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Abstract: In this paper, a bi-level linear fractional programming problem (BILLFPP) with fuzzy interval coefficient (FIC) is contemplated, where all of it is coefficients in the goal function and constraints are fuzzy intervals (FIs). Firstly, to resolve this issue, we are going to construct two LFPP with fuzzy coefficients. Before all else, of these issues is a LFPP where all of coefficients are upper approximations of FIs and the other is a LFPP where all of coefficients are lower approximations of FIs. Secondly, the BILLFPP is transformed to the form of single goal LFPP and QFPP. We address problems with a factorised or non-factorised optimisation problem and homogeneous or non-homogeneous constraints. Our proposed technique is based on a mathematical model that converts the QFPP to a LPP by solving the problem in an algebraic expression with a Taylor series. This technique, which is based on the LPP solution, can be applied to specific problems. NLFPP containing nonlinear constraints, on iterative processes, it decreases the overall processing time. Further explanations of the novel technique for solving BILLFPP are made by taking numerical examples and comparing with Jayalakshmi (2015) and Syaripuddin et al. (2017).

Keywords: LFPP; bi-level linear fractional programming problem; BILLFPP; FBILLFPP; BILLFPP with fuzzy interval; FBILLFPP with FIC; QFPP; Taylor series; a novel technique; fuzzy interval coefficient; FIC.

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1 Introduction

In the many nonlinear programming applications, a function comprising one or perhaps more ratios of functions should be maximised or minimised. Fractional programs are a typical name for such optimisation problems, abbreviating the term fractional functionals program, in their seminal study which was originally revealed to the public by Cooper and Charnes (1962). The term hyperbolic program is also occasionally used. To considering the word ratio program to improve the terminology. However, after well over a thousand journals in the field of nonlinear programming, such a shift may be hard. The goal of this study is to provide a resolution BILLPP with fractional objective functions when the variables are integers. To design a fuzzy model for obtaining the optimum integer resolution to problem under investigation, a membership function is developed (Saad and Hafez, 2011; Mishra and Verma, 2017). Began with an interval-valued LFPP topic and turned everything into a problem of optimisation an interval-valued minimisation problem (Nasseri and Bavandi, 2017). Take the FFLFPP as an example FFLFP. The FFLFPP is first converted to a FFLPP, in which it will be stated in parametric form. Present a simplex type approach for solving FFLPP without switching to a crisp solution counterpart (Loganathan and Ganesan, 2019; Majeed, 2012) recommend a new method for solving LF bounded variables with interval values problems with programming (Das and Hasan, 2012). Offer two fractional transportation issue with interval coefficients solution procedures (Guzel et al., 2012). Application of interval arithmetic for resolving FFL systems with trapezoidal fuzzy numbers (Siahlooei et al., 2018). Non-cooperative game solutions for two-level LFPP with IC in the objective functions were introduced to get non-cooperative game solutions (Borza et al., 2012a, 2012b). For each target, there is an imprecise and ambition level. As a process, the objectives become a hazy goal. Then, by reducing their deviational variables, the objective programming methodology is used to achieve the highest degree of each of the membership objectives (Veeramani and Sumathi, 2016). Sharp LFP is used in the potential methods, as well as new transformation methodology (Das and Mandal, 2017). Presented when using a FGP approach and reformulating fractional constraints into non-fractional nonlinear constraints that are equal but not fractional. An iterative approach is predicated on the altered formulation that really is feasible offer a satisfying the tri-level problem is obviously solution (Kassa and Tsegay, 2018) BIL multi-objective rough nonlinear programming problem, the proposed solution procedure (Elsisy et al., 2021) assumed to be 0–1. The problem is reduced to a BILLFPP with one leader and one follower (Arora and Narang, 2009). Ingenious way for reducing MFLP issues into a single FLPP and solving it using the simplex method with such a linear ranking function (Hamadameen, 2017). Interpolating polynomials are used to replace the multi-choice parameters then, using fuzzy programming, compromise resolution for the converted BILLPP is found. An algorithm is developed to find compromise resolution of BILLPP (Arora and Gupta, 2017) showed that IVLFP may be converted to an optimisation problem with an interval valued goal function and LF function bounds (Effati and Pakdaman, 2012). Divide numerator and denominator in each decision maker fractional objective function to generate two BILQPP from one BILQPP (Rakshit and Kumar, 2016; Hosseini and Kamalabadi, 2013). Hybrid model of TOPSIS and FGP (Dey et al., 2014). Interpolating polynomials are being used to replace the multi-choice parameters. Then, fuzzy programming is employed to determine BILLPP altered compromise resolution (Arora and Gupta, 2017) introduces an IVLFPP (Effati and Pakdaman, 2012). Focus on a version LP with fuzzy numbers and numerous goals with fuzzy coefficients and crisp constraints, then establish necessary efficiency points for a new problem, and aim to identify all of these necessary efficiency points in order to resolve the problem (Nehi and Alineghad, 2008). In comparison to the original problem, nonlinear problem is translated into a LPP with two additional constraints and one additional variable (Borza et al., 2012a). This GA avoids using a penalty function to deal with restrictions by converting the randomly generated starting population into an initial population that meets the constraints. This improves the GA capacity to deal with constraints (Wang et al., 2005). Based on the signed deterministic distance ranking, FMOLPP is translated into the corresponding MOLPP (Ammar and Khalifa, 2020) introduced RILFPP the RILFPP consider rough IC in the goal function coefficients. Ammar and Muamer (2015) provides a method for addressing fully rough interval multi-level multi-objective LFPP in which all of the goal function and constraint coefficients are rough intervals (Osman et al., 2018a, 2018b, 2017) present a method for achieving a satisfying solution to a BILQFPP using fuzzy programming (Mishra, 2016). On IV pseudolinear functions and IV pseudolinear optimisation problems (Zhang et al., 2015; Abo-Sinna and Baky, 2010) solving FRLFPP (Ammar and Eljerbi, 2019). The acronym RILFPP is used for the first time. By including rough intervals in the objective function coefficients, the RILFPP is considered (Khalifa, 2018; Safaei and Saraj, 2012; Solomon et al., 2021; Elsisy and El Sayed, 2019). The MLMOFPP was created at a high level of confidence without increasing the problem fuzziness (Osman et al., 2018b; Khalifa, 2019). Chance constrained linear plus linear fractional BIL (Pramanik et al., 2012) an interval-valued inventory optimisation model is proposed (Kumar, 2015). BILNLFPP with random fuzzy parameters (Amer, 2019) solution method of BILLFPP using of Legendre polynomials (Saad and Farag, 2015), FGP approach for BILLFPP (Pramanik and Dey, 2011).

This paper is organised as follows Sections 2 and 3 briefly presents the required information used in this study. In Section 4, the proposed approach is demonstrated. Section 5 and Section 6 consists of numerical examples we solved them by proposed technique and compare them with Jayalakshmi (2015) and Syaripuddin et al. (2017) and Section 7 the conclusion, respectively.

2 Preliminaries

1 A QFPP is defined as

$$Max.\tilde{Z}_x \text{ or } Min.\tilde{Z}_x = \frac{f(\tilde{x})}{g(\tilde{x})} \tag{1}$$

subject to:

$$\tilde{x} \in \tilde{X} = \left\{ \tilde{x} \in R^n; \left[\tilde{A}\tilde{x} \begin{bmatrix} \le \\ \ge \\ = \end{bmatrix} \tilde{B} \right] \right\}, \tag{2}$$

$$\tilde{x} \ge 0. \tag{3}$$

where $\tilde{x} \in R^n$, an n-dimensional column vector of decision variables, $\tilde{A}$ is an $m \times n$ matrix; $\tilde{B} \in R^m$ (Siviri et al., 2018).

2 Any point $\tilde{x}$ satisfying the constraints (2) and (3) are said a feasible point. The set of all feasible points is called the feasible region, (Siviri et al., 2018) such that

$$\tilde{X} = \left\{ \tilde{x} \in R^n; \left[\tilde{A}\tilde{x} \begin{bmatrix} \le \\ \ge \\ = \end{bmatrix} \tilde{B} \right], \tilde{x} \ge 0 \right\}.$$

3 A feasible point $\tilde{x}^*$ is said the optimum resolution to the optimisation issue (1), $\tilde{Z}_x \le \tilde{Z}_x^*$ for the maximisation problem, $\tilde{Z}_x \ge \tilde{Z}_x^*$ for the minimisation problem for each feasible point $\tilde{x}$ (Siviri et al., 2018).

4 A differentiable function F is continuously differentiable if and only if F is of differentiability class C^1 (Siviri et al., 2018).

5 The first two terms of the Taylor series generated by $F(\tilde{x}) = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n)$ at $\tilde{A}_0 = (\tilde{a}_{01}, \tilde{a}_{02}, ..., \tilde{a}_{0n})$, i.e.

$$F\left(\tilde{A}_0\right) + (\tilde{x}_1 - \tilde{a}_{01})\frac{\partial F\left(\tilde{A}_0\right)}{\partial \tilde{x}_1} + (\tilde{x}_2 - \tilde{a}_{02})\frac{\partial F\left(\tilde{A}_0\right)}{\partial \tilde{x}_2} + \cdots + (\tilde{x}_n - \tilde{a}_{0n})\frac{\partial F\left(\tilde{A}_0\right)}{\partial \tilde{x}_n} = 0.$$

Linearise the function F in n-variables (Siviri et al., 2018).

3 Basic definition on BILLFPP and BILLFPP with FIC

Definition 3.1: Fuzzy LFPP

The MPP for LFPP can be formulated as follows:

$$Max.\tilde{Z}_x \text{ or } Min.\tilde{Z}_x = \frac{\tilde{C}^T \tilde{x} + \tilde{\alpha}}{\tilde{D}^T \tilde{x} + \tilde{\beta}}$$

subject to:

$$\tilde{x} \in \tilde{X} = \left\{ \tilde{x} \in R^n; \left[\tilde{A}\tilde{x} \begin{bmatrix} \le \\ \ge \\ = \end{bmatrix} \tilde{B} \right], \tilde{x} \ge 0 \right\}.$$

where $\tilde{x} \in R^n$, $\tilde{A}$ is an $m \times n$ matrix; $\tilde{C}$ and $\tilde{D}$ are n-vectors; $\tilde{B} \in R^m$, $\tilde{\alpha}, \tilde{\beta}$ are scalar constants. Moreover $\tilde{D}^T \tilde{x} + \tilde{\beta} > 0$, everywhere in $\tilde{X}$ (Cooper and Charnes, 1962).

Definition 3.2: Fuzzy BILLFPP

The MPP for FBILLFPP can be formulated as follows:

$$\text{ULDM } Max.\tilde{Z}_{x_1} \text{ or } Min.\tilde{Z}_{x_1} = \frac{\tilde{C}_1^T \tilde{x} + \tilde{\alpha}_1}{\tilde{D}_1^T \tilde{x} + \tilde{\beta}_1}$$

$$\text{LLDM } Max.\tilde{Z}_{x_2} \text{ or } Min.\tilde{Z}_{x_2} = \frac{\tilde{C}_2^T \tilde{x} + \tilde{\alpha}_2}{\tilde{D}_2^T \tilde{x} + \tilde{\beta}_2}$$

subject to:

$$\tilde{x} \in \tilde{X} = \left\{ \tilde{x} \in R^n; \left[\tilde{A}\tilde{x} \begin{bmatrix} \le \\ \ge \\ = \end{bmatrix} \tilde{B} \right], \tilde{x} \ge 0 \right\}.$$

where $\tilde{x} \in R^n$, $\tilde{A}$ is an $m \times n$ matrix; $\tilde{C}_i$ and $\tilde{D}_i$ are n-vectors; $\tilde{B} \in R^m$, $\tilde{\alpha}_i$, $\tilde{\beta}_i$ are scalar constants and i = 1, 2. Moreover, $\tilde{D}_i^T \tilde{x} + \tilde{\beta}_i > 0$, everywhere in $\tilde{X}$ (Solomon et al., 2021).

Definition 3.3: LFPP with FIC

The MPP for LFPP with FI can be formulated as follows:

$$
\begin{aligned}
&Max.\tilde{Z}_x \text{ or } Min.\tilde{Z}_x = \frac{\left[\tilde{C}_1^T, \tilde{C}_2^T\right]\tilde{x} + \left[\tilde{\alpha}_1, \tilde{\alpha}_2\right]}{\left[\tilde{D}_1^T, \tilde{D}_2^T\right]\tilde{x} + \left[\tilde{\beta}_1, \tilde{\beta}_2\right]} \\
&\text{subject to:} \\
&\qquad \left[\tilde{A}_1, \tilde{A}_2\right]\tilde{x} \begin{bmatrix} \le \\ \ge \\ = \end{bmatrix} \left[\tilde{B}_1, \tilde{B}_2\right], \\
&\qquad \tilde{x} \ge 0.
\end{aligned}
$$

where $\tilde{x} \in R^n$, $\tilde{A}_i$ are an $m \times n$ matrix; $\tilde{C}_i$ and $\tilde{D}_i$ are n-vectors; $\tilde{B}_i \in R^m$, and $\tilde{\alpha}_i$, $\tilde{\beta}_i$ are scalar constants. Moreover, $[\tilde{D}_1^T, \tilde{D}_2^T]\tilde{x} + [\tilde{\beta}_1, \tilde{\beta}_2] > 0$, everywhere in $\tilde{X}$, i = 1, 2 (Calvete and Galé, 2012).

Definition 3.4: BILLFPP with FIC

The MPP for BILLFPP with FI can be formulated as follows:

$$
\begin{aligned}
&\text{ULDM } Max.\tilde{Z}_{x_1} \text{ or } Min.\tilde{Z}_{x_1} = \frac{\left[\tilde{C}_1^T, \tilde{P}_1^T\right]\tilde{x} + \left[\tilde{\alpha}_1, \tilde{\beta}_1\right]}{\left[\tilde{D}_1^T, \tilde{Q}_1^T\right]\tilde{x} + \left[\tilde{\gamma}_1, \tilde{\delta}_1\right]} \\
&\text{LLDM } Max.\tilde{Z}_{x_2} \text{ or } Min.\tilde{Z}_{x_2} = \frac{\left[\tilde{C}_2^T, \tilde{P}_2^T\right]\tilde{x} + \left[\tilde{\alpha}_2, \tilde{\beta}_2\right]}{\left[\tilde{D}_2^T, \tilde{Q}_2^T\right]\tilde{x} + \left[\tilde{\gamma}_2, \tilde{\delta}_2\right]} \\
&\text{subject to:} \\
&\qquad \left[\tilde{A}_1, \tilde{A}_2\right]\tilde{x} \begin{bmatrix} \le \\ \ge \\ = \end{bmatrix} \left[\tilde{B}_1, \tilde{B}_2\right], \\
&\qquad \tilde{x} \ge 0.
\end{aligned}
$$

where $\tilde{x} \in R^n$, $\tilde{A}_i$ are an $m \times n$ matrix; $\tilde{C}_i, \tilde{P}_i$ and $\tilde{D}_i, \tilde{Q}_i$ are n-vectors; $\tilde{B}_i \in R^m$ and $\tilde{\alpha}_i, \tilde{\beta}_i, \tilde{\gamma}_i, \tilde{\delta}_i$ are scalar constants. And moreover, $[\tilde{D}_i^T, \tilde{Q}_i^T]\tilde{x} + [\tilde{\gamma}_1, \tilde{\delta}_i] > 0$, everywhere in $\tilde{X}$, i = 1, 2 (Solomon et al., 2021).

4 Formulation BILLFPP with FIC

In this section, BILLFPP with fully FIC is considered. Formulating a BILLFPP model requires that fuzzy interval values be selected for the model coefficients. The fuzzy values of several of these coefficients are only approximately known. The major advantage of the methodology operations over the existing one is that algorithm deal with uncertainty coefficients which take the form of fully FIC. Now, joining all the data in the BILLFPP with fuzzy interval model is required. Let us consider a BILLFPP with FIC (Solomon et al., 2021).

$$Max.\tilde{Z}_{x_1} \text{ or } Min.\tilde{Z}_{x_1} = \frac{\tilde{P}_1^T \tilde{x} + \tilde{\beta}_1}{\tilde{Q}_1^T \tilde{x} + \tilde{\delta}_1} \qquad Max.\tilde{Z}_{x_1} \text{ or } Min.\tilde{Z}_{x_1} = \frac{\tilde{C}_1^T \tilde{x} + \tilde{\alpha}_1}{\tilde{D}_1^T \tilde{x} + \tilde{\gamma}_1}$$

$$Max.\tilde{Z}_{x_2} \text{ or } Min.\tilde{Z}_{x_2} = \frac{\tilde{P}_2^T \tilde{x} + \tilde{\beta}_2}{\tilde{Q}_2^T \tilde{x} + \tilde{\delta}_2} \qquad Max.\tilde{Z}_{x_2} \text{ or } Min.\tilde{Z}_{x_2} = \frac{\tilde{C}_2^T \tilde{x} + \tilde{\alpha}_2}{\tilde{D}_2^T \tilde{x} + \tilde{\gamma}_2}$$

subject to: or subject to:

$$\tilde{A}_2\tilde{x}\begin{bmatrix} \le \\ \ge \\ = \end{bmatrix}\tilde{B}_2, \qquad \tilde{A}_1\tilde{x}\begin{bmatrix} \le \\ \ge \\ = \end{bmatrix}\tilde{B}_1,$$

$$\tilde{x} \ge 0. \qquad \tilde{x} \ge 0.$$

$$Max.\tilde{Z}_x \text{ or } Min.\tilde{Z}_x = Max.\tilde{Z}_{x_1} + Max.\tilde{Z}_{x_2} \text{ or } Min.\tilde{Z}_{x_1} + Min.\tilde{Z}_{x_2} = \frac{\tilde{P}_1^T \tilde{x} + \tilde{\beta}_1}{\tilde{Q}_1^T \tilde{x} + \tilde{\delta}_1} + \frac{\tilde{P}_2^T \tilde{x} + \tilde{\beta}_2}{\tilde{Q}_2^T \tilde{x} + \tilde{\delta}_2}$$

subject to:

$$\tilde{A}_2\tilde{x}\begin{bmatrix} \le \\ \ge \\ = \end{bmatrix}\tilde{B}_2,$$

$$\tilde{x} \ge 0.$$

Or

$$Max.\tilde{Z}_x \text{ or } Min.\tilde{Z}_x = Max.\tilde{Z}_{x_1} + Max.\tilde{Z}_{x_2} \text{ or } Min.\tilde{Z}_{x_1} + Min.\tilde{Z}_{x_2} = \frac{\tilde{C}_1^T \tilde{x} + \tilde{\alpha}_1}{\tilde{D}_1^T \tilde{x} + \tilde{\gamma}_1} + \frac{\tilde{C}_2^T \tilde{x} + \tilde{\alpha}_2}{\tilde{D}_2^T \tilde{x} + \tilde{\gamma}_2}$$

subject to:

$$\tilde{A}_1\tilde{x}\begin{bmatrix} \le \\ \ge \\ = \end{bmatrix}\tilde{B}_1,$$

$$\tilde{x} \ge 0.$$

Sometimes it may be need to translate $Max.\tilde{Z}_x$ or $Min.\tilde{Z}_x$ LFPP or QFPP for determining the best solution to the problem.

5 The proposed methodology

Consider QFPP given in (1). Our algorithm is given as follows:

Step 1 Choose any initial arbitrary feasible non-zero point $\dot{\tilde{x}} = (\dot{\tilde{x}}_1, \dot{\tilde{x}}_2, ..., \dot{\tilde{x}}_n)$.

Step 2 Linearise the goal function (1) by expanding it to the Taylor series at the chosen point $\dot{\tilde{x}}$ using preliminaries (5).

Step 3 Solve the following LPP.

$$Max.\tilde{Z}_x \text{ or } Min.\tilde{Z}_x = \tilde{C}^T \tilde{x}$$
$$\text{subject to:}$$
$$\tilde{x} \in \tilde{X} = \left\{ \tilde{x} \in R^n ; \left[\tilde{A}\tilde{x} \begin{bmatrix} \le \\ \ge \\ = \end{bmatrix} \tilde{B} \right], \tilde{x} \ge 0 \right\}.$$

where the subscript L is used to show the linearisation of the objective function (1) obtained in step 2, and the constraints are the same as in (2) and (3). Solve this LPP and find a solution the $\ddot{\tilde{x}} = (\ddot{\tilde{x}}_1, \ddot{\tilde{x}}_2, ..., \ddot{\tilde{x}}_n)$.

Step 4 Expand the goal function (1) to the Taylor series at the obtained solution $\ddot{\tilde{x}} = (\ddot{\tilde{x}}_1, \ddot{\tilde{x}}_2, ..., \ddot{\tilde{x}}_n)$.

Step 5 Solve the reconstructed LPP consisting of the linearised objective function at $\ddot{\tilde{x}}$ obtained in step 4 and subject to (2) and (3), and find another solution $\dddot{\tilde{x}} = (\dddot{\tilde{x}}_1, \dddot{\tilde{x}}_2, ..., \dddot{\tilde{x}}_n)$.

Step 6 Check two successive solutions. If the solution $\ddot{\tilde{x}}$ and $\dddot{\tilde{x}}$ overlap, then it is an optimum resolution of the QFPP (1), and stop. Else, assign $\dddot{\tilde{x}}$ to $\ddot{\tilde{x}}$ and go to step 4.

6 Numerical examples

Example 6.1: We consider the following FBILLFPP with FIC.

$$Max.\tilde{Z}_{x_1} = \frac{[0,1]\tilde{x}_1 + [1,2]\tilde{x}_2 + [1,2]}{[1,1]\tilde{x}_1 + [-1,1]\tilde{x}_2 + [1,1]}$$
$$Max.\tilde{Z}_{x_2} = \frac{[1,2]\tilde{x}_1 + [-2,1]\tilde{x}_2 + [-2,-1]}{[1,1]\tilde{x}_1 + [-4,2]\tilde{x}_2 + [2,3]}$$
$$\text{subject to:}$$
$$[-2,-1]\tilde{x}_1 + [1,2]\tilde{x}_2 \le [2,3],$$
$$[1,2]\tilde{x}_1 + [-4,-3]\tilde{x}_2 \le [3,3],$$
$$[-1,1]\tilde{x}_1 + [1,1]\tilde{x}_2 \ge [1,3],$$
$$\tilde{x}_1, \tilde{x}_2 \ge 0.$$

Solution 6.1: Convert FBILLFPP with FI to FBILLFPP after solution a novel technique by using modified simplex technique.

$$
\begin{aligned}
&Max.\tilde{Z}_{x_1} = \frac{\tilde{x}_1 + 2\tilde{x}_2 + 2}{\tilde{x}_1 + \tilde{x}_2 + 1} \\
&Max.\tilde{Z}_{x_2} = \frac{2\tilde{x}_1 + \tilde{x}_2 - 1}{\tilde{x}_1 + 2\tilde{x}_2 + 3} \\
&\text{subject to:} \\
&\qquad -\tilde{x}_1 + 2\tilde{x}_2 \le 3, \\
&\qquad 2\tilde{x}_1 - 3\tilde{x}_2 \le 3, \\
&\qquad \tilde{x}_1 + \tilde{x}_2 \ge 3, \\
&\qquad \tilde{x}_1, \tilde{x}_2 \ge 0.
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
&Max.\tilde{Z}_{x} = \frac{3\tilde{x}_1^2 + 5\tilde{x}_2^2 + 7\tilde{x}_1\tilde{x}_2 + 6\tilde{x}_1 + 10\tilde{x}_2 + 5}{\tilde{x}_1^2 + 2\tilde{x}_2^2 + 3\tilde{x}_1\tilde{x}_2 + 4\tilde{x}_1 + 5\tilde{x}_2 + 3} \\
&\text{subject to:} \\
&\qquad -\tilde{x}_1 + 2\tilde{x}_2 \le 3, \\
&\qquad 2\tilde{x}_1 - 3\tilde{x}_2 \le 3, \\
&\qquad \tilde{x}_1 + \tilde{x}_2 \ge 3, \\
&\qquad \tilde{x}_1, \tilde{x}_2 \ge 0.
\end{aligned} \tag{5}
$$

Steps 1–2 Choose an initial arbitrary feasible non-zero point $\dot{\tilde{x}} = (\dot{\tilde{x}}_1, \dot{\tilde{x}}_2) = (1, 1)$, and linearise the objective (5) by expanding it to the Taylor series at chosen point.

$$
\frac{\partial \tilde{Z}_x}{\partial \tilde{x}_1} = \frac{\begin{pmatrix} \left(\tilde{x}_1^2 + 2\tilde{x}_2^2 + 3\tilde{x}_1\tilde{x}_2 + 4\tilde{x}_1 + 5\tilde{x}_2 + 3\right)\left(6\tilde{x}_1 + 7\tilde{x}_2 + 6\right) \\ -\left(3\tilde{x}_1^2 + 5\tilde{x}_2^2 + 7\tilde{x}_1\tilde{x}_2 + 6\tilde{x}_1 + 10\tilde{x}_2 + 5\right)\left(2\tilde{x}_1 + 3\tilde{x}_2 + 4\right) \end{pmatrix}}{\left(\tilde{x}_1^2 + 2\tilde{x}_2^2 + 3\tilde{x}_1\tilde{x}_2 + 4\tilde{x}_1 + 5\tilde{x}_2 + 3\right)^2}
$$

$$
\frac{\partial \tilde{Z}_x}{\partial \tilde{x}_2} = \frac{\begin{pmatrix} \left(\tilde{x}_1^2 + 2\tilde{x}_2^2 + 3\tilde{x}_1\tilde{x}_2 + 4\tilde{x}_1 + 5\tilde{x}_2 + 3\right)\left(10\tilde{x}_2 + 7\tilde{x}_1 + 10\right) \\ -\left(3\tilde{x}_1^2 + 5\tilde{x}_2^2 + 7\tilde{x}_1\tilde{x}_2 + 6\tilde{x}_1 + 10\tilde{x}_2 + 5\right)\left(4\tilde{x}_2 + 3\tilde{x}_1 + 5\right) \end{pmatrix}}{\left(\tilde{x}_1^2 + 2\tilde{x}_2^2 + 3\tilde{x}_1\tilde{x}_2 + 4\tilde{x}_1 + 5\tilde{x}_2 + 3\right)^2}
$$

Step 3 Construct a LPP as follows:

$$
\begin{aligned}
&Max.\tilde{Z}_x = 0.0556\tilde{x}_1 + 0.1667\tilde{x}_2 \\
&\text{subject to:} \\
&\qquad -\tilde{x}_1 + 2\tilde{x}_2 \le 3, \\
&\qquad 2\tilde{x}_1 - 3\tilde{x}_2 \le 3, \\
&\qquad \tilde{x}_1 + \tilde{x}_2 \ge 3, \\
&\qquad \tilde{x}_1, \tilde{x}_2 \ge 0.
\end{aligned} \tag{6}
$$

The optimum solution of LPP (6) is $\ddot{\tilde{x}}_1 = 15$ and $\ddot{\tilde{x}}_2 = 9$.

Steps 4–5 Expand the goal function (5) to the Taylor series at $\ddot{\tilde{x}}_1 = 15$ and $\ddot{\tilde{x}}_2 = 9$, and the form the LPP as follows:

$$
\begin{aligned}
&Max.\tilde{Z}_x = 0.0102\tilde{x}_1 - 0.0069\tilde{x}_2 \\
&\text{subject to:} \\
&\quad -\tilde{x}_1 + 2\tilde{x}_2 \le 3, \\
&\quad 2\tilde{x}_1 - 3\tilde{x}_2 \le 3, \\
&\quad \tilde{x}_1 + \tilde{x}_2 \ge 3, \\
&\quad \tilde{x}_1, \tilde{x}_2 \ge 0.
\end{aligned}
\tag{7}
$$

The optimum solution of (7) $\dddot{\tilde{x}}_1 = 15$ and $\dddot{\tilde{x}}_2 = 9$ and $\tilde{Z}_x^* = 2.4556.$

Step 6 Because these two successive resolutions $\dddot{\tilde{x}}$ and $\ddot{\tilde{x}}$, overlap, the optimal resolution of (5) found to be is $\tilde{x}_1^* = 15$ and $\tilde{x}_2^* = 9$ and the optimal value is $\tilde{Z}_x^* = 2.4556.$ All solutions of the problem are obtained by using the Python. A novel technique can be applied to the problem solved (Jayalakshmi, 2015) and Wolf technique (Syaripuddin et al., 2017) respectively, and it see that the solutions are identical. Compare of the solutions obtained it presented in Table 1.

Table 1 Compare of the solutions obtained for example 6.1

Techniques	*Example 6.1*	*Number of iterations*
Jayalakshim technique	Non-solution	Non-iteration
Wolf technique	$(\tilde{x}_1, \tilde{x}_2) =$ (2.4,0.6), $\tilde{Z}_x^* =$ 2.0667	10-iterations
A novel technique	$(\tilde{x}_1, \tilde{x}_2) =$ (15, 9), $\tilde{Z}_x^* =$ 2.4556	8-iterations

Example 6.2: We consider the following FBILLFPP with FIC.

$$
Max.\tilde{Z}_{x_1} = \frac{[(-1, 2), (2, 2)]\tilde{x}_1 + [(0, 3), (3, 3)]\tilde{x}_2 + [(-1, 0), (0, 0)]}{[(-1, 1), (1, 1)]\tilde{x}_1 + [(-1, 4), (4, 4)]\tilde{x}_2 + [(1, 6), (6, 6)]}
$$

$$
Max.\tilde{Z}_{x_2} = \frac{[(1, 3), (3, 3)]\tilde{x}_1 + [(0, 4), (4, 4)]\tilde{x}_2 + [(-1, 0), (0, 0)]}{[(2, 6), (6, 6)]\tilde{x}_1 + [(2, 4), (4, 4)]\tilde{x}_2 + [(-1, 3), (3, 3)]}
$$

subject to:

$$
\begin{aligned}
&[(-4, 1), (1, 1)]\tilde{x}_1 + [(-3, 1), (1, 1)]\tilde{x}_2 \le [(-1, 5), (5, 5)], \\
&[(-1, 3), (3, 3)]\tilde{x}_1 + [(-3, 1), (1, 1)]\tilde{x}_2 \le [(2, 10), (10, 10)], \\
&[(-1, 2), (2, 2)]\tilde{x}_1 + [(-1, 1), (1, 1)]\tilde{x}_2 \le [(1, 7), (7, 7)], \\
&\big[(-1, 1), (1, 1)\big]\tilde{x}_1 \le \big[(1, 3)(3, 3)\big], \\
&\tilde{x}_1, \tilde{x}_2 \ge 0.
\end{aligned}
$$

Solution 6.2: Convert FBILLFPP with FI to FBILLFPP after solution a novel technique by using modified simplex technique.

$$Max.\tilde{Z}_{x_1} = \frac{[2, 2]\tilde{x}_1 + [3, 3]\tilde{x}_2 + [0, 0]}{[1, 1]\tilde{x}_1 + [4, 4]\tilde{x}_2 + [6, 6]}$$

$$Max.\tilde{Z}_{x_2} = \frac{[3, 3]\tilde{x}_1 + [4, 4]\tilde{x}_2 + [0, 0]}{[6, 6]\tilde{x}_1 + [4, 4]\tilde{x}_2 + [3, 3]}$$

subject to:

$$[1, 1]\tilde{x}_1 + [1, 1]\tilde{x}_2 \le [5, 5],$$
$$[3, 3]\tilde{x}_1 + [1, 1]\tilde{x}_2 \le [10, 10],$$
$$[2, 2]\tilde{x}_1 + [1, 1]\tilde{x}_2 \le [7, 7],$$
$$[1, 1]\tilde{x}_1 \le [3, 3],$$
$$\tilde{x}_1, \tilde{x}_2 \ge 0.$$

$$Max.\tilde{Z}_{x_1} = \frac{2\tilde{x}_1 + 3\tilde{x}_2}{\tilde{x}_1 + 4\tilde{x}_2 + 6}$$

$$Max.\tilde{Z}_{x_2} = \frac{3\tilde{x}_1 + 4\tilde{x}_2}{6\tilde{x}_1 + 4\tilde{x}_2 + 3}$$

subject to: (8)

$$\tilde{x}_1 + \tilde{x}_2 \le 5,$$
$$3\tilde{x}_1 + \tilde{x}_2 \le 10,$$
$$2\tilde{x}_1 + \tilde{x}_2 \le 7,$$
$$\tilde{x}_1 \le 3,$$
$$\tilde{x}_1, \tilde{x}_2 \ge 0.$$

$$Max.\tilde{Z}_x = \frac{15\tilde{x}_1^2 + 28\tilde{x}_2^2 + 42\tilde{x}_1\tilde{x}_2 + 24\tilde{x}_1 + 33\tilde{x}_2}{6\tilde{x}_1^2 + 16\tilde{x}_2^2 + 28\tilde{x}_1\tilde{x}_2 + 39\tilde{x}_1 + 36\tilde{x}_2 + 18}$$

subject to:

$$\tilde{x}_1 + \tilde{x}_2 \le 5,$$
$$3\tilde{x}_1 + \tilde{x}_2 \le 10, \qquad (9)$$
$$2\tilde{x}_1 + \tilde{x}_2 \le 7,$$
$$\tilde{x}_1 \le 3,$$
$$\tilde{x}_1, \tilde{x}_2 \ge 0.$$

Steps 1–2 Choose an initial arbitrary feasible non-zero point $\dot{\tilde{x}} = (\dot{\tilde{x}}_1, \dot{\tilde{x}}_2) = (1, 1),$ and linearise the objective (9) by expanding it to the Taylor series at chosen point.

$$\frac{\partial \tilde{Z}_x}{\partial \tilde{x}_1} = \frac{\left(\begin{array}{c} \left(6\tilde{x}_1^2 + 16\tilde{x}_2^2 + 28\tilde{x}_1\tilde{x}_2 + 39\tilde{x}_1 + 36\tilde{x}_2 + 18\right)\left(30\tilde{x}_1 + 42\tilde{x}_2 + 24\right) \\ -\left(15\tilde{x}_1^2 + 28\tilde{x}_2^2 + 42\tilde{x}_1\tilde{x}_2 + 24\tilde{x}_1 + 33\tilde{x}_2\right)\left(12\tilde{x}_1 + 28\tilde{x}_2 + 39\right) \end{array} \right)}{\left(6\tilde{x}_1^2 + 16\tilde{x}_2^2 + 28\tilde{x}_1\tilde{x}_2 + 39\tilde{x}_1 + 36\tilde{x}_2 + 18\right)^2}$$

$$\frac{\partial \tilde{Z}_x}{\partial \tilde{x}_2} = \frac{\begin{pmatrix} \left(6\tilde{x}_1^2 + 16\tilde{x}_2^2 + 28\tilde{x}_1\tilde{x}_2 + 39\tilde{x}_1 + 36\tilde{x}_2 + 18\right)\left(56\tilde{x}_2 + 42\tilde{x}_1 + 33\right) \\ -\left(15\tilde{x}_1^2 + 28\tilde{x}_2^2 + 42\tilde{x}_1\tilde{x}_2 + 24\tilde{x}_1 + 33\tilde{x}_2\right)\left(32\tilde{x}_2 + 28\tilde{x}_1 + 36\right) \end{pmatrix}}{\left(6\tilde{x}_1^2 + 16\tilde{x}_2^2 + 28\tilde{x}_1\tilde{x}_2 + 39\tilde{x}_1 + 36\tilde{x}_2 + 18\right)^2}$$

Step 3 Construct a LPP as follows:

$$Max.\tilde{Z}_x = 0.1227\tilde{x}_1 + 0.2494\tilde{x}_2$$

subject to:

$$\begin{aligned} &\tilde{x}_1 + \tilde{x}_2 \le 5, \\ &3\tilde{x}_1 + \tilde{x}_2 \le 10, \\ &2\tilde{x}_1 + \tilde{x}_2 \le 7, \\ &\tilde{x}_1 \le 3, \\ &\tilde{x}_1, \tilde{x}_2 \ge 0. \end{aligned} \tag{10}$$

The optimum solution of LPP (10) is $\ddot{\tilde{x}}_1 = 0$ and $\ddot{\tilde{x}}_2 = 5$.

Steps 4–5 Expand the goal function (9) to the Taylor series at $\ddot{\tilde{x}}_1 = 0$ and $\ddot{\tilde{x}}_2 = 5$ and the form the LPP as follows:

$$Max.\tilde{Z}_x = -0.0417\tilde{x}_1 + 0.0493\tilde{x}_2$$

subject to:

$$\begin{aligned} &\tilde{x}_1 + \tilde{x}_2 \le 5, \\ &3\tilde{x}_1 + \tilde{x}_2 \le 10, \\ &2\tilde{x}_1 + \tilde{x}_2 \le 7, \\ &\tilde{x}_1 \le 3, \\ &\tilde{x}_1, \tilde{x}_2 \ge 0. \end{aligned} \tag{11}$$

The optimum solution of (11) is $\dddot{\tilde{x}}_1 = 0$ and $\dddot{\tilde{x}}_2 = 5$ and $\tilde{Z}_x^* = 1.446$.

Step 6 Because these two successive resolutions $\dddot{\tilde{x}}$ and $\ddot{\tilde{x}}$, overlap, the optimal resolution of (9) found to be is $\tilde{x}_1^* = 0$ and $\tilde{x}_2^* = 5$ and the optimal value is $\tilde{Z}_x^* = 1.446$. All solutions of the problem are obtained by using the Python. A novel technique can be applied to the problem solved (Jayalakshmi, 2015) and Wolf technique (Syaripuddin et al., 2017) respectively, and it see that the solutions are identical. Compare of the solutions obtained it presented in Table 2.

Table 2 Compare of the solutions obtained for example 6.2

Techniques	*Example 6.2*	*Number of iterations*
Jayalakshim technique	Non-solution	Non-iteration
Wolf technique	$(\tilde{x}_1, \tilde{x}_2) = (0, 0),\ \tilde{Z}_x^* = 0$	6-iterations
A novel technique	$(\tilde{x}_1, \tilde{x}_2) = (0, 5),\ \tilde{Z}_x^* = 1.446$	4-iterations

7 Conclusions

In this paper, a brand-new technique is presented, based on solve LPP to minimise the total execution time, that really is useful for iteration operations. This approach can handle QFPP having an optimisation model it is either factorised or non-factorised, homogenous or non-homogenous constraints in problem. A novel approach can easily be applied to resolve any type of QFPP. The QFPP is convert into a LPP with a chosen initial point. The proposed approach differs from other existing methods in terms of computational steps. It is also easier than other methods that can be resolved algebraically. With the proposed approach, the finally resolution can be acquired rapidly, so it would be favourable for computer use. This approach can be implemented for various types of nonlinear FPP containing nonlinear constraints that are more complex. As we see in Tables 1 and 2, the proposed method is more accurate than Wolf technique and also it need a few iterations to complete the optimum solution than other method.

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List of abbreviations

FI	Fuzzy interval
IV	Interval value
IC	Interval coefficients
FIC	Fuzzy interval coefficients
L	Linear
LP	Linear programming
MPP	Mathematical programming problems
LPP	Linear programming problems
NLFPP	Nonlinear fractional programming problems
LFPP	Linear fractional programming problems
QFPP	Quadratic fractional programming problems
LFP	Linear fractional programming
FBILLFPP	Fuzzy bi-level linear fractional programming problems
BILLFPP	Bi-level linear fractional programming problems
BILLPP	Bi-level linear programming problems
$Max.\tilde{Z}_x$	Maximise $\tilde{Z}_x$
$Min.\tilde{Z}_x$	Minimise $\tilde{Z}_x$
FFLFPP	Fuzzy function linear fractional programming problems
FFLPP	Fuzzy function linear programming problems
LF	Linear fractional
FFL	Fuzzy function linear
FGA	Fuzzy goal approach
MFLP	Multi fuzzy linear programming
IVLFP	Interval value linear fractional programming

IVLFPP	Interval value linear fractional programming problems
BIL	Bi-level
BILQPP	Bi-level quadratic programming problems
BILQFPP	Bi-level quadratic fractional programming problems
FGP	Fuzzy goal programming
GA	Genetic algorithm
MOLPP	Multi-objective linear programming problems
MLMOFPP	Multi-level multi-objective fractional programming problems
FMOLPP	Fuzzy multi-objective linear programming problems
RILFPP	Rough interval linear fractional programming problems
FRLFPP	Fuzzy rough linear fractional programming problems
FRILFPP	Fuzzy rough interval linear fractional programming problems
BILNLFPP	Bi-level nonlinear fractional programming problems.