Asset allocation: can technical analysis add value?

Guofu Zhou*

Olin School of Business,
Washington University,
St. Louis, MO, 63130, USA
E-mail: zhou@wustl.edu
*Corresponding author

Yingzi Zhu and Sheng Qiang

School of Economics and Management,
Tsinghua University,
Beijing, 100084, China
E-mail: zhuyz@sem.tsinghua.edu.cn
E-mail: qiangsh.10@sem.tsinghua.edu.cn

Abstract: In this paper, we propose a simple approach to for exploiting optimally the information provided by technical analysis. Our optimal asset allocation strategy is easy to apply in practice and is quite robust to model misspecifications. Empirically, we apply the strategy to the US stock market from January 1926 to March 2011. In addition, we also examine strategy's performances during the recent financial crisis as well as over all the bear markets of the past 85 years. We find that the proposed strategy outperforms the usual fixed asset allocation strategy substantially, and does extremely well during the recent financial crisis.

Keywords: technical analysis; trading rules; asset allocation; bear markets; business cycle.

Biographical notes: Guofu Zhou is a Frederick Bierman and James E. Spears Professor of Finance at Olin Business School of Washington University in St. Louis. He has a BS degree from Chengdu College of Geology, China, and a PhD in Economics from Duke University. Prior to his PhD studies, he has publications in number theory, function theory and numerical solutions to partial differential equations. His current research interests are primarily in asset pricing and investments. He has published numerous papers in *JFE*, *RFS*, *JFQA*, *JF*, *JPM* and *FAJ*. He has won rewards for teaching MBA and MSF students and for conducting research.

Yingzi Zhu is an Associate Professor of Finance in the School of Economics and Management (SEM) at Tsinghua University. She holds an MBA and a PhD both from New York University. She worked in Citigroup NY as the Head of Quantitative Group for six years before joining the SEM in 2003. She had published widely in academic journals including *JFE*, *JFQA*, *FAJ*, and *Journal of Futures Markets*. She consults frequently to major financial institutions in China, such as ICBC and CICC. She teaches in the MBA, IMBA and executive programs of the SEM, and won teaching awards.

Sheng Qiang received his BA in Mathematics and Physics from Tsinghua University, and is now a graduate student in the Masters in Management Science program of both the Engineering Department and the School of Economics and Management, Tsinghua University. His research focuses on the interfaces of finance and operations management, risk management and behavioural economics.

1   Introduction

Asset allocation is one of the most important decisions to make in portfolio management. One has to determine not only the initial investments across asset classes, but also how often to rebalance the allocations over time in respond to new investment needs and market conditions. While there is a huge literature of research and practice about asset allocation (see e.g., Campbell and Viceira, 2002; Grinold and Kahn, 1999; and references therein), little attention has been paid to the use of an important source of information, the information implied by technical analysis from the stock market.

Technical indicators, such as moving averages and price-momentum signals that represent market price reactions to news and events, capture potentially important economic factors which are relevant for future market returns. The construction, use and study of such indicators are known as technical analysis to the investment community. In practice, major brokerage firms and newspapers have been publishing technical commentary on the market for years. With today's technology, technical indicators are readily available from various trading platforms, and are an important component of the real-time information set used by traders and investors (e.g., Billingsley and Chance, 1996; Covel, 2005; Park and Irwin, 2007; Lo and Hasanhodzic, 2010). However, academics have long been sceptical about the usefulness of technical analysis. As a result, there are few academic studies on its effectiveness in either predicting the market returns or improving portfolio performance. Cowles (1933) and Fama and Blume (1966) are examples of earlier empirical studies that find mixed evidence on the investment value of technical analysis. However, Brock et al. (1992), and especially Lo et al. (2000), do find strong evidence of profitability by using technical strategies in the US stock market. Overall, existing studies on technical analysis are mostly about utilising the buy or sell signals generated by technical analysis in ad hoc ways without using a utility maximisation framework. Averting this trend, Zhu and Zhou (2009) provide a theoretical foundation to technical analysis. Emphasising the combination of both investors' utility (investment objective) and the quality of the signals, they show that technical analysis adds value to commonly used allocation rules (that invest fixed proportions of wealth into stocks), and especially so when there is uncertainty about the model used to fit the stock prices.

This paper provides both the theory and empirical applications of using technical analysis optimally in a simple asset allocation model. Theoretically, unlike Zhu and Zhou (2009) who use a continuous-time model of the geometric Brownian motion type, we use a discrete-time generic model and extend their results to our model, making easy applications of the ideas and strategies in practice. Moreover, our proposed optimal strategy is quite robust to model misspecifications. Empirically, we apply the theoretical allocation results, various investment rules, to the US stock market from January 1926 to

March 2011. In addition, we also examine the performances of the rules during the recent financial crisis as well as over all the bear markets of the 85 years.

There are three major empirical findings. First, our proposed generalised moving average (GMA) investment rule, based on the signals from the technical analysis, outperforms substantially over the standard fixed allocation rule over the entire sample period, in terms of both certainty-equivalent (CE) returns and Sharpe ratios. Second, it performs extremely well during the recent financial crisis and over all the bear markets. For example, an investor with a risk aversion parameter of 4 would have incurred a loss of 36.39% CE return during the recent financial crisis, but he would have lost only 8.51% had he used the strategy incorporating technical signals. Third, in terms of the drawdown measure (the percentage drop from top to bottom), the performances of the GMA are far better than standard fixed allocation rule.

There are two important issues about technical analysis that need to be clarified to some practitioners and many academics. First, the profitability of technical analysis requires the predictability of the stock market, but it does not imply in any way that the market is inefficient. If the stock market is entirely unpredictable, then any evidence on the profitability of technical analysis is simply a result of chance because the trading signals are random noises and useless. However, the stock market is indeed predictable with ample evidence. For example, Pástor and Stambaugh (2001) provide strong in-sample evidence of predictability in a predictive system, and Rapach et al. (2010) find that the US stock market can even be consistently predicted out-of-sample by combining forecasts from the common economic variables. Ang and Bekaert (2007), Hjalmarsson (2010), and Henkel et al. (2011) find similar predictability based on macroeconomic variables across countries. Cochrane (2011) explains how this predictability literature profoundly shifts the emphasis of asset pricing theory from expected cash flows to discount rates.[1] Theoretically, the profitability of the stock market arises from the change of the investment opportunity set. Recent asset pricing models, such as Campbell and Cochrane (1999), Bansal and Yaron (2004), and Zhou and Zhu (2011), all imply a significant degree of predictability of the stock market. In practice, due to liquidity reasons, a large purchase may take days to finish, and the stock market may not incorporate all information at once. The bull market takes time to run and to respond to long-term expansion cycles of the economy.

The predictability of the stock market is not equal to market inefficiency. This is because the predictability is far away from being 100% accurate, and there is a significant economic risk associated with it. Hence, there is no guarantee that one can make large risk-adjusted returns out of the predictability found by aforementioned studies beyond transaction costs. However, it is quite possible that, as the case here in this paper, the predictability can help to outperform an investment strategy, on a risk-adjusted basis, that completely ignores it. Indeed, in a rational market, the predictability, though exists, must be small. Theoretically, given an equilibrium asset pricing model, Ross (2005) and Zhou (2010) provide upper bounds on the degree of predictability.

The second issue about technical analysis that is often misunderstood is about its information context. Some researchers believe that technical indicators contain no important economic information about the stock market at all. While it is quite possible that if one has the true information about an asset or the market, one can profit from it without using technical analysis at all which provides no additional useful information in this idealised case. But in the real world, no one is sure either he has the true information or has all the information. Even if he does have all the true information, there is no

guarantee that the market will react the way he expects, as this will depend on also how that information is shared in the market and what investment demand the other investors have. Indeed, the equilibrium market price every day is an aggregate of all the information and reactions of all the participants to this information, and so the result depends on, among other factors, their changing investment objectives and wealth. Therefore, recent prices, which reflect how others react to the information, should be useful for an investor to learn about the market beyond what information he has. Indeed, as shown by Neely et al. (2012), technical indicators can help predict the stock market far better than using available macroeconomic variables alone. Another example is that, prior to the invention of the term systemic risk, technical indicators already capture such a risk during the financial crisis, long before economists and practitioners use measures of it to predict or explain the market. The point is that technical indicators provide a unique source of market information which cannot be replaced by any other well defined economic variables. In short, technical analysis reflects how the market reacts not only to past information, but also to pending future news and anticipated events. It contains unique information about market deriving factors unavailable elsewhere.

The rest of the paper is organised as follows. Section 2 derives the optimal GMA investment rule and discusses its properties. Section 3 studies the performances of the GMA rule, the usual MA rule and the standard fixed allocation rule using the 85 years of stock market data: over the entire period, during the recent financial crisis and over all the bear markets. Section 4 concludes.

2   The GMA rule

In this section, we derive our optimal investment rule that uses technical analysis, primarily the moving average (MA) indicator. Our rule, called the generalised MA or the GMA, is a combination of the standard MA rule and a fixed asset allocation rule. This fixed asset allocation rule is in general different from the standard fixed asset allocation rule which is obtained under the assumption that the stock returns are unpredictable.

For simplicity, we consider the case of a single risky asset and a riskless asset, whose returns resemble the market and the T-bill returns. Let R_{ft} be riskless rate of continuously compounded return, then

$$\tilde{R}_{t+1} = \log\left(S_{t+1} / S_t\right) - R_{ft}$$

is the excess return, where S_t is the price of the risky asset at time t. Assume that

$$\tilde{R}_{t+1} = E_t\left[\tilde{R}_{t+1}\right] + \sigma_t \varepsilon_{t+1}, \tag{1}$$

where $E_t[\tilde{R}_{t+1}]$ is the expected excess return at time t, ε_{t+1} is innovation at time $t + 1$, and σ_t the conditional volatility at time t. In contrast, Zhu and Zhou (2009) assume that the return model is continuous and $E_t[\tilde{R}_{t+1}]$ is mean-reverting. Here we do not make such assumptions. In fact, our set-up is much more general. Both σ_t and ε_{t+1} can follow almost any stochastic processes. As shown below, as long as we assume that the excess return and the MA signals are correlated, the proposed strategy or the technical analysis will add value to investments.

Following the standard technical analysis, we define a MA for a given window length
L as

$$A_t(L) = \frac{1}{L}\sum_{i=0}^{L-1}S_{t-i},\tag{2}$$

which is simply the average price of the past *L* periods up to time *t*. If $L = 200$ and the
time period is one day, then $A_t(L)$ is simply the popular 200-day price MA as reported in
Wall Street Journal and the *Investor's Business Daily*.

Define now the commonly used MA strategy $\tilde{\eta}$, called pure MA here,

$$\tilde{\eta}_t = \begin{cases} 1, & \text{if} \quad S_t > A_t(L); \\ 0, & \text{otherwise}, \end{cases}\tag{3}$$

with *L* is the window or lag length. $\tilde{\eta}_t$ corresponds to the usual simple MA investment
strategy which invests 100% of the wealth into the risky asset when the stock price is
above the MA, and invests 0% otherwise. This pure MA strategy, which takes an
all-or-nothing allocation to the risky asset, is a widely used technical trading rule, but is
suboptimal for two fundamental reasons. First, it does not incorporate investor's
objection or utility into consideration. In particular, it ignores investor's utility and the
level of risk tolerance in particular. Theoretically, any optimal allocation decision should
in general be a function of the risk-aversion parameter. Second, the degree of
predictability must matter. The more reliable the signal, the more it should be followed.
The all-or-nothing allocation fails to take this into consideration.

Intuitively, the more reliable the MA rule, the more allocation to the stock when the
signal is a buy. To quantify this, we define a generalised rule, the GMA rule, as a
combination of the pure MA rule and a fixed allocation rule,

$$GMA(\tilde{\eta}, \gamma) = \xi_{fix} + \xi_{mv} \cdot \tilde{\eta},\tag{4}$$

where ξ_{fix} and ξ_{mv} are constants, which will be determined by an investor's utility function
or investment objective. The intuition is that, if the investor invests an optimal fixed
proportion of his money into the stock market, say 80% unconditional of the MA signal,
he should invest more than 80% when the MA signals a buy, and less otherwise. Thus,
the 100% or 0% allocation of the all-or-nothing strategy is unlikely to be optimal. The fix
rule ξ_{fix} differs from the usual fixed asset allocation rule, which is obtained under the
assumption that the stock returns are unpredictable, because ξ_{fix} is only part of the
allocation here, while the usual fixed asset allocation rule is the entire fixed allocation to
the risky asset no matter what the signals are. For the GMA trading rule, the one-period
mean and variance of the associated portfolio at time *t* are given as

$$\mu_P(t) = (1 + R_f) + (\xi_{fix}E_t[\tilde{R}_{t+1}] + \xi_{mv}E_t[\tilde{\eta}_t\tilde{R}_{t+1}]),\tag{5}$$

$$\sigma_P^2(t) = [\xi_{fix}^2 \text{Var}_t(\tilde{R}_{t+1}) + \xi_{mv}^2 \text{Var}_t(\tilde{\eta}_t\tilde{R}_{t+1}) + 2\xi_{fix}\xi_{mv}\text{Cov}_t(\tilde{\eta}_t\tilde{R}_{t+1}, \tilde{R}_{t+1})].\tag{6}$$

For simplicity, we assume a mean-variance utility

$$V_t = \mu_P(t) - \frac{\gamma}{2}\sigma_P(t)^2.$$

Following Ferson and Siegel (2001) and Zhou (2008), among others, we maximise

$$V = E[V_t] = E\left[\mu_P(t) - \frac{\gamma}{2}\sigma_P(t)^2\right]. \tag{7}$$

Substituting the mean (5) and variance (6) into (7), we obtain

$$V(\xi_{fix}, \xi_{mv}) = 1 + R_f + m_1\xi_{fix} + m_2\xi_{mv} - \frac{\gamma}{2}\left(\xi_{fix}^2 v_1 + \xi_{mv}^2 v_2 + 2\xi_{fix}\xi_{mv}v_{12}\right), \tag{8}$$

where

$$m_1 \equiv E\left[\tilde{R}_t\right], \quad m_2 \equiv E\left[\tilde{\eta}_t\tilde{R}_{t+1}\right], \tag{9}$$

$$v_1 \equiv E\left[\mathrm{Var}_t\left(\tilde{R}_{t+1}\right)\right], v_2 \equiv E\left[\mathrm{Var}_t\left(\tilde{\eta}_t\tilde{R}_{t+1}\right)\right], v_{12} \equiv E\left[\mathrm{Cov}_t\left(\tilde{\eta}_t\tilde{R}_{t+1}, \tilde{R}_{t+1}\right)\right]. \tag{10}$$

Note that m_1 and m_2 are unconditional means of the excess returns of a 100% fix rule and the pure MA rule, v_1 and v_2 are unconditional means of their conditional variances, and v_{12} is the unconditional mean of conditional covariance between them. Although v_1, v_2 and v_{12} are not exactly in the form of unconditional moments, we show in next section that, when time interval is small, they can be well approximated by unconditional moments which can be conveniently estimated from sample moments. In comparison with Zhu and Zhou (2009) under a multi-period dynamic setting, the optimal GMA strategy characterised by ξ_{fix} and ξ_{mv} through optimising (8) is similar to theirs. It turns out that the unconditional mean-variance framework captures the hedging effect to the extent that it provides just enough information so that the rule is equivalent to the approximate GMA1 rule in Zhu and Zhou (2009).

To derive the optimal GMA rule under (8), we have the first order conditions for optimising $V(\xi_{fix}, \xi_{mv})$ in (8) as

$$\frac{dV}{d\xi_{fix}} = m_1 - \gamma\xi_{fix}v_1 - \gamma\xi_{mv}v_{12} = 0,$$

$$\frac{dV}{d\xi_{mv}} = m_2 - \gamma\xi_{mv}v_2 - \gamma\xi_{fix}v_{12} = 0.$$

Solving the above equations, we obtain the optimal GMA rule as

$$\begin{aligned} \xi_{fix} &= \frac{1}{\gamma}\frac{m_1 v_2 - m_2 v_{12}}{v_1 v_2 - v_{12}^2}, \\ \xi_{mv} &= \frac{1}{\gamma}\frac{m_2 v_1 - m_1 v_{12}}{v_1 v_2 - v_{12}^2}. \end{aligned} \tag{11}$$

It will be useful to make several remarks. First, the optimal GMA rule depends on the risk aversion parameter γ, which points to the fact that the MA strategy, or technical analysis in general, should be optimised according to investor's risk preference to be optimal.

Second, the GMA is useful to improve asset allocation performance as long as the MA signal is predictive to the conditional expected return. To see this, let us assume that

the MA signal $\tilde{\eta}_t$ is unconditionally independent of the conditional expected return, $E_t[\tilde{R}_{t+1}]$, then we have

$$
\begin{aligned}
v_{12} &= E\left[Cov_t\left(\tilde{\eta}_t \tilde{R}_{t+1}, \tilde{R}_{t+1}\right)\right] \\
&= E\left[E_t\left(\tilde{\eta}_t \tilde{R}_{t+1}^2\right) - E_t\left(\tilde{\eta}_t \tilde{R}_{t+1}\right) E_t \tilde{R}_{t+1}\right] \\
&= E\left[\tilde{\eta}_t E_t\left[\tilde{R}_{t+1}^2\right] - \tilde{\eta}_t \left(E_t \tilde{R}_{t+1}\right)^2\right] \\
&= E[\tilde{\eta}] \cdot v_1
\end{aligned}
$$

and

$$
m_2 = E[\tilde{\eta}_t \tilde{R}_{t+1}] = E[\tilde{\eta}] \cdot m_1.
$$

In this case, we must have the standard Markowitz allocation rule,

$$
\xi_{\text{fix}} = \frac{1}{\gamma} \frac{m_1}{v_1}, \quad \xi_{\text{mv}} = 0. \tag{12}
$$

Third, as apparent from (11), as long as the MA signal $\tilde{\eta}_t$ and the excess stock return $\tilde{R}_t$ are not independent, the optimal ξ_{mv} will not be zero, that is, the MA must add value to the fixed strategy. In particular, when the autocorrelation of the returns is non-zero, $\tilde{\eta}_t$ and $\tilde{R}_t$ will not be independent because $\tilde{\eta}_t$ contains past price information.

Fourth, to further assess how the MA adds value to the unconditional value function, we assume that the conditional volatility, σ_t, is non-stochastic and constant, that is,

$$
\sigma_t^2 = \mathrm{Var}_t\left(\tilde{R}_{t+1}\right) = \sigma^2,
$$

where σ is a constant. Then the moments can be simplified as

$$
v_1 = E\left[\mathrm{Var}_t\left(\tilde{R}_{t+1}\right)\right] = \sigma^2,
$$

$$
v_2 = E\left[\mathrm{Var}_t\left(\tilde{\eta}_t \tilde{R}_{t+1}\right)\right] = E\tilde{\eta}_t \cdot \sigma^2 = p\sigma^2,
$$

$$
v_{12} = E\left[Cov_t\left(\tilde{\eta}_t \tilde{R}_{t+1}, \tilde{R}_{t+1}\right)\right] = E\tilde{\eta}_t Cov_t\left(\tilde{R}_{t+1}, \tilde{R}_{t+1}\right) = E\tilde{\eta}_t \cdot \sigma^2 = p\sigma^2, \tag{13}
$$

where

$$
p \equiv E[\tilde{\eta}_t]
$$

is the probability that the MA signals a 'buy'. In this case, the portfolio holding of the GMA rule (11) are

$$
\begin{aligned}
\xi_{\text{fix}} &= \frac{1}{\gamma} \frac{m_1 - m_2}{(1-p) \cdot v_1}, \\
\xi_{\text{mv}} &= \frac{1}{\gamma} \frac{m_2 - p \cdot m_1}{p \cdot (1-p) \cdot v_1},
\end{aligned} \tag{14}
$$

where m_1, m_2, v_1 are defined in (9) and (10). Note that m_1 is the unconditional expectation of the daily excess return, and m_2 is the unconditional expectation of the excess return on the pure MA strategy. Intuitively, the great the deviation of ξ_{mv} from 0, the more the value added by the MA rule. In this case, note that the numerator of ξ_{mv} in equation (14) coincides with the unconditional covariance of the MA signal and the expected excess return, i.e.,

$$Cov\left(E_t\tilde{R}_{t+1}, \tilde{\eta}_t\right) = E\left[\tilde{\eta}_t\tilde{R}_{t+1}\right] - E\tilde{R}_{t+1}E\tilde{\eta}_t = m_2 - p\cdot m_1.$$

This confirms our intuition that the more correlated is the MA signal with the expected excess return, the more value added by MA rule. Also note that, the optimal GMA rule of (14) with constant conditional variance implies that the average portfolio holding over time is

$$E\left[\xi_{fix} + \xi_{fix}\eta\right] = \xi_{fix} + p\cdot\xi_{mv} = \frac{1}{\gamma}\frac{m_1}{v_1},$$

which is the same as that of the usual fix allocation rule.[2]

Moreover, consider the value function under constant conditional variance with optimal GMA given in (14). It is easy to verify that the value function, defined in (8), of the optimal GMA is

$$V_{GMA} = 1 + R_f + \frac{1}{2\gamma}\frac{pm_1^2 - 2pm_1m_2 + m_2^2}{p(1-p)v_1}, \tag{15}$$

and the value function of the usual fixed rule is

$$V_{Fix} = 1 + R_f + \frac{1}{2\gamma}\frac{m_1^2}{v_1}. \tag{16}$$

Their difference is thus

$$V_{GMA} - V_{Fix} = \frac{1}{2\gamma}\frac{(pm_1 - m_2)^2}{p(1-p)v_1}. \tag{17}$$

It is clear from this equation that the GMA rule can always improve upon the usual fixed rule, except when the MA signal $\tilde{\eta}_t$ is independent from expected excess return $E_t\tilde{R}_t$, that is, when $pm_1 = m_2$, then the optimal GMA rule coincides with the usual fixed rule.

Finally, the pure MA rule is clearly suboptimal in comparison with the GMA. However, due to its popularity in practice, we examine the conditions under which the pure MA rule ($\xi_{fix} = 0$ and $\xi_{mv} = 1$) is a good rule of thumb. Based on (14), the condition for $\xi_{fix} = 0$ is clearly $m_1 = m_2$; and the condition for $\xi_{mv} = 1$ is $\gamma = 4$, assuming an annualised risk premium of 6% and a market of volatility 18%, which are in general agreement with the S&P 500 data. It turns out that, as shown in the next section, the pure MA is indeed a good rule of thumb when the investor's risk aversion is 4.

3 Empirical applications

In this section, we compare the performances of the three investment rules, the GMA, the pure MA and the usual fixed asset allocation rules in three applications. The first is to apply them to the US stock market from January 1926 to March 2011. The second is to apply them to during the recent financial crisis period, the third is to all the bear markets of the past 85 years.

In the spirit of the common use of asset allocation models, we use the long-term moments of the data to estimate the unconditional parameters and rules. These values correspond to those stylised numbers used in calibration studies. For simplicity, we use all the data to estimate the fundamental moment parameters since they should be roughly the same over any reasonable long periods. For example, we use $\mu = 9.06\%$ and $\sigma = 15.25\%$. For conditional volatilities, we use GARCH (1,1) model for rolling estimates. In addition, the MA signals must vary over time as the prices change when we apply the rules to the real data. Moreover, following the usual practice, the MA signals are computed using daily prices that do not include the dividends, but all returns on the investment rules are computed realistically as the total return including dividends. Treasury bill rates are used as proxy for risk-free rates over time. To avoid real world constraints, for all the rules, both leverage and short-sells are not allowed, and so the allocations to the stock index are always between 0 and 100%.

How do we assess the performances over time? We view the rules an ex ante investment strategies, which are invariant across data or markets, with the obvious exception that the signals must be updated with the prices. Hence, the rule that has the best performance over time or datasets is the best one. The question is how to measure the performance over time. Following many studies (e.g., Tu and Zhou, 2011), we use the CE return as the risk-adjusted performance measure,

$$CE\ return = \hat{\mu}_P - \frac{\gamma}{2}\hat{\sigma}_P^2, \tag{18}$$

where $\hat{\mu}_P$ and $\hat{\sigma}_P^2$ are the annualised sample mean and variance, respectively, of the realised portfolio returns of an investment strategy. The CE return can be perceived as the rate of return of the risk-free investment to which the investor is indifferent to when comparing with the risky portfolio. It can also be interpreted as the realised utility of a mean-variance investor when he uses the strategy repeatedly over time.

Note that the trading signals are computed daily, but the duration of a buy or sell signal is typically longer so that there are only a few trading days in a year to decide whether to include the MA component or not into the fixed allocation rule. Hence, the rebalance frequency of the portfolio can be set at a longer frequency, say monthly or quarterly, with little effect on the comparison between the GMA and the usual fixed strategy since the rebalancing transaction costs of the two should be close. For simplicity, we ignore transaction costs in this paper, and hence, without loss of generality, we can simply rebalance the portfolio daily as if we are working in a continuous-time model.[3]

Table 1 provides the CE returns and other performance measures for the GMA, the pure MA and the usual fixed allocation rules, which are denoted in the table as 'GMA', 'pure MA' and 'fix', respectively. Consider first the case of $\gamma = 4$. Across the MA windows or lags, the popular 200-day MA works the best for both the GMA and the pure MA, both of them earned annually 2% CE returns over the fix rule. This is economically

significant. Given that this is so for over 80 years, the economic significance is even greater. However, in terms of sample returns, the GMA is only about 80 basis points higher, but the risk is much lower. As a result, the Sharpe ratio is 0.63, much greater than 0.46 of the fix rule. The pure MA behaves similarly, in line with early studies of Siegel (1998). Theoretically, the GMA should always outperform the pure MA. But when $\gamma = 4$, they are close for the theoretical reason pointed out earlier in the previous section. In fact, sample errors can play a role too. Hence, the close performance of the GPA and pure MA when $\gamma = 4$ should not be surprising. As expected, when the degree of risk aversion increases from 4 to 7 or 10, the GMA outperforms the pure MA, and outperform the fix too.

Table 1   Performances over the entire sample period

	GMA			Pure MA			Fix
MA lags (days)	20	200	250	20	200	250	
$\gamma = 4$							
CE return	6.25	6.85	6.76	4.41	7.47	6.83	4.70
Mean	8.10	8.57	8.44	6.94	10.16	9.66	7.32
Std	9.61	9.28	9.18	11.26	11.58	11.90	11.44
Sharpe ratio	0.63	0.70	0.69	0.43	0.70	0.64	0.46
Drawdown	42.53	32.29	25.37	47.02	40.85	56.32	67.13
$\gamma = 7$							
CE return	5.00	5.57	5.48	2.50	5.46	4.71	3.58
Mean	7.06	7.84	7.67	6.94	10.16	9.66	5.08
Std	7.67	8.05	7.90	11.26	11.58	11.90	6.54
Sharpe ratio	0.65	0.71	0.71	0.43	0.70	0.64	0.46
Drawdown	29.06	24.36	18.93	47.02	40.85	56.32	44.33
$\gamma = 10$							
CE return	4.29	4.68	4.59	0.60	3.45	2.59	3.13
Mean	6.29	7.12	6.89	6.94	10.16	9.66	4.18
Std	6.33	6.99	6.78	11.26	11.58	11.90	4.58
Sharpe ratio	0.66	0.72	0.71	0.43	0.70	0.64	0.46
Drawdown	20.29	17.94	14.90	47.02	40.85	56.32	31.87

Notes: The table reports the performances of the GMA rule, the pure MA rules, and the usual fix rule from January 1926 to March 2011. The CE returns, means and standard deviations, of the portfolios associated with the investment rules, are annualised and in percentage points.

Today, risk management is becoming increasingly important and the control of the drawdown of a portfolio, defined as the percentage drop from the top to bottom, is the objective of many portfolio managers. In terms of the drawdown, the GMA does in general the best across all lags and risk aversions. Over the entire period, the fix rule suffers a drawdown of 67.13%. In contrast, the GMA has only 32.29% when $L = 200$ and $\gamma = 4$, losing less than half of the money. The drawdown of the pure MA is close to that of the GMA when $\gamma = 4$, but much lager and even greater than the fix when $\gamma = 7$ or $\gamma = 10$. The reason is that, as the risk aversion increases, both the GMA and the fix will

cut their stock allocations, while the pure MA, which is γ independent, invests the same way as before. So the sample statistics of the pure MA remain as large as before, though the ex post risk-adjusted returns vary with the risk version.

Table 2 Market return statistics conditional on 'buy' and 'sell' signals

	'Buy'	*'Sell'*	*Unconditional*
MA lags (days)	$L = 20$		
Mean	8.29	8.44	8.33
Std	14.70	22.23	18.19
Skewness	–0.12	0.04	–0.01
Kurtosis	13.46	18.06	20.01
MA lags (days)	$L = 200$		
Mean	12.27	0.72	8.33
Std	14.27	24.09	18.19
Skewness	–0.27	0.13	–0.01
Kurtosis	15.28	15.59	20.01
MA lags (days)	$L = 250$		
Mean	11.36	2.31	8.33
Std	14.55	24.01	18.19
Skewness	–0.99	0.46	–0.01
Kurtosis	27.66	12.60	20.01

Notes: This table reports the sample statistics of market returns conditional on the MA signal is a 'buy' or 'sell', with a lag length, *L*, of 20-, 200- and 250-days, respectively. The second and third columns report the sample statistics of the returns on 'buy' and 'sell' days, and the fourth column reports the same sample statistics of the unconditional returns. The results are based on the S&P500 data from January 1926 to March 2011. The means and standard deviations are annualised and in percentage points.

Why do the MA rules work? While leaving the economic explanations elsewhere (see, e.g., Han et al., 2011, and references therein), we provide here the statistical intuition. We compute the mean, standard deviation, skewness and kurtosis of the market returns conditional on MA 'buy' and 'sell' signals (when $\tilde{\eta}_t$ is 1 or 0). Table 2 presents the results. When $L = 20$, the sample standard deviation of the 'buy' days are much smaller than that of the 'sell' days. But the striking results occur when the lag is long-term. Let us focus on $L = 200$. The mean of the returns on days with the 'buy' signals is 12.27%, which is about 17 times as large as 0.72%, the sample mean of the return on days with 'sell' signals. In addition, the standard deviation is only about half of the 'sell' signals, while the kurtosis values are similar. Clearly, the signals are quite informative about the ups and downs of the stock market returns, and hence, no matter how one uses the signals, it is likely to be much more profitable compared with a strategy of not using such information at all. For example, one can potentially use them to predict future density of stock returns and then uses it to maximise whatever objective function of an investor with whatever practical constraints.

Table 3   Performances in the recent financial crisis

	GMA			Pure MA			Fix
MA lags (days)	20	200	250	20	200	250	
$\gamma = 4$							
CE return	–18.43	–8.51	–11.18	–23.54	–8.46	–10.38	–36.39
Mean	–16.28	–7.95	–10.50	–18.77	–7.83	–9.63	–24.92
Std	10.38	5.31	5.86	15.45	5.64	6.10	23.95
Sharpe ratio	–1.77	–1.90	–2.15	–1.35	–1.76	–1.92	–1.13
Drawdown	22.64	11.34	14.59	26.30	11.47	14.01	37.20
$\gamma = 7$							
CE return	–10.78	–6.11	–7.32	–27.12	–8.94	–10.93	–20.51
Mean	–9.46	–5.55	–6.73	–18.77	–7.83	–9.63	–13.95
Std	6.13	4.03	4.10	15.45	5.64	6.10	13.68
Sharpe ratio	–1.89	–1.90	–2.16	–1.35	–1.76	–1.92	–1.17
Drawdown	13.75	8.18	9.71	26.30	11.47	14.01	22.30
$\gamma = 10$							
CE return	–7.34	–4.46	–5.14	–30.70	–9.42	–11.49	–14.16
Mean	–6.42	–4.00	–4.69	–18.77	–7.83	–9.63	–9.57
Std	4.29	3.02	2.98	15.45	5.64	6.10	9.58
Sharpe ratio	–1.99	–2.02	–2.28	–1.35	–1.76	–1.92	–1.22
Drawdown	9.52	5.99	6.86	26.30	11.47	14.01	15.74

Notes: The table reports the performances of the GMA rule, the pure MA rules, and the
       usual fix rule during the recent financial crisis. The results are for the time period
       from October 10, 2007 to March 10, 2009, during which the market dropped
       56.21% from the top to bottom. The lag length, L, of the MA is 20-day, 200-day
       and 250-day, respectively. The means and standard deviations are annualised and
       in percentage points.

Consider now the performances of the rules during the recent financial crisis. We focus
on the period from July 10, 2007 to March 10, 2009, during which the market dropped
56.21% from the top to bottom. Table 3 provides the results. When $\gamma = 4$, the usual fix
rule loses 36.39%, the GMA and MA lose only 18.43% and 23.54% when $L = 20$, and
much smaller, 8.51% and 8.47% when $L = 200$. In contrast to the case over the entire
sample period, the results here when $L = 200$ is remarkable. It lost only 1/4 of the value.
Since the wealth or consumption is more valuable when there is scarcity, the low loss of
the GMA and MA strategies should be extremely important and appealing to those real
world investors who have disappointment aversion preferences, such as loss limits on
their wealth or portfolios. The much better CE returns are also reflected in the
drawdowns. While the fix has a drawdown of 37.20%, both the GMA and MA have only
11.34% and 11.47%. The results are similar qualitatively when the risk aversion
increases, but the magnitudes of the relative performances of the GMA and pure MA
over the fix have come down, though still substantially better. Overall, technical analysis
is of great value in reducing the risk during the recent financial crisis. To those who use
it, they would have been able to reduce their losses a few times smaller than those who
completely ignore it by following the standard fixed asset allocation rule.

Table 4 Performances in bear markets

Bear market			CE return		
Begin	*End*	*ΔP/P*	*GMA*	*Pure MA*	*Fix*
1929/09/09	1932/06/02	−86.99	−4.50	−8.22	−15.14
1932/09/08	1933/02/28	−40.81	−6.97	−35.03	−23.93
1933/07/18	1935/03/15	−34.68	−4.71	−40.34	−4.45
1937/03/11	1942/04/29	−60.39	−3.53	−8.45	−2.04
1946/05/31	1949/06/14	−30.84	−2.44	−3.62	−0.27
1956/08/03	1957/10/23	−22.22	−4.19	−4.65	−1.87
1961/12/13	1962/06/27	−28.00	−10.39	−10.39	−13.21
1966/02/10	1966/10/10	−22.38	−8.44	−8.44	−6.99
1968/12/02	1970/05/27	−35.85	−4.46	−4.48	−4.20
1973/01/12	1974/10/04	−48.35	−3.82	−4.95	−7.06
1980/12/01	1982/08/13	−27.02	−1.00	−3.55	0.42
1987/08/26	1987/12/07	−33.67	−19.26	−66.02	−36.22
1990/07/17	1990/10/12	−20.04	−23.75	−34.78	−18.47
2000/03/27	2002/10/10	−49.01	−3.43	−7.41	−5.87
2007/10/10	2009/03/10	−56.21	−4.46	−9.42	−14.16

Notes: The table reports the performances of the GMA rule, the pure MA rules, and the usual fix rule in all the 15 bear markets over 1926 to 2011, which are defined as those periods in which the S&P500 dropped for more than 20% from peak to trough. The first two columns are the starting (peak) and ending (trough) dates. The third column is the percentage of the price change, denoted as $\Delta P/P$. The fourth to sixth columns are the CE Returns of the three rules, respectively. The results are for the conservative case where $L = 200$ and $\gamma = 10$. The CE returns are annualised and in percentage points.

Finally, we examine the performances of the rules over all the bear markets over the January 1926 to March 2011 time period. The bear markets often defined as those periods for the market (the S&P500 index here) to drop more than 20% from peak to trough. Following this definition, the first two columns of Table 4 identify all the bear markets of the past 85 years. The last bear market coincides with the recent financial crisis examined earlier. As we discussed before, a higher risk aversion parameter generally narrows the performances of the GMA and the fix. For brevity, the table reports only the most conservative case when $\gamma = 10$. However, we still use $L = 200$ because of its popularity and being relatively longer-term. Even in this most conservative case, the GMA outperforms the fix most of the time. In general, the greater the drop of the market, the more the GMA outperforms. Note that sometimes there is a large difference between the bear market drops and the CE returns of the fix rule. The reason is that the percentage price changes in the bear market are the returns of the buy-and-hold strategy without the dividends for the entire bear market period. In contrast, the CE return of the fix is the annualised risk-adjusted portfolio return with dividends. Moreover, when $\gamma = 10$, the fix rule has only a small portion of the wealth in the risky asset. Overall, the message of the applications to the bear markets is the same as before that technical analysis adds value to asset allocation, and especially so during a great downturn of the market. In the bear

markets, the GMA outperforms the fix most of the time, and it achieves at least similar performances in the few cases with certain large risk aversion parameters when it does not outperform the fix.

4   Conclusions

Technical analysis provides a unique and important source of information about future stock returns, but this information is largely ignored in asset allocation decisions. Extending Zhu and Zhou's (2009) geometric Brownian motion type model, we propose a simple approach for exploiting optimally the technical analysis information in a generic discrete-time model of the stock returns. Our proposed optimal portfolio strategy is easy to apply in practice and is quite robust to model misspecifications. Empirically, we apply the strategy to the US stock market from January 1926 to March 2011. In addition, we also examine the strategy's performance during the recent financial crisis as well as over all the bear markets of the past 85 years. We find that the proposed strategy outperforms the usual fixed allocation strategy substantially, especially so during the recent financial crisis.

For simplicity, our exploratory study assumes a simple mean-variance utility, while allowing for a generic discrete-time process of the market returns. As long as the MA has predictive power on the stock market, certain structures on the data-generating process may be imposed to solve a general utility maximisation problem, say, a utility of up to the fourth moments, under desired portfolio constraints. On the other hand, the MA is only one of the dozens or more technical indicators. Hence, this paper provides just a lower bound on the value of using technical analysis. Future studies are called for to examine the value of other technical indicators and the value of using them in other markets and asset classes. Theoretically, is technical analysis a tool of learning or a mechanism for herding or both? What equilibrium asset pricing models can one propose that allow for a role of technical analysis? All of these are interesting topics for future research.

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Notes

1   See Rapach and Zhou (2012) for a recent survey on the vast literature about stock market predictability.
2   Note that Zhu and Zhou (2009) obtain the same result for log utility in a continuous-time model.
3   See Han et al. (2011) for an example of incorporating transaction costs into technical trading strategies.