
Mathematical model of one-dimensional penetration stability failure for gaseous coal

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Abstract: Based on the theory of fluid dynamics in porous media, combined with the gas state equation, Darcy's law and the discriminant equation of one-dimensional seepage instability, a mathematical model of one-dimensional penetration stability failure is established to study the seepage damage law of coal seam. Assuming that the background pressure of the coal wall is attenuated according to the exponential law, the mathematical equation is solved by using the finite difference method. Furthermore, the process of coal bed instability which is supposed as a form of a 'sublayer' pushing forward was analysed. That is, the coal bed loses its stability layer by layer. The calculation results showed that the thickness of the failure sublayer decreases with the reduction of coal permeability and the acceleration of dissipation rate of the background pressure. The model provides a method which can analyse the outburst process and its intensity quantitatively.

Keywords: mathematical model; seepage; coal and gas outburst; numerical simulation; outburst intensity.

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1 Introduction

Coal seam properties, ground stress and gas are considered to be the three main factors that cause coal and gas outburst (Yu, 1985; Hu et al., 2007). Many researchers have made a lot of important achievements in the analysis of coal and gas outburst mechanism from different perspectives (Liang et al., 1995; Jiang and Yu, 1998; Lin, 2010; Chen et al., 2014; Jacek, 2014; Yang et al., 2014). The research of coal and gas outburst mechanism involves two aspects:

- 1 the analysis of the outburst process
- 2 the quantitative analysis of the outburst intensity.

At present, because of the complex nature of coal and gas outburst mechanism, the coupled scientific study is short of a mathematical model, which can quantitatively analyse the outburst process and its intensity. Based on the analysis method of soil flow in soil mechanics, Xie et al. (2015) established the formula of penetration stability failure of a coal seam. This method involves the calculation of the pressure change of a coal seam and the thickness of the failure sublayer, which provides an idea for the quantitative prediction of the outburst intensity. According to the formula of penetration stability failure, combining the seepage equation of coal seam, and assuming that the background pressure of the coal wall attenuates exponentially, the paper proposed a simplified mathematical model of one-dimensional seepage failure in the coal seam. Computational program was performed with MATLAB software, through which the influence of the process of coal seam seepage instability and the decay rate of coal wall background pressure on the thickness of the failure sublayer was analysed. On the basis of the research mentioned above, the article also developed the quantitative analysis method for the outburst intensity.

2 Fundament of penetration stability

2.1 Physical basis of penetration stability failure

Coal can be divided into five categories (namely I, II, III, VI and V) in terms of the destruction type. Fundamentally III to V coal seams are vulnerability to outburst. The coal seams prone to burst out are called 'tectonic coal' in coal geology with the characteristics of small diameter and fragile nature. 'Tectonic coal' is the product of physical and chemical

action under tectonic stresses, whose structure includes cataclastic texture, granulated structure, squamose structure, fragmental structure and mylonitic structure. Studies by Xie and Zhao (2012) conclude that 'tectonic coal' belongs to 'granular structure' coal according to the classification of rock mass structure type, and its engineering property is similar to that of the soil. Therefore, coal and gas outburst mechanism can be analysed by applying the analysis method of soil flow and augmenting soil mechanics theories. It is assumed that the coal and gas outburst is the result of the continuous destruction of coal seam under the action of gas seepage pressure. The reason why aforementioned 'tectonic coal' is vulnerable to outburst lies not only in its low strength but also in its granular characteristics, namely, the soil characteristics. Accordingly, coal and gas outburst mechanism can be analysed by taking the analysis method of soil flow in soil mechanics as references.

2.2 Formula of one-dimensional penetration stability failure

Based on the principle of soil mechanics, the seepage failure of sandy soil occurs on the condition that the hydraulic gradient increases towards the critical hydraulic gradient. The calculation formula for the critical hydraulic gradient is governed by

$$i_{cr} = \frac{G_s - 1}{1 + e} \quad (1)$$

where i_{cr} represents the critical hydraulic gradient, G_s represents the specific gravity of soil particles and e represents the void ratio of soil. The formula is derived on the basis of ignoring the force among soil particles, which is only suitable for the condition that the soil pressure and cohesion are small. The force among coal particles cannot be ignored. The main reason is that the underground coal seam is under great in-situ stress. Xie et al. (2015) proposed the formula of one-dimension penetration stability failure of coal seam under osmotic pressure under assumptions where the seepage of coal seam flows along the single direction, the structure of coal seam is homogeneous, the exposure plane of the coal seam is circular and the lateral stress coefficient is 1. The equation can be expressed by

$$i' = \frac{\lambda(P_i - P_a) - \sigma_t}{l} \geq i'_{cr} = \frac{2(c + \sigma \tan \varphi)}{r} \quad (2)$$

where i' represents the correction pressure gradient, P_i represents the gas pressure where l distant from the coal wall, P_a represents the gas pressure of coal wall, r represents the radius of the circular exposure plane, l represents the thickness of the failure sublayer, σ_t represents the tensile strength of coal, c represents the cohesion of coal, φ represents the internal friction angle of coal, σ represents the normal stress and λ represents the correction factor, which is a number no more than 1 and decreases with the increase of permeability. i'_{cr} represents the critical correction pressure gradient.

Equation (2) shows that the effects of in-situ stress on outburst are revealed in two aspects. On the one hand, in-situ stress may decrease the permeability and the strength parameters of coal, including cohesion and internal friction angle, promoting the occurrence of the outburst, on the other hand, the increase of in-situ stress will increase the critical correction pressure gradient. The greater the critical correction pressure gradient, the greater the gas pressure where l is the distance from the coal wall which needs to burst out. Therefore, once the outburst occurs in coal seams under large *in-situ* stress, it often has high outburst intensities. Equation (2) also shows that the coal bed is under the delamination instability, which is in agreement with the experiment data. Moreover, Wang et al. (2016) verified the rationality of the equation through the indoor simulation test. As shown in Figure 1, the coal sample destruction in the experiment reveals the feature that coal sample loses its balance hierarchically, which matches well with the results of laboratory tests conducted by Jiang and Yu (1998) and Jacek (2011).

Figure 1 The stratification phenomenon of coal sample when losing its balance in laboratory simulated experiment (see online version for colours)



Note: The gray slice denotes the portion of outburst masses in laboratory condition.

3 Mathematical model of one-dimensional seepage in coal seam

In order to calculate the modified pressure gradient in the equation (2), the gas pressure of coal seam and coal wall should be calculated first. In the occurrence and development of the outburst, the pressure of coal seam has a constant attenuation, and the background pressure of coal wall also changes progressively. Although the seepage analysis software has been developed for the calculation of coal bed gas pressure in China and abroad, this software lacks the interface to calculate the pressure gradient. For that reason, it is essential that the mathematical model of gas flow in coal seam should be established, and the modified gas pressure gradient can be calculated by programming.

3.1 Ideal gas equation

Assuming that coal seam gas is an ideal gas, we obtain

$$\frac{P}{\rho} = \frac{RT}{\mu'} \quad (3)$$

where P represents the absolute pressure of gas (Pa), ρ is the gas density (g/cm^3), R represents the gas constant ($\text{J}/(\text{mol}\cdot\text{K})$), μ' represents the molecular weight of gas (g/mol) and T represents the absolute temperature of gas (K).

3.2 Gas movement equation (Darcy's law)

Assuming that the gas flow in coal seam is governed by Darcy's law, the equation can be written as

$$V = -\frac{k}{\mu'} \frac{\partial P}{\partial x} \quad (4)$$

where V represents the gas flow velocity (m/s), k represents the medium permeability (m^2), μ' denotes the viscosity coefficient of gas ($\text{Pa}\cdot\text{s}$) and P is absolute pressure of gas (Pa).

3.3 Continuity equation

A cube in coal bed was taken as the research object, considering single direction flow, and the relationship among the three components of velocity can be respectively expressed by $V_z = V_y = 0$, $V_x = V$. According to the principle of fluid dynamics in porous media (Jacab, 1983), the continuity equation which takes the gas source into account can be written as

$$f(t) dx dy dz dt - \frac{\partial(\rho V)}{\partial x} dx dy dz dt = \frac{\partial(\rho \phi)}{\partial t} dx dy dz dt \quad (5)$$

where $f(t)$ represents the item describing the desorption of absorbing gas, ρ represents the gas density (g/m^3), V represents the gas flow velocity (m/s), and ϕ represents the porosity of coal seam. In the preliminary study, this paper only takes the seepage action of free gas into account,

occurs. Additionally, a value of 1.0×10^{-12} is used in the model. According to equation (2), the critical correction pressure gradient is related to in-situ stress and the size of exposure plane. In the exposure plane of the coal seam, the stress is obviously less than the original in-situ stress. R represents the radius of the circular exposure plane, however, the underground coal seam is actually affected by various structural planes. And a critical correction pressure gradient of 30.0 is used in the model. Generally, coal and gas outburst occurs in the process of coal uncovering, coal mining and the expansion of coal mine roadway, and the background pressure of coal wall attenuates rapidly when the fresh face of coal bed is exposed. Different from general coal seam gas seepage calculation, the extenuation process cannot be ignored in the study of coal and gas outburst. The change rule of the background pressure of coal wall should be investigated, and evaluated with experimental data. In this paper, we assume that the background pressure of the coal wall is attenuated according to the exponential law with the start of initial gas pressure, and the formula is expressed as

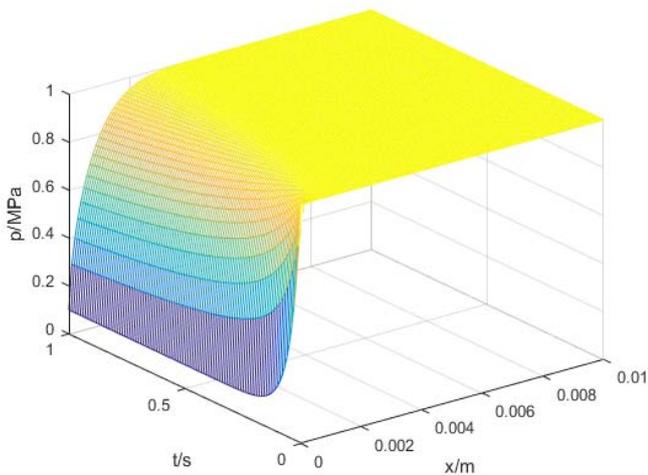
$$P_a = 0.1 + (P_0 - 0.1) \cdot \eta^j \quad (11)$$

where P_a represents background pressure of coal wall roadway, P_0 represents the initial gas pressure of coal wall, η represents the extenuation rate of background pressure and j represents time step.

4.2 Change rule of the coal seam gas pressure and the correction gas pressure gradient

The values of gas pressure at each position in coal seam with time are calculated by using MATLAB software (as shown in Figure 2), indicating that the gas pressure decreases with the increase of time for a certain position.

Figure 2 Temporal and spatial distribution of coal seam gas pressure (see online version for colours)



As shown in Figure 3, the maximum of correction pressure gradient appears at the position located in some distance from coal wall and a time delay. The time-space distribution characteristics of correction pressure gradient indicate that coal and gas outburst has the feature of losing its balance hierarchically.

Figure 3 Temporal and spatial distribution of correction pressure gradient (see online version for colours)

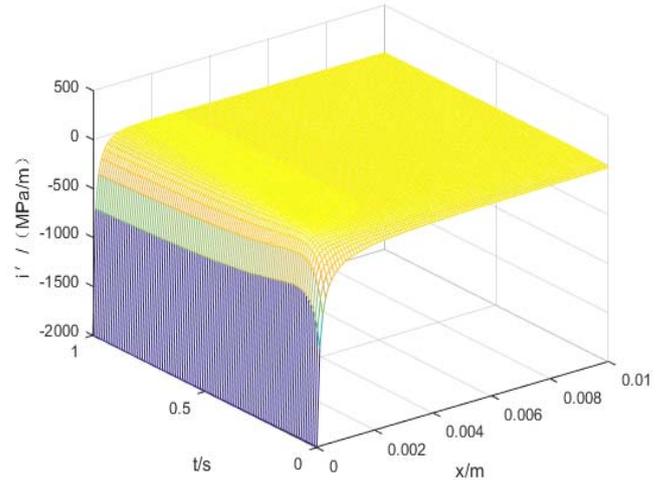


Figure 4 is the distribution curve of correction pressure gradient with a location at the 18th time step, and Figure 5 is the distribution curve of correction pressure gradient with time at the twelfth space step. Both curves present the characteristic that the correction pressure gradient increases firstly and then gradually decreases, which is consistent with Figure 3.

Figure 4 The distribution curve of correction pressure gradient with location at a certain time ($j = 18$) (see online version for colours)

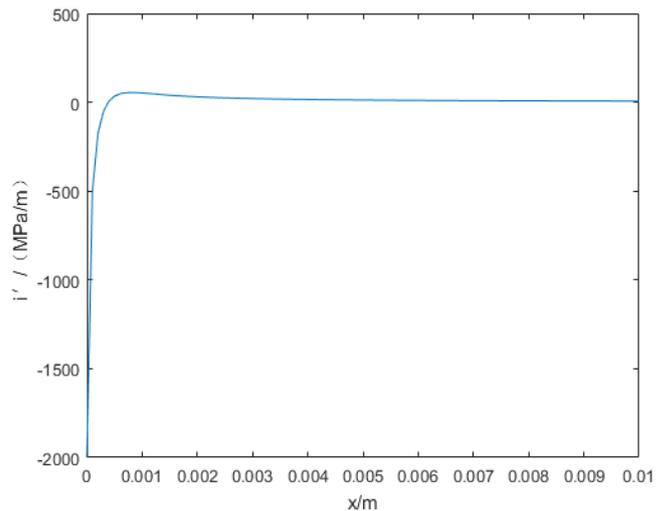
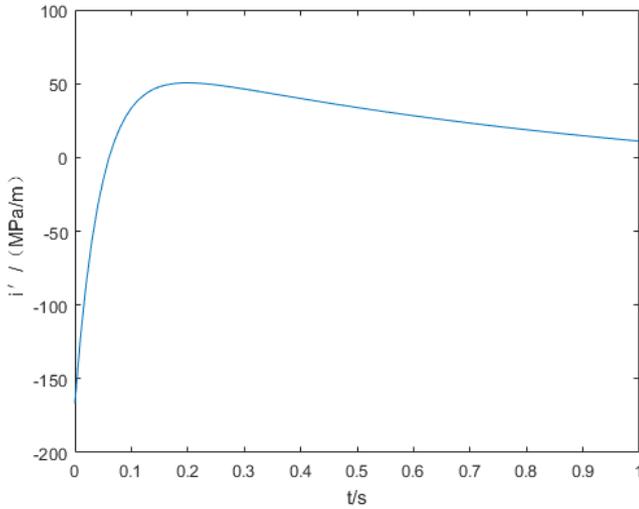


Figure 5 The distribution curve of correction pressure gradient with time at a certain location ($i = 12$) (see online version for colours)



4.3 The calculation method of the first unstable layer thickness and its influencing factors

4.3.1 The calculation method of the first unstable layer thickness

According to equation (2), if the correction pressure gradient is no less than the critical correction pressure gradient, namely $i' \geq i'_{cr}$, then the coal seam may lose its stability. Because the correction pressure gradient (i') changes with time and space, there may be lots of data points which match the criterion. In these data points which satisfy the instability criterion, only the interface corresponded to the shortest time data points is unstable and hierarchical, and the distance from the interface to the coal wall is the unstable layer thickness.

4.3.2 The influence of coal seam permeability

Assuming that $i'_{cr} = 30.0$ MPa/m, if the coal seam permeability is 1.0×10^{-12} m², 2.0×10^{-12} m² and 4.0×10^{-12} m², then the unstable thickness is 0.8 mm, 1.1 mm and 1.3 mm, and the time to lose stability is 0.10 s, 0.11 s and 0.13 s, respectively. Obviously, both the unstable layer thickness and the time to lose stability increases with the increase of permeability.

4.3.3 The influence of extenuation rate of coal wall backgrounds gas pressure

Assuming that $i'_{cr} = 30.0$ MPa/m, $k = 1.0 \times 10^{-12}$ m², if the extenuation rate of background pressure is 0.7, 0.8 and 0.9, then the unstable thickness is 0.5 mm, 10.8 mm and 1.1 mm, and the time to lose stability is 0.07 s, 0.10 s and 0.23 s, respectively. Obviously, both the unstable layer thickness and the time to lose stability increases with the decrease of extenuation rate of background pressure.

4.4 Correction for continuous seepage instability of coal seam

After the first layer loses its stability, the stratified interface becomes the new exposure plane of the coal seam. With some simplifications, the thickness of next layer can be determined. In turn, the thickness of the subsequent unstable layers can be calculated. As a result, the amount of coal outburst can be obtained. Different from the first layer, the gas pressure of subsequent coal seams are the gas pressure that extenuated because of seepage rather than the initial gas pressure. The gas pressure of the coal seam decreases continuously with the ongoing occurrence of the outburst. The outburst will not halt until the gas pressure of the coal seam cannot meet the equation (2). In the ongoing process of the outburst, the extenuation rate of coal wall background gas pressure decreases and coal permeability increases as the aggregation of outbursts on the roadway and the increasing damage of coal structure, resulting in the increase in thickness of the unstable layer. It is not easy to observe the characteristic of the failure process when the thickness of the unstable layer is very small, because of the powdery nature of coal outburst. In addition to calculating the thickness and number of unstable layer, the mathematical model can also obtain the gas quantity released by coal seam outburst. Therefore, the mathematical model provides a method which can analyse the outburst process and its intensity quantitatively.

5 Conclusions

The mathematical model provides a method which cannot only analyse the outburst process but also can analyse the outburst intensity quantitatively. The conclusions are summarised as follows:

- 1 'Tectonic coal' belongs to 'granular structure' coal based on the classification of rock mass structure type, and its engineering property is similar to soil. Therefore, coal and gas outburst mechanism can be analysed by taking the analysis method of soil flow in soil mechanics as references.
- 2 Based on the theory of fluid dynamics in porous media, combined with the gas state equation, Darcy's law and the discriminant equation of one-dimensional seepage instability, as simplified mathematical model of one-dimensional penetration stability failure was established.
- 3 Assuming that the background pressure of the coal wall is attenuated according to the exponential law, the equation was solved by FDM and MATLAB software. And the reason why coal bed loses its stability as a form 'sublayer' was also investigated.

- 4 The influence of coal permeability and dissipation rate of the background pressure on the thickness of the failure sublayer was researched. The calculation results show that the thickness of the failure sublayer decreases with the reduction of coal permeability and the acceleration of dissipation rate of the background pressure.

It should be noted that the process of coal and gas outburst was investigated from the perspective of seepage damage, and the quantitative analysis method of coal and gas outburst was also discussed in this paper. Further investigation may move onto the high-accuracy numerical methods for gas outburst, effects of the desorption of absorbing gas, the attenuation law of the coal wall background pressure and establishment of three-dimensional seepage damage model.

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