
Interpretability and accuracy issues in evolutionary multi-objective fuzzy classifiers

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Abstract: Fuzzy systems have remarkable capability to deal with imprecise and uncertain information existing in the real world complex problems. Evolutionary approaches, i.e., genetic algorithms are utilised to improvise the designing of fuzzy systems. During the design of fuzzy systems, interpretability and accuracy features are considered as an effort toward the improvement of performance and usability. One can only be improved at the cost of the other, leading to a new trade-off called interpretability-accuracy trade-off. On the other hand, the use of interval type-2 fuzzy sets in the development of fuzzy classifier is another dimension of improving it. In this paper, a fuzzy classifier named Engineering Student-Fuzzy Classification System (ES-FCS) is proposed and implemented using type-1 and type-2 fuzzy logic. The accuracy improvement has been studied by the application of linguistic hedges. The interpretability and accuracy assessment and their trade-off are experimentally studied in evolutionary multi-objective framework with both type-1 and type-2 fuzzy sets.

Keywords: fuzzy systems; interpretability-accuracy trade-off; genetic fuzzy systems; GFSs; genetic algorithms; GAs; multi-objective evolutionary algorithms; MOEA.

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1 Introduction

Fuzzy systems (Wang, 1999) are highly capable to deal with imprecision and uncertainty existing in the linguistic information during the modelling of real world applications and are applicable in control, classification and modelling areas. The fuzzy systems are capable to simulate the human reasoning and decision making along with their adaptive behaviour. In the fuzzy systems, more precisely fuzzy knowledge base systems (FKBS) have the knowledge about the problem, encoded in the form of fuzzy if-then rules based on either expert knowledge encoding or using some set of data/machine learning approach.

During the development of fuzzy systems, two important aspects are considered,

- 1 interpretability (Cassilas et al., 2003a; Gacto et al., 2011; Shukla and Tripathi, 2013)
- 2 accuracy (Cassilas et al., 2003b).

Interpretability is the quality of modelled system which shows if its behaviour is human understandable or not by seeing its functioning. On the other hand, accuracy is the feature in which the closeness between real and modelled system is judged. Accuracy and interpretability features are contradictory with each other; one can be improved at the cost of the other. This is called the interpretability-accuracy trade-off (Shukla and Tripathi, 2011, 2012a).

In the subsequent progress of developing the fuzzy systems, genetic algorithms (GAs) (Holland, 1975) are well utilised and leads to a new scope of research called, genetic fuzzy systems (GFSs) (Cordon et al., 2001; Herrera, 2005; Cordon et al., 2004; Herrera, 2008). Various components of fuzzy systems are optimised, learned and adapted using GAs.

Evolutionary multi-objective optimisation (Deb, 2001) is used to deal with well known interpretability-accuracy trade-off and results in a new area called 'evolutionary multi-objective fuzzy systems (EMOFS)' (Ishibuchi, 2007; Ducange and Marcelloni, 2011; Fazzolari et al., 2012; Shukla and Tripathi, 2012b).

The new dimension to improve the FKBS design is the use of type-2 fuzzy logic (Mizumoto and Tanaka, 1976; Mendel and John, 2002) and interval type-2 fuzzy logic (Wu and Mendel, 2002; Wu, 2012). Use of type-2 fuzzy sets make the system computationally expensive and reduction in interpretability. To overcome with this situation, the authors propose the use of interval type-2 fuzzy sets to develop the evolutionary multi-objective fuzzy classifier engineering student-fuzzy classification system ('ES-FCS'). Interval type-2 fuzzy sets are less computationally expensive and have most of the advantages of the type-2 fuzzy sets.

In this paper the authors have proposed 'engineering student fuzzy classification system (ES-FCS)' on which multi-objective evolutionary algorithms-based (MOEA)

studies have been carried out dealing with interpretability-accuracy trade-off as well as interpretability assessment at different levels. Also, accuracy improvement has been studied with the integration of linguistic hedges.

This paper continues with Section 2 that introduces and reviews application of MOEA in fuzzy systems. Section 3 is the introduction and review of applying linguistic hedges in simple and multi-objective GFSs. The proposed system EC-FCS is discussed in Section 4. Experimental details and results are carried out in Section 5. Section 6 is the conclusion of the paper.

2 Type-2 fuzzy theory and systems

A type-2 fuzzy set has one more dimension to deal with the uncertainty not handled in type-1 simple fuzzy sets. Type-2 fuzzy sets are computationally expensive, hence interval type-2 fuzzy sets are used to design the FKBS. In type-2 fuzzy sets, the membership degree is represented by a fuzzy set whereas the interval type-2 fuzzy sets have an interval for the membership degree and results into less computational cost.

Type-2 fuzzy set is represented by A^* and defined by membership function $\mu_{A^*}(x, u)$ where $x \in X$ and $u \in J_x \subseteq [0, 1]$

$$A^* = \{(x, u), \mu_{A^*}(x, u) \mid \forall x \in X, \forall u \in J_x \subseteq [0, 1]\}$$

Here, $0 \leq \mu_{A^*}(x, u) \leq 1$ when all $\mu_{A^*}(x, u) = 1$ then A^* is called interval type-2 fuzzy set. The interval type-2 fuzzy set is represented as follows:

$$A^I = \{(x, u), 1\} \mid x \in X, u \in J_x, J_x \subseteq [0, 1]\}$$

In the above definitions, X is considered as primary domain and J_x is the secondary domain. Here, all secondary grades $\mu_{A^I}(x, u)$ are equal to 1. This is fully expressed by the upper and lower bound.

3 Integration of evolutionary multi-objective optimisation in fuzzy systems to deal with interpretability-accuracy trade-off

Evolutionary algorithms are powerful and robust search mechanism based on simulation of the concept of natural evolution. These are integrated with multi-objective optimisation leading to a new kind of algorithms, denoted by MOEA. The first generation MOEAs are; non-dominated sorting genetic algorithm (NSGA) (Srinivas and Deb, 1994), niched Pareto genetic algorithms (NPGA) (Horn et al., 1994), multi-objective genetic algorithm (MOGA) (Fonseca and Fleming, 1993) and second generation MOEA are; strength Pareto evolutionary algorithms (SPEA) (Zitzler and Thiele, 1999), strength Pareto evolutionary algorithm 2 (SPEA2) (Zitzler et al., 2001), Pareto archived evolutionary strategies (PAES) (Knowles and Corne, 2000), non-dominated sorting genetic algorithms (NSGA-II) (Deb et al., 2002), niched Pareto genetic algorithm II (NPGA-II) (Erickson et al., 2001) and Pareto envelop-based selection algorithm (PESA) (Corne et al., 2000).

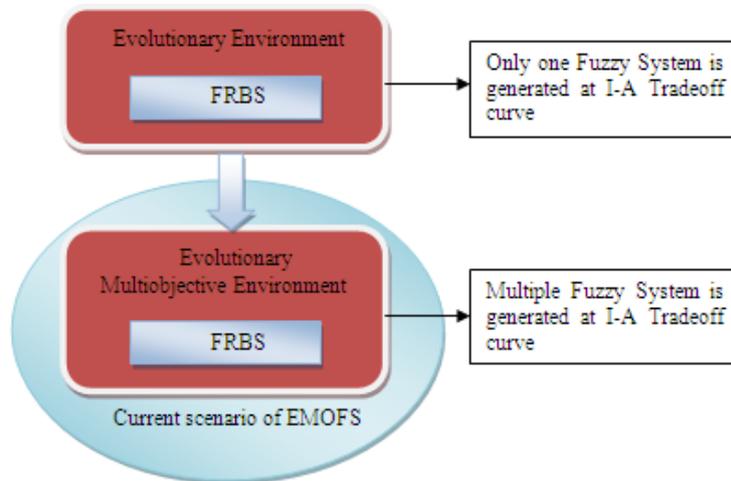
In EMO development environment, the fuzzy systems are characterised by two conflicting objective functions. These are as follows;

- minimise *error* (fs) and complexity (fs)
- maximise *accuracy* (fs) and complexity (fs)

Here, fs is a fuzzy system.

These approaches of integrating GAs with fuzzy system are generating only one fuzzy system on the curve derived in between the accuracy and interpretability. But, further research in this field has proceeded in the new dimension of finding multiple number of fuzzy systems on the curve of accuracy and interpretability. One of these fuzzy systems may be selected as per the preference of user satisfying the problem state. This new dimension leads to a new field, called EMOFS as discussed in Figure 1.

Figure 1 Single and multi-objective GFSs (see online version for colours)



The simple changes in the multi-objective formulation of fuzzy system design are studied in Ishibuchi et al. (2010)

To deal with interpretability and accuracy parameters several multi-objective formulations are developed. In Ishibuchi et al. (1997), the two objectives are; number of correctly classified training patterns (accuracy) and number of selected fuzzy rules (interpretability). In Ishibuchi et al. (2001), the above formulation has been extended with one another objective. The objectives in this formulation are; number of correctly classified training patterns, number of selected fuzzy rules and total number of antecedent conditions [total rule length (TRL)].

Error and complexity are represented by many parameters, like error may be expressed by mean square error (MSE) and root mean square error (RMSE) and interpretability can be expressed by TRL, number of rules (NOR) and average rule length (ARL).

Nauck's (2003) index (NI) has been proposed for the assessment of the interpretability of fuzzy rule-based classifiers. It can be defined as follows;

$$I_{Nauck} = comp \times part \times cov$$

Here, *comp* – number of classes/total number of premises (it measures the complexity),

$$part = \frac{1}{number\ of\ labels - 1}$$

(it is average normalised partition index),

cov is the average normalised coverage degree of the fuzzy partition which is equal to 1 for strong fuzzy partition (SFP).

A global fuzzy index has been introduced in Alonso et al. (2006) and its next version has been discussed in Alonso et al. (2008). In this approach the index has been computed as the output of inference of hierarchical fuzzy system.

4 Linguistic hedges

The linguistic hedges are used and applied on the tuned MFs as discussed in Venturini (1993). The main linguistic hedges are very, more-or-less, extremely, very-very, positively, negatively. Linguistic hedges are playing the role of adjective and adverbs in the languages, responsible to change the qualitative statements.

Several linguistic hedges are proposed and used in the literature. For the basic knowledge of readers these are summarised in Table 1 based on the Zadeh interpretations (Shi and Ward, 2001) and see Venturini (1993) and Zadeh (1972) for more details.

Table 1 Linguistic hedges

Name of linguistic hedges	Mathematical definition
Very	$\mu_{MF}^{very}(x) = (\mu_{MF}(x))^2$ $\mu_{MF}^{very}(x) = (\mu_{MF}(x)) \exp^{(\mu_{MF}(x)-1)}$
Extremely	$\mu_{MF}^{extremely}(x) = (\mu_{MF}(x))^3$
Very very	$\mu_{MF}^{very\ very}(x) = (\mu_{MF}(x))^4$
More-or-less	$\mu_{MF}^{more-or-less}(x) = (\mu_{MF}(x))^{0.5}$ $\mu_{MF}^{more-or-less}(x) = (\mu_{MF}(x))^{1/p} \text{ where } p > 1$
Positively	$\mu_{MF}^{positively}(x) = \begin{cases} 2(\mu_{MF}(x))^2 & \text{if } \mu_{MF}(x) < 0.5 \\ 1 - 2(\mu_{MF}(x))^2 & \text{if } \mu_{MF}(x) \geq 0.5 \end{cases}$
Negatively	$\mu_{MF}^{negatively}(x) = \begin{cases} 2(\mu_{MF}(x))^2 & \text{if } \mu_{MF}(x) < 0.5 \\ 1 - (0.5)^{1-1/n} (1 - \mu_{MF}(x))^{1/n} & \text{if } \mu_{MF}(x) \geq 0.5 \end{cases}$

5 Proposed system

5.1 Engineering student fuzzy classification system

The authors propose 'ES-FCS'. This is the fuzzy classification system that classifies the students of one class into four groups. These four groups are denoted by 1, 2, 3 and 4 numerical digits with following description given in Table 2.

Table 2 Classification of students

1	2	3	4
Excellent	Good	Average	Poor

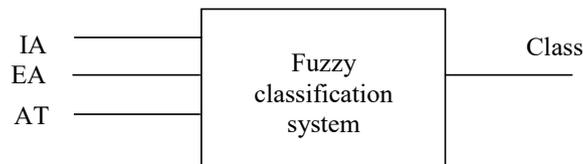
The classification uses three parameters to take decisions for a student to classify him/her in a particular class. The parameters are;

- 1 internal assessment (IA)
- 2 external assessment (EA)
- 3 attendance (AT)

The descriptions of the above parameters are as follows:

- 1 *Internal assessment*: In this assessment, the students are appearing in the internal examination in their department. Three examinations are conducted in the whole semester on the approximate completion of 30%, 60% and 100% completion of the syllabus. Finally the sessional marks of each subject are awarded on behalf of the marks achieved by the students and the assignments/ tutorial sheets submitted to the subject teacher along with the attendance in that particular subject.
- 2 *External assessment*: This assessment is basically the external examination conducted by the affiliating university at the end of the semester. The examinations have subjective paper in each subject with full syllabus. The preparation of the question paper and evaluation of answer sheets are conducted by the affiliating university.
- 3 *Attendance*: Attendance is an important criteria to judge the sincerity and hardworking of the student. So attendance % is calculated at the end of the semester in all theory and lab sessions. The block diagram of the proposed fuzzy classifier is given in Figure 2.

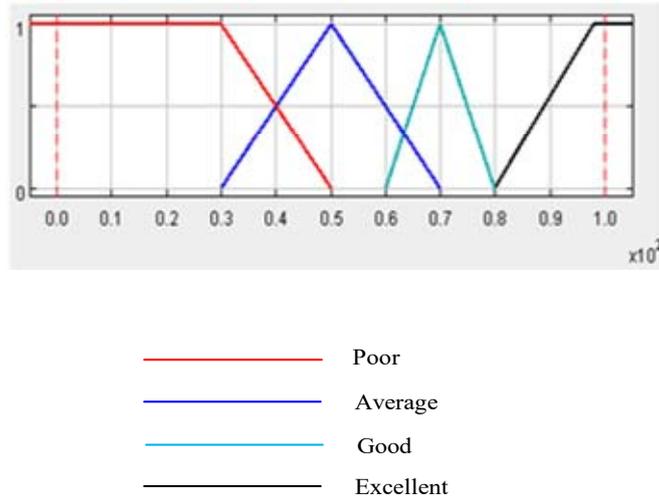
Figure 2 Proposed student classification system



5.2 Type-1 fuzzy set implementation

The membership functions of all three assessment parameters are shown in Figures 3 to 5.

Figure 3 MF of internal assessment (IA) (see online version for colours)



Note: The above linguistic values are applicable on all performance assessment parameters.

Figure 4 MF of external assessment (EA) (see online version for colours)

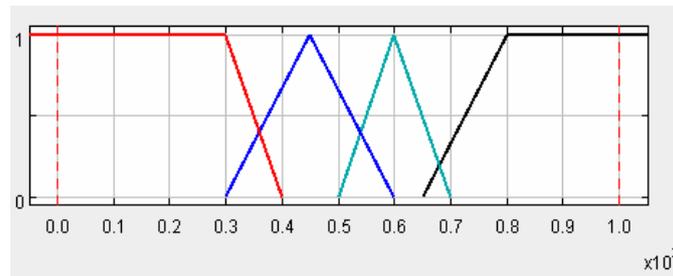
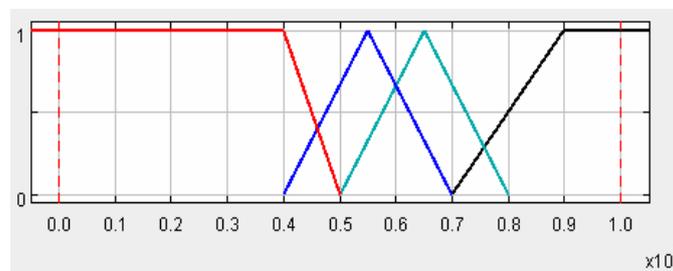


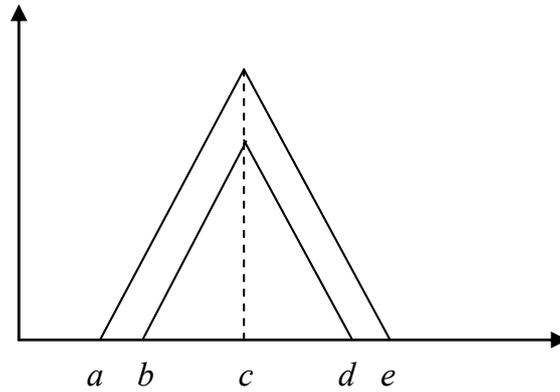
Figure 5 MF of attendance (AT) (see online version for colours)



5.3 Interval type-2 fuzzy set implementation

The interval type-2 fuzzy sets (Shukla and Tripathi, 2014b) are used to implement the proposed system. The used membership function is given in Figure 6.

Figure 6 Interval type-2 fuzzy set



In GAs, the membership function is represented as follows;

Figure 7 Chromosome representing membership function of interval type-2 fuzzy set (see online version for colours)

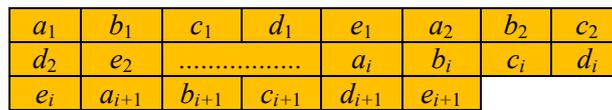


If the rule *R_l* is as follows:

if *x₁* is *MF₁*, *x₂* is *MF₂*,.....and *x_i* is *MF_i* then *y* is *MF_{i+1}*

the encoding of the RB is expressed as given in Figure 8.

Figure 8 Chromosome of knowledge base representation (see online version for colours)



The rule base is represented by decision tree using Thrift’s approach (Thrift, 1991) as given in Figure 9.

Let *A1* and *A2* are the input variables and the output *Y* is as given below.

$$Y = \{E, G, A\}$$

Figure 9 Decision table for knowledge base (see online version for colours)

A1 $\xrightarrow{\hspace{10em}}$ A2

↓	R1 A	R2 A	R3 -	R4 G
	R5 -	R6 A	R7 G	R8 -
	R9 A	R10 G	R11 -	R12 E
	R13 A	R14 G	R15 -	R16 E

The output is encoded as given in Figure 10.

Figure 10 Output encoding

E	G	A
1	2	3

The encoded rule base is expressed by Figure 11.

Figure 11 Encoded rule base (see online version for colours)

0	3	0	2	0	3	2
0	3	0	2	1	3	2
0	1					

The interval type-2 fuzzy membership functions are given in Figures 12 to 14.

Figure 12 MF of internal assessment (see online version for colours)

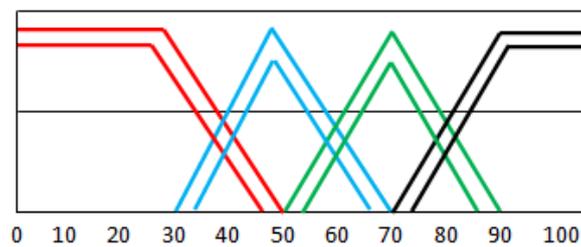
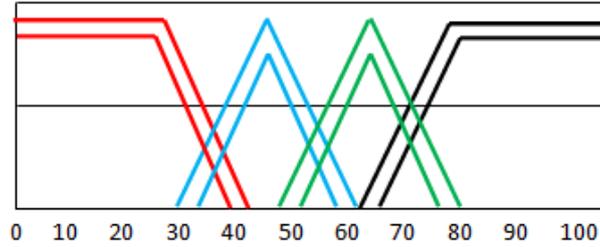
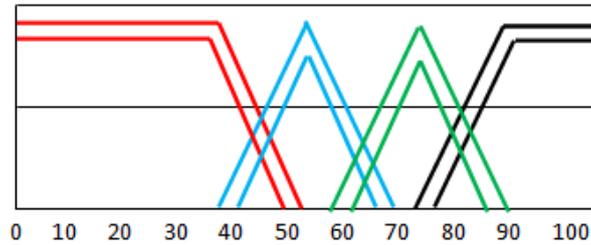


Figure 13 MF of external assessment (see online version for colours)**Figure 14** MF of attendance (see online version for colours)

6 Experiments and result analysis

The proposed ES-FCS has been implemented using the open access software Guaje (Alonso and Magdalena, 2011a, 2011b) with type-1 fuzzy implementation. The EMO experimentations are carried out with the help of MATLAB. The interpretability of the proposed system is assessed in terms of NOR, TRL, ARL, average fired rules (AFR) and NI. On the other hand, accuracy is measured in terms of percentage of correctly classified students (PCCS).

6.1 Type-1 implementation

The results of interpretability and accuracy on different values of NOR are given in Table 3.

Table 3 Interpretability and accuracy parameters

<i>Experiments</i>	<i>Interpretability</i>					<i>Error</i>
	<i>Nauck's index</i>	<i>NOR</i>	<i>TRL</i>	<i>ARL</i>	<i>AFR</i>	<i>PCCS_{1st}</i>
E1	0.056	8	24	3	3.50	71.4%
E2	0.037	12	36	3	4.67	71.4 %
E3	0.026	17	51	3	7.92	76.9%
E4	0.021	21	63	3	7.41	80.9%
E5	0.018	25	73	2.92	9.11	90.8%
E6	0.016	29	85	2.93	9.50	93.8%

On the different input MFs, the effect of using linguistic hedges has been studied. The linguistic hedges are; more-or-less and strictly. The linguistic modifiers have improved the accuracy as per the results in Table 4.

Table 4 Comparative results: linguistic hedges

<i>Condition</i>	<i>Accuracy</i>	<i>Nauck Index</i>	<i>NOR</i>	<i>TRL</i>
Without LH	80.9%	0.021	21	63
With LH	84.1 %	0.021	21	63

Note: LH – linguistic hedges

As discussed in Alcalá et al. (2009) and Shukla and Tripathi (2014a), the multiobjective formulations are as follows:

$$g_1(x) = \text{Percentage error in correctly classified students (PECS)}$$

$$g_2(x) = \text{Number of rules (NOR)}$$

$$g_3(x) = \text{Total rule length (TRL)}$$

- formulation 1:
minimise *PECS* and minimise *NOR*
- formulation 2:
minimise *PECS* and minimise *TRL*

The single objective maximisation formulations are as follows:

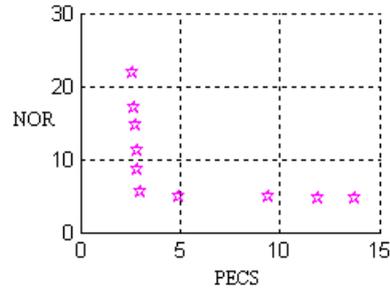
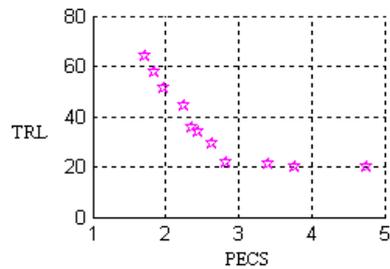
- formulation 1:
 $f_1(S) = g_1(S) - w_1 g_2(S)$
 $f_2(S) = g_2(S) - w_1 g_1(S)$
- formulation 2:
 $f_1(s) = g_1(x) - w_2 g_3(x)$
 $f_2(s) = g_3(x) - w_2 g_1(x)$

The non-dominated solution as per the formulation 1 and 2 are shown in Figures 15 and 16.

It is to be noted that the formulations are indirectly representing

minimise error and minimise interpretability

The error is measured in terms of PECS and interpretability is measured in terms of NOR and TRL.

Figure 15 Pareto front with formulation 1 (see online version for colours)**Figure 16** Pareto front with formulation 2 (see online version for colours)

6.2 Type-2 implementation

The results of interpretability and accuracy on different values of NOR are given in Table 5.

Table 5 Interpretability and accuracy parameters

<i>Experiments</i>	<i>Interpretability</i>				<i>Accuracy</i>
	<i>NOR</i>	<i>TRL</i>	<i>ARL</i>	<i>AFR</i>	<i>PCCS_{1st}</i>
E1	8	24	3	3.69	70.3%
E2	12	36	3	4.89	72.3 %
E3	17	51	3	8.15	77.8%
E4	21	63	3	7.98	82.7%
E5	25	73	2.92	10.15	91.6%
E6	29	85	2.93	10.97	94.5%

7 Conclusions

Interpretability and accuracy are the important features to be considered during the development of fuzzy systems. The trade-off between these two features is handled by evolutionary multi-objective optimisation of fuzzy systems. In this paper the authors have developed a new 'engineering student-fuzzy classification system (ES-FCS)' in evolutionary multi-objective optimisation framework. The proposed system is

implemented using type-1 and interval type-2 fuzzy sets. The interpretability and accuracy measures are evaluated and compared in both implementation of type-1 and interval type 2 fuzzy set. The required Pareto fronts are also generated.

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