Robust linear PID controller for satellite attitude stabilisation and attitude tracking control

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Abstract: In this paper, a linear robust PID controller is presented for satellite attitude stabilisation and attitude tracking control. The PID controller presented in this paper has the advantage that: 1) the Lyapunov function is modified thus the stability of the system is easier to prove; 2) the constraints of the parameters are broadened thus parameters are easier to choose comparing with existing methods; 3) the controller is robust to bounded disturbance torque and the satellite inertia matrix; 4) the controller is linear thus it is convenient for practical application. The proof of the stability and the constraints of the parameters are given in this paper. The simulation results verify the feasibility of the controller presented in this paper.

Keywords: PID controller; robust linear controller; attitude stabilisation control; attitude tracking control.

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1 Introduction

PD control algorithm is the most mature and widely used control algorithm in satellite attitude control system. For satellite attitude stabilisation and tracking control, PD control with negative feedback is global asymptotically stable. In order to improve the accuracy of the system, the integration item is brought into the control law. However, the appearance of the integration item makes it difficult to prove the stability of the system.

Jin and Sun (2009) presented a PID controller for flexible spacecraft attitude tracking in 2009. In his work, a Lyapunov function with coupled item of angular velocity, quaternion and integration item was presented and based on this Lyapunov function the integration item was eliminated in the derivative of Lyapunov function. Thus, the order of the system was reduced and the stability of the system was proved. In his work in 2008 (Jin et al., 2008), he presented a PID controller for decentralised satellite formation control. In this work, the Lyapunov function with coupled item was also used to achieve the goal of reducing the system order. In Kamesh (2004), the author constructed a modified Lyapunov function with integration item and simplified the proof of the system stability. However, this work was mainly focused on rigid spacecraft. In Clarke et al. (1997), the PID controller for attitude tracking issue was discussed. The attitude tracking system was treated as non-autonomous system and its strict stability proof was given. In this work, the stability of discontinuous system was discussed and the stability of control law with sign function was proved. In Vadali and Junkins (1990) and Chaturvedi et al. (2006), the non-autonomous system stability was discussed and the Lyapunov function was constructed in a relatively complicated way to prove the convergence of the system state. It is worth noticing that most of these PID controllers presented were nonlinear. In Li et al. (2014), a parameter optimisation method based on genetic algorithm for PID controller is presented for attitude control systems. In Su et al. (2011), a nonlinear PID controller robust to disturbance torque was presented for quadrotor aircraft and the strict stability proof was given. However, in this work, the accurate inertia matrix was needed to design the controller. In Sakamoto et al. (2006) and Zhang et al. (2010), the fuzzy PID controller and robust PID controller is presented for helicopter attitude control. The author considered the complicated environment and modified traditional PID control algorithm in order to enhance its robustness. In Lu et al. (2008), the author designed PID controller based on the idea of sliding mode for rigid satellite.

In this paper, the attitude stabilisation and attitude tracking issue is discussed for rigid satellite. Inspired by Jin and Sun (2009) and Jin et al. (2008), the Lyapunov function with coupled items is constructed and modified. Thus the proof of stability and the constraints of parameters become easier to achieve. Also, in order to hold the advantage of robustness to inertia matrix of PD controller, the controller designed in this paper does not need the accurate value of satellite inertia matrix. The simulation results examine the feasibility of the algorithm.

2 Dynamic and kinetic model

The dynamic model of satellite could be written as follow:

$$J\dot{\omega} + \omega^{\times} J\omega = u + d \tag{1}$$

where J is the inertia matrix of satellite which is a symmetric matrix, d is the unknown disturbance torque with upper bound $|d|_i < \bar{d}$. The product matrix $r^{\times}$ of vector r is defined as

$$r^{\times} = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix} \tag{2}$$

The product matrix has an important property which will be used in the later part of this paper that the eigenvalues of $r^{\times}$ satisfies

$$\begin{aligned} &\lambda(r^{\times}) = 0, \pm \| r \| i \\ &|\lambda(r^{\times})|_{\max} = \| r \| \end{aligned} \tag{3}$$

The kinetic model of satellite could be written as follow:

$$\dot{q} = \begin{bmatrix} \dot{q}_0 \\ \dot{q}_v \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} q_v^T \omega \\ \frac{1}{2}\left(q_0 I_3 + q_v^{\times}\right)\omega \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -q_v^T \\ q_0 I_3 + q_v^{\times} \end{bmatrix}\omega \tag{4}$$

In satellite attitude tracking issue, the error quaternion and error angular velocity are defined as follow:

$$\begin{aligned} &q_e = q_d^* \otimes q = \begin{bmatrix} q_{d0} q_0 + q_{dv}^T q_v \\ q_{d0} q_v - q_0 q_{dv} - q_{dv} \times q_v \end{bmatrix} \\ &R(q_e) = \left(q_{e0}^2 - q_{ev}^T q_{ev}\right) I_3 + 2 q_{ev} q_{ev}^T - 2 q_{e0} q_{ev}^{\times} \\ &\omega_e = \omega - R(q_e)\omega_d \end{aligned} \tag{5}$$

where q_d and ω_d are desired quaternion and angular velocity.

The model of error angular velocity and quaternion could be written as follow:

$$J\dot{\omega}_e + JR(q_e)\dot{\omega}_d - J\omega_e^{\times} R(q_e)\omega_d + \left[\omega_e + R(q_e)\omega_d\right]^{\times} J\left[\omega_e + R(q_e)\omega_d\right] = u + d \tag{6}$$

$$\dot{q}_e = \begin{bmatrix} \dot{q}_{e0} \\ \dot{q}_{ev} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} q_{ev}^T \omega_e \\ \frac{1}{2}\left(q_{e0} I_3 + q_{ev}^{\times}\right)\omega_e \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -q_{ev}^T \\ q_{e0} I_3 + q_{ev}^{\times} \end{bmatrix}\omega_e \tag{7}$$

3 PID attitude stabilisation control law

The linear PID attitude stabilisation control law presented as follow:

$$u = -k_d \omega - k_p q_v - k_I \vartheta - \bar{d}\,\mathrm{sgn}\left(\beta\omega + l_2 q_v\right) \tag{8}$$

where k_d, k_p, k_I, β, l_2 are all positive scalars and variable ϑ is defined as follow:

$$\dot{\vartheta} = c_1 \omega + c_2 q_v - \frac{1}{2}\alpha F \omega \tag{9}$$

where c_1, c_2, α are positive scalars and F is defined as follow:

$$F = q_0 I_3 + q_v^{\times} \tag{10}$$

The eigenvalues of satisfies following equation:

$$\begin{aligned} &\lambda(F) = q_0,\, q_0 \pm i\sqrt{1-q_0^2} \\ &\left|\lambda(F)\right|_{\max} = 1 \end{aligned} \tag{11}$$

where $\lambda(A)$ represents the eigenvalues of matrix A.

Consider equation (8), the control torque u could be rewritten as follow:

$$\begin{aligned} u &= -k_d\omega = k_p q_v = k_I \int \left(c_1\omega + c_2 q_v\right)dt + \alpha k_I \int \dot{q}_v dt - \bar{d}\,\mathrm{sgn}\left(\beta\omega + l_2 q_v\right) \\ &= -k_d\omega - k_p q_v - k_I \int \left(c_1\omega + c_2 q_v\right)dt + \alpha k_I \int \left(q_v - q_v(0)\right) - \bar{d}\,\mathrm{sgn}\left(\beta\omega + l_2 q_v\right) \end{aligned} \tag{12}$$

Based on equation (12), it can be found that the control law presented in this paper is a linear PID control law without nonlinear item.

The next step is to prove that by choosing proper parameters, system (1), (4), (8) is global asymptotically stable.

Choose Lyapunov function as follow:

$$V = 2l_1\left(1-q_0\right) + \frac{1}{2}\beta\omega^T J\omega + l_2 q_v^T J\omega + l_3\left(\vartheta + \alpha q_v\right)^T\left(\vartheta + \alpha q_v\right) \tag{13}$$

where l_1, l_2, l_3, α, β are positive scalar.

Considering that

$$1 - q_0 \geq \frac{1}{2}\left(1 - q_0^2\right), \quad \forall q_0 \in [-1, 1] \tag{14}$$

It can be got that

$$\begin{aligned} V &\geq l_1 q_v^T q_v + \frac{1}{2}\beta\omega^T J\omega + l_2 q_v^T J\omega + l_3\left(\vartheta + \alpha q_v\right)^T\left(\vartheta + \alpha q_v\right) \\ &\geq l_1 q_v^T q_v + \frac{1}{2}\beta\omega^T J\omega + l_2 q_v^T J\omega \\ &= \begin{bmatrix}\omega^T & q_v^T\end{bmatrix} \begin{bmatrix} \frac{1}{2}\beta\lambda_{\min}(J)I_3 & \frac{1}{2}l_2\lambda_{\max}(J)I_3 \\ \frac{1}{2}l_2\lambda_{\max}(J)I_3 & l_1 I_3 \end{bmatrix} \begin{bmatrix}\omega \\ q_v\end{bmatrix} \end{aligned} \tag{15}$$

Thus, if following inequality is satisfied, V is positive definite.

$$\frac{1}{2}\beta l_1\lambda_{\min}(J) - \frac{1}{4}l_2^2\lambda_{\max}^2(J) > 0 \tag{16}$$

Calculate the derivative of V, it can be got that

$$
\begin{aligned}
\dot{V} &= l_1\omega^T q_v + \beta\omega^T\left(-\omega^{\times}J\omega + u + d\right) + \frac{1}{2}l_2\omega^T JF\omega + l_2 q_v^T\left(-\omega^{\times}J\omega + u + d\right) \\
&\quad + 2l_3\left(\vartheta + \alpha q_v\right)^T\left(c_1\omega + c_2 q_v - \frac{1}{2}\alpha F\omega + \frac{1}{2}\alpha F\omega\right) \\
&= \left(\beta\omega^T + l_2 q_v^T\right)\left(-k_d\omega - k_p q_v - k_I\vartheta\right) + \left(\beta\omega^T + l_2 q_v^T\right)\left(-\bar{d}\,\mathrm{sgn}\left(\beta\omega + l_2 q_v\right) + d\right) \\
&\quad + \frac{1}{2}l_2\omega^T JF\omega - l_2 q_v^T\omega^{\times}J\omega + 2l_3\left(\vartheta + \alpha q_v\right)^T\left(c_1\omega + c_2 q_v\right) + l_1\omega^T q_v \\
&\le -\omega^T\left(\beta k_d I_3 - \frac{1}{2l_2 JF}\right)\omega - q_v^T\left(l_2 k_p I_3 - 2\alpha c_2 l_2 l_3\right)q_v + \left(2c_1 l_3 - \beta k_O\right)\vartheta^T\omega \\
&\quad + \left(2c_2 l_3 - l_2 k_I\right)\vartheta^T q_v + \left(l_1 - \beta k_p - l_2 k_d + 2\alpha c_1 l_3\right)q_v^T\omega - l_2 q_v^T\omega^{\times}J\omega
\end{aligned}
\tag{17}
$$

Choose proper parameters to satisfy following equation.

$$
2l_3c_1 - \beta k_I = 2l_3c_2 - l_2k_I = l_1 - \beta k_p - l_2k_d + 2\alpha c_1 l_3 = 0 \tag{18}
$$

Assume that the norm of angular velocity satisfies $\|\omega\| \le \bar{\omega}$, and consider property (3) and (10), equation (17) could be transformed to

$$
\dot{V} \le -\left[\omega^T \quad q_v^T\right]\begin{bmatrix} \beta k_d I_3 - \frac{1}{2}l_2\lambda_{\max}(J)I_3 & \frac{1}{2}l_2\bar{\omega}\lambda_{\max}(J)I_3 \\ \frac{1}{2}l_2\bar{\omega}\lambda_{\max}(J)I_3 & l_2\left(k_p - \alpha k_I\right)I_3 \end{bmatrix}\begin{bmatrix}\omega \\ q_v\end{bmatrix} \tag{19}
$$

Thus, if the following inequality is satisfied, $\dot{V}$ is negative definite.

$$
l_2\left(\beta k_d - \frac{1}{2}l_2\lambda_{\max}(J)\right)\left(k_p - \alpha k_I\right) - \frac{1}{4}l_2^2\bar{\omega}^2\lambda_{\max}^2(J) > 0 \tag{20}
$$

Thus, if (16), (18) and (20) are satisfied, $V > 0$, $\dot{V} < 0$, the system is global asymptotically stable.

4 PID attitude tracking control law

In this paper, the tracking target is cooperative which means q_d and ω_d are all known. Assume the norm of real angular velocity is upper bounded which means $\|\omega\| \le \bar{\omega}$.

The PID attitude tracking law could be written as follow:

$$
\begin{aligned}
u &= -k_d\omega_e - k_p q_{ev} - k_I\vartheta_e - \bar{d}\,\mathrm{sgn}\left(s_e\right) - \lambda_{\max}(J)s_e \odot \left(R\dot{\omega}d\right) \\
&\quad - \lambda_{\max}(J)s_e \odot \left(\omega_e^{\times}R\omega_d\right) - \bar{\omega}\lambda_{\max}(J)s_e \odot \omega_e
\end{aligned}
\tag{21}
$$

where ϑ_e and s_e satisfies

$$
\begin{aligned}
\dot{\vartheta}_e &= c_1\omega_e + c_2 q_{ev} - \frac{1}{2}\alpha F_e\omega_e \\
F_e &= q_{e0}I_3 + q_{ev}^{\times} \\
s_e &= \beta\omega_e + l_2 q_{ev}
\end{aligned}
\tag{22}
$$

And $\lambda_{\max}(J)$ in equation (21) represents the maximum eigenvalue of J. The operator $\odot$ is defined as follow:

$$a \odot b = \left[\operatorname{sgn}(a_1 b_1) b_1 \quad \operatorname{sgn}(a_2 b_2) b_2 \quad \operatorname{sgn}(a_3 b_3) b_3 \right]^T \tag{23}$$

The next step is to prove the stability of control law (21). Choose Lyapunov function as follow:

$$V_e = 2l_1(1 - q_{e0}) + \frac{1}{1}\beta\omega_e^T J\omega_e + l_2 q_{ev}^T J\omega_e + l_3(\vartheta_e + \alpha q_{ev})^T(\vartheta_e + \alpha q_{ev}) \tag{24}$$

Consider equation (14), it can be got that

$$\begin{aligned} V_e &\geq l_1 q_{ev}^T q_{ev} + \frac{1}{2}\beta\omega_e^T J\omega_e + l_2 q_{ev}^T J\omega_e + l_3(\vartheta_e + \alpha q_{ev})^T(\vartheta_e + \alpha q_{ev}) \\ &\geq l_1 q_{ev}^T q_{ev} + \frac{1}{2}\beta\omega_e^T J\omega_e + l_2 q_{ev}^T J\omega_e \\ &= \begin{bmatrix} \omega_e^T & q_{ev}^T \end{bmatrix} \begin{bmatrix} \frac{1}{2}\beta\lambda_{\min}(J)I_3 & \frac{1}{2}l_2\lambda_{\max}(J)I_3 \\ \frac{1}{2}l_2\lambda_{\max}(J)I_3 & l_1 I_3 \end{bmatrix} \begin{bmatrix} \omega_e \\ q_{ev} \end{bmatrix} \end{aligned} \tag{25}$$

Thus, if following inequality is satisfied, V_e is positive definite.

$$\frac{1}{2}\beta l_1 \lambda_{\min}(J) - \frac{1}{4} l_2^2 \lambda_{\max}^2(J) > 0 \tag{26}$$

Calculate the derivative of V_e, it can be got that

$$\begin{aligned} \dot{V}_e &= -2l_1\dot{q}_{e0} + \beta\omega_e^T J\dot{\omega}_e + l_2\omega_e^T J\dot{q}_{ev} + l_2 q_{ev}^T J\dot{\omega}_e + 2l_3(\vartheta_e + \alpha q_{ev})^T(\dot{\vartheta}_e + \alpha\dot{q}_{ev}) \\ &= l_1 q_{ev}^T\omega_e + l_2\omega_e^T JF_e\omega_e + (\beta\omega_e + l_2 q_{ev})^T J\dot{\omega}_e \\ &\quad + 2l_3(\vartheta_e + \alpha q_{ev})^T\left(c_1\omega_e + c_2 q_{ev} - \frac{1}{2}\alpha F_e\omega_e + \frac{1}{2}\alpha F_e\omega_e\right) \\ &= l_1 q_{ev}^T\omega_e + \frac{1}{2}l_2\omega_e^T JF_e\omega_e + 2l_3c_1\vartheta_e^T\omega_e + 2l_3c_2\vartheta_e^T q_{ev} + 2l_3c_1\alpha q_{ev}^T\omega_e + 2l_3c_2\alpha q_{ev}^T q_{ev} \\ &\quad + (\beta\omega_e + l_2 q_{ev})^T(u + d - JR\dot{\omega}_d + J\omega_e^\times R\omega_d - \omega^\times J\omega) \\ &= -\omega_e^T\left(k_d\beta I_3 - \frac{1}{2}l_2 JF_e\right)\omega_e - q_{ev}^T(l_2k_p - 2l_3c_2\alpha)q_{ev} + (2c_1l_3 - \beta k_I)\vartheta_e^T\omega_e \\ &\quad + (2c_2l_3 - l_2k_I)\vartheta_e^T q_{ev} + q_{ev}^T(l_1 - \beta k_p - l_2k_d + 2\alpha c_1 l_3)\omega_e \\ &\quad + (\beta\omega_e + l_2 q_{ev})^T(\tau + d - JR\dot{\omega}_d + J\omega_e^\times R\omega_d - \omega^\times J\omega) \end{aligned} \tag{27}$$

Choose proper parameters to satisfy following equation

$$2l_3c_1 - \beta k_I = 2l_3c_2 - l_2k_I = l_1 - \beta k_p - l_2k_d + 2\alpha c_1 l_3 = 0 \tag{28}$$

Equation (27) could be transformed to

$$\dot{V}_e = -\omega_e^T \left(k_d \beta I_3 - \frac{1}{2} l_2 J F_e \right) \omega_e - q_{ev}^T \left(l_2 k_p - l_2 k_I \alpha \right) q_{ev} \\ + s_e^T \left(\tau + d - JR\dot{\omega}_d + J\omega_e^\times R\omega_d - \omega^\times J\omega \right) \tag{29}$$

where τ satisfies

$$\tau = -\bar{d}\,\text{sgn}(s_e) - \lambda_{\max}(J) s_e \odot (R\dot{\omega}_d) - \lambda_{\max}(J) s_e \odot (\omega_e^\times R\omega_d) - \bar{\omega}\lambda_{\max}(J) s_e \odot \omega_e \tag{30}$$

Considering the following property

$$\begin{aligned} -s_e^T (s_e \odot r) \pm s_e^T r &= -s_e^T \left[\text{sgn}(s_{e1} r_1) r_1 \quad \text{sgn}(s_{e2} r_2) r_2 \quad \text{sgn}(s_{e3} r_3) r_3 \right]^T \pm s_e^T r \\ &= -\sum s_{ei} r_i \,\text{sgn}(s_{ei} r_1) \pm \sum s_{ei} r_i \\ &= -\sum |s_{ei} r_i| \pm \sum s_{ei} r_i \le 0 \end{aligned} \tag{31}$$

It can be got that

$$\begin{aligned} &-\bar{d} s_e^T \,\text{sgn}(s_e) + s_e^T d \le 0 \\ &-\lambda_{\max}(J) s_e^T s_e \odot (R\dot{\omega}_d) - s_e^T JR\dot{\omega}_d \le -s_e^T s_e \odot (JR\dot{\omega}_d) - s_e^T JR\dot{\omega}_d \le 0 \\ &-\lambda_{\max}(J) s_e^T s_e \odot (\omega_e^\times R\omega_d) + s_e^T J\omega_e^\times R\omega_d \le -s_e^T s_e \odot (J\omega_e^\times R\omega_d) + s_e^T J\omega_e^\times R\omega_d \le 0 \\ &-\bar{\omega}\lambda_{\max}(J) s_e^T s_e \odot \omega_e - s_e^T \omega^\times J\omega \le -s_e^T s_e \odot (\omega^\times J\omega) - s_e^T \omega^\times J\omega \le 0 \end{aligned} \tag{32}$$

Thus, equation (29) could be transformed to

$$\begin{aligned} \dot{V}_e &\le -\omega_e^T \left(k_d \beta I_3 - \frac{1}{2} l_2 J F_e \right) \omega_e - q_{ev}^T \left(l_2 k_p - l_2 k_I \alpha \right) q_{ev} \\ &\le -\omega_e^T \left(k_d \beta - \frac{1}{2} l_2 \lambda_{\max}(J) \right) \omega_e - q_{ev}^T \left(l_2 k_p - l_2 k_I \alpha \right) q_{ev} \end{aligned} \tag{33}$$

Thus, if following inequality is satisfied, $\dot{V}_e < 0$.

$$k_d \beta - \frac{1}{2} l_2 \lambda_{\max}(J) > 0, \; k_p - \alpha k_I > 0 \tag{34}$$

Therefore, the attitude tracking system is global asymptotically stable.

Comparing the necessary parameter constraints in attitude stabilisation and attitude tracking issues, it can be found that the constraints of V positive definite and the constraints to eliminate integration items are all the same. The difference is that the constraint of $\dot{V}$ negative definite. Based on equation (15) and (34), it can be found that the constraint in attitude tracking issue is easier to satisfy.

5 Simulation

Set the system parameters for attitude stabilisation control as follow.

$$
\begin{aligned}
&J = diag\left([50 \quad 75 \quad 100]\right), \bar{\varpi} = 0.2, T = 500, t\_sample = 0.5 \\
&\alpha = 1, \beta = 0.05, k_d = 20, k_p = 5, k_I = 1, \bar{d} = 0.001 \\
&c_1 = 0.25, c_2 = 0.05, l_1 = 0.4, l_2 = 0.01, l_3 = 0.1 \\
&\omega_{initial} = \left[0.1 \quad -0.05 \quad -0.04\right]^T, q_{initial} = \left[0.51 \quad 0.3 \quad 0.4 \quad -0.7\right]^T
\end{aligned}
\tag{35}
$$

The simulation of attitude stabilisation control law is given from Figure 1 to Figure 3.

Based on Figure 1 to Figure 3, it can be found that the angular velocity and attitude quaternion converges to zero. Except the initial ten control cycle, the norm of control torque is constrained in an acceptable range.

Then, set the system parameters for attitude tracking control as follow:

$$
\begin{aligned}
&J = diag\left([50 \quad 75 \quad 100]\right), \bar{\varpi} = 0.2, T = 500, t\_sample = 0.5 \\
&\alpha = 1, \beta = 0.05., k_d = 20, k_p 5, k_I = 1, \bar{d} = 0.001 \\
&c_1 = 0.25, c_2 = 0.05, l_1 = 0.4, l_2 = 0.01, l_3 = 0.1 \\
&\omega_{initial} = [0.05 \quad 0.05 \quad -0.1]^T, q_{initial} = \left[0.5 \quad -0.5 \quad \sqrt{3}/3 \quad \sqrt{6}/6\right]^T \\
&\omega_d = [0.01 \quad -0.005 \quad 0]^T, \dot{\omega}_d = 0_{3\times 1}, q_d = [1 \quad 0 \quad 0 \quad 0]^T
\end{aligned}
\tag{36}
$$

The simulation of attitude tracking control is given from Figure 4 to Figure 6.

Figure 1 Curve of angular velocity (see online version for colours)

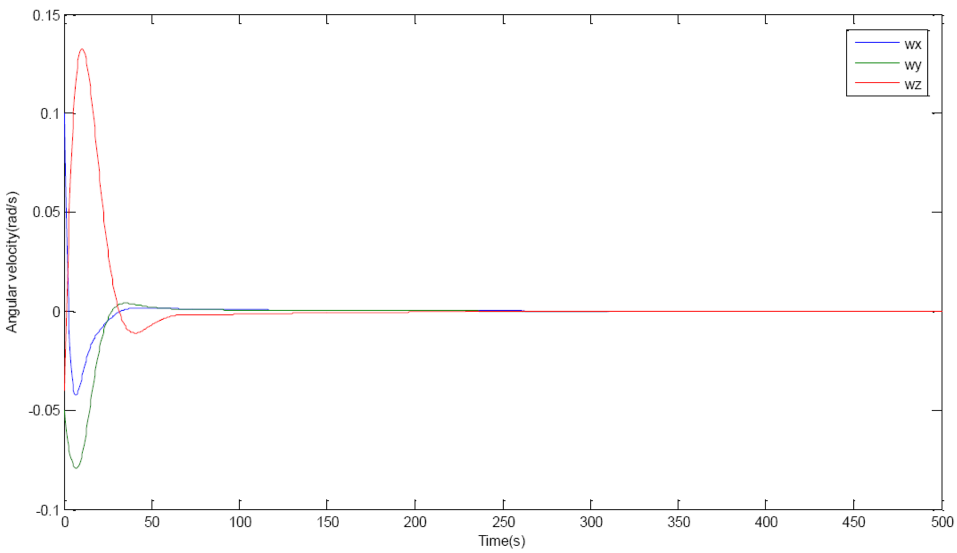


Figure 2 Curve of quaternion (see online version for colours)

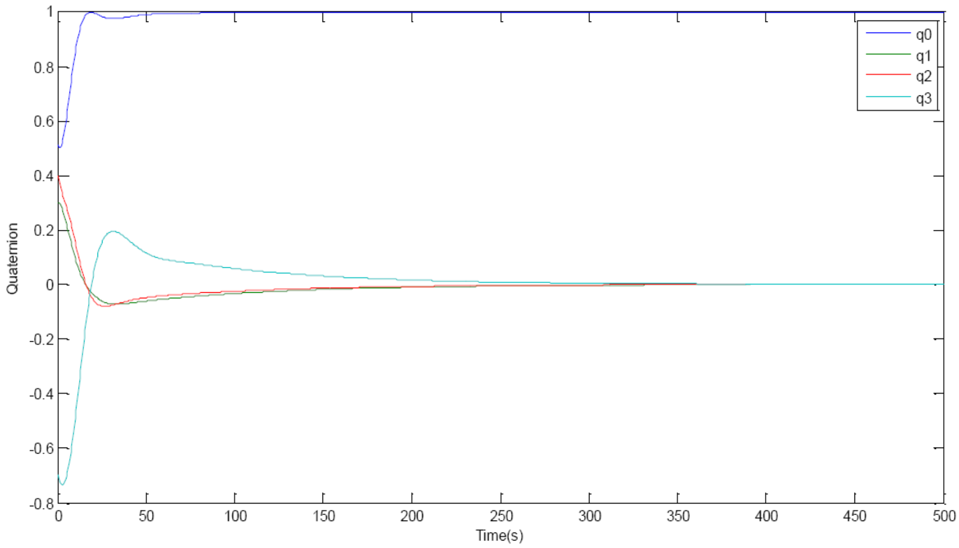
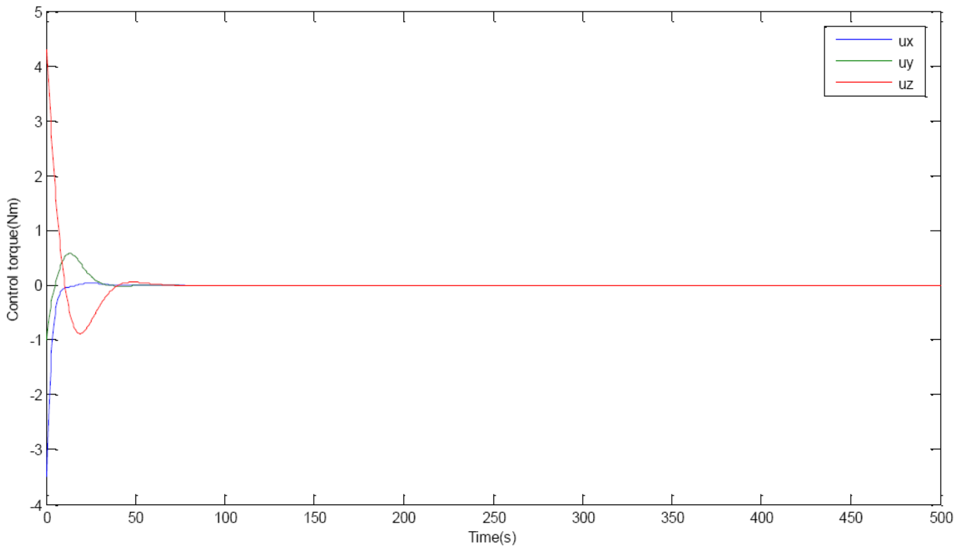


Figure 3 Curve of control torque (see online version for colours)



Based on Figure 4 to Figure 6, it can be found that the error angular velocity and error quaternion converges to zero. Except the initial 20 control cycles, the norm of control torque is constrained in an acceptable range.

Figure 4 Curve of error angular velocity (see online version for colours)

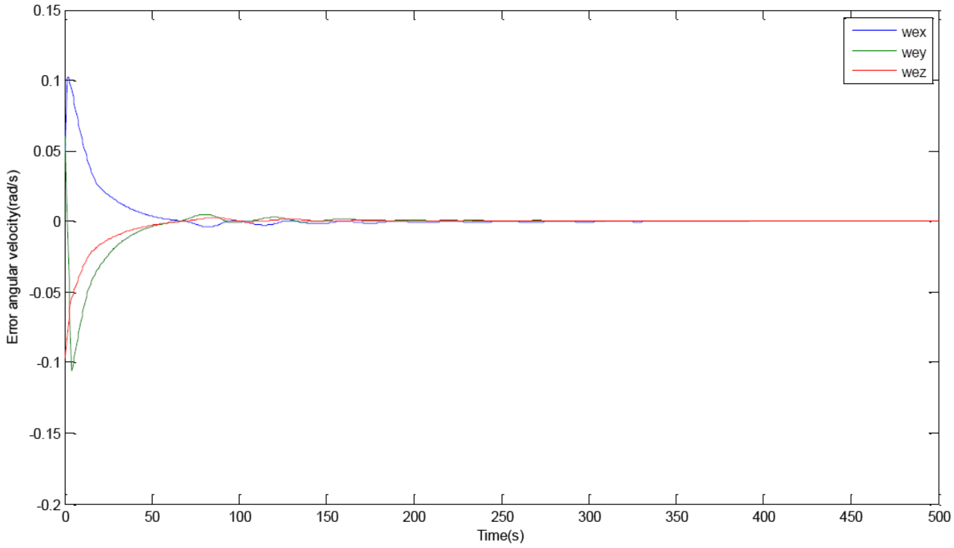


Figure 5 Curve of error quaternion (see online version for colours)

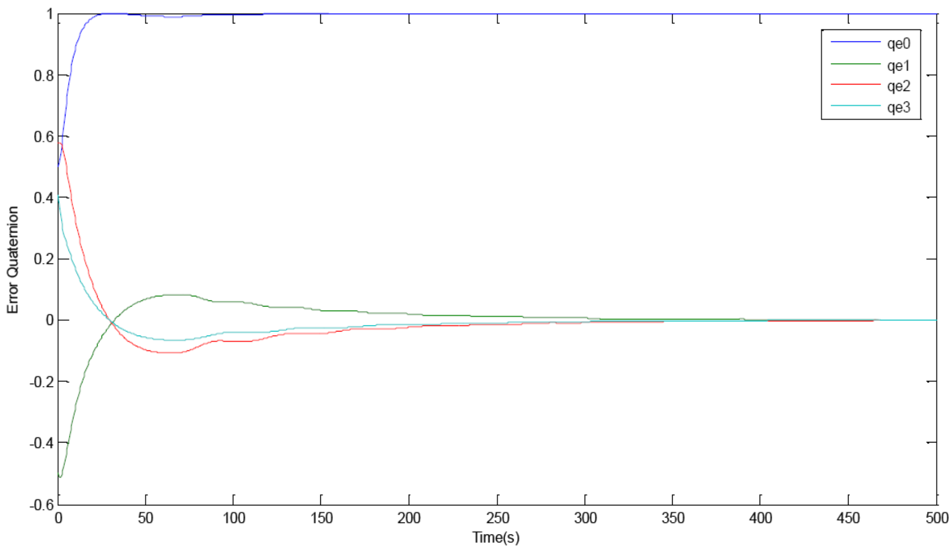
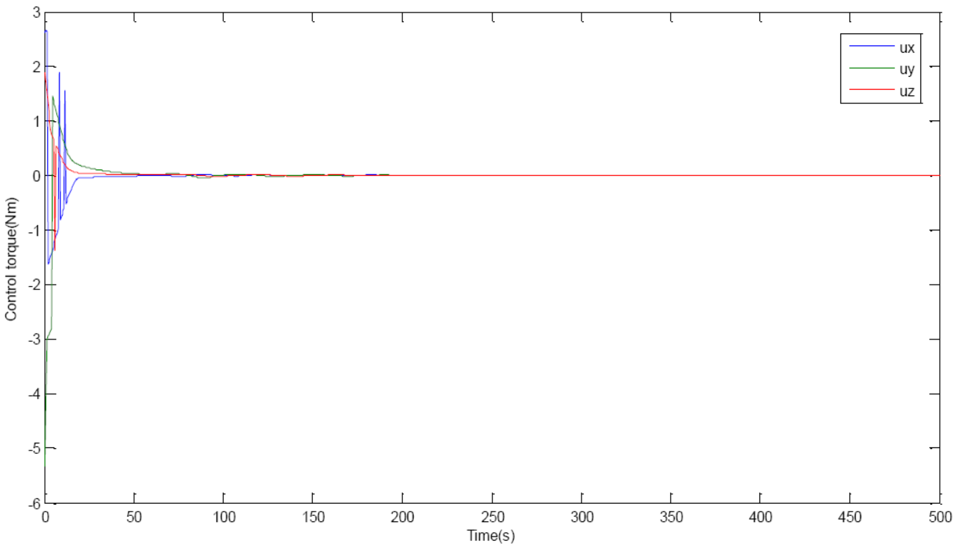


Figure 6 Curve of control torque (see online version for colours)



6 Conclusions

In this paper, a method for design PID attitude stabilisation and attitude tracking control law is given. The traditional Lyapunov function is modified thus the constraints on control parameters are easier to satisfy comparing with existing methods (Jin and Sun, 2009; Jin et al., 2008). Also, the integration item in control law is modified to eliminate the integration items in Lyapunov function. Both of these two improvements make it easier to prove the stability and choose proper parameters.

The stability proof of the control law is given in this paper. Also, the constraints of the system parameters are given in the proof of the system stability. The simulation results indicate that when choosing proper parameters the system is asymptotically stable.

The attitude stabilisation and attitude tracking PID control law in this paper has the advantages that:

1 easier to prove the stability

2 easier to choose the control parameters

3 only the maximum eigenvalue of inertia matrix is needed which means the controller is robust to the inertia matrix uncertainty

4 the controller is linear thus it is convenient for practical application.

References

Chaturvedi, N.A., Sanyal, A.K., Chellappa, M., Valk, J.L. and McClamroch, N.H. (2006) 'Adaptive tracking of angular velocity for a planar rigid body with unknown models for inertia and input nonlinearity', *IEEE Transactions on Control Systems Technology*, Vol. 14, No. 4, pp.613–627.

Clarke, F.H., Ledyaev, Y.S., Sontag, E.D. and Subbotin, A.I. (1997) 'Asymptotic controllability implies feedback stabilization', *IEEE Transactions on Automatic Control*, Vol. 42, No. 10, pp.1394–1407.

Jin, E. and Sun, Z. (2009) 'Robust attitude tracking control of flexible spacecraft for achieving globally asymptotic stability', *International Journal of Robust and Nonlinear Control*, Vol. 19, No. 11, pp.1201–1223.

Jin, E., Jiang, X. and Sun, Z. (2008) 'Robust decentralized attitude coordination control of spacecraft formation', *Systems & Control Letters*, Vol. 57, No. 7, pp.567–577.

Kamesh, S. (2004) 'Nonlinear PID-like control for spacecraft attitude regulation', *The Journal of Astronautical Sciences*, Vol. 52, No. 2, pp.61–74.

Li, X., Qu, S. and Cai, L. (2014) 'Studying of PID parameter optimization method for attitude controller of quadrotor', *Journal of Computational Information Systems*, Vol. 10, No. 18, pp.7805–7812.

Lu, J., Li, C. and Ma, G. (2008) 'Attitude sliding mode PID controller design for satellite regulation', *Journal of Harbin Institute of Technology*, Vol. 40, No. 7, pp.1009–1012.

Sakamoto, T., Katayama, H. and Ichikawa, A. (2006) 'Attitude control of a helicopter model by robust PID controllers', *Proceedings of the 2006 IEEE International Symposium on Intelligent Control*.

Su, J., Fan, P. and Cai, K. (2011) 'Attitude control of quadrotor aircraft via nonlinear PID', *Journal of Beijing University of Aeronautics and Astronautics*, Vol. 37, No. 9, pp.1054–1058.

Vadali, S.R. and Junkins, J.L. (1990) 'Closed loop slewing of flexible spacecraft', *Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 1, pp.57–65.

Zhang, L., Bi, S. and Yang, H. (2010) 'Fuzzy-PID control algorithm of the helicopter model flight attitude control', *2010 Chinese Control and Decisions Conference*.