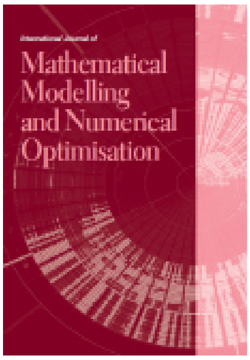


International Journal of Mathematical Modelling and Numerical Optimisation

ISSN online: 2040-3615 - ISSN print: 2040-3607
https://www.inderscience.com/ijmmno

Investigation of the accuracy of an approximate solution of the initial boundary value problem of thermal conductivity with boundary conditions of the third kind

Alexandra S. Nikolaeva, Maxim A. Evtushenko, Luiza A. Manukyan

DOI: 10.1504/IJMMNO.2025.10069967

Article History:

Received:	06 February 2025
Last revised:	13 February 2025
Accepted:	13 February 2025
Published online:	08 April 2025

Investigation of the accuracy of an approximate solution of the initial boundary value problem of thermal conductivity with boundary conditions of the third kind

Alexandra S. Nikolaeva*

Department of Theoretical Mechanics,
Samara National Research University,
Moskovskoe shosse, 34, Samara, Russia
Email: ezhevichka333@gmail.com
*Corresponding author

Maxim A. Evtushenko

Training Laboratory of Ground Testing of Aviation Vehicles,
Samara National Research University,
Moskovskoe shosse, 34, Samara, Russia
Email: m.evtushenko.a@yandex.ru

Luiza A. Manukyan

Department of Space Engineering,
Samara National Research University,
Moskovskoe shosse, 34, Samara, Russia
Email: lu-iza03@mail.ru

Abstract: The paper compares two approximations of the solution of the initial boundary value problem of thermal conductivity with boundary conditions of the third kind for the case of a temperature shock of a homogeneous thin plate. A second-order approximation with respect to the plate thickness is obtained. The results of the numerical simulation are compared with the known first-order approximation with respect to the plate thickness. The results obtained demonstrate good convergence. They can be used in controlling the motion of a small spacecraft with elastic structural elements. It is necessary to level out the effect of temperature shock disturbances on the angular motion of a small spacecraft.

Keywords: thermal conductivity problem; boundary conditions of the third kind; temperature shock; thin plate.

Reference to this paper should be made as follows: Nikolaeva, A.S., Evtushenko, M.A. and Manukyan, L.A. (2025) 'Investigation of the accuracy of an approximate solution of the initial boundary value problem of thermal conductivity with boundary conditions of the third kind', *Int. J. Mathematical Modelling and Numerical Optimisation*, Vol. 15, No. 1, pp.52–63.

Biographical notes: Alexandra S. Nikolaeva is a researcher at the Space Engineering Research Institute. She was awarded the Russian Academy of Sciences Medal for Young Researchers (2022) for her success in research. Since 2023, she has been involved in the implementation of works within the framework of State Assignments to higher education institutions and research organisations in the field of scientific activity (project FSSS-2023-0007). Her areas of interests are theoretical and applied mechanics, heat conduction, initial boundary value problems, modelling the motion of space bodies. She has published many papers in the international journals.

Maxim A. Evtushenko is a Post-Graduate student of the Space Engineering Department. He is an Engineer of the Laboratory of Ground Testing of Aerospace Vehicles. Maxim has been an assistant at the Department of Space Engineering since 2024, where he teaches practical classes for students. For achievements in research, he was awarded the scholarship of the President of the Russian Federation (2024–2027). His areas of interests are applied mechanics, mechanics of composites, vibration theory, computer modelling. He has published many papers in the international journals.

Luiza A. Manukyan is a Space-Engineering specialist at the Department of Space Engineering. Her area of interest is spacecraft motion modelling, measurement data processing. She has presented her results at international conferences, including the Symposium on Applied and Industrial Mathematics. She also participated in the implementation of the Russian Science Foundation (Project No. 23-29-00207). The results of which led to the publication of an article in an international journal.

1 Introduction

From a theoretical point of view, the one-dimensional problem of thermal conductivity is well studied. So, back in 1905, work Einstein (1905) on the thermal motion of suspended particles was published. However, interest in this task has not waned so far. In Muftu (2022), a finite element form was developed for solving one-dimensional equations of thermal conductivity in a steady state with simple boundary conditions. Paper Biswas et al. (2023) is devoted to the search for a new nonlinear shape between two nodal points of a finite element model of one-dimensional thermal conductivity. In Sedelnikov and Orlov (2020), the law of control of a small spacecraft was developed taking into account the temperature effects on it from the Sun in the framework of a one-dimensional thermal conductivity problem. Niu et al. (2021) consider traditional tools for analysing thermal conductivity in reactor systems to be ineffective. In their opinion, these approaches face serious difficulties in developing models, have low accuracy and poor convergence. In Niu et al. (2021), an automatic differentiation method is presented, which allows for automatic numerical calculation of derived functions when solving a one-dimensional thermal conductivity problem. It was about classical problems in the correct formulation according to Hadamard and Morse (1953).

Incorrect tasks according to Hadamard and Morse (1953), first of all, inverse tasks are unlikely to ever lose relevance. It is no coincidence that many works today are devoted to solving incorrect problems. In Jahangiri et al. (2019), a numerical method was used to solve the one-dimensional inverse problem of thermal conductivity, which is a combination of the Haar wavelet collocation (Mehandiratta et al., 2020) and the Tikhonov regularisation approach (Calvetti and Reichel, 2003). In Molhem and Pourgholi (2008), a numerical algorithm is presented for solving the one-dimensional inverse problem of thermal conductivity in a dimensionless form. The algorithm combines the use of the finite difference method with the solution of an ordinary differential equation. In Pourgholi et al. (2013), a numerical method for solving the inverse problem of thermal conductivity based on the Sinc-Galerkin method is proposed. A stable numerical solution to this problem is obtained. It is noted in Wang et al. (2024) that inverse problems of thermal conductivity play an important role in the engineering field. A method of fast Bayesian parallel sampling is proposed for large-scale estimation of the parameters of the reference three-dimensional inverse problem of thermal conductivity. This method allows you to quickly calculate the parameters on a scale of 105 by reducing the dimension of the problem domain, depending on space and time.

Boundary conditions of the third kind should be mentioned separately (Eremin and Gubareva, 2019). They are essentially nonlinear (Sidorov, 1969). This makes it more difficult to obtain an approximate analytical solution to the problem (Sedelnikov et al., 2021).

Even more difficult is the formulation of the thermal conductivity problem for rapidly alternating processes (Choi et al., 2022). One of these processes is temperature shock (Kartashov, 2012). In practice, a temperature shock occurs when a spacecraft with large solar panels is immersed in the shadow and comes out of the shadow (Sedelnikov et al., 2023). In this case, thermal vibrations are excited (Johnston and Thornton, 2000), which can lead to a deterioration in the quality of the spacecraft's solution to its target tasks (Serdakova, 2023). First of all, this applies to Earth remote sensing spacecraft (Khnyryova, 2023). However, when using thin ROSA solar panels (Lee et al., 2013), thermal fluctuations can significantly affect the dynamics of the spacecraft (Shen et al., 2017). During the tests of these panels on the International space Station, thermal fluctuations didn't allow them to collapse (Chamberlain et al., 2021). This shows that a temperature shock can affect the controllability of a spacecraft (Liu et al., 2022). Especially when it comes to small spacecraft (Liu et al., 2019). In Sedelnikov et al. (2024b), new technologies for controlling a small spacecraft are considered, taking into account disturbances from a temperature shock. It is noted that in order to construct an effective control law for a small spacecraft, it is necessary to have approximate analytical dependences of perturbations on a temperature shock. The high load of the on-board computer with the target tasks (Prokopyev et al., 2020) does not allow the installation of resource-intensive applications for the numerical evaluation of these disturbances (Joseph et al., 2018). Therefore, the derivation of approximate analytical dependencies is an important practical task for effective motion control of a small spacecraft.

The purpose of this work is to study the accuracy of the first approximation of solving a one-dimensional thermal conductivity problem with boundary conditions of the third kind for a thin plate under the action of a temperature shock. The first approximation is described in Sedelnikov et al. (2024a) and demonstrates good convergence with the numerical solution in the Ansys package.

2 Setting the task

Let consider the initial boundary value problem of one-dimensional thermal conductivity in the form (Sedelnikov et al., 2021, 2024a, 2024b):

$$\begin{cases} \dfrac{\partial T(z,t)}{\partial t} = a\dfrac{\partial^2 T(z,t)}{\partial z^2}, 0 \le z \le h, t > 0; \\ \left(\lambda \dfrac{\partial T(h,t)}{\partial z}\right) = Q - e\Theta\left(T^4(h,t) - T_C^4\right), z = h, t > 0; \\ \left(\lambda \dfrac{\partial T(0,t)}{\partial z}\right) = -e\Theta\left(T^4(0,t) - T_C^4\right), z = 0, t > 0; \\ T(z,0) = T_0 = const, 0 \le z \le h, t = 0, \end{cases}$$

where

a thermal conductivity coefficient of the material

h plate thickness

λ coefficient of thermal conductivity

e the degree of blackness of the plate material

Θ the Stefan-Boltzmann constant

T_c ambient temperature of the plate

Q eat flow incident on the surface of the plate.

Neglecting the ambient temperature (approximately 3 K (Guo et al., 2024)) in comparison with the temperature of the plate, we rewrite the system of equations as:

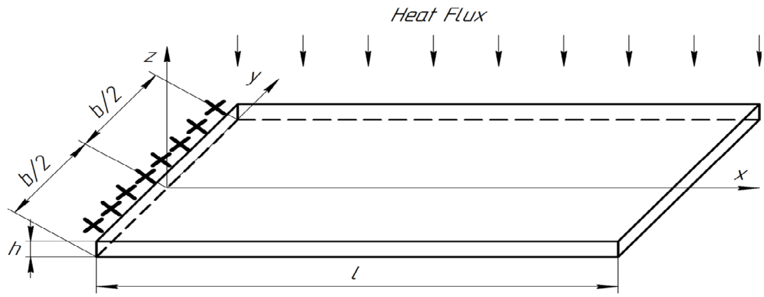
$$\begin{cases} \dfrac{\partial T(z,t)}{\partial t} = a\dfrac{\partial^2 T(z,t)}{\partial z^2}, 0 \le z \le h, t > 0; \\ \left(\lambda \dfrac{\partial T(h,t)}{\partial z}\right) = Q - e\Theta T^4(h,t), z = h, t > 0; \\ \left(\lambda \dfrac{\partial T(0,t)}{\partial z}\right) = -e\Theta T^4(0,t), z = 0, t > 0; \\ T(z,0) = T_0 = const, 0 \le z \le h, t = 0. \end{cases} \tag{1}$$

A homogeneous thin rectangular plate with constant thermophysical properties over the entire temperature range is considered as an object of research. The plate is rigidly fixed with one edge. The other three edges of the plate are free (Figure 1). Heat losses through the side surface of the plate are considered negligible. At the initial moment of time, the temperature field of the plate was homogeneous. At the moment of the thermal shock, the plate had a flat shape. The Q flow is perpendicular to the plate plane at all times.

Such a setting in Sedelnikov and Orlov (2020) and Sedelnikov et al. (2021) is defined as the most dangerous in terms of the effect of temperature shock on the dynamics of a small spacecraft. The flow perpendicular to the plate ensures maximum heating of its surface layer. In this case, the disturbances from the temperature shock will be maximal (Sedelnikov et al., 2024b).

Thus, the problem statement under consideration is relevant from the point of view of estimating the maximum effect of a temperature shock. Based on it, it is possible to make a decision on taking into account the temperature shock when modelling the motion of a small spacecraft. Therefore, the studies conducted in this paper on the accuracy of the approximate solution of the initial boundary value problem (1), presented in Sedelnikov et al. (2024a), are relevant and important.

Figure 1 The appearance of the plate being modelled in operation



3 Output of an approximate solution

For the output, we use the expansion (Sedelnikov et al., 2024a; Korneyev et al., 2019):

$$T_n(z,t) = \sum_{i=0}^{n} \xi_i(t) z^i \tag{2}$$

It is a well-known Fourier decomposition of a function of two variables (Watts, 2012). There are studies on the incorrectness of using such decomposition to describe complex processes (Arinchev, 2022). However, in this paper it is considered suitable for obtaining an approximate analytical dependence for the temperature field of the plate in the framework of the one-dimensional problem of thermal conductivity. This can be a problem for solving two-dimensional (Sedelnikov et al., 2021b) or three-dimensional thermal conductivity problems (Cheng and Ge, 2010).

As in Sedelnikov et al. (2024a), we take into account the first four terms in expansion (2):

$$T_3(z,t) = \xi_0(t) + \xi_1(t)z + \xi_2(t)z^2 + \xi_3(t)z^3 \tag{3}$$

Taking into account the derivatives, substituting (3) into the first equation (1) gives the following equation Sedelnikov et al. (2025):

$$\frac{d\xi_0(t)}{dt} + \frac{d\xi_1(t)}{dt}z + \frac{d\xi_2(t)}{dt}z^2 + \frac{d\xi_3(t)}{dt}z^3 = 2a\left[\xi_2(t) + 3\xi_3(t)z\right] \tag{4}$$

However, in Sedelnikov et al. (2024a), equation (5) was linearised with respect to the variable z. The plate in question is thin. The variable z varies from 0 to h. Therefore, such linearisation is fully justified. In this paper, it is proposed to take into account the terms of the second order relative to z in equation (4). After that, compare the results obtained with the linearised equation (4) and draw conclusions about the correctness and adequacy of the linearised equation (4).

Let's group the terms in (4), neglecting the terms above the second order with respect to *z*:

$$\left[\frac{d\xi_2(t)}{dt}\right]z^2 + \left[\frac{d\xi_1(t)}{dt} - 6a\xi_3(t)\right]z + \left[\frac{d\xi_0(t)}{dt} - 2a\xi_2(t)\right] = 0 \quad (5)$$

The functions in square brackets of equation (5) are generally independent. Therefore, in order to satisfy equation (5), we require that all of them be equal to zero at the same time:

$$\begin{cases} \dfrac{d\xi_2(t)}{dt} = 0; \\ \dfrac{d\xi_1(t)}{dt} - 6a\xi_3(t) = 0; \\ \dfrac{d\xi_0(t)}{dt} - 2a\xi_2(t) = 0 \end{cases} \quad (6)$$

Conditions (6) are in some sense similar to the conditions in the Rayleigh-Ritz method, where each term of the expansion must separately satisfy the boundary conditions (Abdallah and Madjid, 2024). Let's arrange the equations in system (6) by terms of expansion and take the obvious integrals. Then (6) transforms to the form:

$$\begin{cases} \xi_0(t) = a\beta_1 t^2 + \beta_2; \\ \dfrac{d\xi_1(t)}{dt} = 6a\xi_3(t); \\ \xi_2(t) = \beta_1 t, \end{cases} \quad (7)$$

where β_1 и β_2 – integration constants.

One of the integration constants can be found from the initial conditions. The fourth equation of the system of equations (1) for $z = 0$ will take the form: $T(0, 0) = T_0$. According to decomposition (3), for $z = 0$ we have:

$$T_3(0,t) = \xi_0(t). \quad (8)$$

At $t = 0$ get:

$$T_3(0,0) = \xi_0(0) = T_0. \quad (9)$$

Substitute $t = 0$ into the first equation of the system of equations (7) and obtain:

$$\beta_2 = \xi_0(0) = T_0. \quad (10)$$

Then the system of equations (7) will take the final form:

$$\begin{cases} \xi_0(t) = a\beta_1 t^2 + T_0; \\ \dfrac{d\xi_1(t)}{dt} = 6a\xi_3(t); \\ \xi_2(t) = \beta_1 t. \end{cases} \quad (11)$$

If the decomposition functions (3) satisfy the conditions (11), then the decomposition (3) itself will satisfy the equation of thermal conductivity. In the system of equations (11), only the second equation is differential. Next, let us consider the boundary conditions that the functions from expansion (3) must satisfy.

For $z = 0$, the temperature and its derivative with respect to z, based on expansion (3), will have the form:

$$\begin{cases} T(0,t) = \xi_0(t); \\ \left(\dfrac{\partial T(z,t)}{\partial z}\right)_{z=0} = \xi_1(t) \end{cases} \tag{12}$$

Therefore, the third equation of the system of equations (1), taking into account (12), is transformed to the form:

$$\lambda \xi_1(t) = -e\Theta \xi_0^4(t) \tag{13}$$

This form completely coincides with equation (9) of Sedelnikov et al. (2024a).

For $z = h$, the temperature and its derivative with respect to z, based on decomposition (3), will have the form:

$$\begin{cases} T(h,t) = \xi_0(t) + \xi_1(t)h + \xi_2(t)h^2 + \xi_3(t)h^3; \\ \left(\dfrac{\partial T(z,t)}{\partial z}\right)_{z=h} = \xi_1(t) + 2\xi_2(t)h + 3\xi_3(t)h^2. \end{cases} \tag{14}$$

Substituting (14) into the second equation of the system of equations (1) gives a result that coincides with equation (11) of Sedelnikov et al. (2024a) before linearisation:

$$\begin{aligned} &\lambda\left[\xi_1(t) + 2\xi_2(t)h + 3\xi_3(t)h^2\right] = \\ &= Q - e\Theta\left[\xi_0(t) + \xi_1(t)h + \xi_2(t)h^2 + \xi_3(t)h^3\right]^4. \end{aligned} \tag{15}$$

Further in Sedelnikov et al. (2025), a linearisation with respect to h. was performed. In this paper, the second-order terms with respect to h will be retained. After this simplification, (15) is transformed to the form:

$$\begin{aligned} &\lambda\left[\xi_1(t) + 2\xi_2(t)h + 3\xi_3(t)h^2\right] = Q - \\ &-e\Theta\xi_0^2(t)\left[\xi_0^2(t) + 4\xi_0(t)\xi_1(t)h + 4\xi_0(t)\xi_2(t)h^2 + \xi_1^2(t)h^2\right]. \end{aligned} \tag{16}$$

Equation (16) differs significantly from the linearised equation (12) in Sedelnikov et al. (2024a) and is a more accurate approximation of equation (15). We substitute conditions (11) in (16) and perform elementary transformations:

$$\begin{aligned} \frac{d\xi_1(t)}{dt} = &-\frac{12ea\Theta}{\lambda}\left(a\beta_1 t^2 + T_0\right)^2 \xi_1^2(t) - \frac{8ea\Theta}{\lambda h}\left(a\beta_1 t^2 + T_0\right)^3 \xi_1(t) + \\ &+ \frac{2a}{\lambda h^2}\left[Q - 2\lambda\beta_1 th - 4e\Theta h^2\left(a\beta_1 t^2 + T_0\right)^3 \beta_1 t\right]. \end{aligned} \tag{17}$$

Then finally, the functions in decomposition (3), taking into account the second approximation with respect to h, will take the form:

$$\begin{cases} \xi_0(t) = a\beta_1 t^2 + T_0; \\ \dfrac{d\xi_1(t)}{dt} = -\dfrac{12ea\Theta}{\lambda}\left(a\beta_1 t^2 + T_0\right)^2 \xi_1^2(t) - \dfrac{8ea\Theta}{\lambda h}\left(a\beta_1 t^2 + T_0\right)^3 \xi_1(t) + \\ \qquad + \dfrac{2a}{\lambda h^2}\left[Q - 2\lambda\beta_1 th - 4e\Theta h^2\left(a\beta_1 t^2 + T_0\right)^3 \beta_1 t\right]; \\ \xi_2(t) = \beta_1 t; \\ \xi_2(t) = -\dfrac{2e\Theta}{\lambda}\left(a\beta_1 t^2 + T_0\right)^2 \xi_1^2(t) - \dfrac{4e\Theta}{3\lambda h}\left(a\beta_1 t^2 + T_0\right)^3 \xi_1(t) + \\ \qquad + \dfrac{1}{3\lambda h^2}\left[Q - 2\lambda\beta_1 th - 4e\Theta h^2\left(a\beta_1 t^2 + T_0\right)^3 \beta_1 t\right]. \end{cases} \quad (18)$$

The resulting system of functions (18) is a solution to the initial boundary value problem (1). The second equation (18) is solved only approximately.

4 Numerical modelling and comparison of results with a linearised model

Let's perform numerical modelling in the Mathcad mathematical package. The values of the main characteristics of the plate and the temperature impact completely coincide with the work Sedelnikov et al. (2024a). The same values were used in Sedelnikov et al. (2021) for numerical simulation of temperature shock in the ANSYS package. This is done in order to make it possible to correctly compare the obtained simulation results. The main characteristics are shown in Table 1 (Sedelnikov et al., 2024a).

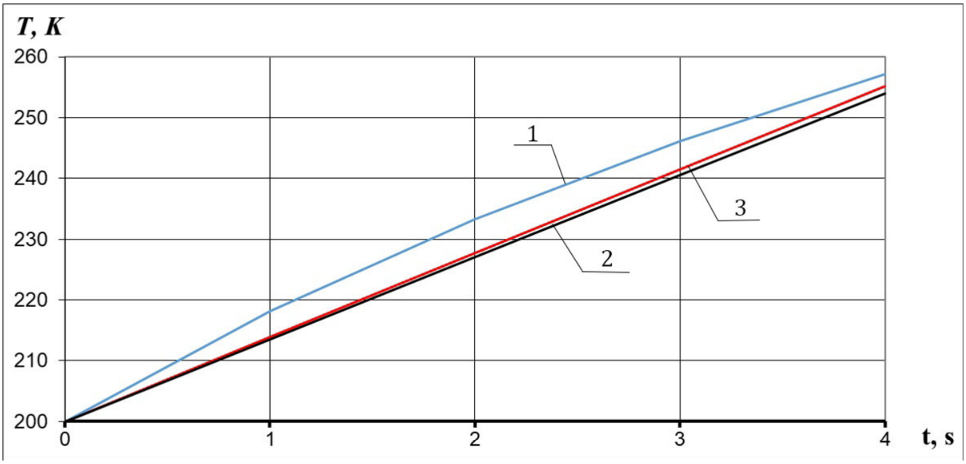
Table 1 The main parameters of the simulated plate

Parameter	*Designation*	*Value*	*Dimension*
Solar panel frame material	–	MA2	–
Coefficient of thermal conductivity	λ	96,3	1400
Stefan-Boltzmann constant	Θ		$W/(m^2.K^4)$
External heat flux	Q	1,400	W/m^2
Initial temperature of the solar panel frame	$T_0 = T(z, 0)$	200	K
Degree of blackness	e	0, 2	-
Specific heat	c	1,130.4	$J/(kg,K)$
Density	ρ	1,780	g/m^3
Young's Module	E	$4{\cdot}10^{10}$	Pa
Shift modulus	μ		Pa
Poisson 's Ratio	v	0, 3	-
Solar panel length	l	1	m
Solar panel width	b	0, 5	m
Solar panel frame thickness	h	1	mm

Source: Sedelnikov et al. (2024a)

The following results were obtained in numerical simulation. They are shown in Figure 2.

Figure 2 Dynamics of surface layer temperature (z = h) (see online version for colours)



Notes: 1 – simulation results in ansys (Sedelnikov et al., 2021); 2 – simulation results based on the linearised model (Sedelnikov et al., 2024a); 3 – simulation results based on the proposed model, taking into account the terms of the second order relative to z.

It can be seen from Figure 2 that there are no significant differences from the linearised model (Sedelnikov et al., 2024a) taking into account the terms of the second order relative to z didn't give. Therefore, the use of a linearised model is fully justified for an approximate assessment of the effect of a thermal shock.

For the surface layer with the coordinate z = 0, the linearisation of equation (4) has no effect on the temperature distribution in the layer. This is a disadvantage not so much of the decomposition (2) as of the Fourier method (Watts, 2012). It is precisely such shortcomings that the authors of Arinchev (2022), as well as a number of other works, for example, Li et al. (2022) and Shenoy et al. (2010) point out.

5 Conclusions

The paper considers an approximate solution to the one-dimensional problem of thermal conductivity using expansion (3), taking into account the terms of the second order relative to z. The results obtained allow us to conclude that the considered approach did not bring a significant increase in accuracy compared to the linearised model in Sedelnikov et al. (2024a). Moreover, accounting for members of a higher order seems irrational. And this isn't due to the fact that the linearised model (Sedelnikov et al., 2024a) is accurate, but due to the disadvantages of the Fourier method itself in the context of the problem being solved. Apparently, the significant nonlinearity of the boundary conditions of the third kind makes the idea of using the decomposition of a function of two variables as a product of two independent functions of one variable less applicable. A similar position can be found in the works Belousova and Serdakova (2020) and Sedelnikov et al. (2023).

Thus, an important result of the work is to verify the reliability of the linearised model (Sedelnikov et al., 2024a). This model, in fact, has exhausted all the possibilities of the Fourier decomposition method (Watts, 2012) and cannot be improved by taking into account a larger number of higher-order terms relative to *z*.

Acknowledgements

This work was supported by the Ministry of Education and Science of the Russian Federation in the framework of the State Assignments to higher education institutions and research organisations in the field of scientific activity (project FSSS-2023-0007).

Declarations

The authors declare that there is no conflict of interest.

References

Abdallah, Z. and Madjid, B.H. (2024) 'Analysis of elastic stresses for a composite disk under the parabolic distribution of temperature using the Rayleigh-Ritz method with trigonometric Ritz functions', *Studies in Engineering and Exact Sciences*, Vol. 5, No. 3, DOI: 10.54021/seesv5n3-091.

Arinchev, S.V. (2022) 'Empirical evidence for incorrectness of using separation of variables according to the fourier method in oscillation theory', *BMSTU Journal of Mechanical Engineering*, Vol. 5, pp.59–68, DOI: 10.18698/0536-1044-2022-5-59-68.

Belousova, D.A. and Serdakova, V.V. (2020) 'Modeling the temperature shock of elastic elements using a one-dimensional model of thermal conductivity', *International Journal of Modeling, Simulation, and Scientific Computing*, Vol. 11, No. 6, DOI: 10.1142/S1793962320500609.

Biswas, A.K., Chakravarti, S. and Das, S. (2023) 'A new nonlinear one-dimensional shape function applied to simulate a steady state heat transfer problem', *Indian Science Cruiser*, Vol. 37, No. 6, pp. 29–32, DOI:10.24906/isc/2023/v37/i6/45876.

Calvetti, D. and Reichel, L. (2003) 'Tikhonov regularization of large linear problems', *BIT. Numerical mathematics*, Vol. 43, No. 2, pp.263–283, DOI:10.1023/A:1026083619097.

Chamberlain, M.K., Kiefer, S.H., LaPointe, M. and LaCorte, P. (2021) 'On-orbit flight testing of the roll-out solar array', *Acta Astronautica*, Vol. 179, pp.407–414, DOI: 10.1016/j.actaastro.2020.10.024.

Cheng, R. and Ge, H. (2010) 'Meshless analysis of three-dimensional steady-state heat conduction problems', *Chinese Physics B*., Vol. 19, No. 9, DOI: 10.1088/1674-1056/19/9/090201.

Choi, Y-S., Park, J-K., Jeon, B.G., Kam, D.H. and Moon, S-K. (2022) 'Application of inverse heat conduction calculation method for fast-transient flow boiling heat transfer analysis', *Journal of Nuclear Science and Technology*, Vol. 59, No. 3, pp.1–16, DOI: 10.1080/00223131.2021.2012287.

Einstein, A. (1905) 'Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen', *Annalen der Physik*, Vol. 322, No. 8, pp.549–560, DOI:10.1002/andp.19053220806.

Eremin, A. and Gubareva, K.V. (2019) 'Analytical solution to the problem of heat transfer using boundary conditions of the third kind', *Vestnik IGEU*, No. 6, pp.67–74, DOI: 10.17588/2072-2672.2019.6.067-074.

Guo, X., Tian, X., Yu, W., Lv, H., Mu, C., Wang, S., Li, Q., Liu, J. and Tang, X. (2024) 'Experimental study on the effect of specimen placement state on temperature control under thermal vacuum environment', *Advances in Engineering Technology Research*, Vol. 12, No. 1, DOI: 10.56028/aetr.12.1.334.2024.

Hadamard, J. and Morse, P.M. (1953) 'Lectures on Cauchy'S Problem in Linear Partial Differential Equations', *Physics Today*, Vol. 6, No.8, p.18, DOI:10.1063/1.3061337.

Jahangiri, A., Mohammadi, S. and Akbari M. (2019) 'Modeling the one-dimensional inverse heat transfer problem using a Haar wavelet collocation approach', *Physica A Statistical Mechanics and its Applications*, Vol. 525, pp.13–26, DOI: 10.1016/j.physa.2019.03.040.

Johnston, J.D. and Thornton, E.A. (2000) 'Thermally induced dynamics of satellite solar panels', *Journal of Spacecraft and Rockets*, Vol. 37, No. 5, pp.604–613, DOI: 10.2514/2.3633.

Joseph, O.C., Agyekum, E.B. and Afornu, B. (2018) 'Analysis of temperature drop along the radial axis in steady state heat transfer of nuclear fuel element using ANSYS APDL', *International Journal of Progressive Sciences and Technologies*, Vol. 6, No. 2, pp.284–291.

Kartashov, E.M. (2012) 'New model representations of dynamic thermoviscoelasticity in the problem of heat shock', *Journal of Engineering Physics and Thermophysics*, Vol. 85, No. 5, pp.1102–1113, DOI: 10.1007/s10891-012-0753-3.

Khnyryova, E.S. (2023) 'Estimation of the angular velocity of rotation of the Aist-2D small spacecraft for remote sensing of the Earth', *Sovremennye Problemy Distantsionnogo Zondirovaniya Zemli iz Kosmosa*, Vol. 20, No. 4, pp.81–88, DOI: 10.21046/2070-7401-2023-20-4-81-88.

Korneyev, V., Korneyev, S. and Shalay, V. (2019) 'The approximate analytic solution of non-stationary heat conduction problem of a deformable plate with time-dependent boundary conditions', *Thermal Processes in Engineering*, Vol. 11, No. 9, pp.424–432.

Lee, B.H., Yamasaki, M. and Murozono, M. (2013) 'Experimental verification of thermal structural responses of a flexible rolled-up solar array', *Transactions of the Japan Society for Aeronautical and Space Sciences*, Vol. 56, No. 4, pp.197–204, DOI: 10.2322/tjsass.56.197.

Li, C., Guo, H., He, T. and Tian, X. (2022) 'Thermally nonlinear non-Fourier piezoelectric thermoelasticity problems with temperature-dependent elastic constants and thermal conductivity and nonlinear finite element analysis', *Waves in Random and Complex Media*, pp.1–38, DOI: 10.1080/17455030.2022.2075953.

Liu, L., Wang, X., Sun, S., Cao, D.Q. and Liu, X. (2019) 'Dynamic characteristics of flexible spacecraft with double solar panels subjected to solar radiation', *International Journal of Mechanical Sciences*, Vol. 151, pp.22–32, DOI: 10.1016/j.ijmecsci.2018.10.067.

Liu, Y., Li, X., Qian, Y. and Yang, H-L. (2022) 'Rigid-flexible coupled dynamic and control for thermally induced vibration and attitude motion of a spacecraft with hoop-truss antenna', *Applied Sciences*, Vol. 12, No. 3, p.1071, DOI: 10.3390/app12031071.

Mehandiratta, V., Mehra, M. and Leugering, G. (2020) 'An approach based on Haar wavelet for the approximation of fractional calculus with application to initial and boundary value problems', *Mathematical Methods in the Applied Sciences*, Vol. 44, No. 1, pp.3195-3213, DOI:10.1002/mma.6800.

Molhem, H. and Pourgholi, R. (2008) 'A numerical algorithm for solving a one-dimensional inverse heat conduction problem', *Journal of Mathematics and Statistics*, Vol. 4, No. 1, pp.60–63, DOI: 10.3844/jmssp.2008.98.101.

Muftu S. (2022) *Finite Element Analysis of one-Dimensional Heat Transfer Problems*, Chapter 9 in Book Finite Element Method. Physics and Solution Methods, Academic Press, pp.291–312, DOI: 10.1016/B978-0-12-821127-4.00011-6.

Niu, Y., He, Y., Xiang, F. and Zhang, J. (2021) 'Automatic differentiation approach for solving one-dimensional flow and heat transfer problems', *Annals of Nuclear Energy*, Vol. 160, No. 10, p.108361, DOI:10.1016/j.anucene.2021.108361.

Pourgholi, R., Molai, A.A. and Hoularit, T. (2013) 'Resolution of an inverse parabolic problem using Sinc-Galerkin method WMS', *Journal of Applied and Engineering Mathematics*, Vol. 3, No. 2, pp.160–181.

Prokopyev, A.I., Dolgov, I.I., Borodyuk, A.A. and Pankrushin, V.N. (2020) 'The model of architecture fault tolerance of the spacecraft computing system', *Aerospace Instrument-Making*, No. 1, pp.49–54, DOI: 10.25791/aviakosmos.01.2020.1139.

Sedelnikov, A.V. and Orlov, D.I. (2020) 'Development of control algorithms for the orbital motion of a small techno-logical spacecraft with a shadow portion of the orbit', *Microgravity Science and Technology*, Vol. 32, No. 5, pp.941–951, DOI: 10.1007/s12217-020-09822-y.

Sedelnikov, A.V. and Orlov, D.I. (2021a) 'Analysis of the significance of the influence of various components of the disturbance from a temperature shock on the level of microaccelerations in the internal environment of a small spacecraft', *Microgravity Science and Technology*, Vol. 33, No. 2, p.22, DOI: 10.1007/s12217-020-09867-z).

Sedelnikov, A.V., Nikolaeva, A.S., Serdakova, V.V. and Evtushenko, M.A. (2024a) 'Approximate solution of initial-boundary value problem of one-dimensional heat conduction for the thermal shock of thin plate', *WSEAS Transactions on Applied and Theoretical Mechanics*, Vol. 19, No. 22, pp.200–205, DOI: 10.37394/232011.2024.19.22.

Sedelnikov, A.V., Nikolaeva, A.S., Serdakova, V.V. and Khnyryova, E.S. (2024b) 'Technologies for increasing the control efficiency of small spacecraft with solar panels by taking into account temperature shock', *Technologies*, Vol. 12, No. 10, p.207, DOI: 10.3390/technologies12100
207.

Sedelnikov, A.V., Orlov, D.I., Bratkova, M.E. and Khnyryova, E.S. (2023) 'Estimating the inertia tensor components of an asymmetrical spacecraft when removing it from the operational orbit at the end of its active life', *Sensors*, Vol. 23, No. 23, p.9615, DOI: 10.3390/s23239615.

Sedelnikov, A.V., Serdakova, V.V. and Khnyreva E.S. (2021b) 'Construction of the criterion for using a two-dimensional thermal conductivity model to describe the stress-strain state of a thin plate under the thermal shock', *Microgravity Science and Technology*, Vol. 33, No. 6, p.65, DOI: 10.1007/s12217-021-09912-5.

Serdakova, V.V. (2023) 'Estimation of maximum temperature deformations of solar panels of earth remote sensing small satellite «Aist-2D»', *Advances in Machinery, Materials Science and Engineering Application*, Vol. 11, pp.411–417, DOI: 10.3233/ATDE230488.

Shen, Z., Li, H., Liu, X. and Hu, G. (2017) 'Thermal shock induced dynamics of a spacecraft with a flexible deploying boom', *Acta Astronautica*, Vol. 141, pp.1–6, DOI: 10.1016/j.actaastro.2017.10.004.

Shenoy, S., Barrera, E.V. and Bayazitoglu, Y. (2010) 'Non-Fourier 3-omega method for thermal conductivity measurement', *Nanoscale and Microscale Thermophysical Engineering*, Vol. 14, No. 3, pp.123–136, DOI: 10.1080/15567265.2010.484010.

Sidorov, E.A. (1969) 'The approximation of boundary conditions of the third kind in certain problems of steady-state heat conduction', *Journal of Engineering Physics*, Vol. 17, No. 1, pp.886–889, DOI: 10.1007/BF00828399.

Wang, C., Heng, Y., Luo, J. and Wang, X. (2024) 'A fast Bayesian parallel solution framework for large-scale parameter estimation of 3D inverse heat transfer problems', *International Communications in Heat and Mass Transfer*, Vol. 155, p.107409, DOI: 10.1016/j.icheatmass transfer.2024.107409.

Watts, R.G. (2012) *The Fourier Method: Separation of Variables*, in book: Essentials of Applied Mathematics for Engineers and Scientists, pp.11–29, DOI: 10.1007/978-3-031-02404-7_2.