Proposing a new multi objective mathematical model for university course timetabling problem regarding optimisation of the quality of lecturers

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Abstract: Scheduling of activities is considered in all daily tasks and many researchers have referred to this as an important factor to enhance productivity of activities. One of the issues in which scheduling is important is to create a university course timetable that is usually employed in universities for scheduling of lessons and assignment of the existing resources to them. Resources including classes, lecturers and so on are normally assigned to the university course timetable. Given the importance of university course timetable, a new model is presented in the current study. In this study, constraints of research models are identified from research background and then a new model is proposed by aggregation of previous constraints and considering many new ones. Paying attention to quality of teaching is one of the most important constraints that has been represented as the model's objective. Finally, the model is solved through a numerical example.

Keywords: mathematical model; course timetabling; quality; university.

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1 Introduction

Education in all countries has different levels that some of these levels are mandatory and higher levels are optional. Given that benefits of education at high levels are in public and social form besides individual benefits, its expenses are usually universal. Considering this, optimal use of resources is very important and it will be possible to help improve the education quality and as a result, increase its productivity using a suitable plan. This issue has compelled organisations, educational institutions and even the governments to pay more attention to programming and assignment of resources in educational institutions. In this regard, operations research has been considered as an effective tool (Johnes, 2014).

Different educational, health, and sports organisations are faced with the problem of scheduling in different parts of the world and this has created many challenges for most of them. Therefore, an appropriate decision-making tool helps the decision-maker adopt a suitable decision and a flexible timetable (Zhang et al., 2010). The university course timetabling problem is one of the problems that universities are faced with each year and given the differences among the universities, this has various constraints and objectives in each university (Santiago-Mozos et al., 2005). Given the different studies that have been carried out about university course timetable, there are generally two classes of constraints for such problems. They are divided into hard and soft constraints. Hard constraints are the constraints which make it possible to achieve a feasible answer and soft constraints are those which do not involve achieving a feasible answer but are effective on improving the quality of the answer (Burke and Petrovic, 2002).

The timetable contains optimal assignment of a set of events (lessons, tests, sports and health events), resources (lecturers, observers, nurses and physicists) to spaces (class, exam hall, and doctor's office) and time. For instance, in university course timetable, a number of lessons are assigned to a group of classes, lecturers, and time periods and this assignment can be fulfilled with different objectives and various constraints can be considered for it (Zhang et al., 2010).

High level programming of educational institutions and assignment of resources are not the only problems that have been considered by researchers of operations research and especially since the time when the computer has influenced academic environments, it has been tried to explore and solve university course timetabling problem automatically. Academic researchers, however, have realised its complexity and encountered with its problems. Despite the extensive range of studies in this regard during these years, this scope has still several gaps in research field and researchers have tried to use operations research for this reason (Causmaecker et al., 2009).

The university course timetabling problem is not new and several researchers have conducted studies and have presented different models and algorithms. This high volume of studies shows high range of the above problem (MirHassani, 2006). University course timetabling problem is simply understood but it has many complexities too and contains an extensive range of solutions at different levels (Shiau, 2011). This problem includes programming of a set of course sessions, students, lecturers, and a group of time periods that are programmed given certain constraints and conditions (MirHassani, 2006).

Considering the importance of university course timetable, a new model will be proposed in this study. For this purpose, the constraints are first determined by reviewing the research literature and then a new model is designed using the modelling techniques. The new model in this research considers all previous soft and hard constraints and also quality of lecturers as an important index in course timetabling is considered. Finally, it will be solved via a numerical example.

In the second section, the research background is presented. Section three explains the university course timetabling problem. Section four deals with explaining this problem in the current study and a new mathematical model is represented in section five. In section six, a numerical example is proposed and solved. Conclusion will be presented in the final section.

2 Literature review

The studies about university course timetable have been initiated since previous years and many researchers have done this given the problem and the constraints through different methods. The most important difference in previous studies is the difference in the intended objectives and constraints. Some of the related studies in 10 recent years are mentioned in this section.

Al-Yakoob and Sherali (2006) proposed a mathematical programming model in their survey for the assignment of lecturers to classrooms. This was done at the Department of Mathematics and Computer at Kuwait University. In this model, some constraints such as presenting all courses, class capacity, and constraint of presenting consecutive courses to lecturers were referred. They enhanced their model in another study and mentioned the constraints of holding separate classes (45 min and 75 min) for girls and boys given the lesson plan at Kuwait University.

MirHassani (2006) represented a mathematical model which was based on programming for university students. It was solved via deterministic methods. The constraints in this model given certain conditions of the place for programming were presenting the courses for 200 students, presenting general courses for all students, constraint of taking courses for students at each day, programming of separate courses for second shift students and compilation of lecturers' program based on their attendance. The objective function of the problem intended to minimise unfeasibility of soft constraints.

Head and Shaban (2007) proposed a mathematical programming model and aimed to minimise the associated costs that are related to difficulty level of students' programming. Whatever this value is higher, it shows that the program is more difficult for students. The constraints that were considered in this model were the number of classes and lecturers with regard to the courses, the required possibilities for lecturers, and class capacity besides common constraints in the university course timetable.

Beligiannis et al. (2008) presented a computational optimisation algorithm for university course timetabling problem in a high school in Greece. The obtained results were compared to the results of optimisation techniques and showed that the proposed algorithm offers a feasible answer close to optimum value more conveniently and in a shorter time. The objectives in this paper were minimisation of idle hours of lecturers, minimisation of non-uniform distribution of lecturers' idle hours, minimisation of lecturers' idle hours nonuniformly, minimisation of non-uniform teaching in time periods, and minimisation of presenting similar courses in a day.

Broek et al. (2009) proposed a model for university course timetable at Eindhoven University of Technology. They presented four sub-problems and proposed an integer programming model for each sub-problem. The objectives of each sub-problem were maximisation of the assigned lessons with urgency, minimisation of the required students for minimum capacity of each class, and maximisation of the assigned work volume.

Studenovský (2009) solved the university course timetabling problem in a survey and proposed a model in two sub-problems of accessible time scheduling and place scheduling. The model's assumption is that place scheduling is done based on room types and each room includes a group of similar rooms. In this problem, first, the problem of accessible time scheduling is done and then, place scheduling will be done using polynomial algorithm.

Causmaecker et al. (2009) presented a decomposed meta-heuristic approach for university course timetabling problem in a study and it was focused on reduction of interference in time periods. Several constraints including assignment of just one lecturer, student, and class in each time period, considering the required equipment for each lesson, non-interference of resources, and scheduling of all required courses were considered.

Zhang et al. (2010) presented a SA algorithm with a new neighbourhood structure to solve the university course timetabling problem. In this model, constraints such as non-interference of lecturers, classes, the time when lecturers can attend at the university, and succession of presenting courses for lecturers have been considered. In another study, Lü and Hao (2010) presented a model given the existing constraints for university course timetable and then solved it via tabu search algorithm. The constraints that were considered in this study included not presenting two courses simultaneously, limited capacity of classes, non-dispersion of courses in weekdays, and compression of lesson plans besides the constraint of program interference of lecturers and classes.

Soza et al. (2011) solved the university course timetabling problem in a study by means of traditional meta-heuristic algorithms. Their basic idea referred to how the data was extracted and it was solved through the optimisation algorithm and simulated annealing. The proposed model was based on programming for students and the constraints were more focused on students' program. They were non-interference of students' program, non-interference of classes, minimisation of holding classes at the final time period of each day, and minimisation of daily program of students with one lesson.

Wu (2011) presented an expert system and programming based on the constraints via the artificial intelligence approach to solve the problem of university course timetabling. Besides considering the common constraints of interference of lecturers and classes and class capacity, constraints such as compression of the presented courses, not allocating more than four courses to each lecturer in a day, and considering the required equipment of each course were considered.

Shiau (2011) proposed a meta-heuristic algorithm on the basis of PSO algorithm in his study. Considering the discrete space of answer in university course timetabling problem, thus, an algorithm was proposed for local search. Modelling in this study was based on the program of lecturers and students simultaneously. The objective was achieving optimal satisfaction for each lecturer and class based on their priority of arrangements. The constraints were non-interference of lecturers, the required equipment, the time for lunch and rest, non-interference of students' program, minimisation of students' displacement, and considering a certain model for 3-h courses.

Sabar et al. (2012) proposed a mathematical model in a study to determine university course timetable and solved it via bees algorithm. This model was based on students' program and the constraints were determined according to it that included non-interference of program of students and classes, capacity of classes, minimisation of presenting the courses at the final time interval of each day, minimisation of consecutive courses in students' program, and minimisation of days with only one class in lecturers' program.

Abdullah and Turabieh (2012) employed memetic algorithm on the basis of tabu search with the process of neighbourhood search to solve the university course timetabling problem. The constraints were presentation of all courses, not presenting two courses simultaneously, non-interference of the program of lecturers and classes, capacity of classes, compression of lesson plans, and presenting different courses in one class.

Gunawan et al. (2012) explored two sub-problems of course timetabling including assignment of lecturers and lessons' scheduling simultaneously in a study. To this end, they proposed a hybrid mathematical model. Also they presented a hybrid algorithm and a hybrid approach of Lagrange estimation to solve it. They used this approach because they could obtain a suitable primary answer by means of it. The constraints were non-interference of lecturers and classes, maximum number of courses for each lecturer and non-compression of lecturers' program.

Kaviani et al. (2013) proposed a mathematical model for university course timetabling and intended to reduce overlapping of lecturers' schedule. Moreover, they considered the constraints in the model. Then, they solved this model via LINGO software and a numerical example. The obtained results showed that the proposed model in this paper can cover educational needs of the university.

Cacchiani et al. (2013) represented a new method to calculate low limit for the problem of timetabling based on education program and assigned the courses and lecturers to classes using it. They presented the programming based on students' lesson plans and lecturers' program and considered constraints such as presenting all courses, non-interference of common courses of lesson plans, non-interference of the program of lecturers and classes and capacity of classes, compression of lesson plans, and minimisation of lecturers and students' displacement.

Basir et al. (2013) presented a mathematical model for university course timetabling and solved it via simulated annealing. The constraints in their study were non-interference of the program of students, lecturers and classes. In another study, Al-Yakoob and Sherali (2015) proposed a two-phase mixed integer programming model for the university course timetabling problem and tested the model using a numerical example of Kuwait's educational system. In the first phase of this model, weekly time periods were determined for classes and in the second phase, lecturers were assigned to classes.

Phillips et al. (2015) proposed a new integer programming model for university course timetable in a study that can be presented for several cases. They used the data of the University of Auckland to confirm validity of their model and the university courses were programmed using it.

Vermuytenet al. (2015) proposed an integer programming model for the university course timetable. Its major purpose was to minimise student flow. This model explores educational preferences in the first phase and then it assigns rooms to courses and minimises student flow in the second phase. The model was then executed for course timetabling at Brussels School of Economics.

In a study, Lewis and Thompson (2015) proposed a mathematical behaviour based on NP-Hard problems for the university course timetabling problem. They presented a two-phase algorithm that was focused on the connection of answer space of the problem; hence, the neighbourhood space will be improved to search the answer. According to the results, quality of the proposed answer has been improved using this algorithm.

Soria-Alcaraz et al. (2016) presented an algorithm based on local search repetition in a study. For this purpose, the university course timetabling problem was considered in two general states. It was solved by means of the proposed algorithm. Soft and hard constraints were considered in this problem. In hard constraints, non-interference of classes, students' program, and lecturers, and not presenting different courses simultaneously were mentioned. Soft constraints refer to not presenting the courses at the final time interval of each day, programming of not more than three courses in a day for students, not programming one course at each day for students, class capacity, compression of days of classes, and presenting all sections of each course in one class.

Bellio et al. (2016) explored university course timetabling problem in a survey. To this end, they used the simulated annealing. Their most important innovation had two sections. In the first innovation, they used the annealing simulation in a steady state to solve university course timetabling problem. Another innovation of them was calculation and production of the required parameters in simulated annealing.

Song et al. (2017) proposed an algorithm for university course timetabling via the approach of increased efficiency of *energy* consumption. In this algorithm, energy consumption of the university to prepare the university course timetable was focused. The purpose was to minimise energy consumption by the university. Experimental results of the proposed algorithm showed that the proposed timetable decreases 40% of heating and cooling energy comparing with the previous timetable.

Borchani et al. (2017) proposed an algorithm to solve university course timetabling problem. The purpose was to decrease the number of time slots in the university timetable as well as the number of single lessons in the timetable of each group of students. In this algorithm, eleven special neighbouring structures were considered. Six structures were taken into account to modify time slots and five structures were used to regulate the time of the selected lesson. This algorithm was implemented at a university in Tunisia and its practicality was tested. The obtained results revealed that the timetable has a better performance when more than 50% time slots are omitted.

As it is obvious, the university course timetabling problem contains a series of resources like lecturers, classes, equipment, and students that are assigned to the required courses according to the type of programming. Researchers have considered different constraints in their modelling that are shown briefly in Table 1.

Table 1 The constraints in previous studies

The author's name (year)	*Presenting all courses*	*Programming the courses for lecturers on the basis of their*	*Non-interference of lecturers' program*	*Considering the essential equipment for each course*	*Non-interference of the place where classes are held*	*Capacity of classes*	*Non-interference of students' program*	*Not holding two courses simultaneously*	*Preferring the faculty members to visiting lecturers*	*Quality of lecturers*	*Considering the capacity of lecturers for teaching*	*Compression of classes during a week*	*Not presenting the courses at the final time interval of each day*
Al-Yakoob and ….	✓		✓								✓		
MirHassani			✓				✓						
Head and …		✓	✓	✓			✓						

Table 1 The constraints in previous studies (continued)

The author's name (year)	*Presenting all courses*	*Programming the courses for lecturers on the basis of their*	*Non-interference of lecturers' program*	*Considering the essential equipment for each course*	*Non-interference of the place where classes are held*	*Capacity of classes*	*Non-interference of students' program*	*Not holding two courses simultaneously*	*Preferring the faculty members to visiting lecturers*	*Quality of lecturers*	*Considering the capacity of lecturers for teaching*	*Compression of classes during a week*	*Not presenting the courses at the final time interval of each day*
Beligiannis and …			✓		✓			✓			✓		
Studenovský					✓	✓	✓						
Causmaecker and …	✓		✓		✓		✓						
Zhang, and …		✓	✓		✓								
Lü, and …		✓	✓			✓	✓						
Soza, and …				✓	✓	✓	✓						
Wu			✓		✓	✓		✓					
Shiau		✓	✓	✓	✓		✓						
Sabar and …				✓	✓	✓	✓						
Abdullah and …	✓	✓	✓		✓	✓							
Gunawan and …	✓	✓	✓								✓		
Cacchiani and …	✓	✓	✓		✓	✓	✓						
Basir, N. and …			✓		✓		✓						
Al-Yakoob and …			✓								✓		
Soria-Alcaraz et al.			✓		✓		✓	✓				✓	✓

University course timetabling is one of the daily affairs of universities which is carried out during different educational terms. Given that it has a direct effect on lecturers and students' satisfaction, it will be highly important. On the other hand, university course timetable can be effective on education quality too. Various studies have already been carried out in this regard and several models have been proposed to prepare the university course timetable. But paying attention to education quality in the timetable has been considered less in these models. The distinctive point of this study with other related studies is preparing a university course timetable via the approach of enhancing educational quality.

As it can be observed in previous studies, researchers have paid attention to various constraints considering their own problem. If more constraints can be considered in a model, more appropriate results of timetabling will be obtained. There are other constraints for university course timetabling in the real world which can be investigated. Some of these constraints are as below:

- presenting all courses
- programming the courses for lecturers on the basis of their attendance at the university
- non-interference of lecturers' program
- non-interference of classes
- capacity of classes
- not holding two courses simultaneously
- preferring the faculty members to visiting lecturers
- quality of lecturers
- considering the capacity of lecturers for teaching.

Given the above constraints, a mathematical modelling is done that is mentioned below.

3 University course timetabling problem

By timetables, it is meant to create a scheduling program in which it is determined that each task must be done at which time slot and who should do it. The process of programming and timetabling is referred to as the problem of timetabling (Soria-Alcaraz et al., 2014).

The university course timetabling problem is a special type of timetabling that has been considered by various researchers since 1996. This problem contains an extensive range such as education (for instance lesson plan timetable), health institutions (for instance shift schedule for nurses), transportation (for instance train schedule), sports (for instance scheduling of races) and etc. The timetable includes programming for a group of resources which have constraints and are programmed to be assigned to a series of time periods. Various objectives are usually followed in such programming but these objectives are often directed to minimise the programming expenses (Basir et al., 2013). Some expressions that are normally used in university course timetabling problems are presented in Table 2.

Table 2 Basic expressions that are used in timetabling

Expressions	*Definitions*
Event	A programmed activity, for example lessons and tests
Empty time (period)	A time interval in which any event can be placed
Resource	The required resources for events, for example a class
Constraint	A constraint for programming of events, for example capacity of classes
Interference	If two events are assigned to one person or presented at one time, then they have interference with each other

Source: Abdullah (2006)

University course timetable is highly important for suitable and effective use of human resources and accessible space of universities. These human resources include students, lecturers, and employees while spatial resources of the university include classes, laboratories, workshops and so on. Generally speaking, the university course timetabling problem contains assignment of these resources to a class of time periods. In this programming, a group of constraints is considered according to the opinion of programmers, the university, etc. (Carter and Laporte, 1998).

This problem has been solved traditionally and manually by trial and error method but this method does not guarantee an optimal and authentic program and does not often consider some constraints. Another solution that many researchers have employed is innovative solutions for solving the above problem that deals with the assignment of a group of lecturers, classes, and different courses to accessible time periods by means of these methods. Various innovative algorithms have been used in different studies that were carried out to solve this problem among which Genetic Algorithm, Simulated Annealing, Tabu Search, graph colouring, optimisation algorithms, etc. can be mentioned (Basir et al., 2013).

In the proposed model in this study, some constraints with regard to the quality of educational planning have been considered besides the common constraints in university course timetabling models. Thus, 'educational need', 'educational place' and 'organising' are considered as elements of educational planning.

Educational need as one of the dimensions of quality of assessment of educational program is considered with two constraints:

- offering all groups of a course
- minimisation of the presented courses at the final period of each day.

With regard to educational place, two below constraints are considered:

- non-interference of the place of holding classes
- capacity of the place of holding classes.

With regard to the element of organising, the below constraint is considered:

- considering capacity of lecturers for teaching.

Besides these, the constraint of 'quality of lecturers' is taken into account to consider teaching quality of lecturers.

4 Mathematical model

For mathematical modelling of the university course timetabling problem, model constraints and conditions should first be considered in order to propose a model. These constraints and conditions are shown in Section 4. Indexes of the model are considered that are as Table 3.

Table 3 Indexes of the mathematical model of university course timetable

I	$i = \{1, 2, \ldots, I\}$	Set of courses
J	$j = \{1, 2, \ldots, J\}$	Set of days
K	$k = \{1, 2, \ldots, K\}$	Set of time periods in division of (midday) *r*th by day *j*th
M	$m = \{1, 2, \ldots, M\}$	Set of rooms
P	$p = \{1, 2, \ldots, P\}$	Set of lecturers
R	$r = \{1, 2, \ldots, P\}$	Divisions of each day (midday)
V	$v = \{1, 2, \ldots, V\}$	Number of units

Given the indexes and constraints, the parameters and variables are determined below in Table 4.

Table 4 Parameters and variables of the mathematical model of university course timetable

Parameters	
MK_{rv}	The number of time periods in daily divisions (midday) r for v-unit courses
MT_{pjrkv}	If lecturer p in day j, daily divisions (midday) r, period k is accessible for v-unit courses, it is equal to1; otherwise it is equal to zero.
CT_{pi}	If lecturer p can teach the course i, it is equal to 1; otherwise it is equal to zero.
C_m	Room capacity m
S_i	Maximum students who can take the course i
a_v	Number of units of a v-unit course
Ma_p	Maximum allowable unit number of lecturer p
Mi_p	Minimum allowable unit number of lecturer p
L_i	The required group from course i
Q_{pi}	Quality of lecturer p in course i
B_{iv}	If course i is in v-unit form, it is equal to 1; otherwise it is equal to zero.
Variables	
$C_{ijrkvmp}$	If v-unit course i in day j, midday r, period k is assigned to class m and lecturer p, it is equal to 1; otherwise it is equal to zero.

The mathematical model is proposed using the introduced indexes and parameters in Tables 2 and 3 that is as below. Each constraint and objective is explained in the following:

$$max = \frac{\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{v=1}^{V}\sum_{p=1}^{P} C_{ijrkvmp}}{\sum_{v=1}^{V} MK_{rv}} \quad \forall j, r, m \tag{1}$$

$$max = \frac{\sum_{i=1}^{I}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{v=1}^{V}\sum_{m=1}^{M} C_{ijrkvmp} * CT_{pi}}{\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{v=1}^{V} MT_{pjrkv}} \quad \forall j, p \tag{2}$$

$$min = \frac{C_m - \sum_{i=1}^{I}\sum_{v=1}^{V}\sum_{p=1}^{P} C_{ijrkvmp} * S_i}{C_m} \quad \forall j, r, k, m \tag{3}$$

$$max = \frac{\sum_{i=1}^{I}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{v=1}^{V} C_{ijrkvmp} * CT_{pi}}{\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{v=1}^{V} MT_{pjrkv}} \quad \forall j, m, p \tag{4}$$

$$max = \left(\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{v=1}^{V}\sum_{m=1}^{M}\sum_{p=1}^{n} C_{ijrkvmp} * CT_{pi}\right) - \left(\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{v=1}^{V}\sum_{m=1}^{M}\sum_{p=n+1}^{P} C_{ijrkvmp} * CT_{pi}\right) \tag{5}$$

$$max = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{v=1}^{V}\sum_{m=1}^{M}\sum_{p=1}^{P} C_{ijrkvmp} * Q_{pi}}{\sum_{i=1}^{I}\sum_{p=1}^{P} Q_{pi}} \tag{6}$$

s.t

$$\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{v=1}^{V}\sum_{m=1}^{M}\sum_{p=1}^{P} B_{iv} * C_{ijrkvmp} = L_i \quad \forall i \tag{7}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{v=1}^{V}\sum_{m=1}^{M} C_{ijrkvmp} * CT_{pi} * a_v \le Ma_p \quad \forall p \tag{8}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{v=1}^{V}\sum_{m=1}^{M} C_{ijrkvmp} * CT_{pi} * a_v \ge Mi_p \quad \forall p \tag{9}$$

$$\sum_{i=1}^{I}\sum_{v=1}^{V}\sum_{m=1}^{M} C_{ijrkvmp} * CT_{pi} \le 1 \quad \forall j, r, k, p \tag{10}$$

$$\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{p=1}^{P} \left(C_{ijrkvmp} + C_{ijrkv'mp}\right) \le 1 \quad \forall r, j, v, v', m \quad v \ne v' \tag{11}$$

$$\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{p=1}^{P}C_{ijrkvmp} \leq MK_{rv} \quad \forall\, m, j, r, v \tag{12}$$

$$C_{ijrkvmp} = 0 \quad \forall\, i, j, r, m, p \quad v \geq 2\ ; k > 2 \tag{13}$$

$$S_i * C_{ijrkvmp} \leq C_m \quad \forall\, i, j, r, k, v, m, p \tag{14}$$

$$\sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{v=1}^{V}C_{ijrkvmp} \leq 1 \quad \forall\, i, j, r, k \tag{15}$$

$$\sum_{i=1}^{I}\sum_{m=1}^{M}C_{ijrkvmp} \leq MT_{pjrkv} \quad \forall\, j, r, k, v, p \tag{16}$$

$$C_{ijrkvmp} \leq B_{iv} \quad \forall\, i, j, r, k, v, m, p \tag{17}$$

Objective 1: Maximum use of the existing rooms in each division (midday) shall be done every day. In other words, maximum use of every room shall be done every day and midday.

Objective 2: Maximisation of the use of a lecturer is in days and periods of his/her attendance at the university. In other words, it is tried to minimise lecturers' idle hours in a day.

Objective 3: Minimisation of surplus capacity of rooms is done in a way that those courses whose capacity is more than that of the room are prevented to be presented.

Objective 4: Maximisation of use of a room is done by a lecturer. In other words, this relation helps minimisation of lecturers' displacement among the rooms.

Objective 5: Maximisation of the faculty members is done; that is, given that lecturers *n* are the faculty members, thus relation 5 helps assign specialised courses more to the faculty members.

Objective 6: Maximisation of quality of the assigned lecturers to courses is done; that is, this relation shows that the courses are assigned to higher quality lecturers in that course.

Constraint 7: All required courses must be assigned.

Constraint 8: The allowable unit of each lecturer in every week must be observed; that is, each lecturer is allowed to teach Ma_p units maximally every week (term).

Constraint 9: The allowable unit of each lecturer in every week must be observed; that is, each lecturer is allowed to teach Mi_p units minimally every week (term).

Constraint 10: Each lecturer shall be assigned to just one course in each time period in every division (midday).

Constraint 11: In each division of the day (midday) only one of the v-unit courses shall be assigned to each class.

Constraint 12: In each division (midday) of the day and in each room, only v-unit courses shall be assigned to the number of v-unit courses maximally.

Constraint 13: According to this relation for courses that their units are equal to 3 or more than it, no programming should be done for the periods after the second one in midday. For example, for 3-unit courses that it is only possible to present two 3-unit courses in every class, no lesson will be presented in the third period ($C_{ijr32mp} = 0$). Also for 4-unit courses and higher than it that can just be presented in each midday, no course will be presented in the second period and after it ($C_{ijr23mp} = 0$ and $C_{ijr33mp} = 0$) ($v = 2$ for 3-unit courses and v = 3 for 4-unit courses).

Constraint 14: The number of assigned students to each class should not be more than the class capacity.

Constraint 15: One class and one lecturer should maximally be assigned to each course.

Constraint 16: In each day *j*, midday *r*, time period *k* and for v-unit courses, courses should be assigned to the lecturers who attend at the university.

Constraint 17: Course *i* in day *j*, midday *r*, time period *k* in class *m* is presented by lecturer *p* if the course i is in v-unit form.

As it has been shown in the above model, the number of courses, capacity of lecturers for teaching, class capacity, preferring the faculty members to visiting lecturers, and quality of lecturers who teach each course were considered in the proposed mathematical model.

5 Numerical example

In this step, the mathematical model is solved by means of a numerical example. First a numerical example is designed which has 5 courses including two types of v-unit courses as well as 4 educational days that have 2 middays and 3 time periods in each midday *r*, 3 classrooms, and 4 lecturers. The information is briefly shown in Table 5.

Table 5 Briefly information of numerical example

I	Set of courses	5
J	Set of days	4
K	Set of time periods in division of (midday) *r*th by day *j*th	3
M	Set of rooms	3
P	Set of lecturers	4
R	Divisions of each day (midday)	2
V	Number of units	2

Now parameters of the numerical example are determined. To this end, the model parameters for courses, lecturers, classes, and so on are introduced in Tables 6 and 7.

Given the parameters of the problem and by means of Lingo software, the model is solved. The results including the university course timetable are displayed in Table 8.

Table 6 Parameters of numerical example

				CT_{pi}			
				I			
		1	*2*	*3*	*4*	*5*	
p	1	1	0	1	0	0	
	2	0	0	1	0	1	
	3	0	1	0	1	0	
	4	1	1	0	0	1	
			Q_{pi}				
				I			
		1	2	3	4	5	
p	1	0.84		0.82			
	2			0.78		0.87	
	3		0.65		0.9		
	4	0.72	0.8			0.75	
	MK_{rv}			Ma_p		Mi_p	
		V		Ma(1)	12	Mi(1)	8
		1	2	Ma(2)	18	Mi(2)	12
R	1	3	2	Ma(3)	12	Mi(3)	8
	2	3	2	Ma(4)	16	Mi(4)	10
	B_{iv}			C_m		a_v	
				C(1)	35	a(1)	2
		V		C(2)	40	a(2)	3
		1	2	C(3)	30	L_i	
I	1	1	0	S_i		L(1)	5
	2	0	1	S(1)	35	L(2)	2
	3	1	0	S(2)	32	L(3)	9
	4	0	1	S(3)	28	L(4)	4
	5	0	1	S(4)	40	L(5)	2
				S(5)	30		

Table 7 Continuation of parameters of numerical example

								MT_{pjrkv}			
								P			
								1	*2*	*3*	*4*
J	1	r	1	*v*	1	*k*	1	1	0	1	0
							2	1	0	1	0
							3	1	0	1	0
					2	*k*	1	1	0	1	0
							2	1	0	1	0
							3	0	0	0	0
			2	*v*	1	*k*	1	0	1	0	1
							2	0	1	0	1
							3	0	1	0	1
					2	*k*	1	0	1	0	1
							2	0	1	0	1
							3	0	0	0	0
	2	r	1	*v*	1	*k*	1	0	1	1	1
							2	0	1	1	1
							3	0	1	1	1
					2	*k*	1	0	1	1	1
							2	0	1	1	1
							3	0	0	0	0
			2	*v*	1	*k*	1	1	0	0	0
							2	1	0	0	0
							3	1	0	0	0
					2	*k*	1	1	0	0	0
							2	1	0	0	0
							3	0	0	0	0
	3	r	1	*v*	1	*k*	1	1	1	1	0
							2	1	1	1	0
							3	1	1	1	0
					2	*k*	1	1	1	1	0
							2	1	1	1	0
							3	0	0	0	0
			2	*v*	1	*k*	1	0	0	0	1
							2	0	0	0	1
							3	0	0	0	1
					2	*k*	1	0	0	0	1
							2	0	0	0	1
							3	0	0	0	0

Table 7 Continuation of parameters of numerical example (continued)

MT_{pjrkv}										
							P			
							1	*2*	*3*	*4*
4	r	1	*v*	1	*k*	1	0	1	0	1
						2	0	1	0	1
						3	0	1	0	1
				2	*k*	1	0	1	0	1
						2	0	1	0	1
						3	0	0	0	0
		2	*v*	1	*k*	1	1	0	1	0
						2	1	0	1	0
						3	1	0	1	0
				2	*k*	1	1	0	1	0
						2	1	0	1	0
						3	0	0	0	0

Table 8 University course timetabling problem (see online version for colours)

Day			*Morning (r = 1)*										*Afternoon (r = 2)*								
$j = 1$	$v = 1$	Section		1			2			3				1			2			3	
		Room	1	2	3	1	2	3	1	2	3		1	2	3	1	2	3	1	2	3
		Courses											1	3							3
		Lecturers											1	2							1
	$v = 2$	Section		1			2							1			2				
		Room	1	2	3	1	2	3					1	2	3	1	2	3			
		Courses	2					5													
		Lecturers	4					2													
$j = 2$	$v = 1$	Section		1			2			3				1			2			3	
		Room	1	2	3	1	2	3	1	2	3		1	2	3	1	2	3	1	2	3
		Courses			3	1				3			1								
		Lecturers			1	1				1			1								
	$v = 2$	Section		1			2							1			2				
		Room	1	2	3	1	2	3					1	2	3	1	2	3			
		Courses												2	5						
		Lecturers												4	2						

Table 8 University course timetabling problem (see online version for colours) (continued)

Day			*Morning (r = 1)*									*Afternoon (r = 2)*								
$j = 3$	$v = 1$	Section		1			2			3			1			2			3	
		Room	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
		Courses	3								3						3	1		
		Lecturers	2								2						2	4		
	$v = 2$	Section		1			2						1			2				
		Room	1	2	3	1	2	3				1	2	3	1	2	3			
		Courses		4									4							
		Lecturers		1									3							
$j = 4$	$v = 1$	Section		1			2			3			1			2			3	
		Room	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
		Courses			3				3			1								
		Lecturers			2				2			4								
	$v = 2$	Section		1			2						1			2				
		Room	1	2	3	1	2	3				1	2	3	1	2	3			
		Courses					4						4							
		Lecturers					3						3							

6 Conclusion

The scheduling problem is one of the existing problems in different jobs and activities and is fulfilled via various methods. Scheduling in educational institutions is one of the effective factors on improvement of the education level. For this reason, several researchers have considered this in previous years and have tried to present various methods for it. One of the mostly considered methods is mathematical modelling that researchers have proposed different models considering type of the problem and their own constraints in the educational environment under study.

Having studied the research literature in this study, a list of constraints was identified and modelling was conducted via these constraints. The major novelty of this study is that many constraints that were employed by previous researchers were considered and modelling was carried out based on them. Then a numerical example was used to solve the model and coding of the mathematical model was performed via Lingo software.

In the proposed model in this study, constraints including presenting all courses, programming the courses for lecturers on the basis of their attendance at the university, non-interference of lecturers' program, non-interference of the place where classes are held, class capacity, not holding two courses simultaneously, preferring the faculty members to visiting lecturers, quality of lecturers, considering the lecturers' capacity for teaching in the form of soft and hard constraints were taken into account. Using these constraints, the proposed model becomes closer to existing reality in educational systems of universities. The obtained results revealed that a suitable university course timetable can be achieved by means of the proposed model in this study with enhanced quality of programming.

Given that the proposed model has a feasible answer, it can be claimed that this model responds the universities' need for the university course timetabling in various courses in a specific case study.

As it was mentioned earlier, several models have been proposed to determine university course timetable but the most important innovation of the proposed model in this study is to consider quality of course planning. Dimensions of educational quality are quality of lecturers in presenting each course and not holding any class during final hours of a day.

Future researchers can pay attention to constraints such as prerequisite and post-requisite relations. On the other hand, attendance of at least one lecturer for counselling in the university can be programmed which will enhance educational quality. One of the problems of the current study is that definite solution of the model may not be possible in applied cases with the increase of days, classes, and courses and hence, researchers can solve it through other methods such as meta-heuristic algorithms like Tabu Search, simulated annealing, Genetic Algorithm, etc.

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