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Application of Bayesian methods in the analysis of dynamic conditional correlation multivariate GARCH models

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Abstract: The study investigates the use and performance of the multivariate generalised autoregressive conditional heteroscedastic (MGARCH) model, specifically the dynamic conditional correlation (DCC)-MGARCH model in Bayesian framework. It uses a Markov chain Monte Carlo strategy and the Metropolis-Hastings algorithm for effective posterior sampling. The model is found to be more flexible and can describe uncertainties and volatilities of the error distribution. The sensitivity test shows that posterior results are more reliable when prior parameters are randomly sampled from the beta distribution.

Keywords: Bayesian inference; dynamic conditional correlation; DCC; generalised error distribution; GED; Markov chain Monte Carlo; MCMC; Metropolis-Hastings; generalised autoregressive conditional heteroscedastic; skewed distributions.

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1 Introduction

Bollerslev's (1990) proposed a constant conditional correlation (CC) generalised autoregressive conditional heteroscedastic (CCC-GARCH) model, a multivariate extension, maintains a constant correlation between series of returns over time. The extension of CCC-GARCH model known as dynamic conditional correlation GARCH (DCC-GARCH), proposed by Engle (2002) and Tsay (2014), is a widely used model that considers the correlation between variant volatilities over time. Recently, the Bayesian approach, gaining popularity due to its improved interpretation and incorporation of prior information into parameters, has experienced significant growth in recent years (Nascimento et al., 2019). The DCC-GARCH becomes more popular to use the Bayesian methods of analysis, since the parameter estimates are generated from the distributions that give a lot of information about the estimate than the point estimate in classical methods (Haddad and Heidari, 2020). Moreover, the proportionality and positivity of the estimates can be easily handled based on the restrictions of the priors with out further assumptions. This procedure is usually considered to analyse macroeconomic time series assuming stochastic volatility models.

Corresponding to the proposal by Fioruci et al. (2014a), sampled priors from truncated normal distributions, the newly proposed prior is from the beta distribution as alternative that are assumed to be a priori independent and beta distributed intervals. Accordingly, the use of likelihood function, prior and posterior distribution in solving the standard problems of statistical inference, that is, point and interval estimation are compared. A simulation study is conducted to access the finite sample performance of the procedure proposed here, under the presence of long-memory in the volatility. The multivariate skewed normal, skewed student-t and skewed generalised error distribution (GED) models are considered in the comparison, so as to accommodate asymmetry and heavy tails in the distribution of the innovation process from the fitted DCC-MGARCH processes.

The choice of the sampling algorithm is the first issue when dealing with MCMC methods and it depends on the nature of the problem under study. To generate samples from the joint posterior distribution for the parameters and to conducting computational posterior inference in the context of the multivariate skew distributions of interest, MCMC Metropolis-Hastings (M-H) algorithm was applied. These samples are generated from all conditional posterior distributions for each parameter given all the other parameters and the data.

The sampling algorithm involves draws from standard distributions, and can be implemented fairly and easily. Bayes factors of the multivariate GED was computed against skew normality and student-t distribution assumptions of the error terms. Based on the computational techniques developed in this paper, it is possible to perform full Bayesian inference on all parameters of the model, derive marginal posterior distributions, and compute Bayes factors. Further, the DCC-GARCH model has a clear computational advantages in that the number of parameters to be estimated in the correlation process is independent of the number of series to be correlated. Thus potentially very large correlation matrices can be estimated. The Monte Carlo simulation techniques on the estimation of the DCC-GARCH model parameters is used. This procedure can be considered to analyse financial time series assuming stochastic volatility exists among residual series (Meyer and Yu, 2000).

A sensitivity analysis is performed by considering different prior density functions and by integrating (or not) the knowledge on the true parameter values to select the hyper parameter values. Sensitivity analysis entails checks on the model settings to see if one choose several priors to study the sensitivity of the analysis to prior specification changes in posterior inferences. This may take the form of comparing models with plausible but different priors, whether different parameter values for the same distribution or different distributions to give more confidence in the final selection.

The remaining sections are organised as follows. Section 2 review some related literature to the Bayesian method in the analysis of DCC-MGARCH. In Section 3, as the methodology part, source of data and definition of some terms are presented in Subsection 3.1; the main properties of DCC-MGARCH processes, definitions of multivariate skewed densities and the Bayesian inference scheme and techniques of sampling by M-H algorithm are described in Subsections 3.2, 3.3 and 3.4, respectively. In Section 4 tests of existence of time varying correlation from the preDCC-MGARCH model fitted to the dataset, MCMC simulation results from the posterior distribution based on prior parameters from the fitted preDCC-MGARCH model and some sensitivity analysis are presented wit their respective Subsections 4.1, 4.2 and 4.3. Lastly some concluding remarks and possible directions for future work are given in Section 5.

2 Review of literature

Multivariate GARCH (MGARCH) models are frequently used in the analysis of dynamic covariance structure for multiple asset returns of financial time series (Bauwens et al., 2006; Silvennoinen and Terasvirta, 2009). More recently, interest has focused on Bayesian GARCH models. Bollerslev (1990) introduced a MGARCH model, referred as the CCC-GARCH model, in which univariate GARCH models are related to one another with a constant correlation matrix. A major drawback of the assumption of constant CC in volatility modelling is that the correlation coefficient tends to change over-time in a real application.

Advances in computing facilities and computational methods have dramatically increased the ability to solve complicated problems. Engle (2002) relaxed the assumption of constant correlation to a DCC. The advances have also extended the applicability of many existing econometric and statistical methods. Such achievements include the MCMC method and data augmentation. Ardia and Lennart (2010) outline the MCMC method for GARCH models. Jun (2015), Fioruci et al. (2014a) and Takaishi (2007) employed the Hamilton Monte Carlo algorithm and Ardia (2008) used Metropolis Hasting scheme in DCC-MGARCH with student-t innovations. They all demonstrated the MCMC methods as very useful in estimating parameters in the DCC-MGARCH models in conditional volatilities that accommodate asymmetry as well as heavy tailed distribution.

Oyeleke et al. (2022) extended their examination of the policy regime's dominance (fiscal versus monetary) to include the calculation of inflation in Nigeria between 1981 and 2016. The research uses MCMC sample drawing and Bayesian time-varying parameter vector auto regression (TVP-VAR) with stochastic volatility to produce impulse response functions. The findings indicate that Nigeria's economy lacks a clear dominant policy framework. Similary, Semuel and Nurina (2014) employed interest rates, currency rates, and inflation as GDP's supporting variables and concluded that the

GDP is significantly correlated with interest rates in a negative way and with exchange rates in a positive way. However, the GDP is not significantly impacted by inflation. Dickson (2021) study on Ghana's economic growth from 2006–2019 found a strong positive correlation between interest rates and inflation, significantly influencing the country's economic growth. Rodrik (2008), also evaluated the link between exchange rate and growth on a database of 188 countries. Based on their findings, there is a positive correlation between growth and real exchange rate.

Asai (2006) compared different MCMC methods, and found that the best method is the so-called 'tailored' approach based on the acceptance-rejection Metropolis-Hastings (ARM-H) algorithm, with respect to the mixing, efficiency and computational requirement. Similarly, Vrontos et al. (2000, 2003) also used the M-H algorithm in the application MGARCH models at different time.

Prass et al. (2016) used Bayesian inference for fractionally integrated exponential generalised autoregressive conditional heteroskedastic (FIEGARCH) models using MCMC methods, by considering the GED for the innovation process with different prior density functions: Gaussian, beta and uniform prior for sensitivity analysis and conclude that the absolute percentage error of estimation for parameters only became smaller than 10% when the beta prior was considered and the true value of the parameter was used to select the hyperparameter.

The main focus of applying sensitivity analysis in the Bayesian framework is to address the problem of describing the acceptable parameter structure. Ardia (2008) modelled the errors as Gaussian distributed with zero mean and unit variance while the priors are chosen as Gaussian and a M-H algorithm is used to draw samples from the joint posterior distribution and has also performed a sensitivity analysis of the prior means and scale parameters and concluded that the initial priors in this case are vague enough. Ausin and Lopes (2010) modeled the errors of a GARCH model with a mixture of two Gaussian distributions. The Gaussian and Student-t distributions are common choices while more sophisticated parameterisations such as the skewed student-t or the mixture of Gaussian distributions allow to model skewness and fat tails in the conditional distribution of returns (Ausin and Galeano, 2007).

Some of the contribution of Bayesian inference in the analysis of dynamic relationships between volatility processes of multiple macroeconomic time series observations to the existing literature is that it simplify some practical difficulties faced when dealing with the maximum likelihood estimation (MLE) of GARCH-type models are the following. First, the complicated nonlinear inequality constraints in maximisation of the likelihood function must be achieved since all model parameters must be positive to ensure a positive conditional variance and the covariance stationarity condition holds. This leads to the optimisation procedure cumbersome. Second, since optimisation results are often sensitive to the choice of starting values, the convergence of the optimisation is hard to achieve if the true parameter values are close to the boundary of the parameter space and if the GARCH process is nearly non-stationary. Third, if inference is needed for the parameters using the MLE, one worries about the use of the Hessian matrix for standard errors, and the asymptotic normality assumption. Fourth, the MLE estimates the volatilities σ_t^2 at the MLE point estimate of the parameters. This is another more important issue for the difference in inference on in-sample and future volatilities (Geweke, 1989). Lastly, since the standard applications of GARCH-type models are highly complicated nonlinear functions of the parameters, the asymptotic argument would require a very large number of data to hold by simulation (Fileccia and Sgarra, 2015).

Fortunately, the Bayesian simulation methods solve those problems and difficulties. First, any constraints on the model parameters can be incorporated in the modelling through appropriate prior specifications that produce the expected non-normal posterior distributions. Second, appropriate Markov chain Monte Carlo (MCMC) procedures can explore the joint posterior distribution of the model parameters (Haddad and Heidari, 2020). These techniques avoid local maxima (i.e., non-convergence or convergence to the wrong values) encountered via ML estimation of sophisticated GARCH-type models. Third, exact distributions of nonlinear functions of the model parameters can be obtained at low cost by simulating from the joint posterior distribution. Fourth, the Bayes methods delivers, by simulation, the entire posterior density of each σ_t^2 as well as of the parameters in GARCH models (Berger, 2006).

In certain situations, the practical application of Bayesian approaches may be restricted due to their computing complexity, particularly when dealing with huge datasets and sophisticated models. In situations where prior knowledge is scarce or nonexistent, the method may also be more difficult to comprehend and implement than frequentist methods (Louis and Carlin, 2000; Robert, 2001; Wasserman, 2004). Further, even though they provide new insights but they also require large amounts of data.

3 Methodology

3.1 Definition of macroeconomic variables under study and source of data

3.1.1 The annual GDP growth rate

The annual GDP growth rate is the average change in GDP at market prices for a national economy over a specified period, expressed as a proportion of the previous period's GDP. The real GDP growth rate measures economic growth, adjusted for inflation or deflation, by analysing changes in the value of goods and services produced by a country. It can also be measured as a dollar or a percentage by calculating changes in real GDP in a selected interval in time, often a year (Leamer, 2009; Semuel and Nurina, 2014). Accordingly, the annual percentage growth rate of Ethiopian GDP at market prices is based on a constant local currency, Ethiopian Birr, with aggregates based on a constant 2010 US dollar.

3.1.2 Exchange rate

The value of one currency relative to another is known as the exchange rate (Krugman, 1993; Semuel and Nurina, 2014). The difference in the rates of inflation in domestic and foreign countries drives changes in exchange rates, according to the purchasing power parity (PPP) theory. The exchange rate will increase and vice versa when changes in foreign prices are outpaced by local inflation. Based on the nominal exchange rate, the real exchange rate can be defined as the ratio of foreign prices to local prices (Ethiopian Birr per unit of USD). The National Bank of Ethiopia (central bank), follows a managed floating exchange rate regime where the local currency Birr is pegged to the US Dollar.

3.1.3 Inflation rate

Inflation is the increase in the Consumer Price Index (CPI) over a year, indicating the cost of goods and services. It is a broad measure, encompassing both the overall increase in prices and the cost of living in a country. According to Brue and McConnell (2014) and Semuel and Nurina (2014), inflation is best defined as a general price increase that reduces the purchasing power of a currency. A few factors lead to inflation, whereby the cost of goods and services rises when aggregate demand grows more quickly than aggregate supply.

3.1.4 Interest rate

An interest rate is the annual cost of borrowing or lending money, determining the percentage paid for its use over a period. Real interest rates, adjusted for inflation, are financial rates used in monetary policy setting, while nominal interest rates are unadjusted rates used by central banks to target specific rates. These rates will reflect the interaction between exchanges of money (Harswari and Hamza, 2017; Patterson and Lygnerud, 1999). The terms and conditions attached to lending rates differ by country, however, limiting their comparability.

3.1.5 Data

Data on gross domestic product growth rate (GDPGR), inflation rates (INFR), exchange rate (EXR) and interest rate (INTR) were obtained from National Bank of Ethiopia on a yearly level for the period 1990 to 2015. While the first three variables are well known in business cycle literature the exchange rate is included because it plays a prominent role in more open economies so that National Bank find it useful to target it. The unavailability of high frequency macroeconomic time series data, like GDP growth is a common problem in some countries that makes it difficult for end users of the data to analyse short-term movements in output. One way to overcome this problem is temporal disaggregation method that involves the conversion of low frequency data (like an annual data series) into high frequency data (namely quarterly or monthly series) and often aim to solve the common problems of distribution and interpolation.

Table 1 Comparison of the observed yearly level and disaggregated quarterly data series

Variable	*Yearly*		*Quarterly*	
	Mean	*St. deviation*	*Mean*	*St. deviation*
GDPR	7.062	5.363	6.946	5.53
INFR	10.408	10.965	10.207	11.32
EXR	9.604	5.084	9.315	5.018
INTR	0.059	0.027	0.059	0.026
T		26		104

Accordingly, a total of 26 yearly level data set of each variable is disaggregated in to 104 quarterly data for the period 1990(1) to 2015(4) following average temporal disaggregation method by Sax and Steiner (2013). The method of averaging of the resulting high frequency, quarterly, series is consistent with low frequency, yearly, series using the optimise function *td*, under package *stats* in R, to solve the one-dimensional optimisation problem at the core of the Chow-Lin and Litterman methods (Silva and Cardoso, 2001). In this methods, either the sum, the average, the first or the last value of the resulting high frequency series is consistent with the low frequency series, as shown in Figure 1 and have almost the same mean and standard deviations when approximated in one or two decimal place as shown in Table 1. It performs a simple interpolation that meets the temporal additivity constraint and uses an efficient and numerically stable algorithm that is based on the *qr*-decomposition (Paige, 1979).

Figure 1 Time series plot of yearly data (vertical lines) and disaggregated quarterly data (doted points) for the study variables (see online version for colours)

Analysing and forecasting economic growth indicating macroeconomic variables has always been an issue for economic researchers and policymakers (Ammouri et al., 2021). The long-run equilibrium interrelationship between real economic growth rate, inflation rate, interest rate and exchange rate in case of Ethiopia was analysed by fitting a vector error correction (VEC) model by adopted Johansen's trace and max-eigenvalue co-integration tests. The VEC model was applied due to the fact that the variables were found to be co-integrated. For model checking, we apply the Ljung-Box test statistics for the residuals of the fitted VEC model. Volatility clustering is a phenomenon where large changes in asset prices cluster together, with large changes likely to follow large changes and small changes expected to follow small changes. This serial dependency occurs when periods of high and low volatility persist (Ruey, 2005). Figure 7, in Appendix part, shows the time plots of the corresponding residuals. It is clearly shows that the time varying nature of the residuals series observed and there is also some evidence of volatility clustering over time, that is there are period of high volatility followed by periods of low volatility and vis versa.

Moreover, it is shows stationary and evolves in time in a continuous manner, that is no volatility jumps exists in the residual series. In other words, the residuals have conditional heteroscedasticity, which can be handled by multivariate volatility models. Such empirical regularities suggest the behaviour of financial time series that may captured by MGARCH models which provide a convenient framework for modelling correlations (Tsay, 2005). Therefore, fitting DCC-MGARCH(1, 1) model seem to be adequate to capture the heteroskedastic effect of time varying CC. Accordingly, the application of Bayesian DCC-MGARCH model analysis is proposed based on the existing literature and properties of these marginally distributed standardised series of estimated residual using multivariate skew-normal, skew-GED and student-t distributions.

3.2 DCC-MGARCH model

In the DCC-MGARCH model specification and estimation, consider the stochastic vector of a k-dimensional time series macroeconomic process $\mathbf{Y}_t = (y_{1t}, ..., y_{kt})'$, the model is given by

$$\boldsymbol{\varepsilon}_t | \boldsymbol{\Omega}_{t-1} = \boldsymbol{\Sigma}_t^{1/2} \hat{\boldsymbol{\eta}}_t, \tag{1}$$

where the vector $\hat{\boldsymbol{\eta}}_t = (\hat{\boldsymbol{\eta}}_{1t}, ..., \hat{\boldsymbol{\eta}}_{kt})' \overset{iid}{\sim} f(\hat{\boldsymbol{\eta}}; \mathbf{0}, \mathbf{I}_k)$ is a $T \times 1$ element random vector of white noise residuals of the GARCH process, such that expectation $E[\boldsymbol{\varepsilon}_t | \boldsymbol{\Omega}_{t-j}] = \mathbf{0}$ and $E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' | \boldsymbol{\Omega}_{t-j}) = \boldsymbol{\Sigma}_t = [\boldsymbol{\sigma}_{ijt}]$ is positive definite conditional covariance matrix; where $\mathbf{I}_k$ is the identity matrix and both matrices have dimension $k \times k$ (Tsay, 2005).

With correlation matrix, $\boldsymbol{\rho}_t$, each conditional covariance is then given by the time evolution of $\boldsymbol{\sigma}_{ij,t} = \rho_{ij}\sqrt{\sigma_{ii,t}\sigma_{jj,t}}$ is governed by that of the conditional variances $\sigma_{ij,t}$ and the elements $\boldsymbol{\rho}_{ij,t}$ of $\boldsymbol{\rho}_t$, where $1 \leq j < i \leq k$ (Engle, 2002; Fioruci et al., 2014a; Tsay, 2005). Therefore, to model the volatility of $\boldsymbol{\varepsilon}_t$, it suffices to consider the conditional variances and correlation coefficients of each $\varepsilon_{i,t}$ for $i = 1, ..., k$.

The process $\{\boldsymbol{\eta}_t\}_{t \in Z}$ is called a DCC-GARCH process, proposed by Engle (2002), with conditional covariance matrix of the form $\boldsymbol{\Sigma}_t = \mathbf{D}_t \boldsymbol{\rho}_t \mathbf{D}_t$ and the diagonal volatility matrix $\boldsymbol{\rho}_t$ and $\mathbf{D}_t$ satisfies

$$\boldsymbol{\rho}_t = \mathbf{J}_t \mathbf{Q}_t \mathbf{J}_t \tag{2}$$

$$\mathbf{Q}_t = (1 - \theta_1 - \theta_2)\bar{\mathbf{Q}} + \theta_1 \mathbf{Q}_{t-1} + \theta_2 \hat{\boldsymbol{\eta}}_{t-1} \hat{\boldsymbol{\eta}}_{t-1}', \tag{3}$$

where $\mathbf{J}_t = diag\left\{q_{11}^{-1/2}, ..., q_{kk}^{-1/2}\right\}$ with $q_{ii,t}$ being the $(i,i)^{\text{th}}$ element of $\mathbf{Q}_t$ and $\overline{\mathbf{Q}}$ is the estimated sample unconditional covariance matrix of $\left(\mathbf{D}_t^{-1}\hat{\boldsymbol{\eta}}_t\right)_{t \in \mathbf{Z}}$. The DCC parameters $\theta_1 \geq 0$ and $\theta_2 \geq 0$ satisfying $\theta_1 + \theta_2 < 1$, to capture the effects of previous standardised shocks and DCCs on current DCCs, respectively. where $\boldsymbol{\rho}_t$ is the CC matrix of $\boldsymbol{\varepsilon}_t$, and $\mathbf{D}_t$ is a $k \times k$ diagonal matrix consisting of the conditional standard deviations of elements of $\boldsymbol{\varepsilon}_t$ (i.e., $\mathbf{D}_t = diag\left\{\sqrt{\sigma_{11,t}}, ..., \sqrt{\sigma_{kk,t}}\right\}$.

$$\sigma_{k,t}^2 = \omega_k + \sum_{i=1}^{q} \alpha_{ki} \varepsilon_{k,t-i}^2 + \sum_{j=1}^{p} \beta_{kj} \sigma_{k,t-j}^2, \tag{4}$$

where $\omega_k > 0$, $\alpha_{ki} \geq 0$, $\beta_{kj} \geq 0$ and $\alpha_{ki} + \beta_{kj} < 1$ for $i = 1, ..., q$ and $j = 1, ..., p$. It is clear that $\boldsymbol{\Sigma}_t$ is positive definite if and only if $\sigma_{i,t}^2 > 0$, $i = 1, ..., k$ and $\boldsymbol{\rho}_t$ is positive definite matrix. $\mathbf{Q}_t$ is a positive-definite matrix and $\mathbf{J}_t$ is simply a normalisation matrix.

The dynamic dependence of the correlations is governed by equation (3) with two parameters θ_1 and θ_2 (see Engle, 2002; Tsay, 2005 for details). Three distributions namely multivariate skew normal, skew student-t and skewed GED for innovations are described here below.

3.3 Multivariate skewed densities

Now, defining the standardised white noise processes from the VEC model as:

$$\boldsymbol{\eta}_{it} = (\boldsymbol{\varepsilon}_{it} - \mu_{\gamma_i})/\sigma_{\gamma_i},$$

then the general method of describing as multivariate skew distribution from a symmetric one indexed by a skewness parameter $\gamma_i > 0$ for $i = 1, ..., k$, which describes the degree of asymmetry, is given by

$$p(\boldsymbol{\eta}_t|\gamma_i) = 2^K \left(\prod_{i=1}^{k} \frac{\sigma_{\gamma_i}}{\gamma_i + 1/\gamma_i} \right) f^*(\boldsymbol{\eta}_t), \tag{5}$$

where

$$f^*(\boldsymbol{\eta}_t) = f\left(\frac{\boldsymbol{\eta}_t}{\gamma_i}\right) I\left(\boldsymbol{\eta}_t \geq \frac{-\mu_{\gamma_i}}{\sigma_{\gamma_i}}\right) + f(\gamma_i \boldsymbol{\eta}_t) I\left(\boldsymbol{\eta}_t < \frac{-\mu_{\gamma_i}}{\sigma_{\gamma_i}}\right),$$

where $\boldsymbol{I}$ stands for the indicator function over the given interval. The parameters γ_i control the degree of skewness on each margin, right (left) marginal skewness corresponding to $\gamma_i > 1 (\gamma_i < 1)$ and the allocation of probability mass to each side of the mode is also determined just by γ_i. This can also be seen from $\gamma_i^2 = Pr(\eta_{it} > 0)/Pr(\eta_{it} < 0)$ (Fernandez and Steel, 1998; Bauwens et al., 2006).

Fernandez and Steel (1998) showed that the r^{th} order moment of equation (5) can be computed as:

$$E(\boldsymbol{\eta}_t^r|\gamma_i) = m_r \frac{\gamma_i^{r+1} + \frac{-1^r}{\gamma_i^{r+1}}}{\gamma_i + \gamma_i^{-1}}. \tag{6}$$

Thus the mean and variance of $p(\boldsymbol{\eta}_t|\gamma_i)$ depend on γ_i and are given by

$$\mu_{\gamma_i} = m_1 \left(\gamma_i - \frac{1}{\gamma_i}\right) \text{ and } \sigma_{\gamma_i}^2 = (m_1 - m_1^2)(\gamma_i^2 + 1/\gamma_i^2) + 2m_1^2 - m_2, \tag{7}$$

where

$$m_r = 2\int_0^{\infty} \boldsymbol{\eta}_t^r f(\boldsymbol{\eta}_t) d\boldsymbol{\eta}_t.$$

3.3.1 Multivariate skewed normal distribution

The stochastic vector of a k-dimensional innovation process $\boldsymbol{\eta}_t$, defined in equation (1), is assumed to be distributed as standardised multivariate normal with $E(\boldsymbol{\eta}_t) = \mathbf{0}$ and $cov(\boldsymbol{\eta}_t) = \mathbf{I}_k$, that is, with probability density function (pdf):

$$f(\boldsymbol{\eta}_t|\boldsymbol{\Omega}_{t-1}) = \frac{1}{(2\pi)^{k/2}} \exp\left[-\frac{1}{2}\boldsymbol{\eta}_t' \boldsymbol{\eta}_t\right]. \tag{8}$$

Accordingly, the skewed version the multivariate normal distribution indexed by a skewness parameter $\gamma_i > 0$ for $i = 1, ..., k$, that describes the degree of asymmetry for each of the i^{th} marginally distributed standardised error, $\boldsymbol{\eta}_t$ defined in equation (1), is given by

$$f\left(\boldsymbol{\eta}_t|\gamma_i\right) = 2^k \left(\prod_{i=1}^{k} \frac{\gamma_i \sigma_{\gamma_i}}{1+\gamma_i^2}\right) \frac{1}{(2\pi)^{k/2}} \exp\left[-\frac{1}{2}\boldsymbol{\eta}_t'\boldsymbol{\eta}_t\right], \tag{9}$$

where $\boldsymbol{\eta}_{it} = (\sigma_{\gamma_i}\boldsymbol{\varepsilon}_{it} + \mu_{\gamma_i})/\sigma_{\gamma_i}$ if $\boldsymbol{\varepsilon}_{it} \geq -\mu_{\gamma_i}/\sigma_{\gamma_i}$ and $\boldsymbol{\eta}_{it} = (\sigma_{\gamma_i}\boldsymbol{\varepsilon}_{it} + \mu_{\gamma_i})\sigma_{\gamma_i}$ if $\boldsymbol{\varepsilon}_{it} \leq -\mu_{\gamma_i}/\sigma_{\gamma_i}$ (Fernandez and Steel, 1998; Fioruci et al., 2014a; Bauwens et al., 2006). This is then the standardised multivariate skew skew normal density, which allows for a tail behaviour heavier than a multivariate symmetric normal distribution.

In the univariate case, the excess of (unconditional) kurtosis has been most commonly accommodated with student-t distributed errors (Baillie and Bollerslev, 1989). A natural alternative in the multivariate case is then the multivariate student-t distribution which has the extra degrees of freedom parameter ν to be estimated Fiorentini et al. (2003) discussed in the next subsection.

3.3.2 Multivariate skewed student-t distribution

When a k-dimensional stochastic vector of innovation process $\boldsymbol{\eta}_t$, defined in equation (1), is assumed to be distributed as multivariate student-t distribution, with estimated extra degrees of freedom parameter, $\nu > 2$, the univariate skew densities has a pdf

$$f\left(\boldsymbol{\eta}_t|\nu, \boldsymbol{\Sigma}_{\boldsymbol{\eta}}\right) = \frac{\Gamma\left((\nu+k)/2\right)}{\pi(\nu-2)^{k/2}\Gamma(\nu/2)} \left[1 + (\nu-2)^{-1}\, \boldsymbol{\eta}_t'\boldsymbol{\eta}_t\right]^{-(\nu+k)/2}. \tag{10}$$

We assume that the degree of freedom $\nu > 2$ so that $\boldsymbol{\Sigma}_{\boldsymbol{\eta}}$ can always be interpreted as a conditional covariance matrix.

The skewed multivariate student-t distribution version with the same index of a skewness parameter $\gamma_i > 0$ for $i = 1, ..., k$ is given in the form:

$$f\left(\boldsymbol{\eta}_t|\gamma_i\right) = 2^k \left(\prod_{i=1}^{k} \frac{\gamma_i \sigma_{\gamma_i}}{1+\gamma_i^2}\right) \frac{\Gamma\left((\nu+k)/2\right)}{\left[\pi(\nu-2)\right]^{k/2}\Gamma(\nu/2)} \left[1 + (\nu-2)^{-1}\, \boldsymbol{\eta}_t'\boldsymbol{\eta}_t\right]^{-(\nu+k)/2}. \tag{11}$$

We notice that $\nu \to \infty$ would be equivalent to assuming a standard multivariate normal density. The moment for the multivariate skewed student-t density is given by

$$m_i = \frac{\Gamma\left((\nu-1)/2\right)\sqrt{\nu-2}}{\sqrt{\pi}\Gamma(\nu/2)} \left(\boldsymbol{\gamma}_i - \frac{1}{\boldsymbol{\gamma}_i}\right), \tag{12}$$

and

$$\sigma_{\gamma_i}^2 = \left(\boldsymbol{\gamma}_i^2 + \frac{1}{\boldsymbol{\gamma}_i^2} - 1\right) - m_i^2, \tag{13}$$

(Fioruci et al., 2014a).

And we may also notice that $\gamma_i = 1$ yields the symmetric distribution as $f(\boldsymbol{\eta}_t|\gamma_i = 1) = f(\boldsymbol{\eta}_t)$, and values of $\gamma_i > 1 (< 1)$ indicate right (left) skewness (Fioruci et al., 2014a). Also, the mode of this density remains at zero irrespective of the particular value of γ_i.

3.3.3 Multivariate skewed GED distribution

Another heavy tailed multivariate distribution considered is the multivariate GED also known as multivariate exponential power distribution. Box and Tiao (1973) and Nadarajah (2005) The joint distribution of k independent random variables with additional shape parameter $\delta > 0$ is then given by,

$$f\left(\boldsymbol{\eta}_t|\delta\right) = \left[\frac{\Gamma\left(3/\delta\right)}{\Gamma\left(1/\delta\right)}\right]^{k/2} \frac{1}{\left[2\Gamma\left((\delta+1)/\delta\right)^k\right]}$$
$$\exp\left\{-\left[\frac{\Gamma\left(3/\delta\right)}{\Gamma\left(1/\delta\right)}\right]^{\delta/2} \sum_{i=1}^{k} |\boldsymbol{\eta}_t|^{\delta}\right\}, \tag{14}$$

is standardised skewed GED distribution as $\boldsymbol{\eta}_t$ defined under equation (9) is standardised and thus it follows that $E(\boldsymbol{\eta}_t) = \mathbf{0}$ and $Var(\boldsymbol{\eta}_t) = I_k$ and we can use the method proposed in Bauwens and Laurent (2005) to introduce asymmetry in the multivariate distribution. We refer to this multivariate distribution as $GED(\mathbf{0}, \mathbf{I}_k, \delta)$.

The multivariate skewed GED distribution version is the case where distributions defined in equation (14) indexed by similar skewness parameter $\gamma_i > 0$ for $i = 1, ..., k$, which describes the degree of asymmetry for each of the i^{th} marginal distribution and for standardised error, $\boldsymbol{\eta}_t$. The density of this standardised skewed version of the GED, denoted $SSGED(\mathbf{0}, \mathbf{I}_k, \gamma, \delta)$, is given by:

$$f\left(\boldsymbol{\eta}_t|\delta\right) = 2^k \left(\prod_{i=1}^{k} \frac{\gamma_i \sigma_{\gamma_i}}{1+\gamma_i^2}\right) \left[\frac{\Gamma\left(3/\delta\right)}{\Gamma\left(1/\delta\right)}\right]^{k/2}$$
$$\exp\left\{-\left[\frac{\Gamma\left(3/\delta\right)}{\Gamma\left(1/\delta\right)}\right]^{\delta/2} \frac{\sum_{i=1}^{k} |\boldsymbol{\eta}_t|^{\delta}}{(2/\delta)^k \left[\Gamma(1/\delta)\right]^k}\right\}. \tag{15}$$

We note that this approach entirely separates the effects of the skewness and shape parameters thus making prior independence between the two a plausible assumption, and hence facilitates the choice of their prior distributions.

3.4 Bayesian inference using MCMC simulation

3.4.1 Basics of Bayesian inference

Let $\boldsymbol{\Theta}$ be the vector of the unknown parameters of the DCC-MGARCH model. Any knowledge about these parameters can be expressed as prior distributions, which is denoted by $P\left(\boldsymbol{\Theta}\right)$. According to Haddad and Heidari (2020), the prior knowledge is important in the parameter estimation together with the likelihood of the data. Bayesian inference is known to combine these two sources of information. With the likelihood function of the data given by $f\left(\mathbf{Y}|\boldsymbol{\Theta}\right)$, the posterior distribution $f\left(\boldsymbol{\Theta}|\mathbf{Y}\right)$ is defined as:

$$f\left(\boldsymbol{\Theta}|\mathbf{Y}\right) = \frac{f\left(\boldsymbol{\Theta}, \mathbf{Y}\right)}{f\left(\mathbf{Y}\right)} = \frac{f\left(\mathbf{Y}|\boldsymbol{\Theta}\right) P\left(\boldsymbol{\Theta}\right)}{f\left(\mathbf{Y}\right)}, \tag{16}$$

where $f\left(\mathbf{Y}\right)$ is the marginal distribution which can be obtained by

$$f(\mathbf{Y}) = \int f(\boldsymbol{\Theta}, \mathbf{Y})\, d\boldsymbol{\Theta} = \int f(\mathbf{Y}|\boldsymbol{\Theta})\, P(\boldsymbol{\Theta})\, d\boldsymbol{\Theta}. \tag{17}$$

The marginal distribution $f(\mathbf{Y})$ in equation (17) serves as the normalisation constant (Asai, 2016; Greenberg, 1995; Gamerman, 2006).

The posterior distribution in equation (16) can also be expressed as proportion:

$$f(\boldsymbol{\Theta}|\mathbf{Y}) \propto f(\mathbf{Y}|\boldsymbol{\Theta})\, P(\boldsymbol{\Theta}). \tag{18}$$

Thus it is an unnormalised posterior density that can be interpreted as the updated knowledge about $\boldsymbol{\Theta}$ after having observed $\mathbf{Y}$.

3.4.2 *Likelihood function*

Considering the three most typically elliptical standardised multivariate skewed distributions that have been applied so far, the conditional likelihood function of model in equation (1), for a sample of observations $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, ..., \varepsilon_{kt})'$ can be written as:

$$\begin{aligned} L(\boldsymbol{\theta}) &= \prod_{t=1}^{T} |\boldsymbol{\Sigma}_t|^{-1/2} f_\eta \left(|\boldsymbol{\Sigma}_t|^{-1/2} \boldsymbol{\varepsilon}_t \right) \\ &= \prod_{t=1}^{T} \left[\prod_{i=1}^{k} \sigma_{ii}^{-1/2} \right] |\boldsymbol{\rho}_t|^{-1/2} f_\eta \left((\mathbf{D}_t \boldsymbol{\rho}_t \mathbf{D}_t)^{-1/2} \boldsymbol{\varepsilon}_t \right), \end{aligned} \tag{19}$$

where f_η is the joint density function for $\boldsymbol{\eta}_t$ (Fioruci et al., 2014a).

3.4.3 *Prior and posterior distributions*

The Bayesian estimation of the model parameters of DCC-MGARCH model is applied by sampling from the joint posterior distribution using a M-H algorithm discussed in the previous subsection. The list of parameters in DCC-MGARCH models is $\boldsymbol{\theta}_1 = \{\omega_i, \gamma_i, \alpha_i, \beta_i, \theta_1, \theta_2, \rho_{i-1,i}\}$ for skew normal case, $\boldsymbol{\theta}_2 = \{\omega_i, \gamma_i, \alpha_i, \beta_i, \theta_1, \theta_2, \rho_{i-1,i}, \nu\}$ for skew student-t and $\boldsymbol{\theta}_3 = \{\omega_i, \gamma_i, \alpha_i, \beta_i, \theta_1, \theta_2, \rho_{i-1,i}, \delta\}$ for GED case for each $i = 1, ..., k$, where ω_i's, α_i's and β_i's are parameters of the constant variances, coefficient for lags of the squares of residuals and for lags of variances in GARCH model in equation (4), respectively. γ_i's are the skewness parameters, θ_1 and θ_2 are DCC parameters in equation (3), $\rho_{i-1,i}$'s are correlation coefficients and ν and δ are the degree of freedom and the shape parameters in equations (10) and (11), respectively.

In order to move forward towards the Bayesian estimation, we need to establish the prior distributions for all of the model parameters. We consider prior distributions that are non-informative and have known forms that can be understood intuitively, also we assume independence between the set of all model parameters. Here, completing the model specification by specifying the prior distributions of all parameters of interest is required. The prior distribution allows us to include any information that a researcher has about the parameters being studied in a systematic way and thus plays a great role in determining the posterior distribution specifically for small samples. Since $0 \leq \boldsymbol{\alpha}, \boldsymbol{\beta}, \theta_1, \theta_2 \leq 1$, the prior should allow these parameters to take on any value in that interval and not allow it to fall outside that interval.

Fioruci et al. (2014a) proposed truncated normal distributions. The truncated normal density function, defined as:

$$f\left(x;\mu,\sigma,a,b\right)=\begin{Bmatrix}\frac{1}{\sigma}\frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right)-\Phi\left(\frac{a-\mu}{\sigma}\right)}, & \text{if } a\le x\le b\\ 0, & \text{otherwise}\end{Bmatrix},$$

where $\phi(.)$ and $\Phi(.)$ are, respectively, the probability density and cumulative distribution functions of the standard normal distribution; a, $b \in \boldsymbol{R}$ are, respectively, the lower and upper limits of the distribution's support; μ and σ denote, respectively, the mean and variance of the distribution's (non-truncated version) (Prass et al., 2016). However, the newly proposed prior is the beta distribution as alternative that is assumed to be a priori independent and beta distributed intervals.

The reason why beta distribution was chosen is that first, it is defined in the relevant range of intervals. Second, it is capable of producing a wide varieties of shapes based on the values of a and b in sampling from the $Beta(a, b)$ distribution. Third, better results of this distribution were obtained provided that the parameters $a > 1$ and $b > 1$ since the beta distribution can be completely enveloped by the truncated normal distribution in the interval (0, 1) as shown by Ahrens and Dieter (1974) in the following proposition including its proof.

Proposition: If $a > 1$, $b > 1$ and $0 \le x \le 1$, then

$$\left(\frac{x}{a-1}\right)^{a-1}\left(\frac{1-x}{b-1}\right)^{b-1}(a+b-2)^{a+b-2}$$
$$\le \exp\left\{-\left(x-\frac{a-1}{a+b-2}\right)^{2}2(a+b-2)\right\}. \tag{20}$$

The left hand side of equation (20) is proportional to the density function of $Beta(a, b)$ distribution and the right hand side is proportional to the density of normal distribution with mean $\frac{a-1}{a+b-2}$ and standard deviation of $\frac{1}{2\sqrt{a+b-2}}$. The inequality in equation (20) is valid for all $x \in (0, 1)$. Thus the assertion that the beta distribution can be completely enveloped by a normal distribution is true.

Depending on the choice of a and b, the $Beta(a, b)$ can capture beliefs that indicate x is symmetric if $a = b > \frac{3}{2}$, in case where the inequality in equation (20) becomes

$$(4x(1-x))^{a-1}\le \exp\left\{-\left(x-\frac{1}{2}\right)^{2}4(a-1)\right\}. \tag{21}$$

The beta-prior distribution θ_i's of $\boldsymbol{\theta}$ is:

$$f(\boldsymbol{\theta_i})=\frac{\Gamma\left(\alpha_0+\beta_0\right)}{\Gamma\left(\alpha_0\right)\Gamma\left(\beta_0\right)\boldsymbol{\theta_i}^{\alpha_0-1}\left(1-\boldsymbol{\theta_i}\right)^{\beta_0-1}},$$

where $\alpha_0 > 0$, $\beta_0 > 0$ are are hyper parameters such that $0 \le \boldsymbol{\theta_i} \le 1$, for each parameter θ_i.

The shape of a beta distribution can be understood by examining its mean and variance:

$$E(\boldsymbol{\theta_i})=\frac{\alpha_0}{\alpha_0+\beta_0} \text{ and } var(\boldsymbol{\theta_i})=\frac{\alpha_0\beta_0}{\left(\alpha_0+\beta_0\right)^2\left(\alpha_0+\beta_0+1\right)}.$$

From these expressions it can be observed that the mean is 1/2 if $\alpha_0 = \beta_0$, a larger $\alpha_0(\beta_0)$ shades the mean toward $1(0)$, and the variance decreases as α_0 or β_0 increases. Further more if $\alpha_0 = \beta_0 \to \infty$, then $Beta(\alpha_0, \beta_0)$ approaches to normal (Casella and Berger, 2001).

A third reason for choosing this distribution is that the beta prior in combination with the likelihood function of equation (19) yields a posterior distribution that has a standard form, which is convenient for analysing the properties of the posterior.

A random variable $\boldsymbol{\theta}$ has a Gamma distribution with positive parameters a and b if its probability density function is

$$f(\boldsymbol{\theta_i}|a, b) = \frac{b^{-a}}{\Gamma(a)} \boldsymbol{\theta_i}^{a-1} e^{\boldsymbol{\theta_i} b}$$

where $\Gamma(a)$ is a gamma function. For this distribution, $E(\boldsymbol{\theta}) = a/b$ and $E(\boldsymbol{\theta}_i) = a/b^2$.

A random variable $\boldsymbol{\theta}$ has an inverse Gamma distribution if its probability density function is

$$f(\boldsymbol{\theta_i}|a, b) = \frac{b^a}{\Gamma(a)} \frac{1}{\boldsymbol{\theta_i}^{a+1}} e^{\boldsymbol{\theta_i}/b}.$$

For this distribution, $E(\boldsymbol{\theta}) = \frac{b}{a-1}$ if $a > 1$ and $E(\boldsymbol{\theta}_i) = \frac{b^2}{(a-1)^2(a-2)}$ if $a > 2$.

The posterior distribution $f(\boldsymbol{\theta}|\mathbf{Y})$ is proportional to the product of likelihood function $L(\boldsymbol{\theta})$ and the prior distribution $f(\boldsymbol{\theta})$, when independent uninformative priors are selected (Jun, 2015).

3.4.4 M-H algorithm

The M-H algorithm is a popular MCMC algorithms used to obtain a sequence of random samples from a target distribution, typically a posterior distribution, for which direct sampling is difficult. The algorithm is first proposed by Metropolis et al. (1953) and extended by Hastings (1970). The approach constructs a Markov chain by generating draws from the target distribution $f(\boldsymbol{\theta}|\mathbf{Y}_t)$; the candidate draw is then accepted (or rejected) based on an acceptance probability. If the candidate is accepted, the chain moves to the new value, otherwise the chain stays in the current state. After a burn-in period, which is required to make the influence of initial values negligible, draws from the Markov chain are considered as (correlated) draws from the joint posterior distribution of interest. M-H algorithm is easy to implement since it does not require knowing the normalisation constant described in equation (17) and is also easy to apply since it does not require the fine tuning of a proposal density and effectively generates the posterior probabilities of the models, as well as the parameter draws of each model (Greenberg, 1995). In addition to the parameter draws, the algorithm jumps to another candidate model with a certain probability.

In the M-H algorithm, we need a proposed distribution $q(.|\boldsymbol{\theta})$ defined on the parameter space of $\boldsymbol{\theta}$. Assuming the current state $\boldsymbol{\theta} = \boldsymbol{\theta}^{(i)}$, we propose a new value $\boldsymbol{\theta}'$ for the next state from the proposed distribution $q(\boldsymbol{\theta}'|\boldsymbol{\theta})$ and accept it with a probability

$$p = \min\left\{1, \frac{f(\boldsymbol{\theta}'|\mathbf{Y})\, q(\boldsymbol{\theta}|\boldsymbol{\theta}')}{f(\boldsymbol{\theta}|\mathbf{Y})\, q(\boldsymbol{\theta}'|\boldsymbol{\theta})})\right\}. \tag{22}$$

We sample a random variable u from a standard uniform distribution, and accept $\boldsymbol{\theta}'$ if $u < p$ and set $\boldsymbol{\theta}^{i+1} = \boldsymbol{\theta}'$. Otherwise, we reject $\boldsymbol{\theta}'$ and stay at current state $\boldsymbol{\theta}^{i+1} = \boldsymbol{\theta}^{i}$. We repeat these steps until a sufficient number of samples are obtained. The M-Hs sampling procedure consists if the following steps (Gamerman, 2006; Givens and Hoeting, 2013; Jun, 2015).

Algorithm 1 M-H algorithm

Input: An initial value $\boldsymbol{\theta}^{(0)}$ such that $f(\boldsymbol{\theta}^{(0)}) > 0$ and the number of iterations N.
for $i = 1 : (N-1)$ **do**
  current state $\boldsymbol{\theta}^{i}$;
  propose $q(\boldsymbol{\theta}|\cdot)$;
  sample u from uniform distribution: $u \sim \mathbf{U}(0,1)$;
  if $u < p = \min\left\{1, \frac{f(\boldsymbol{\theta}'|\mathbf{Y}).q\left(\boldsymbol{\theta}^{i}|\boldsymbol{\theta}'\right)}{f(\boldsymbol{\theta}^{i}|\mathbf{Y}).q\left(\boldsymbol{\theta}'|\boldsymbol{\theta}^{i}\right)}\right\}$ **then**
    $\boldsymbol{\theta}^{i+1} = \boldsymbol{\theta}'$;
    else $\boldsymbol{\theta}^{i+1} = \boldsymbol{\theta}^{i}$;
  end
end
Return the simulations $\boldsymbol{\theta}^{1}, \boldsymbol{\theta}^{2}, ..., \boldsymbol{\theta}^{N}$

4 Results and discussion

4.1 Fitting DCC-MGARCH model

In fitting the class of MGARCH models based on the decomposition of the conditional covariance matrix into conditional standard deviations and correlations, the correct order selection can be identified using AIC values of the GARCH(p, q) process. Too large values of the ARCH and GARCH orders p and q, respectively lead to an identification problem because several parameterisations yield the same representation of the model and also in most empirical applications it turns out that the simple specification $p = q = 1$ is able to reproduce the volatility dynamics of financial data (Engle and Sheppard, 2001; Tsay, 2015). To overcome these numerical difficulties GARCH(1, 1) model was fitted to vector residual series. Moreover, we check for a couple of misspecification tests proposed by Nakatani and Teräsvirta (2010). The null and alternative hypothesis are stated as follows: $H_0 : \alpha_{ki}$ and β_{kj} in equation (4) are jointly diagonal elements against H_1: at least one of off-diagonal elements of α_{ki} and β_{kj} is non-zero. We proceed to apply the proposed test to our data series. Since the number of dimensions of series is k = 4, the value of the test statistic under the null of no causality in conditional variance, is a χ^2_{24} = 28.17 with $2k(k-1)$ = 24 degrees of freedom and associated p-value of 0.253 for NT test. Thus the null hypothesis is not rejected at 5% of significance levels in the tests. Therefore, it is recommended to use the diagonal version of the conditional variance in estimating the parameters in DCC-MGARCH model fitting.

4.1.1 Testing existence of dynamic and time-varying correlation in volatility models

Although due to its computational simplicity, the assumption of constant correlation volatility model is widely used in empirical research, its major drawback is that the correlation coefficient tends to change over time in a real application (Bauwens et al., 2006). Thus, we applied Engle and Sheppard (2001) method to test the null hypothesis of 'constant probability' of the correlation of standard residuals series estimated by constant CC-MGARCH model. The test results rejected the null hypothesis with a test statistic of 21.584 and p-value of (0.000). This is a strong evidence for existence of time-varying correlations between the four series variables under study at any significance level and against the assumption of constant CCs. Thus, further insights of CC can be gained by allowing the correlation matrix to vary over time as discussed above.

Following the procedure to build DCC models by Tsay (2005), first we apply preDCC-MGARCH(1, 1) to obtain estimates of the condition mean, $\hat{\boldsymbol{\mu}}_t$, for the Box-Cox transformed data series, $\mathbf{Y}_t$, and the estimated residuals $\hat{\boldsymbol{\varepsilon}}_t = \mathbf{Y}_t - \hat{\boldsymbol{\mu}}_t$. The estimated volatility series of a univariate GARCH, $\hat{\boldsymbol{\Sigma}}_{it} = \{\hat{\boldsymbol{\sigma}}_{ij,t}\}$ models to each component series $\hat{\boldsymbol{\varepsilon}}_{it}$ is

$$\begin{bmatrix} \boldsymbol{\sigma}^2_{GDPGR,t} \\ \boldsymbol{\sigma}^2_{INFR,t} \\ \boldsymbol{\sigma}^2_{EXR,t} \\ \boldsymbol{\sigma}^2_{INTR,t} \end{bmatrix} = \begin{bmatrix} 0.0014 \\ 0.0013 \\ 0.000002 \\ 0.000 \end{bmatrix} + \begin{bmatrix} 0.734 \\ 0.519 \\ 0.455 \\ 0.549 \end{bmatrix}' \begin{bmatrix} \boldsymbol{\varepsilon}^2_{GDPGR,t-1} \\ \boldsymbol{\varepsilon}^2_{INFR,t-1} \\ \boldsymbol{\varepsilon}^2_{EXR,t-1} \\ \boldsymbol{\varepsilon}^2_{INTR,t-1} \end{bmatrix} + \begin{bmatrix} 0.00087 \\ 0.462 \\ 0.516 \\ 0.402 \end{bmatrix}' \begin{bmatrix} \boldsymbol{\sigma}^2_{GDPGR,t-1} \\ \boldsymbol{\sigma}^2_{INFR,t-1} \\ \boldsymbol{\sigma}^2_{EXR,t-1} \\ \boldsymbol{\sigma}^2_{INTR,t-1} \end{bmatrix}. \tag{23}$$

From the volatility series of the prior four models shown in rquation (23) obtained by fitting the preDCC-MGARCH(1, 1) model, we obtain the marginally standardised residual series $\hat{\boldsymbol{\eta}}_t$ shown in Figure 8 in Appendix.

In order to fit Engle's (2002) DCC-MGARCH model of the volatilities, further insights of the correlation matrix, $\boldsymbol{\rho}_t$, can be gained by allowing it to vary with time so that the dynamic nature of the correlation can be captured. In this case, following Tse and Tsui (2002), Tsay (2005) and Engle (2002) a two-stage estimation procedure: first, a conventional univariate GARCH parameter estimation was made for each zero mean series by fitting a preDCC-MGARCH model discussed above, and the next stage is using estimates of a marginally standardised innovation vector, $\hat{\boldsymbol{\eta}}_{it}$ from the first stage the DCC parameters, $\hat{\theta}_1$ and $\hat{\theta}_2$ are estimated as shown in Table 6 in Appendix.

To handle heavy tails behaviour of macroeconomic time series variables, we used multivariate Student-t distribution for the innovations and the fitted model as given in equations (24) and (25).

$$\mathbf{Q}_t = (1 - 0.670 - 0.080)\bar{\mathbf{Q}} + 0.670\mathbf{Q}_{t-1} + 0.080\hat{\boldsymbol{\eta}}_{t-1}\hat{\boldsymbol{\eta}}'_{t-1}, \tag{24}$$

$$\boldsymbol{\rho}_t = \mathbf{J}_t\mathbf{Q}_t\mathbf{J}_t, \tag{25}$$

where $\mathbf{J}_t = diag\left\{q_{11,t}^{-1/2}, q_{22,t}^{-1/2}, q_{33,t}^{-1/2}, q_{44,t}^{-1/2}\right\}$ with $q_{ii,t}$ being the $(i,i)^{\text{th}}$ element of $\mathbf{Q}_t$, and $\overline{\mathbf{Q}}$ is the estimated sample unconditional covariance matrix of $\hat{\boldsymbol{\eta}}_t$ given by:

$$\overline{\mathbf{Q}} = \begin{bmatrix} 1.000 & -0.149 & 0.148 & 0.236 \\ -0.149 & 1.000 & 0.028 & 0.010 \\ 0.148 & 0.028 & 1.000 & 0.362 \\ 0.236 & 0.010 & 0.362 & 1.000 \end{bmatrix}.$$

As stated, in the theoretical description of the DCC-MGARCH model, the estimates of both scalar parameters $\hat{\theta}_1$ = 0.670 and $\hat{\theta}_2$ = 0.080, shown in Table 6, and as fitted in equation (24), are to capture the effects of previous DCCs and previous standardised shocks on current DCCs, $\mathbf{Q}_t$ and are significant at 5% level with t-ratios of 4.743 and 2.134, respectively. The assumptions $\hat{\theta}_1$ and $\hat{\theta}_2$ have to be individually larger than zero and their sum $\hat{\theta}_1 + \hat{\theta}_2$ = 0.750 has to be strictly less than a unity are both satisfied. Accordingly, the results of estimates of the DCC-parameters have high persistence that directly enforces both the necessary and sufficient conditions for positive definiteness and covariance stationarity assumption of $\hat{\boldsymbol{\Sigma}}_t$ and $\hat{\boldsymbol{\rho}}_t$ matrices. Thus, it is necessary to apply the DCC-MGARCH model to the existing dataset understudy.

From the fitted DCC-MGARCH in equation (24) the $\hat{\theta}_1$ estimates are typically much higher than $\hat{\theta}_2$ estimates, which implies that a higher long-run persistence in the CC contributed from the re-normalised CC, $\mathbf{Q}_t$, component at each time index t than CC contributed from $\hat{\boldsymbol{\eta}}_{t-1}\hat{\boldsymbol{\eta}}'_{t-1}$. Additionally, the estimated degrees of freedom for a multivariate student-t distribution, $\hat{\nu}$ = 13.86, is greater than 2 and is also significant with t-ratio of 2.007.

A visual representation of the estimated dynamic and time-varying CCs models in equations (24) and (25) were fitted to the estimated standardised residuals series of preDCC-GARCH model, $\hat{\eta}_t$. The interdependency of inflation rate and GDP growth rate are most negative throughout the study period, which is strong indications of the unstable relationship between the two macroeconomic variables. A negative correlation implies that long-run costs due to a period of low inflation rates will tend to be slightly offset by a period of high GDP growth rate. Conversely, long-run benefits during a period of high inflation rates will be offset by low economic growth rate. The estimates of dynamic and time-varying CC, between real GDP growth with both exchange and interest rates are positive. The correlation structure of the DCC(1, 1) MGARCH model has a clear interpretation: there is a non-constant interaction of the past CC that has a significant impact on current CC with a lag of 1. This interaction effect would be neglected if the time-series of macro-variables under consideration were modeled in isolation, using a univariate GARCH models.

4.2 Fitting the Bayesian DCC-MGARCH model

The Bayesian framework inference is based on computationally intensive methods, MCMC, to obtain the joint posterior distribution of the parameters of high dimensional volatility modelling, such as the class of DCCs. The MCMC sampler implemented in the package *bayesGARCH* (Fioruci et al., 2014b) is based on the approach of Ardia (2008). A sample obtained by the method being described will probably present significant correlation. However, due to the ergodicity property of the Markov chain, the estimation of the mean is not affected by the correlation in the sample. Therefore, to avoid unnecessary computational work, which ultimately would not lead to improvement in terms of parameter estimation, thinning is not implemented. The R-package 'MTS' implemented by Tsay (2015) was adopted with modified priors from standardised distributions.

The prior is specified with the help of parameters called hyperparameters which are initially assumed to be known and constant. For the GARCH(1, 1) coefficients in equation (4), we have the prior distributions proposed for ω_i for i = 1, 2, 3, 4 and ν in simulation are both the same in all cases. The priors for the lag coefficients $\alpha_i \sim Be\,(2,2)$ and $\beta_i \sim 0.1 * Be\,(2,2)$, $i = 1,2,...,k$ assigned so that it can be elicited by controlling the priors $\beta_i, \alpha_i > 0$ and $(\beta_i + \alpha_i)$ on the interval $(0,1)$. Similarly, the DCC parameters, $\theta_1 \sim Be\,(2,2)$ and $\theta_2 \sim 0.1 * Be\,(2,2)$, $i = 1,2,...,k$ assigned so that it can be elicited by controlling the priors $\theta_1, \theta_2 > 0$ and $(\theta_1 + \theta_2)$ on the interval (0, 1). For the skewed-t distribution the prior for the degree of freedom parameter assigned as ν = 8 was considered to ensures the conditional variance to be finite. In the case of the skewness parameters, we use beta distributions on positive values, i.e., $\gamma_i \sim Be\,(2,2)$ for $i = 1,...,k$. Using a beta distribution as prior facilitates the insertion of information in certain regions of the parameter space that satisfies the assumptions in the model since the hyper-parameters no longer represent the mean and variance but still control the region of higher probability mass. We emphasise the fact that only positivity constraints are implemented in the M-H algorithm, no stationarity conditions are imposed in the simulation procedure.

As of the variance parameters, we find it reasonable to choose a prior that is centered around the symmetric version of the skewed distribution and gives approximately equal weights to left and right skewness. If we choose the hyper-parameters a = 2 and b = 1 such that controlling the prior variance so small is a reasonable choice, since positivity of the variance is also mandatory that can be satisfied by the distribution chosen. Thus, the priors $\sigma^2_{\omega_i} = \sigma^2_{\alpha_i} = \sigma^2_{\beta_i} = \sigma^2_{\delta} = \sigma^2_{\theta_1} = \sigma^2_{\theta_2} = 0.1 * Gamma(2,1)$ are randomly chosen. Optionally, the inverse Gamma is was also checked in the sensitivity test.

Tables 2, 3 and 4 present the summary statistics for the samples obtained from the posterior distribution for each parameter of the DCC-MGARCH(1, 1) model by MCMC simulations for the model with multivariate skew normal, student-t and GED errors, respectively. The statistics reported in each table are the sample mean, the sample standard deviation and the 95% credibility interval for the parameter of the transition kernel density considered at N = 100,000 simulated realisations of the process and 10,000 burn in with out thinning in each case using M-H sampling steps. A large number of iterations is chosen because of the complexity of the model. As shown in Figure 2 (see also Tables 2, 3 and 4), mean estimates of the posterior distribution, ω_i, are nearly zero and do not change significantly when the entire sample is considered for skew normal, skew-t and skew-GED distributions. Figure 2 also reinforces the idea that the entire chain gives better estimates for the density function. That means, the greater efficiency is achieved through a prior belief in the curves smoothness of the kernel density of the posterior joint distribution of the model parameters. The means of the marginal posterior distributions are very similar to the estimates obtained by classical method discussed earlier. Note that the distribution of the MGARCH parameters α_i and β_i for each variables appear to be almost symmetric, as their means almost equal to the median (50%) summarises it well. They are clearly positive in view of the 95% credibility intervals.

Empirical results for the error terms, including the symmetric version of the distribution, were then compared according to the deviance information criterion (DIC). The bottom row of each Tables 2, 3 and 4 present the fitted MLE computed values of DIC and log-likelihood. The values signify the GED distribution is much better in fitting the data considered in the study. Since the DIC is subject to Monte Carlo sampling error (it is a function of stochastically simulated quantities), this might cast doubt whether the inclusion of the shape and skewness parameters δ and γ, respectively, is substantially improving model fit. Figure 2 illustrates the three fitted density curves to the simulated data from posterior densities of the skewness parameters $\hat{\gamma}_i$ for the multivariate skew normal, skew-t and skew-GED distributions. Graphically, it also confirms clearly that the density curve of the GED distribution provides the best fit to the data compared to the other two densities.

Table 2 Summary of the simulated Bayesian DCC-MGARCH model using M-H sampling for multivariate skewed-normal distribution

Variable	*Parameters*	*Mean*	*Std. dev.*	*2.5%*	*50%*	*97.5%*
GDPGR	γ_1	1.09	0.146	0.797	1.087	1.383
	ω_1	0.050	0.022	0.017	0.046	0.104
	α_1	0.173	0.558	0.056	0.175	0.274
	β_1	0.723	0.235	0.677	0.723	0.769
	$\alpha_1+\beta_1$	0.896	0.055	0.779	0.899	0.989
INFR	γ_2	0.767	0.158	0.516	0.749	1.122
	ω_2	0.037	0.014	0.016	0.035	0.070
	α_2	0.353	0.109	0.166	0.346	0.577
	β_2	0.487	0.106	0.260	0.494	0.674
	$\alpha_2+\beta_2$	0.840	0.091	0.624	0.848	0.984
EXR	γ_3	1.416	0.277	1.008	1.366	2.082
	ω_3	0.000	0.000	0.000	0.000	0.000
	α_3	0.217	0.043	0.142	0.217	0.300
	β_3	0.666	0.087	0.462	0.682	0.783
	$\alpha_3+\beta_3$	0.883	0.087	0.683	0.901	0.989
INTR	γ_4	0.760	0.097	0.576	0.756	0.959
	ω_4	0.007	0.002	0.004	0.007	0.011
	α_4	0.117	0.044	0.037	0.115	0.207
	β_4	0.532	0.137	0.198	0.548	0.751
	$\alpha_4+\beta_4$	0.649	0.137	0.330	0.665	0.861
	θ_1	0.107	0.027	0.055	0.106	0.162
	θ_2	0.657	0.105	0.438	0.672	0.805
	$\theta_1+\theta_2$	0.764	0.105	0.553	0.782	0.888
Log-likelihood = –352.611; DIC = –469.830						

Thus, we focus on the interpretation of parameter estimation results from GED model. From Table 4 one observes that the skewness parameter γ_i is well estimated. That is, inclusion of the shape parameters δ and γ_i is substantially improving model fit. Posterior means, medians and 95% credible intervals of the skewness parameter γ_i indicate high asymmetry (right skewed) for the GDP growth rate and exchange rate, while for the inflation rate and interest rates there is a slight skewness to the left.

Table 3 Summary of the simulated Bayesian DCC-MGARCH model using M-H sampling for multivariate skewed student-t distribution

Variable	*Parameters*	*Mean*	*Std. dev.*	*2.5%*	*50%*	*97.5%*
GDPGR	γ_1	1.028	0.123	0.792	1.024	1.287
	ω_1	0.025	0.009	0.242	1.164	1.432
	α_1	0.185	0.053	0.077	0.188	0.277
	β_1	0.731	0.026	0.679	0.730	0.780
	$\alpha_1 + \beta_1$	0.916	0.053	0.800	0.921	0.994
INFR	γ_2	0.754	0.141	0.511	0.745	1.046
	ω_2	0.063	0.019	0.033	0.061	0.106
	α_2	0.336	0.109	0.141	0.329	0.563
	β_2	0.530	0.103	0.317	0.533	0.720
	$\alpha_2 + \beta_2$	0.866	0.086	0.662	0.887	0.990
EXR	γ_3	1.128	0.153	0.888	1.107	1.472
	ω_3	0.000	0.000	0.000	0.000	0.000
	α_3	0.209	0.047	0.118	0.209	0.302
	β_3	0.645	0.103	0.363	0.666	0.779
	$\alpha_3 + \beta_3$	0.854	0.107	0.562	0.878	0.983
INTR	γ_4	0.849	0.101	0.654	0.848	1.054
	ω_4	0.015	0.0035	0.009	0.015	0.022
	α_4	0.105	0.043	0.024	0.103	0.195
	β_4	0.543	0.133	0.249	0.554	0.770
	$\alpha_4 + \beta_4$	0.648	0.133	0.347	0.659	0.874
	θ_1	0.103	0.030	0.046	0.102	0.164
	θ_2	0.655	0.103	0.432	0.664	0.825
	$\theta_1 + \theta_2$	0.532	0.0.095	0.546	0.769	0.904
	ν	7.760	0.196	7.366	7.762	8.141
Log-likelihood = –257.469; DIC = –464.277						

Lag coefficients for ARCH and GARCH parameters α_i and β_i, respectively, are all significant. The ARCH parameter is highest for inflation rate and lowest for interest rate while the GARCH parameter is highest for GDP growth rate and lowest for inflation rate. This indicates that inflation rate is more affected by the squares of lagged innovations while GDP growth rate is more affected by the lagged variances. In general, the hypothesis ARCH and GARCH parameters α_i and β_i for $i = 1, ..., 4$, respectively, are all zero is rejected, and that indicates the assumptions of positive definiteness of the correlation as well as covariance matrices are met since $\alpha_i + \beta_i < 1$ for $\forall i = 1, ..., 4$. High persistence $\alpha_i + \beta_i$ = 0.915 is observed for GDP growth rate and less persistent, 0.621 for interest rate.

The estimates of θ_1 and θ_2 in the bottom of Table 4 are significant and these indicate that a constant CC model hypothesis ($\theta_1 = \theta_2 = 0$) can be rejected on the basis of the marginal posterior distributions of θ_1 and θ_2. The posterior distribution of

$\theta_1 + \theta_2 = 0.780$ indicates more strong persistence in equation (3) with more load to θ_2, that is, higher persistence to CC contributed from $\hat{\eta}_{t-1}\hat{\eta}'_{t-1}$ than the re-normalised CC, $\mathbf{Q}_t$, component at each time index t. Lastly, the estimates of the shape parameter $\delta = 1.568$ last row of Table 4 for Skew-GED distribution is significant indicating that an asymmetric distribution is appropriate for modelling the error terms under this distribution. The length of 95% credible interval for δ is 0.775. This signifies the shape parameter for GED is more preferable for modelling fat tail behaviour heavier of the macroeconomic time series data set under study.

Table 4 Summary of the simulated Bayesian DCC-MGARCH model using M-H sampling for multivariate skewed GED distribution

Variable	*Parameters*	*Mean*	*Std. dev.*	*2.5%*	*50%*	*97.5%*
GDPGR	γ_1	1.157	0.124	0.925	1.151	1.419
	ω_1	0.041	0.022	0.009	0.037	0.096
	α_1	0.186	0.048	0.089	0.188	0.274
	β_1	0.729	0.025	0.680	0.729	0.779
	$\alpha_1 + \beta_1$	0.915	0.048	0.810	0.920	0.993
INFR	γ_2	0.746	0.167	0.477	0.723	1.110
	ω_2	0.039	0.016	0.014	0.036	0.077
	α_2	0.352	0.110	0.159	0.346	0.590
	β_2	0.508	0.108	0.283	0.513	0.708
	$\alpha_2 + \beta_2$	0.860	0.088	0.657	0.872	0.989
EXR	γ_3	1.293	0.206	0.953	1.267	1.788
	ω_3	0.000	0.000	0.000	0.000	0.000
	α_3	0.213	0.046	0.125	0.212	0.304
	β_3	0.650	0.102	0.406	0.669	0.775
	$\alpha_3 + \beta_3$	0.863	0.112	0.623	0.885	0.985
INTR	γ_4	0.767	0.099	0.591	0.761	0.982
	ω_4	0.007	0.002	0.004	0.007	1.011
	α_4	0.101	0.043	0.031	0.108	0.199
	β_4	0.520	0.129	0.226	0.533	0.741
	$\alpha_4 + \beta_4$	0.621	0.128	0.344	0.644	0.843
	θ_1	0.102	0.029	0.049	0.100	0.160
	θ_2	0.678	0.089	0.472	0.690	0.821
	$\theta_1 + \theta_2$	0.780	0.080	0.584	0.794	0.896
	δ	1.568	0.169	1.257	1.559	1.929
Log-likelihood = –416.871; DIC = –472.746						

4.3 Sensitivity analysis

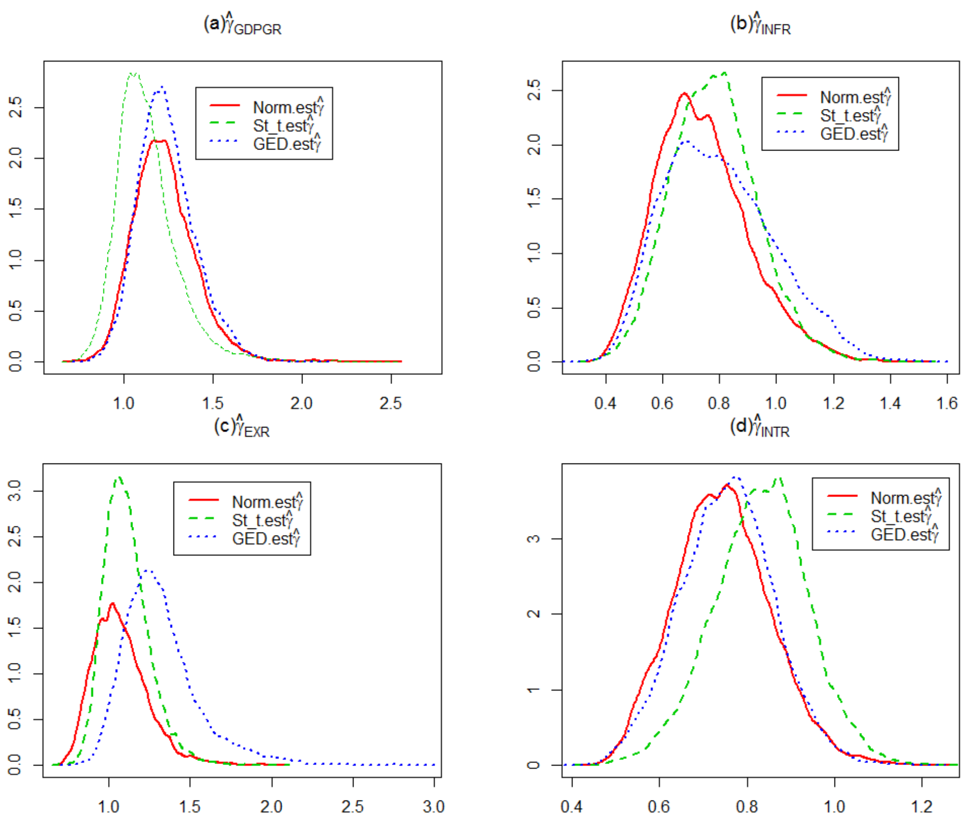
Sensitivity analysis is concerned with understanding how changes in the model inputs, priors, influence the outputs, the posterior distribution. This may be motivated simply by a wish to understand the implications of a complex model but often arises because there is uncertainty about the true values of the priors that should be used for a particular application. To evaluate the robustness of the Bayesian model in the presence of uncertainty, the key assumptions of the model fitted to multivariate skewed normal, student-t and GED, four different sensitive assumptions for the prior parameters, $\boldsymbol{\theta_3} = \{\gamma_i, \alpha_i, \beta_i, \theta_1, \theta_2, \rho_{i-1,i}, \delta\}$ under each model are computed and compared.

The different prior distribution assumptions for set of parameters $\boldsymbol{\theta_3}$ includes: Beta(2, 2), Truncated Normal(0, 1), Uniform(0, 1), and Beta(0.5, 0.5) were considered under three cases and the results displayed in Figures 3–5 and the summary statistics is given in Table 5 below for each case.

Case 1

Following Fioruci et al. (2014a) method the prior distribution for set of parameters $\boldsymbol{\theta_3}$ to be randomly generated from a truncated normal distribution in the interval (0, 1), but with the same variance to the reference model [previously randomly generated from Gamma(1, 2)]. The estimated posterior distribution of γ_i is given in Figure 3.

Figure 2 Posterior densities of skewness parameters for the, (a) GDPGR (b) INFR (c) EXR (d) INTR using the skew multivariate normal (solid red line), student-t (dotted blue points) and GED (dashed green line) distributions

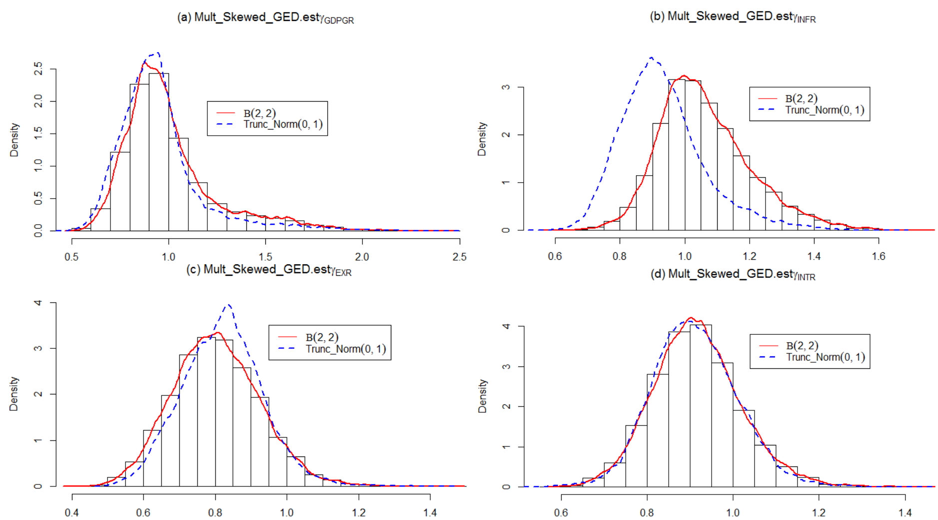


The result shows that the prior parameters from truncated Gaussian are with almost the same mean value and standard deviation but with higher kurtosis and more skewed than Beta(2, 2) prior for the set of parameters $\boldsymbol{\theta}_3$. The Beta(2, 2) prior improves and degrades the estimation performance for γ_i as observed for each variable in Figure 3 and the summary statistics in Table 5.

Case 2

The prior distribution for set of parameters $\boldsymbol{\theta_3}$ to be randomly generated from a uniform distribution in the interval (0, 1), but with the same variance to the reference model. Sampling posterior distributions of γ_i under GED using different prior distribution for set of parameters $\boldsymbol{\theta_3}$ is given in Figure 4.

Figure 3 Sampling distributions of γ_i under GED using different priors: Beta(2, 2) (solid line), Truncated Normal(0, 1) (dashed line) for, (a) GDPGR (b) INFR (c) EXR (d) INFR (see online version for colours)



The result shows the sensitivity test in case of GED model, the prior parameters from Beta(2, 2) and Uniform(0, 1) are with almost the same mean, kurtosis and skewness but with smaller standard deviation for the later, see Figure 4 and Table 5. This indicates that the beta prior for parameter set $\boldsymbol{\theta}_3$ neither improves nor degrades the estimation performance in uniform priors.

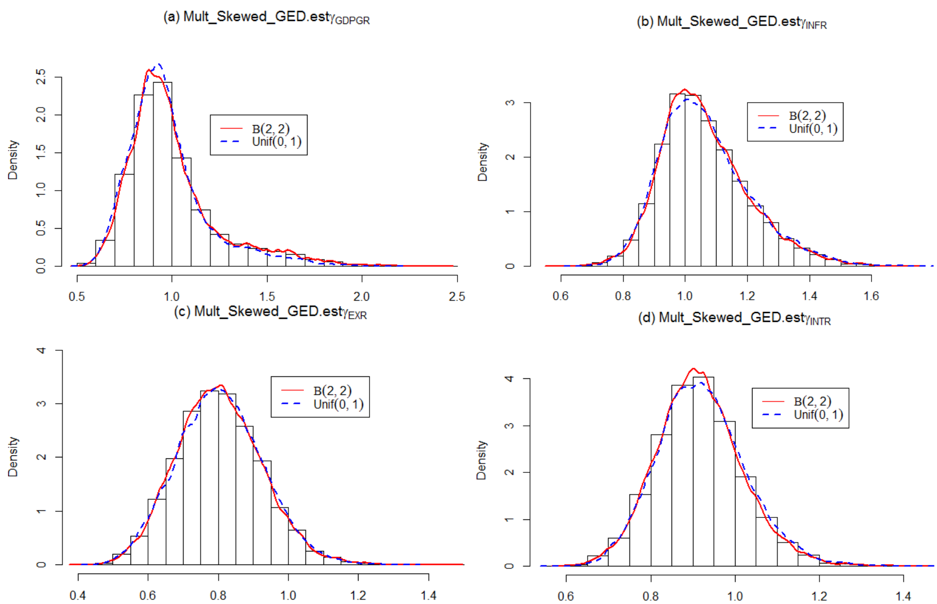
Case 3

$\boldsymbol{\theta_3}$ to be generated from the Beta(0.5, 0.5) again with the same variance to the reference model to observe the effect of the beta prior parameters in the interval (0, 1). As for the $Beta(a, b)$ prior taking $a = b$ is reasonable to choose a prior that gives approximately symmetric distribution with equal weights to left and right skewness. Sampling posterior distributions of γ_i under GED using different prior distribution is given in Figure 5.

The results of case 3 also shows that the beta prior for parameter set $\boldsymbol{\theta}_3$ does not affected by the beta prior parameters (a, b) restricted to the interval (0, 1). Beta prior for the set of parameters $\boldsymbol{\theta}_3$ improves the estimation performance, compared to either the truncated Gaussian prior or the uniform prior for γ_i in the Bayesian

DCC-MGARCH model fitted to multivariate skewed distributional models. Almost all of Bayesian analysis considered are insensitive to the priors change. Posterior distributions of scale and shape parameters, ν and δ for the multivariate skewed student-t and GED distributions, respectively, under different priors are reported in Figures 6 and **??**. The result shows the estimated posterior distributions are all quite close except some slight shift of the shape parameter δ to the left in case of the truncated normal prior. It is to be noted that posterior results for the multivariate skewed student-t model were not extremely sensitive with respect to the prior parameter ν. However, more sensitive with respect to the prior parameter δ for the multivariate skewed GED model.

Figure 4 Sampling distributions of γ_i GED using different priors: Beta(2, 2) (solid line) and Uniform(0, 1) (dashed line) for, (a) GDPGR (b) INFR (c) EXR (d) INFR (see online version for colours)



4.4 Discussion

The estimates of dynamic and time-varying CC, between real GDP growth with both exchange and interest rates are positive, which are in line with Rodrik (2008) and Bader and Malawi (2010) the study conducted in Jordan and also by Dickson (2021) in Ghana's economic growth.

There is strong evidence of high volatility both in magnitude and persistence of co-movements between macroeconomic variables under study. The estimates for DCC parameters are statistically significant. This makes it clear that the assumption of constant CC is not supported empirically. The correlation structure of the fitted models have a clear interpretation that there is a non-constant interaction of the past CC that has a significant dynamic and time-varying impact on current CC. Thus, ignoring the extension of the CC-MGARCH model to the DCC-MGARCH models have an impact that would induce model misspecification. This confirms the recent empirical evidences by Engle (2002), Tse and Tsui (2002), Tsay (2005) and Greenberg (1995). Furthermore, by inclusion of local correlations to update the CC matrices and the smoothing parameter the time-varying CC fits the data better.

Figure 5 Sampling distributions of γ_i under multivariate skewed GED distribution using different priors: Beta(2, 2) (solid line) and Beta(0.5, 0.5) (dashed line) for, (a) GDPGR (b) INFR (c) EXR (d) INFR (see online version for colours)

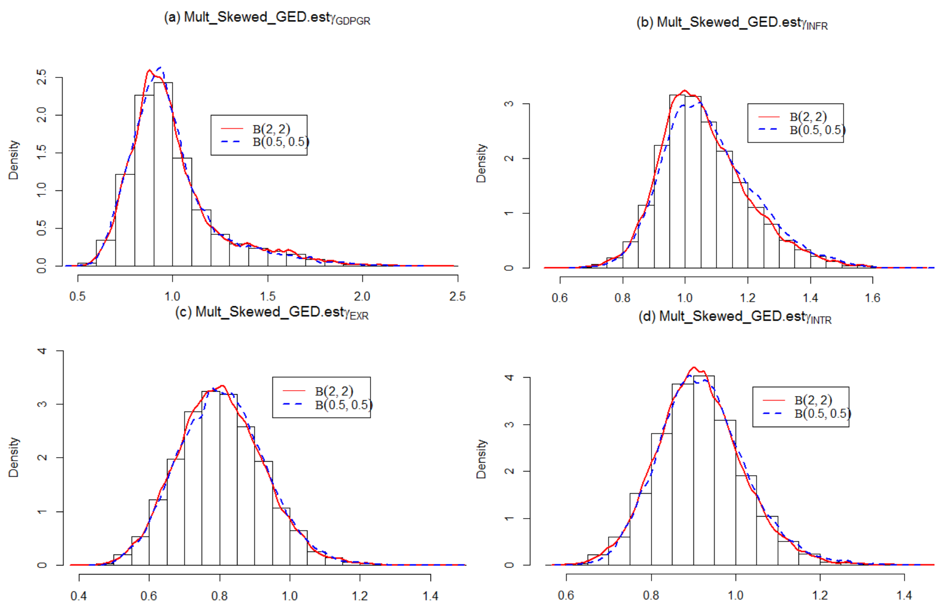
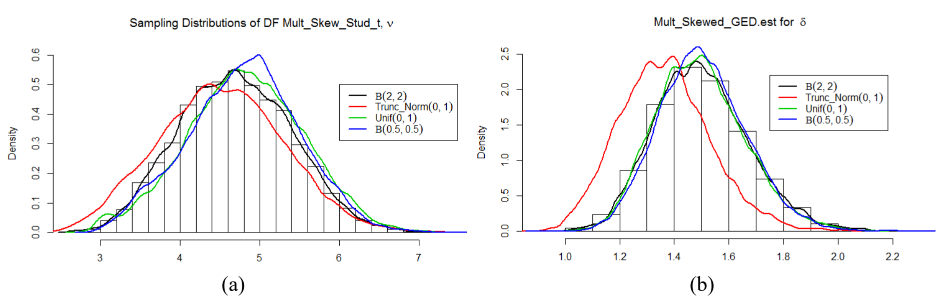


Figure 6 Sampling distributions of, (a) ν (b) δ under multivariate skewed GED distribution using different priors: Beta(2, 2) (black), Truncated Normal(0, 1) (red), Uniform(0, 1) (blue) and Beta(0.5, 0.5) (see online version for colours)



Empirical results concerning the distribution of conditionally volatile error terms that accommodate asymmetry as well as heavy tailed distribution, the Metropolis Hastings algorithm in MCMC methods are very useful in estimating parameters fitting Bayesian DCC-MGARCH models, that confirm with the existing literature Fiorentini et al. (2003), Meyer and Yu (2000), Ardia (2008) and Asai (2006).

In summary, since the DIC is subject to Monte Carlo sampling error in Bayesian methods, it select the beta prior, that is; the inclusion of the shape and skewness parameters δ and γ, respectively, which substantially improving model fit. Further, the results signify that the GED distribution is much better when beta prior density functions was considered in the selection of the true value of the hyper-parameters in Bayesian DCC-MGARCH model fitting, which is consistent with the conclusion made by Prass et al. (2016) in the literature.

Table 5 Summary statistics of γ_i in the sensitivity analysis for different prior distributions assumptions

Skewed mult. distribution	*Assumed prior distribution*	*Mean*	*Standard deviation*	*Skewness*	*Kurtosis*
Normal	Tranc. N(0, 1)	1.143	0.240	0.711	0.354
	Unif(0, 1)	1.152	0.241	0.724	0.357
	Beta(0.5, 0.5)	1.160	0.242	0.678	0.288
	Beta(2, 2)	1.162	0.247	0.746	0.423
Student-t	Tranc. N(0, 1)	0.911	0.084	-0.168	0.707
	Unif(0, 1)	0.887	0.116	0.695	2.816
	Beta(0.5, 0.5)	0.892	0.105	0.185	0.980
	Beta(2, 2)	0.884	0.101	0.104	1.378
GED	Tranc. N(0, 1)	0.949	0.221	1.826	5.270
	Unif(0, 1)	0.972	0.013	1.395	2.865
	Beta(0.5, 0.5)	0.991	0.231	1.466	2.843
	Beta(2, 2)	0.998	0.241	1.497	2.782

5 Concluding remarks

This paper explores the technical challenges faced by Bayesian econometricians in enhancing forecasting models for macroeconomic variables using Bayesian inference. Recent Bayesian study on GARCH model, promising for stochastic simulation techniques to explore multivariate skewed distributions and heavy-tailed error properties. The study enhances MGARCH model usage and performance by incorporating Bayesian frame work and efficient sampling methods for modelling asymmetric error terms. The DCC-GARCH model is proposed to incorporate new priors, scale parameters, and shape parameters using a Bayesian approach and an efficient sampling method from non-normal posterior distributions. They come together to form a versatile family of distributions that can handle heavy-tailed, asymmetric data, which is commonly seen in financial time series.

The analysis results shows the posterior estimations of the parameters are found to be reliable under the beta prior distribution. There are some right skewness for the posterior distributions of errors of both GDP growth and exchange rates. However, for the inflation and interest rates, slight skewness to the left are signified. Thus, asymmetric distribution is appropriate for modelling the error terms of such macroeconomic data. Both log-likelihood and DIC method suggest that the multivariate GED model fits best to the data. The sensitivity test under GED model shows that the Bayesian analyses are insensitive to the prior choices of uniform and beta distributions. The truncated normal provides slightly different results than the two priors. The Bayesian GED model is promising as analysing such economic data considered in this study. The new approach is more flexible and can better describe the uncertainties in volatilities than the classical methods.

Further research could compare other uni- and MGARCH models, such as the models that directly model the conditional covariance matrix like VEC and the Baba, Engle, Kraft and Kroner (BEKK) MGARCH models. One may still looking into using skewed distributions with fat tails in addition to the multivariate student-t and GED in their ongoing research.

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Appendix

Figure 7 Time plots of residuals series of the fitted VEC model (see online version for colours)

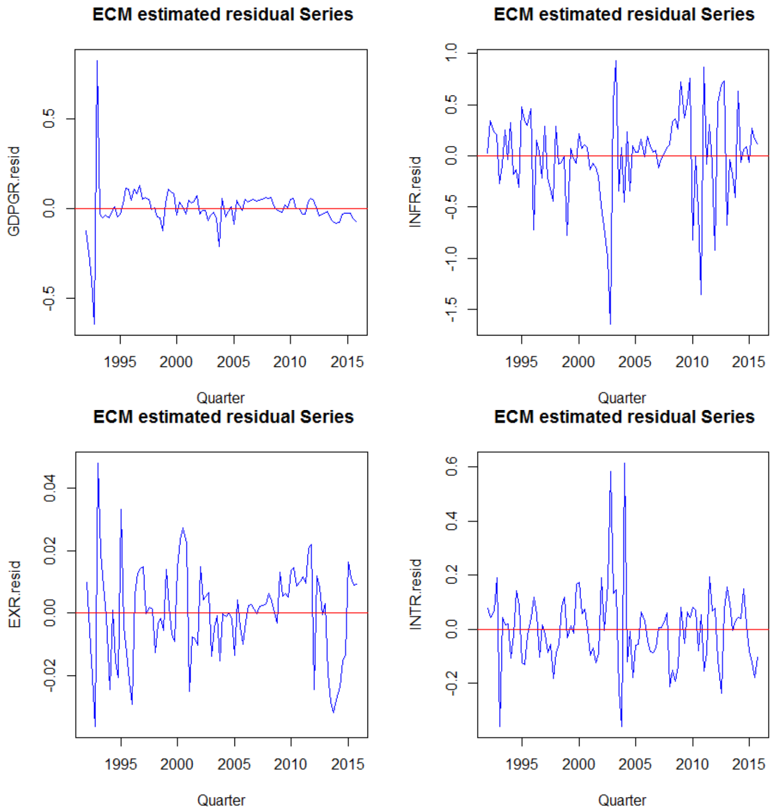

Table 6 The estimation of DCC-MGARCH model parameters

Variable	*Parameters*	*Estimate*	*Std. error*	*t-value*	*Significance*
GDPGR	$\hat{\varpi}_1$	0.0014	0.000316	4.391	0.000***
	$\hat{\alpha}_1$	0.734	0.187	3.917	0.000***
	$\hat{\beta}_1$	0.00087	0.005	0.174	0.431
INFR	$\hat{\varpi}_2$	0.0013	0.00068	1.98	0.024***
	$\hat{\alpha}_2$	0.519	0.175	2.917	0.002***
	$\hat{\beta}_2$	0.462	0.099	4.633	0.000***
EXR	$\hat{\varpi}_3$	0.000002	0.000001	1.303	0.096
	$\hat{\alpha}_3$	0.455	0.199	2.280	0.011***
	$\hat{\beta}_3$	0.516	0.169	3.038	0.001***
INTR	$\hat{\varpi}_4$	0.000	0.000	1.142	0.126
	$\hat{\alpha}_4$	0.549	0.180	3.043	0.001***
	$\hat{\beta}_4$	0.402	0.191	2.103	0.017***
	$\hat{\theta}_1$	0.670	0.141	4.743	0.000***
	$\hat{\theta}_2$	0.080	0.0374	2.134	0.016***
	$\hat{\nu}$	13.86	6.672	2.077	0.018***
Log-likelihood = –16.95					

Figure 8 Estimated standardised residual plot of fitted preDCC-MGARCH(1, 1) models for, (a) GDPGR (b) INFR (c) EXR (d) INTR (see online version for colours)