1) \right] (1 - \theta) \left[e^{\left(1 - \frac{i}{i_0}\right)} \right] + \frac{\Omega h_d}{2\theta} [e^{\theta T} - 1] (1 - \theta) T \pi e^{\left(1 - \frac{i}{i_0}\right)} \end{aligned} \quad (3)$$

Therefore, the average total profit for the deteriorating inventory model is given by the following.

$$\begin{aligned}
 ATP(T, i) &= \frac{TR(T, i) - TC(T, i)}{T} \\
 &= \left(\frac{e^{\theta T}}{T} - \frac{1}{T} \right) \left\{ (1 - \theta) \left(\frac{\Omega}{\theta} \right) \left[P_g - (P_g - P_d + f) \pi e^{\left(\frac{1-i}{i_0} \right)} - i - \frac{u}{(1-\theta)} \right] \right\} \\
 &\quad - \frac{A}{T} - \frac{\Omega h}{\theta} \left[\frac{e^{\theta T} - 1}{\theta T} - 1 \right] - \frac{\Omega h_d}{2\theta} [e^{\theta T} - 1] (1 - \theta) \pi e^{\left(\frac{1-i}{i_0} \right)}
 \end{aligned} \quad (4)$$

We find that equation (4) is an exponential function of two variables, making it difficult to find a closed solution. In the following section, we present an alternative method of obtaining an approximate solution.

4 Solution procedures and theoretical results

The following property is needed to find the approximate optimal solution for the average total profit.

Property 1: Fixed T , the average total profit $ATP(T, i)$ is concave in i .

Proof: One fixes T and then conducts the first- and second-order partial differentiation for equation (4) concerning i to obtain the following:

$$\begin{aligned}
 \frac{\partial ATP(T, i)}{\partial i} &= \left(\frac{e^{\theta T}}{T} - \frac{1}{T} \right) \left\{ (1 - \theta) \left(\frac{\Omega}{\theta} \right) \left[-1 - (P_g - P_d + f) \pi e^{\left(\frac{1-i}{i_0} \right)} \cdot \left(-\frac{1}{i_0} \right) \right] \right\} \\
 &\quad - \frac{\Omega h_d}{2\theta} [e^{\theta T} - 1] (1 - \theta) \pi e^{\left(\frac{1-i}{i_0} \right)} \cdot \left(-\frac{1}{i_0} \right)
 \end{aligned} \quad (5)$$

$$\begin{aligned}
 \frac{\partial^2 ATP(T, i)}{\partial i^2} &= \left(\frac{e^{\theta T}}{T} - \frac{1}{T} \right) \left\{ (1 - \theta) \left(\frac{\Omega}{\theta} \right) \left[-(P_g - P_d + f) \pi x e^{\left(\frac{1-i}{i_0} \right)} \cdot \left(\frac{1}{i_0^2} \right) \right] \right\} \\
 &\quad - \frac{\Omega h_d}{2\theta} [e^{\theta T} - 1] (1 - \theta) \pi e^{\left(\frac{1-i}{i_0} \right)} \cdot \left(\frac{1}{i_0^2} \right) \\
 &< 0
 \end{aligned} \quad (6)$$

Equation (6) is less than 0. We yield $ATP(T, i)$, given that a fixed value of T is concave in i . This completes the proof.

Property 1 illustrates that a unique solution maximising equation (3) exists when the replenishment time is given. Thus, equation (5) is set to 0 and the closed-form solution for the optimal unit-packaging cost is as follows:

$$i = i_0 \left[1 - \ln \left(\frac{i_0}{\pi(P_g - P_d + f + h_d T/2)} \right) \right] \quad (7)$$

Since $i > i_0$, which is an assumption based on a practical situation, we know that the following holds:

$$\ln \left(\frac{i_0}{\pi(P_g - P_d + f + h_d T/2)} \right) > 0 \quad (8)$$

It implies the following:

$$T > \frac{2((i_0/\pi) - P_g + P_d - f)}{h_d} \quad (9)$$

Generally, the right-hand side in equation (9) is always negative because the selling price for good items (P_g) is significantly larger than that of others. In addition, the replenishment time is always larger than 0. We, therefore, set the lower boundary of the optimal replenishment time $T_L = \max \left(0, \frac{2((i_0/\pi) - P_g + P_d - f)}{h_d} \right)$. We should also

mention that the demand rate is expressed in years. Under normal conditions, this means that the maximum replenishment cycle for retailers is 1 year. Therefore, we set the replenishment cycle's upper limit to one year. Combining the above discussions, we determine the occurrence of the optimal replenishment time to be at $(T_L, 1]$. Substituting equation (7) into equation (4) yields the following result:

$$\begin{aligned} ATP(t) = & \left(\frac{e^{\theta T}}{T} - \frac{1}{T} \right) \left\{ (1-\theta) \left(\frac{\Omega}{\theta} \right) \left[P_g - \frac{u}{(1-\theta)} - i_0 \right. \right. \\ & \left. \left. + i_0 \ln \left(\frac{i_0}{\pi(P_g - P_d + f + h_d T/2)} \right) \right] \right\} \\ & - \frac{A}{T} - \frac{\Omega h}{\theta} \left[\frac{e^{\theta T} - 1}{\theta T} - 1 \right] - i_0 \left(\frac{e^{\theta T}}{T} - \frac{1}{T} \right) \left(\frac{\Omega}{\theta} \right) (1-\theta) \end{aligned} \quad (10)$$

The following theorem is required to demonstrate that equation (10) has an optimal replenishment time that exists and is unique.

Theorem 1: There exists a unique T that maximises $ATP(T)$.

Proof: The first- and second-order differentiation of equation (10) concerning T yields the following results:

$$\begin{aligned}
 \frac{dATP(T)}{dT} = & \left[\left(\frac{\theta e^{\theta T}}{T} \right) - \left(\frac{e^{\theta T} - 1}{T^2} \right) \right] \left[(1 - \theta) \left(\frac{\Omega}{\theta} \right) \left(P_g - \frac{u}{(1 - \theta)} - i_0 \right) \right] \\
 & + i_0 (1 - \theta) \left(\frac{\Omega}{\theta} \right) \left\{ \left[\left(\frac{\theta e^{\theta T}}{T} \right) - \left(\frac{e^{\theta T} - 1}{T^2} \right) \right] \ln \left(\frac{i_0}{\pi (P_g - P_d + f + h_d T / 2)} \right) \right. \\
 & \left. - \left(\frac{1}{P_g - P_d + f + h_d T / 2} \right) (h_d / 2) \left(\frac{e^{\theta T} - 1}{T} \right) \right\} \\
 & + \frac{A}{T^2} - \frac{\Omega h}{\theta} \left[\frac{e^{\theta T}}{T} - \frac{1}{\theta T^2} (e^{\theta T} - 1) \right] \\
 & - i_0 (1 - \theta) \left(\frac{\Omega}{\theta} \right) \left[\left(\frac{\theta e^{\theta T}}{T} \right) - \left(\frac{e^{\theta T} - 1}{T^2} \right) \right]
 \end{aligned} \tag{11}$$

$$\begin{aligned}
 \frac{d^2 ATP(T)}{dT^2} = & \left[\left(\frac{\theta^2 e^{\theta T}}{T} \right) - \left(\frac{2\theta e^{\theta T}}{T^2} \right) + \left(\frac{2(e^{\theta T} - 1)}{T^3} \right) \right] \\
 & \times \left[(1 - \theta) \left(\frac{\Omega}{\theta} \right) \left(P_g - \frac{u}{(1 - \theta)} - 2i_0 \right) \right] \\
 & + i_0 (1 - \theta) \left(\frac{\Omega}{\theta} \right) \left\{ \left[\left(\frac{\theta^2 e^{\theta T}}{T} \right) - \left(\frac{2\theta e^{\theta T}}{T^2} \right) + \frac{2(e^{\theta T} - 1)}{T^3} \right] \right. \\
 & \times \ln \left(\frac{i_0}{\pi (P_g - P_d + f + h_d T / 2)} \right) \\
 & - h_d \left(\frac{1}{P_g - P_d + f + h_d T / 2} \right) \left[\left(\frac{\theta e^{\theta T}}{T} \right) - \left(\frac{e^{\theta T} - 1}{T^2} \right) \right] \\
 & \left. + (h_d / 2)^2 \left(\frac{e^{\theta T} - 1}{T} \right) (P_g - P_d + f + h_d T / 2)^{-2} \right\} \\
 & - \frac{2A}{T^3} - \frac{\Omega h}{\theta} \left[\left(\frac{\theta e^{\theta T}}{T} \right) - \left(\frac{2e^{\theta T}}{T^2} \right) + \frac{2(e^{\theta T} - 1)}{T^3} \right]
 \end{aligned} \tag{12}$$

Because $T = (T_L, 1]$, $T > \frac{2((i_0/\pi) - P_g + P_d - f)}{h_d}$, and $-\frac{2A}{T^3} \ll 0$, equation (12) in general is less than 0, indicating the existence of a unique optimal solution. This completes the proof.

We have not yet obtained the optimal replenishing and logistics packaging policies. Although Theorem 1 states that the optimal replenishment time exists and is unique, we cannot obtain a closed-form expression from equation (11). The following algorithm is then used to obtain optimal replenishing and packaging policies that maximise the average total profit.

- Step 1 Let ε be a tiny real number (i.e., $\varepsilon = 0.000001$).
- Step 2 Choose any initial value between $(T_L, 1]$ and then employ bisection method or Newton-Raphson method to obtain candidate optimal replenishment time T , which results equation (11) $< \varepsilon$.
- Step 3 Substitute T obtained from Step 2 into equation (12) to compute its value. If the value of equation (12) is less than 0, the optimal replenishment time T is obtained. Otherwise, STOP (no feasible solution exists).
- Step 4 Substitute T obtained from Step 2 into equation (7), and we have the optimal unit-packaging cost i .
- Step 5 Substitute T obtained from Step 2 and i obtained from Step 4 into equation (4), we have the optimal (T, i) which maximises equation (4).
- Step 6 STOP.

We note that if the retailer does not implement any improved logistics packaging policy and the items do not deteriorate, we know $i = i_0$ and $\theta = 0$. Then, equation (4) reduces to the EOQ model with a return rate:

$$ATP(T) = \Omega P_g(1 - \pi) + \Omega P_d \pi - \left\{ \frac{A}{T} + (u + i_0 + \pi f) \Omega + \frac{\Omega}{2} (h + \pi h_d) T \right\} \quad (13)$$

We already know that maximising equation (11) equals minimising the last part (i.e., the curly brackets) in equation (11). Letting the last part in equation (10) be $ATC(T)$, we have

$$ATC(T) = \frac{A}{T} + (u + i_0 + \pi f) \Omega + \frac{\Omega}{2} (h + \pi h_d) T \quad (14)$$

We conduct the first- and second-order differentiation for equation (14) concerning T to obtain the following results:

$$\frac{dATC(T)}{dT} = \frac{-A}{T^2} + \frac{\Omega}{2} (h + \pi h_d) \quad (15)$$

$$\frac{d^2 ATC(T)}{dT^2} = \frac{2A}{T^3} > 0 \quad (16)$$

Equation (16) demonstrates the existence of a unique optimal replenishment time that minimises equation (14). As a result, we set equation (15) to zero and obtain the following result:

$$T_{ret} = \sqrt{\frac{2A}{\Omega(h + \pi h_d)}} \quad (17)$$

If the return rate is 0, equation (17) is reduced to the replenishment time of the traditional EOQ model, which is given as follows.

$$T_{trad} = \sqrt{\frac{2A}{\Omega h}} \quad (18)$$

The replenishment time in equation (17) is less than the time in equation (18) for the traditional EOQ model. Because returned items cause ordered quantities to be depleted more quickly, replenishment times must become shorter to meet customer demand.

5 Numerical example and sensitivity analysis

For the proposed model, 3C retailers in China provide a good example, as illustrated in the introduction section, by enhancing their packaging policy to reduce the return rate. The following data are used to show that the proposed model is operational. Let $\theta = 0.001$, $h = 3$, $h_d = 1$, $f = 2$, $\pi = 0.2$, and $\Omega = 6,000$, $A = 150$, $u = 7$, and $i_0 = 2$, $P_g = 16$, and $P_d = 3$. Using the algorithm developed in Section 4 and the above parameters, we obtain the optimal replenishment time $T^* = 0.1787821$, the unit-packaging cost $i^* = 2.822814$, and the expected total profit per unit time $ATP(T^*, i^*) = 21,711.13$. If the items do not have the deterioration property (i.e., $\theta = 0$), the optimal replenishment time T^* is 0.2482971, the unit-packaging cost i^* is 2.827415, and the expected total profit per unit time $ATP(T^*, i^*)$ is 23,272.42. By comparing the above results, we can determine that the unit-packaging cost is lower and that the average total profit for items with no deterioration are higher than that for items with deterioration, demonstrating that deterioration can have a significant impact on managers' decision-making. We further note that the replenishment time of the EOQ model with a return rate is $T_{ret}^* = 0.125$, which is less than the replenishment time of the traditional EOQ mode ($T_{rad}^* = 0.129099$).

Table 1 The values of T^* , i^* and ATP^* corresponding to 16 combinations of A , Ω , h and θ

A	Ω	h	θ	T^*	i^*	ATP^*
150	6,000	3	0.001	0.1264156	2.819340	22,694.11
			0.002	0.1264969	2.819346	22,628.55
		6	0.001	0.09032134	2.816943	21,745.81
			0.002	0.09035020	2.816945	21,679.78
	12,000	3	0.001	0.08938815	2.816881	46,778.60
			0.002	0.08944675	2.816884	46,646.35
		6	0.001	0.06386679	2.815183	45,437.29
			0.002	0.06388777	2.815185	45,304.61
300	6,000	3	0.001	0.1787821	2.822814	21,711.13
			0.002	0.1788939	2.822821	21,646.20
		6	0.001	0.1277338	2.819428	20,370.01
			0.002	0.1277730	2.819430	20,304.42
	12,000	3	0.001	0.1264156	2.819340	45,388.21
			0.002	0.1264969	2.819346	45,257.09
		6	0.001	0.09032134	2.816943	43,491.61
			0.002	0.09035020	2.816945	43,359.57

We then ran a sensitivity analysis to determine how these parameters affected the replenishing and packaging policies of e-tailors. Because many decision situations are uncertain, sensitivity analysis can aid decision-making to a great extent. We explore only four important parameters: A , Ω , h and θ . All four parameters are set to two levels (low vs. high): $A = (150, 300)$; $\Omega = (6,000, 12,000)$; $h = (3, 6)$; and $\theta = (0.001, 0.002)$. All other parameters remain unchanged except for the four parameters stated. Table 1 displays the optimal solution for 16 different combinations of A , Ω , h and θ . The following are the findings drawn from Table 1.

- 1 As expected, both T^* and i^* increase as A , whereas ATP^* decreases as A decreases. It is understood that the higher the set-up cost, the lower the average total profit. We note that the set-up cost has a significant impact on the replenishment time because the higher set-up cost increases the retailer's operating expenses. Hence, they expand replenishment to minimise the number of set-ups. Simultaneously, the set-up cost has a minor effect on the new unit-packaging cost, thus reducing the average total profit to a bare minimum.
- 2 T^* and i^* all decrease with Ω , whereas ATP^* increases with Ω . A significant decrease exists in ATP^* , but a slight decrease exists in i^* with an increasing Ω . In general, if the annual demand increases, the retailer increases the order quantity, which means enlarging the replenishment time, to meet the customer's needs. Conversely, as the number of orders increases, the retailer's unit-packaging cost must increase as little as possible to maximise total profit. As a result, as annual demand grows, the unit-packaging cost decreases slightly. We observe that the annual demand rate significantly affects the replenishment time but only a minor impact on unit-packaging cost, thus increasing the average total profit.
- 3 T^* , i^* and ATP^* all decrease with h . It is intuitive to know that the higher the holding cost for items, the higher the operating expenses of the e-tailer. Therefore, e-tailors strive to reduce inventory to the hilt to avoid additional storage costs. The holding cost has a significant impact on the replenishment time. Simultaneously, the unit-packaging cost decreases slightly as the holding cost increases to reduce unnecessary expenses. Integrating the effects of T^* and i^* , we obtain the fact that the average total profit decreases as the holding cost increases.
- 4 Similar to the results in (1), T^* and i^* increase with increasing θ , whereas ATP^* decreases with decreasing θ . The reasons for these results are similar to those discussed in (1). However, we note that the decrease in the average total profit caused by the deterioration rate is significantly smaller than that in result (1) as is the increase in the replenishment time and unit-packaging cost. These results show that when compared with the deterioration rate, the set-up cost has a much larger effect on the optimal solution and the average total profit.
- 5 From Table 1, we find that none of the second-order interactions between A , Ω , h , and θ are significant. Therefore, an additive model seems appropriate to model the relationship between T^* (i^* and ATP^*) and the four parameters (A , Ω , h and θ).

6 Conclusions

This study proposed an instantaneously deteriorating inventory model in an e-commerce environment where the retailer uses an improved logistics packaging policy (techniques, materials, or methods) to reduce the rate of damage to goods in transit. This study determines the replenishing and logistics packaging policies that maximise the average total profit. Because of the difficulty in finding a closed-form solution, an efficient algorithm was developed to find the optimal solution for the proposed model. The theoretical results show that the deteriorating rate could impact the optimal solution and the average total profit. Meanwhile, the EOQ model with a return rate has a shorter replenishment time than the traditional EOQ model. We can see from the numerical example that incorporating the logistic packaging policy into the traditional EOQ model can significantly impact managerial decisions. The numerical examples show that:

- 1 the replenishing and logistics packaging policies are affected by the deteriorating rate and the return rate of items
- 2 the operating cost rises if deterioration occurs
- 3 the four parameters (i.e., set-up cost, demand rate, holding cost, and deteriorating rate) explored insensitivity analysis provide flexibility in determining replenishing and logistics packaging policies
- 4 the set-up cost has a significant impact on the replenishment time and the average total profit
- 5 except for the demand rate, all parameters in the sensitivity analysis increase costs, resulting in a decrease in the average total profit.

The proposed model can be further enriched by considering more practical problems to match various real-world situations, such as quantity discounts, probability demand, and non-instantaneously deteriorating items.

Acknowledgements

The authors sincerely thank the anonymous referees for their very helpful comments and for the improvement of the paper.

References

- Almathkour, F. and Benkherouf, L. (2023) 'Optimal policies for finite horizon model with time-varying demand rate, non-instantaneous deterioration and backlogging', *International Journal of Systems Science: Operations & Logistics*, Vol. 10, No. 1, p.2041128.
- Bakker, M., Riezebos, J. and Teunter, R.H. (2012) 'Review of inventory systems with deterioration since 2001', *European Journal of Operational Research*, Vol. 221, No. 2, pp.275–284, <https://doi.org/10.1016/j.ejor.2012.03.004>.
- Chonhenchob, V., Sittipod, S., Swasdee, D., Rachtanapun, P., Singh, S.P. and Singh, J. (2009) 'Effect of truck vibration during transport on damage to fresh produce shipments in Thailand', *Journal of Applied Packaging Research*, Vol. 3, No. 1, pp.27–38.

- Covert, R.B. and Philip, G.S. (1973) 'An EOQ model for items with Weibull distribution deterioration', *AIIE Transaction*, Vol. 5, No. 4, pp.323–326.
- Daryanto, Y., Wee, H.M. and Astanti, R.D. (2019) 'Three-echelon supply chain model considering carbon emission and item deterioration', *Transportation Research Part E: Logistics and Transportation Review*, Vol. 122, pp.368–383.
- Das, D., Samanta, G.C., Barman, A., De, P.K. and Mohanta, K.K. (2022) 'A recovery mathematical model for the impact of supply chain interruptions .the lockdown in COVID-19 using two warehouse perishable inventory policies', *Results in Control and Optimization*, Vol. 9, p.100184, <https://doi.org/10.1016/j.rico.2022.100184>.
- Dave, U. and Patel, L.K. (1981) '(T, Si) policy inventory model for deteriorating items with time proportional demand', *Journal of Operational Research Society*, Vol. 32, No. 2, pp.137–142.
- Defraeye, T., Lambrecht, R., Delele, M.A., Tsige, A.A., Opara, U.L., Cronje, P., ..., Nicolai, B. (2014) 'Forced-convective cooling of citrus fruit: cooling conditions and energy consumption in relation to package design', *Journal of Food Engineering*, Vol. 121, pp.118–127.
- Delele, M.A., Ngcobo, M.E.K., Getahun, S.T., Chen, L., Mellmann, J. and Opara, U.L. (2013) 'Studying airflow and heat transfer characteristics of a horticultural produce packaging system using a 3-D CFD model. Part II: effect of package design', *Postharvest Biology and Technology*, Vol. 86, pp.546–555.
- Fadiji, T., Coetzee, C., Chen, L., Chukwu, O., Opara U.L. (2016) 'Susceptibility of apples to bruising inside ventilated corrugated paperboard packages during simulated transport damage', *Postharvest Biology and Technology*, Vol. 118, pp.111–119, <https://doi.org/10.1016/j.postharvbio.2016.04.001>.
- Ghare, P.M. and Schrader, G.P. (1963) 'A model for an exponentially decaying inventory', *Journal of Industrial Engineering*, Vol. 14, pp.238–243.
- Goyal, S.K. and Giri, B.C. (2001) 'Recent trends in modeling of deteriorating inventory', *European Journal of Operational Research*, Vol. 134, No. 1, pp.1–16.
- Han, J.W., Zhao, C.J., Yang, X.T., Qian, J.P. and Fan, B.L. (2015) 'Computational modeling of airflow and heat transfer in a vented box during cooling: optimal package design', *Applied Thermal Engineering*, Vol. 91, pp.883–893.
- Hariga, M.A. (1996) 'Optimal EOQ models for deteriorating items with time-varying demand', *Journal of Operational Research Society*, Vol. 47, No. 10, pp.1228–1246.
- Idah, P.A., Yisa, M.G., Chukwu, O. and Morenikeji, O.O. (2012) 'Simulated transport damage study on fresh tomato', *Agric. Eng. Int.: CIGR Journal*, Vol. 14, No. 2, pp.119–126.
- Jaber, M.Y., Peltokorpi, J. and Smunt, T.L. (2022) 'The lot size problem and the learning curve: a review of mathematical modeling (1950's -2020)', *Applied Mathematical Modelling*, Vol. 105, pp.832–859, <https://doi.org/10.1016/j.apm.2022.01.007>.
- Jain, M. and Singh, P. (2022) 'Optimal inspection and advance payment policy for deteriorating items using differential evolution metaheuristic', *Applied Soft Computing*, Vol. 128, p.109475, <https://doi.org/10.1016/j.asoc.2022.109475>.
- Jarimopas, B., Singh, S.P. and Saengnil, W. (2005) 'Measurement and analysis of truck transport vibration levels and damage to packaged tangerines during transit', *Packaging Technology and Science: An International Journal*, Vol. 18, No. 4, pp.179–188.
- Khakzad, A. and Gholamian, M.R. (2020) 'The effect of inspection on deterioration rate: an inventory model for deteriorating items with advanced payment', *Journal of Cleaner Production*, Vol. 254, p.120117.
- Li, G.P., He, X.L., Zhou, J. and Wu, H. (2019) 'Pricing, replenishment and preservation technology investment decisions for non-instantaneous deteriorating items', *Omega*, Vol. 84, pp.114–126, <https://doi.org/10.1016/j.omega.2018.05.001>.

- Liao, J.J., Huang, K.N., Chung, K.J., Lin, S.D., Chuang, S.T. and Srivastava, H.M. (2020) 'Optimal ordering policy in an economic order quantity (EOQ) model for non-instantaneous deteriorating items with defective quality and permissible delay in payments', *Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas*, Vol. 114, No. 1, pp.1–26.
- Lowalekar, H. and Basu, S. (2020) 'Theory of constraints based mafia offer for supply chains of deteriorating products', *International Journal of Production Research*, Vol. 58, No. 14, pp.4421–4449.
- Maihami, R. and Abadi, I.N.K. (2012) 'Joint control of inventory and its pricing for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging', *Mathematical and Computer Modelling*, Vol. 55, Nos. 5–6, pp.1722–1733, <https://doi.org/10.1016/j.mcm.2011.11.017>.
- Maihami, R., Ghalekhondabi, I. and Ahmadi, E. (2021) 'Pricing and inventory planning for non-instantaneous deteriorating products with greening investment: a case study in beef industry', *Journal of Cleaner Production*, Vol. 295, p.126368, <https://doi.org/10.1016/j.jclepro.2021.126368>.
- Maihami, R., Govindan, K. and Fattahi, M. (2019) 'The inventory and pricing decisions in a three-echelon supply chain of deteriorating items under probabilistic environment', *Transportation Research Part E: Logistics and Transportation Review*, Vol. 131, pp.118–138, <https://doi.org/10.1016/j.tre.2019.07.005>.
- Mashud, A.H.M., Wee, H.M. and Huang, C.V. (2021) 'Preservation technology investment, trade credit and partial backordering model for a non-instantaneous deteriorating inventory', *RAIRO-Operations Research*, Vol. 55, p.51–77.
- Meena, P., Meena, S., Sharma, A.K., Singh, P.T. and Kumar, G. (2022) 'Managing inventory of non-immediately degrading items with partial backlog and discounting cash flow under inflation', *Materials Today: Proceedings*, Vol. 50, No. 2, pp.155–162, <https://doi.org/10.1016/j.matpr.2021.11.168>.
- Mishra, U., Wu, J.-Z., Tsao, Y.-C., Tseng, M.-L. (2020) 'Sustainable inventory system with controllable non-instantaneous deterioration and environmental emission rates', *Journal of Cleaner Production*, Vol. 244, p.118807, <https://doi.org/10.1016/j.jclepro.2019.118807>.
- National Retail Federation (2022) *Retail Returns Increased to \$761 Billion in 2021 as a Result of Overall Sales Growth* [online] <https://nrf.com/media-center/press-releases/retail-returns-increased-761-billion-2021-result-overall-sales-growth> (accessed 18 May 2023).
- Pando, V., San-José, L.A. García-Laguna, J. and Sicilia, J. (2018) 'Optimal lot-size policy for deteriorating items with stock-dependent demand considering profit maximization', *Computers & Industrial Engineering*, Vol. 117, pp.81–93, <https://doi.org/10.1016/j.cie.2018.01.008>.
- Resimović, L., Kovačević, D. and Brozović, M. (2022) 'Design of sustainable packaging for natural cosmetics', *Journal of Applied Packaging Research*, Vol. 14, No. 1, p.2.
- Sebatjane, M. (2022) 'The impact of preservation technology investments on lot-sizing and shipment strategies in a three-echelon food supply chain involving growing and deteriorating items', *Operations Research Perspectives*, Vol. 9, p.100241, <https://doi.org/10.1016/j.orp.2022.100241>.
- Singh, S.P. (1992) 'New package system for fresh berries', *Packaging Technology and Science*, Vol. 5, No. 1, pp.3–10.
- Soni, H.N. (2013) 'Optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment', *International Journal of Production Economics*, Vol. 146, No. 1, pp.259–268.
- Tavassoli, S., Manavizadeh, N. Rezaei, A. and Rabbani, M. (2022) 'A lot-sizing model for non-instantaneous deteriorating products under advance payment and non-linear partial backlogging', *Iranian Journal of Management Studies*, Vol. 15, No. 1, pp.85–110.

- Teng, J-T., Cárdenas-Barrón, L.E., Chang, H-J. and Wu, J. (2016) 'Inventory lot-size policies for deteriorating items with expiration dates and advance payments', *Applied Mathematical Modelling*, Vol. 40, Nos. 19–20, pp.8605–8616.
- Teng, J-T., Chang, H-J. Dye, C-Y., Hung, C-H. (2002) 'An optimal replenishment policy for deteriorating items with time-varying demand and partial backlogging', *Operational Research Letter*, Vol. 30, No. 6, pp.387–393.
- Tiwari, S., Ahmed, W. Sarkar, B. (2019) 'Sustainable ordering policies for non-instantaneous deteriorating items under carbon emission and multi-trade-credit-policies', *Journal of Cleaner Production*, Vol. 240, p.118183, <https://doi.org/10.1016/j.jclepro.2019.118183>.
- Tiwari, S., Cárdenas-Barrón, L.E. Malik, A.I. and Jaggi, C.K. (2022) 'Retailer's credit and inventory decisions for imperfect quality and deteriorating items under two-level trade credit', *Computers & Operations Research*, Vol. 138, p.105617, <https://doi.org/10.1016/j.cor.2021.105617>.
- Tiwari, S., Cárdenas-Barrón, L.E., Goh, M. and Shaikh, A.A. (2018) 'Joint pricing and inventory model for deteriorating items with expiration dates and partial backlogging under two-level partial trade credits in supply chain', *International Journal of Production Economics*, Vol. 200, pp.16–36, <https://doi.org/10.1016/j.ijpe.2018.03.006>.
- Tripathy, M., Sharma, G. and Sharma, A.K. (2022) 'An EOQ inventory model for non-instantaneous deteriorating item with constant demand under progressive financial trade credit facility', *Opsearch*, Vol. 59, pp.1215–1243, <https://doi.org/10.1007/s12597-022-00573-5>.
- Vladić, G., Kecman, M., Kašiković, N., Pál, M. and Stančić, M. (2015) 'Influence of the shape on the consumers perception of the packaging attributes', *Journal of Graphic Engineering and Design*, Vol. 6, No. 2, p.27.
- Zhang, T., Hao, Y. and Zhu, X. (2022) 'Consignment inventory management in a closed-loop supply chain for deteriorating items under a carbon cap-and-trade regulation', *Computers & Industrial Engineering*, Vol. 171, p.108410, <https://doi.org/10.1016/j.cie.2022.108410>.
- Zou, Q., Opara, L.U. and McKibbin, R. (2006) 'A CFD modeling system for airflow and heat transfer in ventilated packaging for fresh foods: I. Initial analysis and development of mathematical models', *Journal of Food Engineering*, Vol. 77, No. 4, pp.1037–1047.