Active vibration control of an elevator system using magnetorheological damper actuator

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Abstract: We investigated the horizontal response of a vertical transportation with nonlinearities under excitation by guide rail deformations. A LQR control strategy was used in order to improve the comfort of passengers. To this end, a magnetorheological damper (MR damper) was used. The control force of the damper is a function of the voltage applied in the coil of the MR damper that is based on the force given by the controller. Numerical simulations were performed to investigate the nonlinear behaviour of the adopted mathematical model. Moreover, other issues such as robustness of the control technique were evaluated considering parametric errors and noise measurement. The results show that the LQR control strategy using MR damper can be effective in reducing the vibration of the vertical transport.

Keywords: optimal feedback control; vertical transportation; MR damper; high-speed elevator; parametric uncertainties; wavelet-based scale index; nonlinear model; chaos; LQR control; nonlinear dynamics.

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1 Introduction

Nowadays, due to construction of high buildings, it is necessary the design of high-speed elevators, however, high speeds can result in decreases of quality of ride (Mitsui and Nara, 1971), this being one of the main problems in a high-speed elevator system (Nai et al., 1994). Thus, it is necessary to improve the travel quality, without losing the efficiency of elevator where there are limits on levels of horizontal and vertical vibrations, lateral and longitudinal acceleration and jerk, necessary to ensure good ride quality to passengers (Fortune, 1997). Kaczmarczyk and Picton (2013) used a traction drive elevator systems with long ropes and cables in order to analyse the high-rise structures. Benosman and Fukui (2014) have studied numerically the problem of elevator rope with sway motion due to external force disturbances through an active control, by using nonlinear controllers based on Lyapunov theory, to stabilise the rope sway dynamics. A model for transversal vibrations of an elevator cable system is studied in Sandilo and van Horssen (2015). Chang et al. (2011) investigated a high-speed elevator system in order to examine the characteristics of the excitations and analyse the dynamic responses due to horizontal vibration generated from the elevator wheels running on rough and winding guide rails. Yang et al. (2014) proposed an active control with time variant states using correlation filtered-X least mean square (co-FXLMS) algorithm and moving band pass filter (MBPF) has been proposed to control the cabin noise of the high-speed elevator. Arrasate et al. (2014) conducted a study of vertical vibrations caused by torque ripple generated at the elevator drive system and its influence in passenger comfort during an elevator travel. In Venkatesh et al. (2002) is presented a methodology for designing high-performance of the linear time-invariant (LTI) controller by elevator vertical motion for high-rise buildings with high-speed.

Motivated by necessity to improve passenger's comfort level, this work presents a form of the controlling the dynamic horizontal behaviour of a three degrees of freedom model of a vertical transportation system excited through guide rail deformations by a controlled magnetorheological (MR) damper.

MR dampers, are widely used in the modern industry nowadays, and has capable of generating a force sufficient for rapid response in various applications (Truong and Ahn, 2011; Spelta et al., 2009). The magnetic properties permit its use as a damper, controlled by an electrical current (Tusset et al., 2009). Additionally, these devices offer highly reliable operations and their performance is relatively insensitive to temperature fluctuations or impurities in the fluid (Carlson and Weiss, 1994).

The MR damper can be used in the suppression of unwanted oscillations, which it can be controlled through the electrical current or voltage, which changes the viscosity of the fluid's internal damper. The damping force will depend on the speed of the piston of the damper and the density of internal fluid.

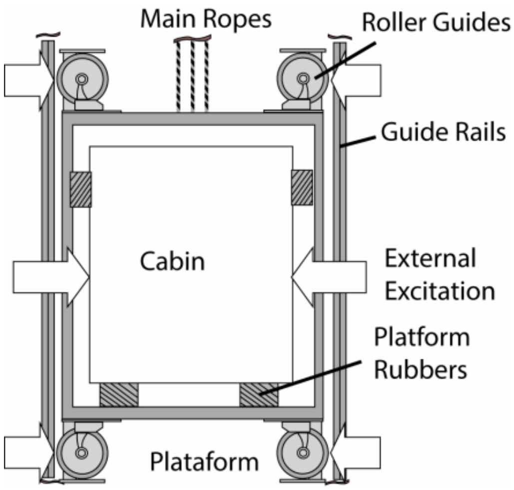
The feedback of state variables by means of sensors can provide information about the behaviour of the controlled system, over time and thereby on the optimal intensity of force being applied by the MR damper. Here, we will utilise a mathematical model to transform the values of control force in electric current signal, this model was proposed by Tusset et al. (2012).

The control strategy proposed in this paper contributes to the research in modelling and control of dynamic systems through the numerical display of control that uses an estimation strategy of the control signal and another technique for converting this control signal into an electrical signal that is applied on the physical actuator, it contributes directly to practical applications as the conversion of the estimated damping force signal through the application of optimal control (LQR) into electrical current signal enabling its application in the MR damper because the force control damping depends on the electric current applied to its coil.

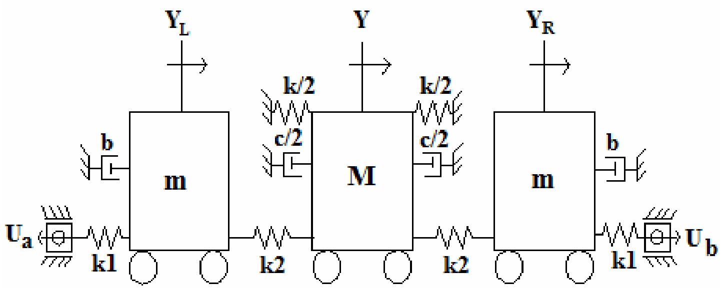
2 Mathematical modelling

The schematic diagram of the cabin elevator is shown in Figure 1(a). The system consists of a roller guide supported by a platform with springs to avoid the transmission of the external excitation caused by misalignment and deformation of guide rails. Figure 1(b) shows an equivalent physical model to represent the horizontal motions of an elevator system.

Figure 1 (a) Schematic diagram of cabin structure (b) Equivalent model for the horizontal motion of the elevator (see online version for colours)



(a)



(b)

Source: (a) Adapted from Funai (2004); (b) Adapted from Lopez et al. (2010)

The system consists of a mass M related to the cabin [kg], m is the mass of the suspension system [kg], b is the linear damping coefficient of the suspension [N.s/m], c is the linear damping coefficient of the cabin [N.s/m], k_1 is the stiffness coefficient of the guide rollers [N/m], k_2 is the stiffness coefficient of the suspension [N/m], k is the stiffness coefficient of the spring equivalent to tilting motion of the cabin [N/m], Y is the displacement of the cabin [m], Y_L is the displacement of the left suspension system [m], Y_R is the displacement of the right suspension system [m], and U_a and U_b are external excitations caused by guide rails deformations, defined as follows:

$$U_a = U_b = a \sin(\omega t) \tag{1}$$

where a and ω are the amplitude [m] and the external excitation frequency [rad/s] of the forced oscillation, respectively.

The equations of motion that represent the vertical transportation system can be formulated by considering the balance of forces acting on the masses as follows:

$$\begin{aligned} &m\ddot{Y}_L + b\dot{Y}_L + (k_1 + k_2)Y_L - k_2 Y = k_1 U_a \\ &M\ddot{Y} + c\dot{Y} + 2k_2 Y + kY^3 - k_2 Y_L - k_2 Y_R = 0 \\ &m\ddot{Y}_R + b\dot{Y}_R + (k_1 + k_2)Y_R - k_2 Y = k_1 U_b \end{aligned} \tag{2}$$

The dimensional system of equations for the vertical transportation system can be written in state space form in the following form:

$$\begin{aligned} &\dot{x}_1 = x_2 \\ &\dot{x}_2 = -\alpha_2 x_1 - \alpha_1 x_2 + \alpha_3 x_3 + \alpha_4 U_a \\ &\dot{x}_3 = x_4 \\ &\dot{x}_4 = \beta_3 x_1 - \beta_2 x_3 - \beta_5 x_3^3 - \beta_1 x_4 + \beta_4 x_5 \\ &\dot{x}_5 = x_6 \\ &\dot{x}_6 = \alpha_3 x_3 - \alpha_2 x_5 - \alpha_1 x_6 + \alpha_4 U_b \end{aligned} \tag{3}$$

where the system's parameters are defined as follows:

$$x_1 = Y_L,\ x_2 = \dot{Y}_L,\ x_3 = Y,\ x_4 = \dot{Y},\ x_5 = Y_R,\ x_6 = \dot{Y}_R,\ \alpha_1 = \frac{b}{m},\ \alpha_2 = \frac{(k_1 + k_2)}{m},$$
$$\alpha_3 = \frac{k_2}{m},\ \alpha_4 = \frac{k_1}{m},\ \beta_1 = \frac{c}{M},\ \beta_2 = \frac{2k_2}{M},\ \beta_3 = \frac{k_2}{M},\ \beta_4 = \frac{k_2}{M},\ \beta_5 = \frac{k}{M}.$$

3 A wavelet-based scale index

In recent years some classical systems, such as the logistic map, Henon map, Bonhoeffer-van der Pol oscillator (Benítez et al., 2010), Rössler system (Akhshani et al., 2014), SMA oscillators (Piccirillo et al., 2016) and the heartbeat dynamics (Behnia et al., 2013) have been analysed by the Scale index.

The wavelet transform of a one-dimensional (1D) signal consists of the development into a basis constructed via solutions like functions called wavelet, using various internal transformations and shifts (Awrejcewicz et al., 2009). Given $f \in L^2(\mathbb{R})$, the continuous wavelet transform (CWT) of f at time v, scale s and time location t is defined as

$$Wf(v,s) := \langle f, \psi_{v,s}^* \rangle = \int_{-\infty}^{+\infty} f(t)\psi_{v,s}^*(t)\,dt \tag{4}$$

where

$$\psi_{v,s} = \frac{1}{\sqrt{s}}\psi\left(\frac{t-v}{s}\right), v \in \mathbb{R}, s > 0 \tag{5}$$

and $Wf(v, s)$ provides the frequency component of the signal of f at time v and scale s with respect to some analysing wavelet $\psi_{v,s}$.

The scalogram of f, $\wp$, is defined as follows (Benítez et al., 2010):

$$\wp(s) := \left\| Wf(v, s) \right\| = \left(\int_{-\infty}^{+\infty} \left| Wf(v, s) \right|^2 dv \right)^{1/2} \tag{6}$$

$\wp(s)$ is the energy of the CWT of f at scale s. The scalogram is a useful tool for studying a signal, since it allows the detection of its most representative scales (or frequencies). Then, the innerscalogram of f at scale s can be defined by (Benítez et al., 2010):

$$\wp^{inner}(s) := \left\| Wf(v, s) \right\|_{J(s)} = \left(\int_{c(s)}^{d(s)} \left| Wf(v, s) \right|^2 dv \right)^{1/2} \tag{7}$$

where $J(s) = [c(s), d(s)] \subseteq I$ is the maximal subinterval in I for which the supported of $\psi_{u,s}$ is included in I for all $u \in J(s)$ (Benítez et al., 2010). As regards the length of $J(s)$ depends on the scale s, so that the values of the inner scalogram at different scales cannot be com-pared. Therefore, the inner scalogram should be normalised as follows (Benítez et al., 2010):

$$\overline{\wp}^{inner}(s) = \frac{\wp^{inner}(s)}{\left(d(s) - c(s) \right)^{1/2}} \tag{8}$$

The Scale index in the scale interval $[s_0, s_1]$ can be defined by the quotient (Benítez et al., 2010)

$$i_{scale} := \frac{\wp(s_{\min})}{\wp(s_{\max})} \tag{9}$$

where $s_{\max}$ is the smallest scale such that $\wp(s) \leq \wp(s_{\max})$ for all $s \in [s_0, s_1]$, and $s_{\min}$ the smallest scale such that $\wp(s_{\min}) \leq \wp(s)$. Note that for compactly supported signals only the normalised inner scalogram will be considered (Benítez et al., 2010).

From its definition, the scale index i_{scale} is such that $0 \leq i_{scale} \leq 1$ and it can be interpreted as a measure of the degree of non-periodicity of the signal: the scale index will be zero or close to zero for periodic sequences and close to one for highly non-periodic sequences (Benítez et al., 2010).

3.1 Numerical simulation

The parameters used in numerical simulations were experimentally obtained in Lopez et al. (2010), and are given in Table 1.

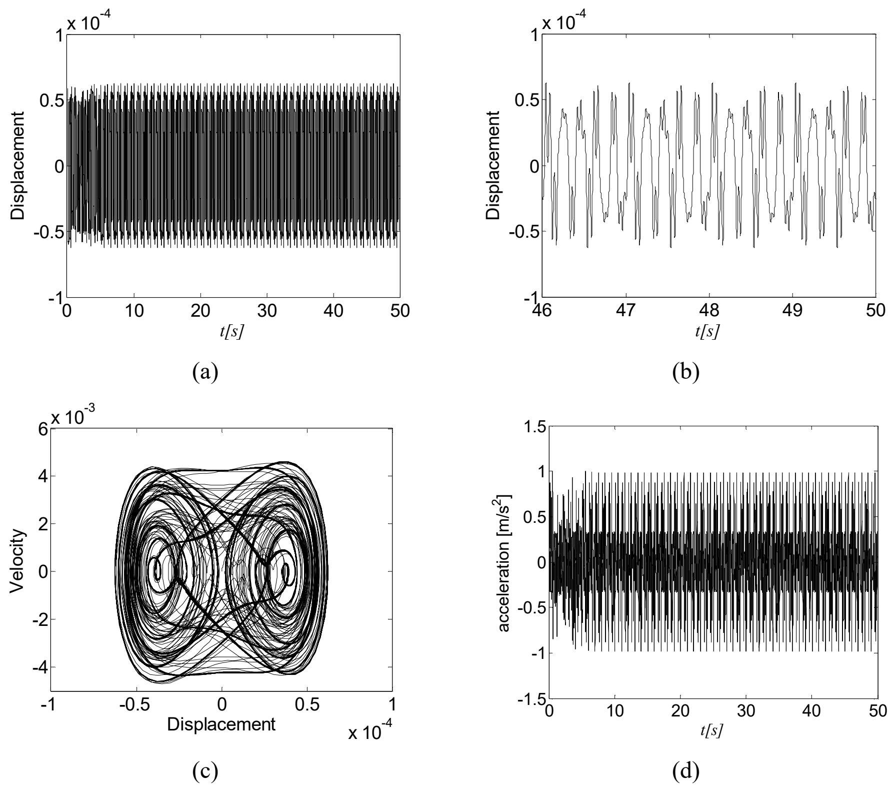
Table 1 Parameters for numerical simulations

Parameters	*Unit*	*Description*	*Value*
M	kg	Mass of the cabin	1,120
m	kg	Mass of the suspension system	17.5
b	N.s/m	Damping coefficient of the suspension	668.21
c	N.s/m	Damping coefficient of the cabin	2,058.2
k_1	N/m	Stiffness coefficient of the guide rollers	250,000
k_2	N/m	Stiffness coefficient of the suspension	19,027
k	N/m	Stiffness coefficient of the spring equivalent to tilting motion of the cabin	19,027
a	m	External excitation amplitude	0.01
ω	rad/s	External excitation frequency	31.4159

Source: Lopez et al. (2010) and Santo et al. (2014)

Figure 2(a) shows the time history of the displacement of the cabin, and we can see in Figure 2(b) an irregular structure for a magnification of Figure 2(a), for the parameters shown in the table.

Figure 2 (a, b) Time history of the displacement (x_3) (c) Phase plane (x_3 and x_4) (d) Time history of acceleration ($\dot{x}_4$)



Here, scale index was obtained using the Daubechies eight-wavelet (db8) function and the scale interval from s_0 = 1 to s_1 = 512. As can be seen from the figure that the displacement of (x_3), has a non-periodic behaviour comproved by the index scale (i_{scale} = 0.1134), and the acceleration which passengers are exposed is close to 1.02[m/s^2].

As proposed by Griffin (1990), the peak acceleration values between 0.01[m/s^2] and 10[m/s^2] can cause some discomfort to the human body. After analysing the results obtained in Figure 2 we can see that the peak value is within the range of interest of vibration on the human body.

3.2 Analysis of the level of exposition to vibration

Griffin (1990), suggests that the level of exposure to vibration can be estimated by calculating the root mean square (rms):

$$rms = \sqrt{\left(\frac{1}{N}\sum X_i^2\right)} \tag{10}$$

On the other hand, for the evaluation of the severity of all movements (deterministic or random, stationary or non-stationary, transient or shock), the standard (ISO 2631, 1997), suggests adopt vibration dose value (VDV) given by equation (11), which takes into account the relationship between the magnitude and duration of all the periods.

$$VDV = \left(\frac{T}{N}\sum_{i=1}^{N} a_w{}^4\right)^{1/4} \tag{11}$$

where:

VDV = vibration dose value [$m/s^{1.75}$]

a_w = compensated acceleration [m/s^2]

T = total period of exposure [s].

The compensated acceleration can be obtained from equation (12):

$$a_w = \left[\sum_i (W_d a_i)^2\right]^{\frac{1}{2}} \tag{12}$$

where: W_d is the compensating factor for lateral displacements, a_i is the acceleration in [rms] for each variation of the excitation frequency. The weights of W_d for different frequencies can be obtained from Table 2.

Considering equation (4) for T = 50[s] and N = 5,000, by applying this procedure we obtain an acceleration in rms $\left(\ddot{x}_{3rms} = 0.2714[m/s^2]\right)$, with an (VDV = 0.0351[$m/s^{1.75}$]) given by equation (5), for a frequency of 5 [Hz] given by Table 1 (ω= 31.4159[rad/s]).

Table 2 Compensation factors *Wd*

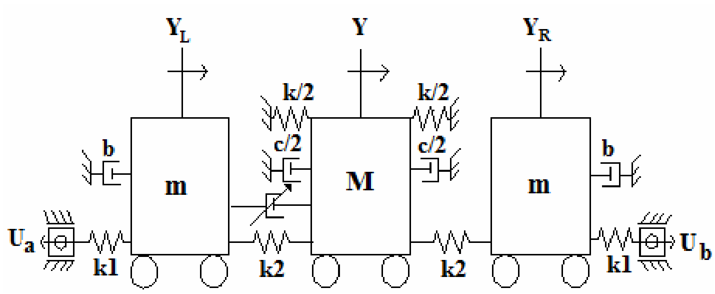
Frequency [Hz]	W_d *(×10*^{3}*)*	*Frequency [Hz]*	W_d *(×10*^{3}*)*
1	1,011	10	212
1.25	1,008	12, 5	161
1.6	968	16	125
2	890	20	100
2.5	776	25	80
3.15	642	31, 5	63.2
4	512	40	49.4
5	409	50	38.8
6.3	323	63	29.5
8	253	80	21.1

Source: ISO 2631-1 (1997)

4 Reduction of the level of exposition to vibration by MR damper

The interest in use of MR dampers in vibrational systems is take advantage their ability to dissipate energy in order to control the dynamic behaviour of the system. The aim of the present study is to show the potential of MR damper as a actuator to reduction the vibration of the cabin of the elevator. Consider now the introduction of the MR damper in parallel with the spring roller guides as shown in Figure 3.

Figure 3 Equivalent model for the horizontal motion of the elevator with MR damper



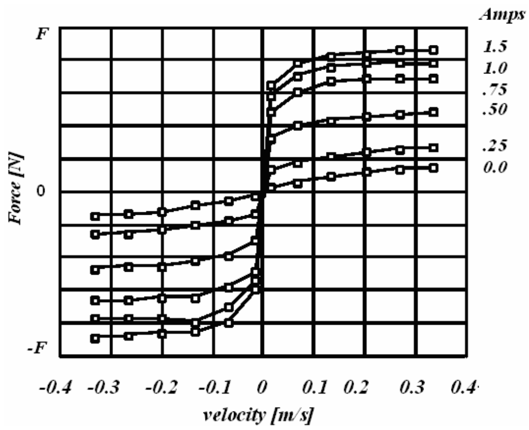
The system of equation (3) with the introduction of active control can be represented by the following system:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\alpha_2 x_1 - \alpha_1 x_2 + \alpha_3 x_3 + \alpha_4 U_a + \delta_1 u \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \beta_3 x_1 - \beta_2 x_3 - \beta_5 x_3^3 - \beta_1 x_4 + \beta_4 x_5 - \delta_2 u \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= \alpha_3 x_3 - \alpha_2 x_5 - \alpha_1 x_6 + \alpha_4 U_b
\end{aligned}
\tag{13}
$$

where u represent the MR damper force, and $\delta_1 = \frac{1}{m}$ and $\delta_2 = \frac{1}{M}$.

Figure 4 shows the characteristic curves of the force of the damper in function of both velocity and electric current applied to the damper coil, it is possible to note that this force unit is [N], (i) is the electric current [A] and v is the velocity of the displacement of the piston of the damper [m/s] (McManus et al., 2002; Tusset et al., 2012).

Figure 4 Characteristics of the normalised force-velocity of a MR damper in function of the current



Source: Tusset et al. (2012)

We have considered that the force is normalised and the velocity will be bounded on $v \leq 0.4$ [m/s], and the electrical current to be applied can be determined by solving numerically the following function:

$$C = \begin{cases} f(i)\left|v\right|^{-g(i)} \tanh(0.43v) - F, \mathit{if}\ v \neq 0 \\ 0, \mathit{if}\ v \approx 0 \end{cases} \tag{14}$$

where: C is the electrical current applied in the damper coil, F is the estimate force developed by the MR damper, and $f(i)$ and $g(i)$ can be written as follows (Tusset et al., 2012):

$$f(i) = \frac{2.6}{\left(3.7058e^{-3.2934i}\right) + 1} \tag{15}$$

$$g(i) = \frac{0.9}{\left(1.1548e^{-6.8239i}\right) + 1} \tag{16}$$

5 Force determination of the MR damper by feedback control

The corresponding matrix form of system (13) is given by as follows:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{GU} \tag{17}$$

where:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\alpha_2 & -\alpha_1 & \alpha_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \beta_3 & 0 & -\beta_2 & -\beta_1 & \beta_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \alpha_3 & 0 & -\alpha_2 & -\alpha_1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ \delta_1 \\ 0 \\ -\delta_2 \\ 0 \\ 0 \end{bmatrix} \text{ and } \mathbf{G} = \begin{bmatrix} 0 \\ \alpha_4 U_a \\ 0 \\ -\beta_5 x_3^2 \\ 0 \\ \alpha_4 U_b \end{bmatrix}$$

The damping force control can be obtained through the LQR control, given by:

$$\mathbf{u} = -\mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{Px} \tag{18}$$

where Q and R are positive definite matrices, the matrix P is obtained solving the Riccati equation given by:

$$\mathbf{PA} + \mathbf{A}^T\mathbf{P} - \mathbf{PBR}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = \mathbf{0} \tag{19}$$

so that the feedback of the force output (18) is able to minimise the performance index J as following:

$$J = \frac{1}{2}\int_{t_0}^{\infty}\left[\mathbf{x}^T\mathbf{Qx} + \mathbf{u}^T\mathbf{Ru}\right]dt \tag{20}$$

Therefore, the minimisation of the functional (20) implies the minimisation of the states x, and the force (u) applied by MR damper. The control signal u is determined using the matrices A and B, and we define the positives definite matrices Q and R, as follows;

$$\mathbf{Q} = 10^5 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1000000 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{R} = 10^{-1}$$

Substituting the matrices A, B, Q and R in equation (10), we obtain the matrix P:

$$\mathbf{p} = \begin{bmatrix} 1.9048(10^7) & 784.5849 & 1.9345(10^6) & 8.6144(10^5) & 8.0549(10^4) & -1.5830(10^4) \\ 784.5849 & 1.3293(10^3) & 7.6146(10^4) & 8.2427(10^4) & 1.9614(10^4) & 50.5085 \\ 1.9345(10^6) & 7.6146(10^4) & 1.0018(10^{11}) & 1.1260(10^8) & 1.6852(10^6) & 640.4774 \\ 8.6144(10^5) & 8.2427(10^4) & 1.1260(10^8) & 1.1716(10^8) & 3.3662(10^6) & 6.7063(10^3) \\ 8.0549(10^4) & 1.9614(10^4) & 1.6852(10^6) & 3.3662(10^6) & 2.1187(10^7) & 2.5678(10^3) \\ -1.5830(10^4) & 50.5085 & 640.4774 & 6.7063(10^3) & 2.5678(10^3) & 1.3755(10^3) \end{bmatrix}$$

By substituting the matrices R, B and P into the equation (18), we obtain

$$u = 7243.1294x_1 - 23.6365x_2 + 961849.1073x_3 + 998972.4480x_4 + 18847.0373x_5 + 31.0160x_6 \tag{21}$$

Figures 5 and 6 show the behaviour of the elevator cabin, considering the proposed control.

Figure 5 Comparison between uncontrolled and controlled cabin response, (a) (b) history of the displacement (x_3) (c) phase plane ($x_3 \times x_4$) (see online version for colours)

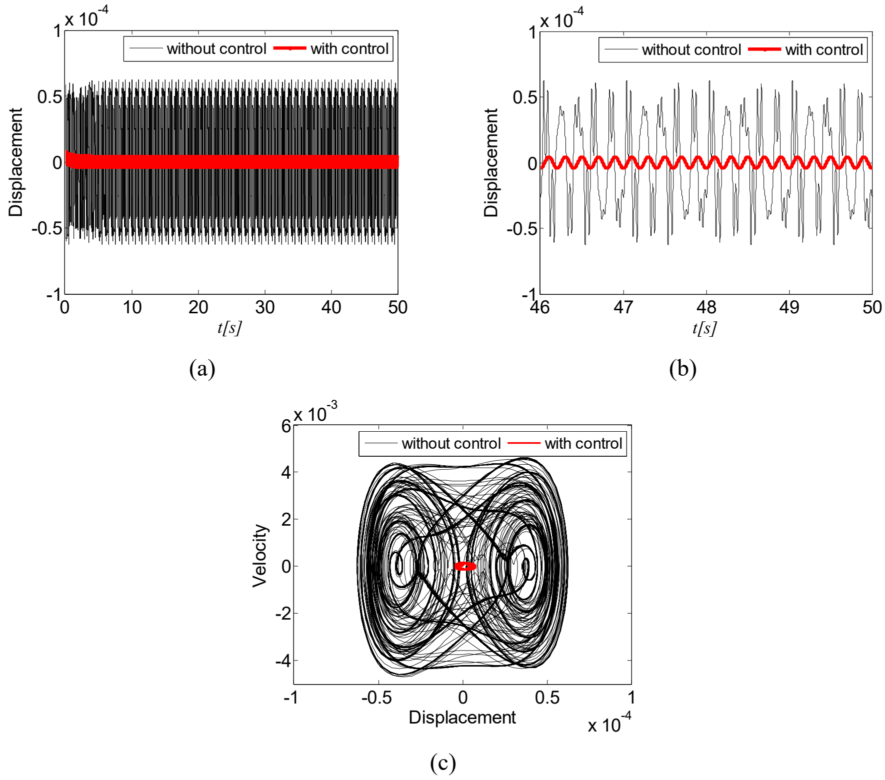
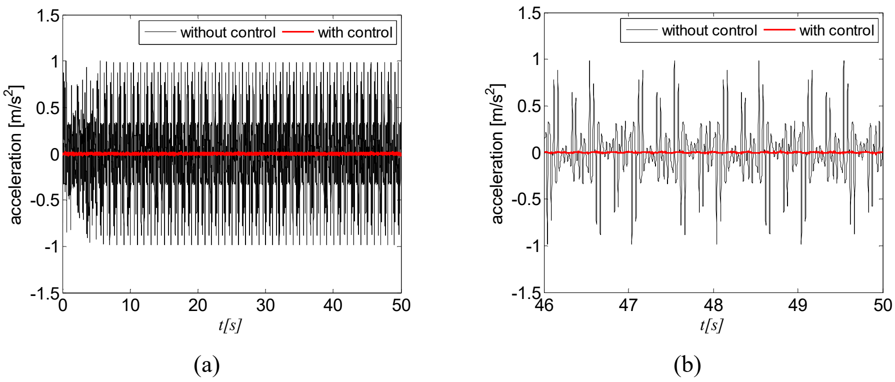


Figure 6 Comparison between uncontrolled and controlled cabin response (a) (b) history of acceleration $\left(\dot{x}_4\right)$ (see online version for colours)

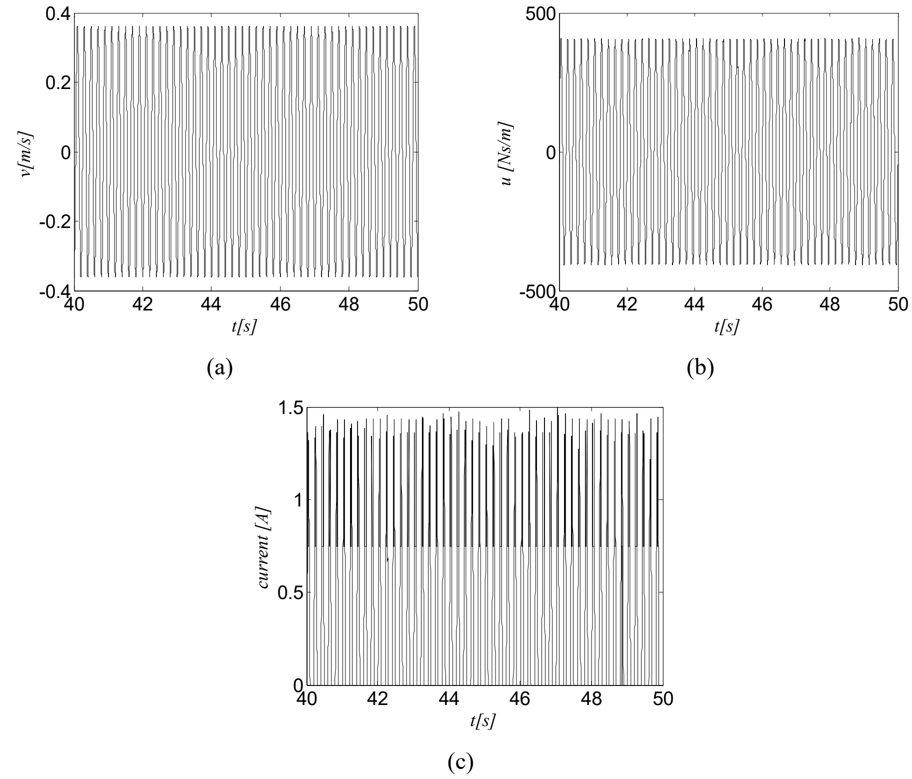


It is possible to observe that the proposed control reduces the system oscillation amplitude (Figure 5). The rms of the magnitude acceleration peak has been reduced in about 98.6367% (Figure 6), where the uncontrolled system acceleration $\left(\ddot{x}_{3_{rms}} = 0.2714\right)$ was reduced to $\left(\ddot{x}_{3_{rms}} = 0.0037\right)$ when the MR damper was introduced into the system.

At the same time the decrease in the *VDV* is observed, that is, the $VDV = 0.0047[m/s^{1.75}]$ for the proposed control system (17), and thus reducing the (VDV) of the uncontrolled system in 98.661%.

Figure 7 presents the velocity of the MR damper piston obtained through ($v = x_4 - x_2$) and the variation of the MR damper force obtained through the equation (21), considering the variation the current applied to the coil of MR damper obtained from equation (14).

Figure 7 (a) Velocity v (b) Force control u (c) The current applied in the damper



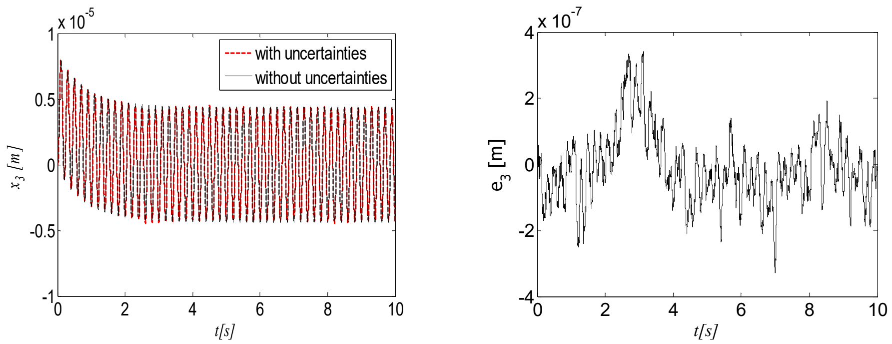
As shown in Figure 7(b), the value of the force control is less than 420 Ns/m, being a common value of the force developed by MR dampers. Also we can see in Figure 6(c), the electric current required to control the MR damping force.

6 Controlled system in the presence of parametric errors

In order to consider the effect of uncertainties on the control performance, the parameters used in the matrix of state variables A (α_1, α_2, α_3, β_1, β_2, β_3 and β_4), will be considered as a random error of ±20%, this strategy is similar to previously described method used in (Nozaki et al., 2013; Tusset et al., 2016; Balthazar et al., 2014).

In the Figure 8 it is possible observe the robustness of the control, where $e_3 = x_3 - \hat{x}_3$ represents the trajectory deviation obtained by the control without uncertainties (x_3) and trajectory obtained by control with parametric uncertainties $(\hat{x}_3)$.

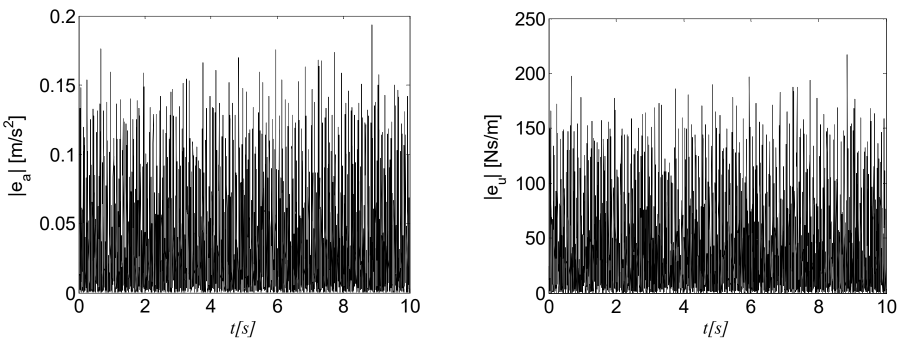
Figure 8 (a) Displacement controlled x_3 with and without uncertainties parameters in matrix *a* (b) Trajectory deviation $(e_3 = x_3 - \hat{x}_3)$ (see online version for colours)



(a) (b)

In Figure 9 we observe the robustness of the control where $|e_a| = |\dot{x}_4 - \dot{\hat{x}}_4|$ represents the acceleration deviation obtained by control without uncertainties $(\dot{x}_4)$ and acceleration obtained by control with parametric uncertainties $(\dot{\hat{x}}_4)$, and $|e_u| = |u - \hat{u}|$ represents the signal control obtained by control without uncertainties (u) and signal control with parametric uncertainties $(\hat{u})$.

Figure 9 (a) Acceleration deviation $(|e_a| = |\dot{x}_4 - \dot{\hat{x}}_4|)$ (b) Signal control deviation $(|e_u| = |u - \hat{u}|)$



(a) (b)

As can be seen in Figure 9, the proposed control is not sensitive to errors in the parameters of the matrix A, maintaining the level of acceleration and displacement below the uncontrolled system. As the acceleration in rms for the parametric error case is $\left(\ddot{x}_{3_{rms}} = 0.0449[m/s^2]\right)$, is possible to note that occurred a reduction of approximately (83.456%) relative to system without control, and the error was $e_{3_{rms}} = 1.0729 \times 10^{-7}[m]$ in rms.

7 Conclusions

Chaos suppression of the horizontal behaviour of a vertical transportation system harmonically excited by guide rails was studied using LQR control. We propose a control strategy whereby the MR damping is chosen as the actuator in this system. Then, the damping force control is found by minimising a defined functional.

Our results reveal that the control technique used here proved to be effective for case without parametric error in state matrix *A*, reducing the vibration of the cabin and contributing on prevention of elevators components integrity. Also, the amplitude of the acceleration has been reduced, increasing the comfort level of the passengers during the movement of the elevator.

Therefore, the proposed control has proved to be efficient for both comfort and safety. Through the equation (21) it was possible control the coil voltage of the MR damper [Figure 5(c)]. We can conclude that in general, the proposed active control is appropriate and efficient.

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