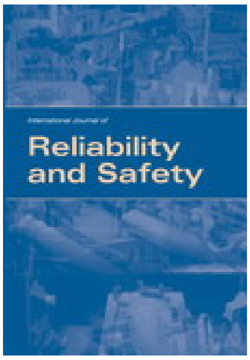

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Abstract: Joint decision-making for preventive maintenance and spare parts inventory in multi-component systems is crucial for industrial applications, especially as many expensive, complex equipments can be repaired and reused. This study investigates this joint decision-making using a discrete multi-state degradation model, focusing on the unique characteristics of repairable systems, including their structure and maintenance strategies. First, the operational interactions among production, maintenance, and inventory are analysed to derive state transition probability models for degradation and ordering processes. Subsequently, a sequential decision model is developed to minimise the total system cost, identifying preventive maintenance thresholds, inspection periods, and order batch sequences. To address the problem, a combination of global dynamic programming and genetic algorithms is employed. Numerical experiments with wind turbine spindles validate the decision model, demonstrating its effectiveness in addressing maintenance and inventory optimisation in repairable multi-unit systems while ensuring an optimal dynamic combination of decision variables.

Keywords: multi-unit repairable system; discrete multi-state degradation; imperfect maintenance; spare parts inventory; sequential decision; joint optimisation.

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1 Introduction

With the advancement of social technology, industrial systems are undergoing continuous upgrades and transformations, leading to increasingly complex functional structures. To minimise expected downtime and total costs, it is crucial to develop scientific and rational maintenance strategies for these integrated industrial systems (Zheng et al., 2021). Industrial systems are typically composed of multiple interconnected sub-systems or components, and there exist various correlations among these components, such as economic, structural and stochastic correlations. Consequently, research focusing on decision-making for preventive maintenance of multi-component systems has received significant attention (Rasmekomen and Parlikad, 2016; Poppe et al., 2018).

The execution of maintenance activities directly creates a demand for maintenance spare parts. The sufficiency of spare parts has a direct impact on the operation of maintenance activities. Therefore, the joint decision-making of preventive maintenance and spare parts inventory has become a hot research topic in the field of industrial application. For instance, Zhang and Zeng (2017) and Zhang et al. (2021) examined the joint decision-making of Condition-Based Maintenance (CBM) and spare parts management for multi-component systems by developing a deterioration state-space partitioning model. Zhang et al. (2022) investigated the joint decision-making of CBM and spare parts inventory in series-parallel systems, considering both soft and hard failure modes. Li et al. (2022) explored a joint optimisation model for preventive maintenance and spare parts inventory in multi-component systems, accounting for the impact of stochastic shocks. Wakiru et al. (2021) conducted a study on the joint optimisation problem of remanufacturing, maintenance, and spare parts inventory policies for multi-component systems. The effectiveness of the proposed model was demonstrated using a turbocharger as an example. Mjirda et al. (2016) introduced a joint optimisation model addressing both periodic preventive maintenance and spare parts inventory problems.

Although the aforementioned studies have mainly focused on the joint decision-making of maintenance and spare parts for multi-component systems, it is important to note that most of them assume that the components in the system are non-repairable (Lin et al., 2020). Whether it is maintenance after a system failure or preventive maintenance before a failure, components are typically replaced and restored to a state as good as new. However, much industrial equipment is expensive and involves complex manufacturing processes. As a result, numerous critical or expendable components can be reused after maintenance to reduce overall system operating costs. Examples of such components include rechargeable lithium battery systems, wind turbine spindles that can be reused after on-site brush plating, and rolls in steel rolling systems that can be ground or welded for reuse.

In recent years, scholars have conducted research on the maintenance decision-making of repairable multi-component systems. For example, Ait Mokhtar et al. (2023) proposed an original approach for modelling the maintenance efficiency of multi-component systems. Zhang et al. (2024) proposed a hybrid condition-based opportunistic maintenance strategy for Multi-Component Repairable Systems (MCRS) with economic dependence. They also modelled the maintenance within a finite time horizon. Fu and Wang (2022) investigated a novel periodic maintenance policy for multi-component systems. This policy involved joint implementation of component reallocations and system overhauls at fixed intervals. Additionally, components were minimally repaired immediately after failures throughout their life cycle. Furthermore, scholars have actively conducted research on the joint decision-making of maintenance and spare parts inventory for repairable multi-component systems in recent years. De Pater and Mitici (2021) conducted a joint optimisation study on predictive maintenance strategies for repairable components, which integrates Time-Based Maintenance (TBM), CBM and inventory strategies. Chemweno et al. (2015) presented a discrete event simulation modelling study that evaluated the impact of various factors on unreliable repairable systems. Guo et al. (2019) focused on modelling spare parts allocation for an isolated system consisting of multiple types of components, while considering backorder dependence.

The decision-making for maintenance and spare parts inventory in repairable multi-component systems differs significantly from that of irreparable systems. In terms of maintenance strategy, repairable components can adopt a repair by replacement maintenance strategy, which ensures continuous production and allows for the simultaneous repair of components. This requires an offline spare parts inventory sub-system and a maintenance sub-system to support it, inevitably increasing the structural complexity of the system. It is necessary to consider the interrelationships between the production, maintenance, and inventory sub-systems, as well as the imperfect characteristics of the components after repair, to develop a combined strategy for maintenance and spare parts inventory. Regarding the modelling of system state transitions, the probability of failures in the production sub-system directly determines the maintenance requirements of the maintenance sub-system and the ordering requirements for spare parts in the inventory sub-system. Simultaneously, the maintenance capability of the maintenance sub-system also impacts the replenishment capability of spare parts in the inventory sub-system, thereby influencing the availability of spare parts during the maintenance process. Discarded irreparable parts make it necessary to supplement the system with new spare parts to meet maintenance requirements. In this case, both new and old components coexist in the inventory sub-system. Using different types of spare parts during the repair process leads to the restoration of the production system to different states. Therefore, it is necessary to integrate the analysis of interrelationships between various sub-systems to model the state transitions of the production, maintenance and inventory sub-systems. In terms of maintenance effectiveness, factors such as maintenance technology and resources have a direct impact on maintenance services compared to direct replacement. The repaired components often cannot be restored to a brand-new state after repair, but instead fall into an imperfect state between the brand-new state and the state before repair. Some scholars have studied maintenance decision-making under imperfect maintenance conditions (Dong et al., 2020; Mercier and Castro, 2019; Yan et al., 2020). This leads to the presence of multiple deteriorating components in the inventory sub-system, which in turn indirectly affects the ongoing deterioration state of the production system and generates new maintenance requirements. In terms of decision modelling approaches, it would be more realistic to integrate the stochastic dynamic characteristics of degradation, maintenance and sequencing throughout the entire lifecycle of the system with its dynamic operation.

The fundamental reason for maintenance requirements is the inevitable degradation that occurs during the operation of the system. Therefore, modelling the system's degradation serves as the basis for making maintenance-related decisions. The degradation process of the system is typically a continuous stochastic process. However, in practical industrial systems, degradation does not significantly impact the operation of the system until the degradation state reaches a specific threshold. Dividing the system's state into regions allows for an accurate reflection of its performance, as slight changes in certain state values may not be significant. On one hand, discrete multi-state modelling can effectively represent degradation patterns. On the other hand, it reduces the burden of collecting and processing continuous state data. Markov models (Wang and Chen, 2023) and their derivative models (Gámiz et al., 2023) are effective modelling methods for multi-state systems. Tyagi and Ram (2023) and Ram and Manglik (2016), respectively modelled the states and biological characteristics of hybrid power generation systems as Markov models to evaluate the availability and reliability of the systems.

This study focuses on the off-line maintenance characteristics of repairable multi-component systems. Under imperfect maintenance conditions, the mutual influence of degradation, maintenance and spare parts inventory is considered. A joint sequential strategy (p, T, q) is developed based on the condition-based maintenance strategy (T, p) with non-periodic inspections combined with the (T, q) inventory strategy. First, the degradation process of components in the production sub-system is modelled using a multi-state degradation model based on Markov processes. The main contributions of this paper are as follows:

1) A joint strategy of condition-based repair by replacement maintenance and spare parts inventory management is proposed to ensure the continuity of production in repairable multi-component systems by utilising non-periodic inspections and unified maintenance thresholds.

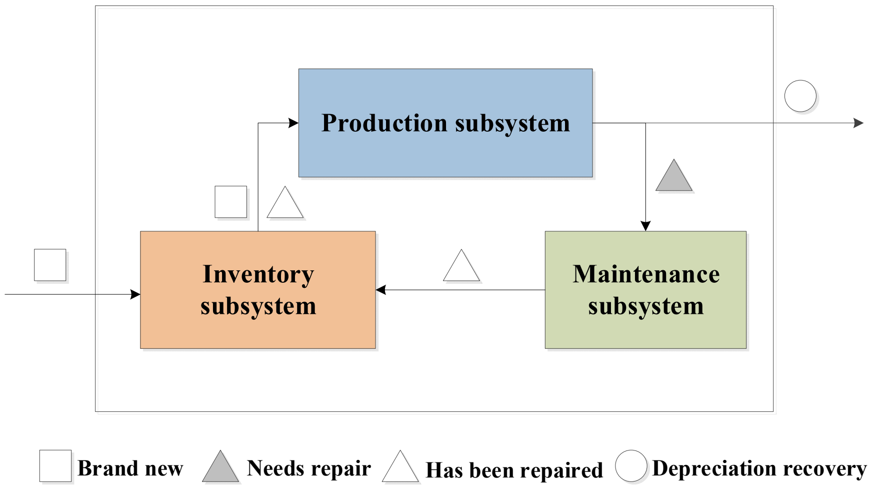
2) The proposed strategy relies on the support of offline spare parts inventory and maintenance sub-systems. Consequently, considering the interaction between the production, maintenance, and ordering sub-systems, the state transition probability models for degradation, replacement, ordering and repair processes in repairable systems were derived.

3) A sequential decision model was constructed with the objective of minimising the total cost of the system within a finite time horizon and the optimal solution was determined using a combination of dynamic programming and genetic algorithm.

2 System description

In systems with high production continuity requirements, it is necessary to first replace defective and degraded components before performing maintenance on the replaced components. The ability to perform replacements directly influences the subsequent maintenance arrangements. In this study, the industrial system is divided into three sub-systems: the production sub-system, the maintenance sub-system and the inventory sub-system, which are used to describe the degradation and replacement of the entire system, maintenance of defective components, ordering and storage of spare parts. The interrelationships between these sub-systems are illustrated in Figure 1.

Figure 1 Relationship diagram of three sub-systems within the system (see online version for colours)



During the operation of the system, components in the production sub-system gradually degrade as production progresses. When they reach a certain level of degradation and can no longer meet production requirements, they need to be replaced with components from the inventory sub-system. Depending on the level of deterioration, some components are sent to the maintenance sub-system for repairs, while others are deemed depreciated and recycled. Meanwhile, due to the existence of depreciation and recycling of components, the inventory sub-system orders a certain quantity of replenishment components as needed. The repaired and ordered components are both stored in the inventory sub-system.

2.1 Production sub-system and degradation process model

The production sub-system consists of M identical and independent components undergoing degradation processes. Each component gradually deteriorates in performance during the production process and needs to be replaced when it reaches a certain level of degradation. The degradation process can be modelled as a continuous-time discrete-state Markov process, where the state space of the components is defined as $Z = \{1, 2, ..., z\}$. The transition characteristics between states are illustrated in Figure 2. Let $x_i(t)$ represent the degradation state of any component i at time t. The degradation process exhibits the following characteristics:

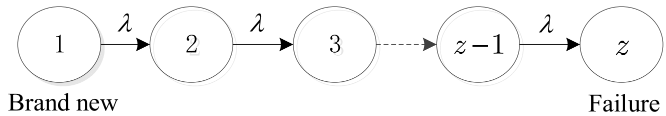
1) Degradation state $x_i(t) \in Z$;

2) When $t = 0$, the component state is new, $x_i(0)=1$;

3) When $x_i(t) = z$, the component has failed;

4) The duration in any state $x_i(t)$ $x_i(t) \neq z$ follows an exponential distribution with parameter λ.

Figure 2 Schematic diagram of degradation state transition of components (see online version for colours)



2.2 Modelling of maintenance sub-system and maintenance capability

The maintenance sub-system is responsible for repairing defective components that have been replaced from the production sub-system and subsequently delivering the repaired components to the inventory sub-system. It is composed of *N* identical repair stations, each capable of independently performing maintenance activities. Only one component can be repaired at a time at each repair station, and the maintenance activities for any two components are carried out simultaneously. The maintenance activities at different repair stations are independent and do not affect each other. The maintenance capability of a maintenance system can be assessed by two measurement metrics: repair rate and repair effectiveness. The duration of the maintenance activity is typically influenced by factors such as the degradation state of the components to be repaired and the maintenance techniques employed. Consequently, the duration of a single component's maintenance activity is assumed to follow an exponential distribution with a parameter μ_{m}, where its probability density function is given by

$$f_{T_m}\left(t_m\right)=\begin{cases}\mu_m e^{-\mu_m t_m} & ,t_m \geq 0\\ 0 & ,t_m<0\end{cases} \tag{1}$$

Maintenance effectiveness refers to the degree of component recovery during the maintenance process. For multi-state systems, the number of state transitions *t* before and after repair can be used to characterise repair effectiveness. It is modelled as a Poisson distribution with an intensity parameter λ_1, and the probability of the component transitioning from state u to state v can be represented as:

$$p_{u,v}=\frac{\lambda_1^{u-v}}{(u-v)!}e^{-\lambda_1},\left(1<v\leq u\leq z\right) \tag{2}$$

The probability of the component being restored to a state as good as new is:

$$p_{u,1}=1-\sum_{w=2}^{u}\frac{\lambda_1^{u-w}}{(u-w)!}e^{-\lambda_1} \tag{3}$$

2.3 Modelling of inventory sub-system and lead time for ordering

The inventory sub-system is responsible for storing ordered and repaired components and supplying available components to the production sub-system. In this paper, the inventory level is not restricted, and the lead time τ for component ordering is modelled by a truncated normal distribution with a probability density function defined by equation (4).

$$f_\tau(t) = \begin{cases} \dfrac{e^{-\frac{(t-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma\left(1-\phi(0)\right)}, t \geq 0 \\ 0 \qquad\qquad\qquad\quad , t < 0 \end{cases} \tag{4}$$

where $\phi(\cdot)$ represents a normal distribution function with parameter μ, σ^2

3 Joint strategy of maintenance and spare parts inventory

For the described system, a joint sequential strategy $\left(p, T_k, q_k\right)$ is developed based on the repair by replacement condition-based maintenance strategy (T_k, p) with non-periodical inspection intervals and a unified maintenance threshold, and circular inventory strategy (T_k, q_k). In this strategy, the lengths of the $k\left(k = 0,1,...\right)$ inspection cycle and the order quantity are denoted as T_k and q_k, respectively, while p represents the unified preventive maintenance threshold for all cycles. The sequence combination $\left(p, T, q\right)$ consists of strategies for any K periods, while $T = \left\{T_1, T_2, ..., T_K\right\}$, $q = \left\{q_1, q_2, ..., q_K\right\}$. The specific strategy is as follows:

1) *Aperiodic inspections*: Inspect the three sub-systems based on an interval length sequence $\left\{T_1, T_2, ..., T_k, ...\right\}$. At *k*-th $\left(k = 0,1,...\right)$ inspection point. The degradation status of components in the production, maintenance and inventory sub-systems can be detected, and the state vectors can be respectively recorded as $x_k = \left(X_1^k, X_2^k, ..., X_z^k\right)$, $y_k = \left(Y_1^k, Y_2^k, ..., Y_z^k\right)$, $\mathbf{s}_k = \left(S_1^k, S_2^k, ..., S_z^k\right)$, where X_u^k, Y_u^k, S_u^k $\left(u = 1, ..., z\right)$ represents the number of components in state u in each sub-system at the *k*-th inspection point. The inspection time is negligible, and the total cost for a single detection is C_{ins}. Define the operation $a_{u_1:u_2}^k = \sum_{u=u_1}^{u_2} a_u^k, a = \left\{a_1^k, a_2^k, ..., a_z^k\right\}$ as the total number of components in the vector a that transition from state u_1 to state u_2, while $u_1, u_2 \in Z$, $\mathbf{a}$ can represent the state vectors of production, maintenance or inventory sub-systems.
2) *Component replacement strategy*: At *k*-th inspection point, maintenance needs are determined based on the degradation status x_i^k of components i in the production sub-system,

   a) If $x_i^k = z$, the component failed, the depreciated recovery is carried out with a probability of ξ, and the return of depreciation recovery per unit is C_{d}; The component transported to the maintenance sub-system for corrective maintenance with probability of $(1-\xi)$,the cost of a single repair for one component is C_{c},the post-repair state randomly transitions to a certain state between the current state and the new state, with transition probabilities defined by equations (2) and (3).

b) If $p \le x_i^k < z$, the component is defective, and is transported to the maintenance sub-system for preventive maintenance. The cost of a single repair for one component is C_p. Its effect is similar to the that of corrective maintenance, is also random and imperfect. Considering that the degree of degradation increases, the difficulty of repair also increases, it is generally recognised that $C_p < C_c$.

c) Otherwise, if the component is in normal condition, no action will be taken.

Assuming that the transportation time and cost between sub-systems for the component are negligible, the repair time of the component follows an exponential distribution.

3) *Component maintenance and ordering strategies*: According to the strategy in (2), determine the corrective maintenance, preventive maintenance needs and the number of depreciated and recovered components, after the *k*-th inspection, the following activities are to be scheduled:

a) *Replacement*: If the inventory level in the inventory sub-system is 0, or if the replacement demand in the production sub-system is 0, no replacement occurs in the system. If there is a replacement demand in the production sub-system and the inventory level in the inventory sub-system is greater than the replacement demand, all components will be replaced. Otherwise, priority will be given to replacing components with a higher degree of deterioration until all the components in the inventory sub-system are exhausted, while the remaining components are not replaced. During the replacement process, priority is given to using components in better condition from the inventory. The replacement cost includes a one-time replacement preparation cost C_{rset}, which is independent of the number of components being replaced, and a unit replacement cost C_{r}. In addition, if the defective component is not replaced in time, it will cause the performance of the production system to deteriorate, and the penalty fee for each unit of operation time of the defective part is C_{f}.

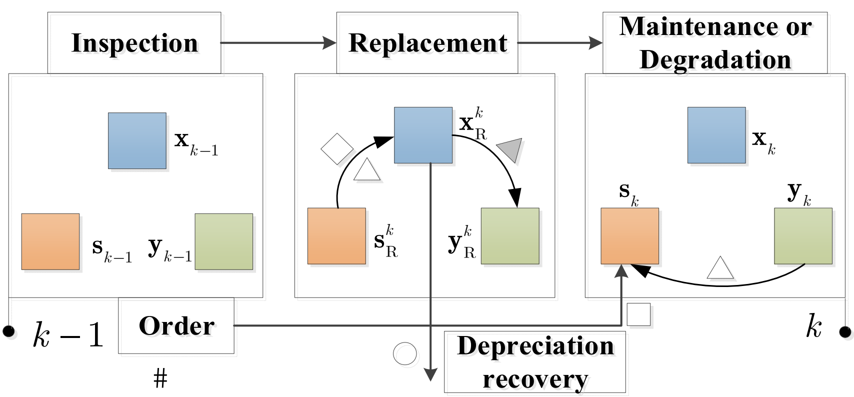
b) *Maintenance*: After the components arrive at the repair sub-system, if there is an available repair station, they will be repaired immediately. Otherwise, they will have to wait for repairs, and the cost of accumulating wait time per unit of time for each component is C_{ms}. When multiple components arrive simultaneously, priority is given to repairing the components with a lower degree of degradation.

c) *Ordering and holding*: Ordering q_k new components to replenish inventory, with a unit purchase price of C_{io}, a one-time ordering cost of C_{iset}, the holding cost per unit of inventory per unit of time for the component is C_{is}.

4 Decision modelling

This paper aims to construct a sequential decision model based on dynamic programming for imperfect maintenance and joint inventory management. The objective is to minimise the total cost of the system within a finite time horizon to optimise the detection interval sequence, order quantity sequence and preventive maintenance threshold for the system. The establishment of the decision model is based on the corresponding state transition model, where the state transition depends on the activities experienced. The entire operation process of the system can be divided into multiple operating cycles, and the state changes of the entire operation process of the system can be characterised by the state transition between adjacent inspection points. Figure 3 illustrates the activities experienced and the corresponding state changes during the *k*-th cycle of the system.

Figure 3 Transition of system activities and states during the *k*-th cycle (see online version for colours)



In the figure, the states of three sub-systems x_{k-1}, y_{k-1}, s_{k-1} are inspected based on the (*k*–1)-th inspection point. After calculating the maintenance requirements and inventory quantity according to strategy (2), the system enters an intermediate state x_R^k, y_R^k, s_R^k following the replacement and depreciation recovery based on strategy (3). The system continues to degrade while performing maintenance and ordering activities during the cycle. At the end of the cycle, the states of each sub-system are transformed into x_k, y_k, s_k, which represents the state at the *k*-th inspection point.

4.1 *State transitions during the k-th cycle*

Since inspections do not change the states of each sub-system, subsequent analysis will focus on the state transitions during replacement, maintenance, degradation and ordering activities.

4.1.1 *State transitions before and after replacement*

Replacement leads to changes of states in production, inventory and maintenance sub-system. However, the implementation of replacement is influenced by the states of the production system and inventory, x_{k-1} and s_{k-1}. The probability of no replacement and no state change occurring at $k-1$ inspection point is

$$\begin{aligned} P_{NR}^{k-1} &= P\left(x_{p:z}^{k-1}=0 \text{ or } s_{1:z}^{k-1}=0\right) \\ &= P\left(x_{p:z}^{k-1}=0\right)+P\left(s_{1:z}^{k-1}=0\right)-P\left(x_{p:z}^{k-1}=0, s_{1:z}^{k-1}=0\right) \end{aligned} \tag{5}$$

If the system undergoes replacement and state change, there are two scenarios:

1) *Scenario 1*: All components are replaced. In this case, the replacement threshold a can be determined, indicating that during the replacement process, at least all the components in the inventory sub-system that in $a-1$ state will be depleted, and at most all the components in state a will be depleted. The replacement process is illustrated in Figure 4. While the probability of depleting all components in state a

$$P_1^{k-1}(a)=P\left(0<X_{p:z}^{k-1}\leq S_{1:a}^{k-1}\right), \quad 1\leq a\leq z \tag{6}$$

The probabilities of replacement scenarios for different critical values a are as follows:

$$P_{\mathrm{R\_1}}^{k-1}(a)=\begin{cases} P_1^{k-1}(a), & a=1 \\ P_1^{k-1}(a)-P_1^{k-1}(a-1), & 1<a\leq z \end{cases} \tag{7}$$

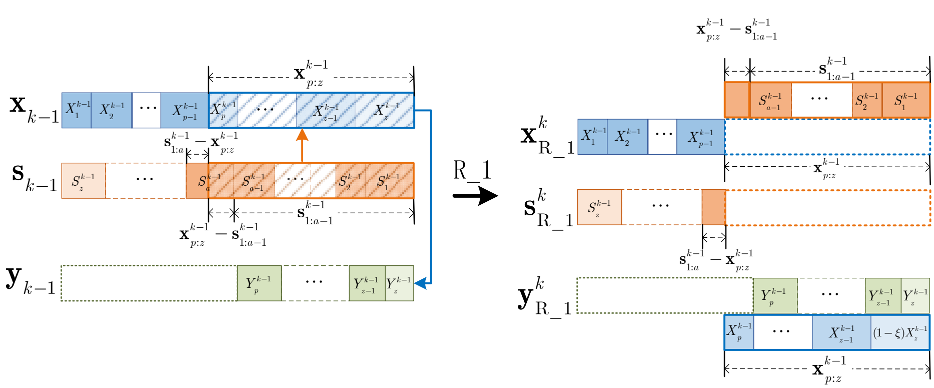
Define three matrix operation functions $before(A,w)$, $after(A,w)$ and $remain(A,w)$, which respectively indicate setting the first w columns of matrix A to zero, setting the last w columns of matrix A to zero, and setting all columns except the w column to zero. When the replacement threshold a can be determined, the state changes of the three sub-systems can be represented as follows:

$$\begin{aligned} x_{R\_1}^{k}(a) &= after\left(x_{k-1}, z-p+1\right)+after\left(s_{k-1}, z-a+2\right) \\ &\quad +\left(x_{p:z}^{k-1}-s_{1:a-1}^{k-1}\right) remain(I,a) \end{aligned} \tag{8}$$

$$s_{R\_1}^{k}(a)=before\left(s_{k-1}, a\right)+\left(s_{1:a}^{k-1}-x_{p:z}^{k-1}\right) remain(I,a) \tag{9}$$

$$y_{R\_1}^{k}(a)=y_{k-1}+before\left(x_{k-1}, p-1\right)-\xi\cdot remain\left(x_{k-1}, z\right) \tag{10}$$

Figure 4 Transition of three sub-systems states if all components are replaced (see online version for colours)



Here, and in the following text, I refers to a row vector of all-one of the corresponding dimensions.

2) *Scenario 2*: All components are not replaced. In this case, the replacement threshold b can be determined, indicating that during the replacement process, at least all the components in the inventory sub-system with b+1 state will be depleted, while at most all the components in state b will be depleted, the replacement process is illustrated in Figure 5.

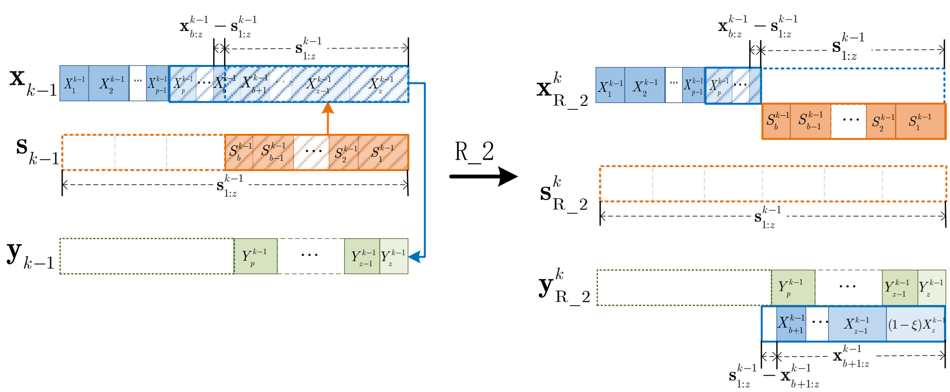
The probability of replacing at most all components of state b in the production sub-system is given by equation (11).

$$P_2^{k-1}(b) = \begin{cases} P\left(0 < s_{1:z}^{k-1} \leq x_{z:z}^{k-1}\right), b = z \\ P\left(x_{b+1:z}^{k-1} < s_{1:z}^{k-1} \leq x_{b:z}^{k-1}\right), p < b < z \\ P\left(x_{b+1:z}^{k-1} < s_{1:z}^{k-1} < x_{b:z}^{k-1}\right)), b = p \end{cases} \tag{11}$$

The probabilities of replacement scenarios for different critical values b are as follows:

$$P_{\mathrm{R\_2}}^{k-1}(b) = \begin{cases} P_2^{k-1}(b), & b = z \\ P_2^{k-1}\left(b-1\right) - P_2^{k-1}(b), & p < b \leq z \end{cases} \tag{12}$$

Figure 5 Transition of three subsystems states if all components are not replaced (see online version for colours)



When the replacement threshold b can be determined, the corresponding changes of states for the three sub-systems can be expressed using equations (13) to (15).

$$\begin{aligned} x_{R\_2}^{k}(b) &= after\left(x_{k-1}, z-b\right) + s_{k-1} \\ &\quad + \left(x_{b:z}^{k-1} - s_{1:z}^{k-1}\right) remain\left(I, b\right) \end{aligned} \tag{13}$$

$$s_{R\_2}^{k}(b) = 0 \tag{14}$$

$$\begin{aligned} y_{R\_2}^{k}(b) &= y_{k-1} + before\left(x_{k-1}, b-1\right) \\ &\quad -\xi \cdot remain\left(x_{k-1}, z\right) + \left(s_{1:z}^{k-1} - x_{b+1:z}^{k-1}\right) remain\left(I, b\right) \end{aligned} \tag{15}$$

After considering various replacement scenarios, the states of the individual sub-systems after replacement are as follows:

$$x_R^k = P_{R\_1}^{k-1}(a)\,x_{R\_1}^k(a) + P_{R\_2}^{k-1}(b)\,x_{R\_2}^k(b) + P_{NR}^{k-1}\,x_{k-1} \tag{16}$$

$$y_R^k = P_{R\_1}^{k-1}(a)\,y_{R\_1}^k(a) + P_{R\_2}^{k-1}(b)\,y_{R\_2}^k(b) + P_{NR}^{k-1}\,y_{k-1} \tag{17}$$

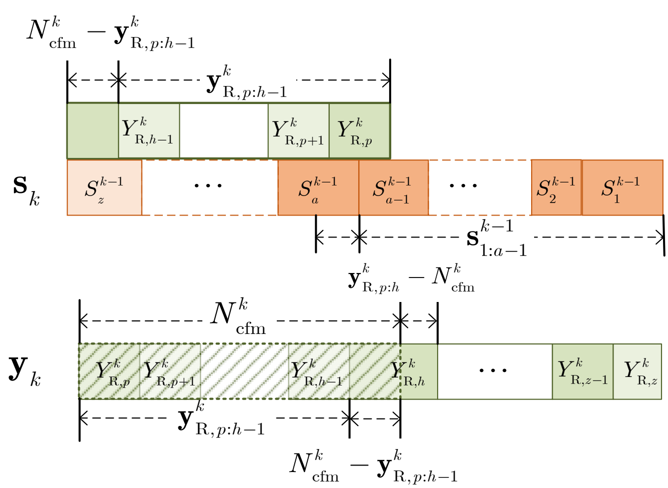
$$s_R^k = P_{R\_1}^{k-1}(a)\,s_{R\_1}^k(a) + P_{R\_2}^{k-1}(b)\,s_{R\_2}^k(b) + P_{NR}^{k-1}\,s_{k-1} \tag{18}$$

where a, b is a vector composed of all possible replacement threshold values a, b.

4.1.2 Transition of states before and after maintenance

1) Changes of state y_k in the maintenance sub-system: To determine the system state transitions, it is necessary to identify which components' maintenance activities have been completed and the post-maintenance states of these components. Here, a maintenance threshold h is set, indicating that during the maintenance process, at least the components in state $h-1$ in the system should be repaired, and at most the components in state h should be repaired. As shown in Figure 6.

Figure 6 Transition of system states before and after maintenance (see online version for colours)



Let N_{cfm}^k represent the number of components repaired during the maintenance period. The probability of at most complete repair of all components in state $p \sim h$ is given by the following equation.

$$P_3^k(h) = \begin{cases} P\left(N_{cfm}^k \le y_{R,p:p}^k\right), & h = p \\ P\left(y_{R,p:h-1}^k < N_{cfm}^k \le y_{R,p:h}^k\right), & p < h < z \\ P\left(N_{cfm}^k > y_{R,p:z}^k\right) & , h = z \end{cases} \tag{19}$$

The repair probabilities under different threshold values h are as follows:

$$P_m^k(h) = \begin{cases} P_3^k(h), & h = p \\ P_3^k(h-1) - P_3^k(h), & p < h \le z \end{cases} \tag{20}$$

After the repair of N_{cfm}^k components, the change in the degradation state in the maintenance sub-system from state $p - h$ is as follows:

$$\begin{aligned} y_{fm}^k(h) = &\ after\left(before\left(y_R^k, p-1\right), z-h+2\right) \\ &+ \left(N_{cfm}^k - y_{R,p:h-1}^k\right) remain(I, h) \end{aligned} \tag{21}$$

Considering all possibilities, the change in the degradation state in the maintenance sub-system from state $p \sim h$ after repair is as follows:

$$y_{fm}^k = y_{fm}^k(h) P_m^k(h) \tag{22}$$

where h is a vector composed of all possible replacement threshold values h.

After maintenance, the state of the maintenance sub-system at the next inspection point is as follows:

$$y_k = y_R^k - y_{fm}^k \tag{23}$$

1) *Changes of state s_k in the inventory sub-system*: The maintenance process may result in the restoration of the condition of the repaired component, according to the assumption of imperfect effectiveness described in equation (2) of Section 2, refer to preventive maintenance as approach $\lambda_1 = \lambda_p$ and corrective maintenance as approach $\lambda_1 = \lambda_c$. When the system is in a better condition, the maintenance effectiveness is also expected to be higher. Therefore, we assume that $\lambda_p < \lambda_c$, and the probability matrix for maintenance effectiveness is denoted as $P = \left\{p_{u,v}\right\}_{z \times z}$, in which

$$p_{u,v} = \begin{cases} \dfrac{\lambda_p{}^{u-v}}{(u-v)!} e^{-\lambda_p} & p \le u < z, 1 < v < u \\ 1 - \displaystyle\sum_{w=1}^{u-1} \dfrac{\lambda_p{}^{u-w}}{(u-w)!} e^{-\lambda_p} & p \le u < z, v = 1 \\ \dfrac{\lambda_c{}^{u-v}}{(u-v)!} e^{-\lambda_c} & 1 < v \le u = z \\ 1 - \displaystyle\sum_{w=1}^{u-1} \dfrac{\lambda_c{}^{u-w}}{(u-w)!} e^{-\lambda_c} & u = z, v = 1 \\ 0 & else \end{cases} \tag{24}$$

The status of the inventory sub-system is dependent on the status of the maintenance sub-system. After receiving the repaired parts, the status of the inventory sub-system changes as follows:

$$s_k = s_R^k + y_{fm}^k \cdot P \tag{25}$$

4.1.3 The state transition before and after degradation

Degradation only occurs in the production sub-system and only causes changes in the state of the production sub-system. Let matrix $D_k = \left(d_{u,v}^k\right)_{z \times z}$ represent the state transition characteristics of components during the degradation process. Let $d_{u,v}^k$ represent the probability of transitioning from state u to state v during the k-th period, then

1) The state transition count of degenerate process follows Poisson process with the same parameter λ, then

$$d_{u,v}^k = \frac{\left(\lambda T_k\right)^{u-v}}{\left(u-v\right)!} e^{-\lambda T_k} \tag{26}$$

2) The sum of the probabilities of transitioning to all states is 1, and the maximum for the degenerate state is $d_{u,z}^k$, satisfying

$$d_{u,z}^k = 1 - \sum_{v=u}^{z-1} d_{u,v}^k \tag{27}$$

3) If there is no maintenance intervention after degenerating into state z, it will remain in that state indefinitely, that $d_{z,z}^k = 1$.

4) The degeneration process exhibits monotonic non-decreasing characteristics, when $u > v$, $d_{u,v}^k = 0$, thus D_k forming an upper triangular matrix.

Based on the above, the state transition probability matrix of the component during its own degeneration process can be represented as follows:

$$D_k = \left(d_{u,z}^k\right)_{z \times z} = \begin{cases} \dfrac{\left(\lambda T_k\right)^{v-u}}{\left(v-u\right)!} e^{-\lambda T_k} & u \le v < z \\ 1 - \displaystyle\sum_{w=u}^{z-1} \dfrac{\left(\lambda T_k\right)^{v-w}}{\left(v-w\right)!} e^{-\lambda T_k} & u < v = z \\ 1 & u = v = z \\ 0 & u > v \end{cases} \tag{28}$$

The state transition of the production sub-system in the k-th cycle can be represented by:

$$x_k = x_R^k \cdot D_k \tag{29}$$

4.1.4 The state transition before and after the arrival of ordered components

The arrival of ordered components also causes a state transition in the inventory sub-system. Let's assume that the quantity of ordered components arrived during the cycle is q_k^{reach}. The state of the inventory sub-system after the arrival of ordered components can be represented as

$$s_k = s_k + remain(I,1) \cdot q_k^{reach} \tag{30}$$

$$\begin{aligned} q_k^{reach} &= \sum_{i=1}^{k-1} q_i \cdot P\left(\sum_{j=j+1}^{k-1} T_j \le \tau < \sum_{j=j+1}^{k} T_j \right) \\ &= \sum_{i=1}^{k-1} q_i \cdot \int_{\sum_{j=i+1}^{k-1} T_j}^{\sum_{j=i+1}^{k} T_j} f_\tau(t)dt \end{aligned} \tag{31}$$

4.2 The cost model of the system during the k-th cycle

As shown in Figure 3, the cost of the system during the *k*-th cycle is composed of *t* the costs associated with inspection, replacement, maintenance, inventory activities, as well as related penalty and waiting costs.

1) *Inspection cost*: The system is detected at each inspection time point, so detection costs C_{ins} will be always incurred during the *k*-th cycle.

2) *Replacement cost*: The cost of replacement includes preparation costs and component replacement costs. The cost of preparation is shown as follows:

$$C_{rset}^k = P\left(s_{1:z}^{k-1} > 0, x_{p:z}^{k-1} > 0\right) C_{rset} \tag{32}$$

The cost of component replacement is shown as

$$C_{rr}^k = E\left[\min\left\{s_{1:z}^{k-1}, x_{p:z}^{k-1}\right\}\right] \cdot C_r \tag{33}$$

The total replacement cost incurred during the cycle is

$$C_{\text{r}}^k = C_{\text{rset}}^k + C_{\text{rr}}^k \tag{34}$$

3) *Penalty cost*: These costs are incurred due to the failure to replace defective components in the production sub-system in a timely manner during the current cycle.

$$C_{nr}^k = E\left[\sum_{u=1}^{z}\left(X_{R,u}^k \int_0^{T_k} \int_0^t \frac{(\lambda t)^{z-u}}{(z-u)!} \lambda e^{-\lambda t} dtd\left(T_k - t\right)\right)\right] \cdot C_f \tag{35}$$

4) *Depreciation income*: The number of depreciated and recovered components required in this cycle is

$$E\left[N_{d_k}\right] = \xi \cdot E\left[\min\left\{s_{1:z}^{k-1}, x_{z:z}^{k-1}\right\}\right] \tag{36}$$

The corresponding generated depreciation income is

$$C_d^k = E\left[N_{d_k}\right] \cdot C_d \tag{37}$$

5) *Maintenance cost*: The maintenance costs incurred during this period depend on the number of components repaired through preventive maintenance and corrective maintenance within it. Assuming the number of components requiring preventive maintenance and corrective maintenance are N_{pm}^k and N_{cm}^k, they are determined by the states of the sub-system after replacement and maintenance.

$$N_{pm}^k = y_{R,p:z-1}^k \tag{38}$$

$$N_{cm}^k = y_{R,z:z}^k \tag{39}$$

As the maintenance time for each individual component follows the same exponential distribution, let $N_m(t)$ represent the number of components repaired within a duration of time *t*, and it follows Poisson process with a parameter μ_m. Thus, the total number of components that can be repaired within the *k*-th period is

$$N_{\mathrm{cfm}}^k = N_m\left(T_k\right) \tag{40}$$

The number of components completed for preventive maintenance during the cycle is

$$N_{\mathrm{fpm}}^k = P\left(N_{\mathrm{cfm}}^k \le N_{\mathrm{pm}}^k\right) N_{\mathrm{cfm}}^k + P\left(N_{\mathrm{cfm}}^k > N_{\mathrm{pm}}^k\right) N_{\mathrm{pm}}^k \tag{41}$$

The number of components completed for corrective maintenance during the cycle is

$$N_{\mathrm{fcm}}^k = P\left(N_{\mathrm{cfm}}^k > N_{\mathrm{pm}}^k\right) \cdot \min\left\{N_{\mathrm{cfm}}^k - N_{\mathrm{pm}}^k, N_{\mathrm{cm}}^k\right\} \tag{42}$$

Based on the above, the maintenance cost incurred during the cycle is

$$C_{\mathrm{main}}^k = C_{\mathrm{c}} \cdot E\left[N_{\mathrm{fcm}}^k\right] + C_{\mathrm{p}} \cdot E\left[N_{\mathrm{fpm}}^k\right] \tag{43}$$

6) *The cost of waiting for maintenance*: The costs incurred due to delayed repairs during the cycle can be classified into two scenarios. The first scenario involves components that have repair requirements but were not addressed during the cycle. The number of components in this scenario is

$$N_{\mathrm{nnm}}^k = \max\left\{N_{\mathrm{cm}}^k + N_{\mathrm{pm}}^k - N_{\mathrm{cfm}}^k, 0\right\} \tag{44}$$

The corresponding costs incurred in this scenario is

$$C_{\mathrm{ms1}}^k = E\left[N_{\mathrm{nnm}}^k\right] \cdot C_{\mathrm{ms}} T_k \tag{45}$$

The second scenario involves components that can be repaired during the cycle but incur waiting costs due to a lack of availability at the repair station. To calculate the costs, it is necessary to determine the waiting time for each individual component firstly, and then sum them up. In the maintenance sub-system, the number of components that can be simultaneously repaired at N repair station is N .The number of maintenance batches required for N_{cfm}^k components is $\left\lceil N_{\mathrm{cfm}}^k / N \right\rceil$. Since the duration of each individual repair follows exponential distribution with a parameter μ_{m}, the waiting time for any batch w follows the distribution $\Gamma\left(w-1, \mu_{m}\right)$. Therefore, the average total duration of waiting for maintenance is

$$E\left[T_{wtime}^{k}\right]=\sum_{w=2}^{\left\lfloor N_{\mathrm{cfm}}^{k} / N\right\rfloor} N\left(w-1\right) \mu_{m} + \left(N_{\mathrm{cfm}}^{k} \% N\right)\left\lfloor N_{\mathrm{cfm}}^{k} / N\right\rfloor \mu_{m} \tag{46}$$

Then, the accumulated cost generated due to delays in maintenance within a cycle is

$$C_{\mathrm{ms}}^{k}=C_{\mathrm{ms}} \cdot E\left[T_{wtime}^{k}\right] \tag{47}$$

7) *Inventory cost*: The inventory cost within this cycle includes two parts: holding cost and ordering cost. The arrival of repaired components and ordered components will both lead to changes in the inventory system. The completed components are delivered to the inventory sub-system in batches after the repairs are completed, and the total holding time is as follows:

$$E\left[T_{stime}^{k}\right]=\sum_{w=2}^{\left\lfloor N_{\mathrm{cfm}}^{k} / N\right\rfloor} N\left(T_{k}-(w-1) \mu_{m}\right)+ \left(N_{\mathrm{cfm}}^{k} \% N\right)\left(T_{k}-\left\lfloor N_{\mathrm{cfm}}^{k} / N\right\rfloor \mu_{m}\right) \tag{48}$$

The holding time after the arrival of the ordered parts is

$$t_{new}^{k}=\int_{0}^{T_{k}} \int_{0}^{t} f_{\tau}(t) dt d\left(T_{k}-t\right) \tag{49}$$

The total holding cost is

$$C_{\mathrm{is}}^{k}=C_{\mathrm{is}} \cdot\left(E\left[T_{stime}^{k}\right]+E\left[q_{k} t_{new}^{k}\right]\right) \tag{50}$$

The order preparation cost is

$$C_{\mathrm{iset}}^{k}=C_{\mathrm{iset}} \cdot P\left(q_{k}>0\right) \tag{51}$$

Components ordering cost is

$$C_{\mathrm{io}}^{k}=q_{k} C_{\mathrm{io}} \tag{52}$$

Furthermore, regarding the ordering of components, the ordered components should be able to function properly when replacement is needed, thus avoiding any penalty costs. Therefore, the potential loss that can be reduced throughout the entire cycle of the system is as follows:

$$C_{\mathrm{op}}^{k}=\left(p-1\right) q_{k} C_{\mathrm{f}} / \lambda \tag{53}$$

The inventory cost incurred during this cycle can be represented as

$$C_{\text{inventory}}^{k}=C_{\mathrm{io}}^{k}+C_{\mathrm{is}}^{k}+C_{\mathrm{iset}}^{k}-C_{\mathrm{op}}^{k} \tag{54}$$

The total cost of the system for the k-th cycle can be summarised as follows:

$$C_k = C_{\text{ins}} + C_{\text{r}}^k + C_{\text{nr}}^k + C_{\text{d}}^k + C_{\text{main}}^k + C_{\text{ms}}^k + C_{\text{inventory}}^k \tag{55}$$

4.3 Joint sequential decision model for multi-period maintenance and spare parts inventory

According to the state transition model, the system state at the (k–1)-th inspection point is defined as $X_{k-1} \overset{def}{=\!=} \{x_{k-1}, y_{k-1}, z_{k-1}\}$.The activity experienced during the k-th cycle are defined as $U_k \overset{def}{=\!=} f(p, T_k, q_k)$. The state transition relationship during this cycle is defined as $X_k \overset{def}{=\!=} F[X_{k-1}, U_k]$. The decision model is established based on the finite time horizon T_{total}, consisting of K cycles, are as follows:

$$\begin{aligned}
&\min_{(p,T,q)} G(p,T,q) = \min \sum_{k=1}^{K} C[X_k, U_k] \\
&s.t. \\
&\quad \sum_{k=1}^{K} T_k = T_{total} \quad, K = 1, 2, \ldots \\
&\quad X_k = F[X_{k-1}, U_k] \\
&\quad C[X_0, U_0] = 0
\end{aligned} \tag{56}$$

where $C[X_k, U_k]$ represents the minimum average cost rate of the system from the initial time to the k-th inspection point. The cost rates of subsequent cycles satisfy.

$$C[X_k, U_k] = \min\left\{\left(C[X_{k-1}, U_{k-1}] + C_k\right) / \left(\sum_{ki=1}^{k-1} T_{ki} + T_k\right)\right\} \tag{57}$$

4.4 Model solution

The decision model constructed in this study is based on the corresponding state transition model, which depends on the activities experienced by the system between two adjacent inspection points. It constitutes a multi-stage decision process, following the principles of dynamic programming. The cost rate model in an individual period is a constrained, mixed variable, non-linear single-objective optimisation model. Genetic Algorithm (GA) is an efficient and powerful heuristic search optimisation method (Bhunia et al., 2017), widely applied in various fields such as bioinformatics, engineering, economics and physics (Ram, 2013). It can handle linear, non-linear, constrained, unconstrained, discrete, continuous and mixed search spaces. Considering the robustness of genetic algorithms, they are chosen to solve the model for each cycle. The optimal solution for multiple cycles is obtained using the dynamic programming approach.

5 Numerical experiments

The spindles are one of the key components that affect the availability of wind turbines. In this paper, a numerical experiment is conducted using a multi-component repairable system consisting of multiple spindles of wind turbines as a case study. Assuming the system consists of 10 wind turbines, and $M = 10$. The temperature (°C) of the spindles can be approximately used to characterise its operating performance, it is divided into ten different deterioration levels, $z = 10$. The dwell time of the spindles in each temperature state is fitted to an exponential distribution with a parameter of $\lambda = 0.2$. It is assumed that at the initial moment, all components in the production sub-system are in a brand-new state, while there are no components in the maintenance sub-system or inventory sub-system. Take $T_{\text{total}} = 150$, and approximate the truncation of ∞ with 100,000. The parameters of the production sub-system are set as $C_{\text{f}} = 1500$, $\xi = 0.8$, $C_{\text{d}} = 20$, $C_{\text{rset}} = 1500$, $C_{\text{r}} = 20$. The parameters of the maintenance sub-system are set as $N = 5$, $\mu_{\text{m}} = 0.4$, $C_{\text{c}} = 250$, $C_{\text{p}} = 90$, $\lambda_{\text{c}} = 5$, $\lambda_{\text{p}} = 3$, $C_{\text{ms}} = 20$. The parameters of the inventory sub-system are set as $\mu = 20$, $\sigma = 10$, $C_{\text{iset}} = 500$, $C_{\text{is}} = 2$, $C_{\text{io}} = 1200$. The cost unit is measured in thousands of Yuan.

5.1 Analysis of the effectiveness of the decision model

Firstly, the effectiveness of the decision model is validated through three different perspectives. One is to validate that the sum of probabilities of all replacement/maintenance scenarios occurring during the replacement/maintenance process equals 1. The second is to analyse the impact of different system parameters on the system's state. The third is to validate the existence of the model's solution by analysing how the total system cost changes with variations in the decision variables.

5.1.1 The probability models for maintenance and replacement

Given the decision variable $p = 5$, $T = 35$, $q = 5$, the probabilities associated with different replacement and maintenance scenarios are shown in Tables 1 and 2.

1) The probability model for replacement activity: As shown in Table 1, the total probabilities of all possible replacement activities in the three cases are very close to 1: no replacement, complete replacement and partial replacement. This verifies the effectiveness of the sub-system states after replacement, confirming the feasibility of the solving process from equations (6) to (18).

2) *The probability model for maintenance activities*: Similarly, as demonstrated in Table 2, the sum of probabilities for all possible maintenance activities in the three cases, namely no component maintenance, complete component maintenance and partial component maintenance, are also very close to 1. This validates the effectiveness of obtaining sub-system states after maintenance and further confirms the feasibility of the solving process from equations (19) to (23).

Table 1 Replacement scenarios and corresponding probabilities

Replacement scenarios		*Probabilities*
No replacement (NR)		*0.147481*
Complete replacement R_1	1	0.006560
	2	0.037386
	3	0.070438
	4	0.069450
	5	0.038143
	6	0.165462
	7	0.191317
	8	0.131170
	9	0.059015
	10	0.050370
Partial replacement R_2	5	0.004661
	6	0.010194
	7	0.004152
	8	0.003817
	9	0.003476
	10	0.006905
Probability and Sum		0.999997

Table 2 Maintenance scenarios and corresponding probabilities

Maintenance scenarios		*Probabilities*
No component maintenance		0.000003
Maintenance scenarios *m*	5	0.086147
	6	0.193186
	7	0.229643
	8	0.156591
	9	0.105495
	10	0.148340
All components have been maintenance.		0.080593
Probability and Sum		0.999998

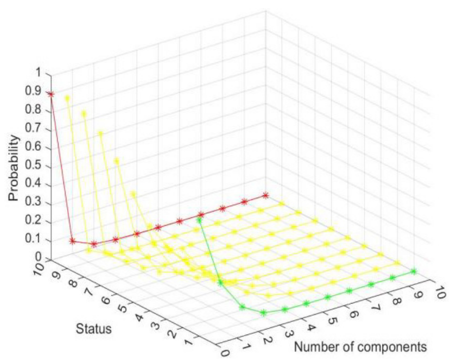
3) Probability distribution of system states: Taking a three-period system as an example, after determining the decision variables $p = 6$, $T = [30, 20, 10]$, $q = [3, 5, 7]$, we will individually modify one of the three decision variables and analyse the impact of different decision variables on the probability distribution of system states. The states of the three sub-systems are interrelated, and variation of the values of p, T, q shown in Figures 7 to 15 reflects the effectiveness of the decision results in terms of the probability distribution of different quantities of maintenance required components in the sub-systems. In the figures, the red, yellow

and green colours represent corrective maintenance, preventive maintenance and no maintenance required, respectively.

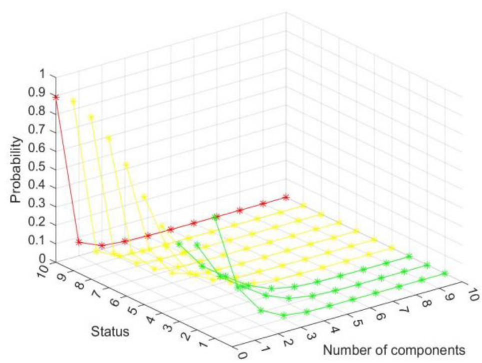
5.1.2 The influence of preventive maintenance threshold p on the probability of system states

As shown in Figure 7, under the given condition of T, q, the number of states in the preventive maintenance area gradually decreases with the increase of p due to the quantity of components that require preventive maintenance decreases. During specific maintenance cycles, the probability of defective components occurring in the production sub-system gradually increases due to a lack of timely repairs. Simultaneously, the probability of component depreciation, recycling and replacement also increases with the increasing number of defective components. Consequently, this increases the probability of having a greater number of components in a good condition within the system.

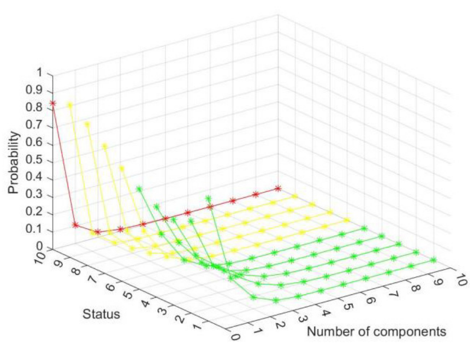
Figure 7 The influence of preventive maintenance threshold on the probability of production sub-system state (see online version for colours)



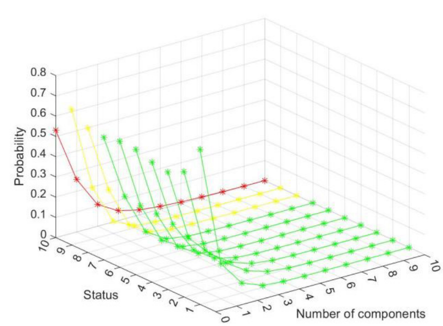
(a) $p = 2$



(b) $p = 4$



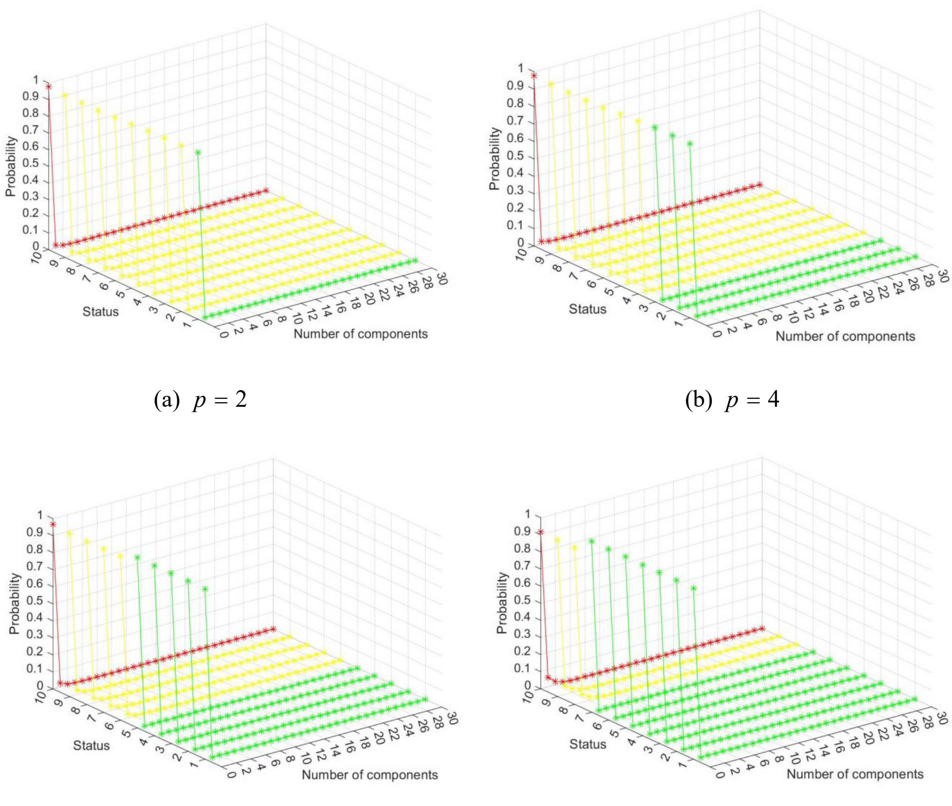
(c) $p = 6$



(d) $p = 8$

As shown in Figure 8, under the given condition T, q, as the p increases, the probability of a component in the maintenance sub-system being in a failed state decreases due to the reduced need for post-failure maintenance in the production system. At the same time, the increasing number of components scrapped due to delayed repairs, as well as the continuous replenishment of new components, results in a higher probability of preventive maintenance in the maintenance sub-system. Regardless of the p variation, the number of components in the maintenance sub-system that are in the normal zone is always zero. This is because components in the production sub-system are only replaced and transferred to the maintenance sub-system when their state is not less than p.

Figure 8 The influence of preventive maintenance threshold on the probability of maintenance sub-system state (see online version for colours)

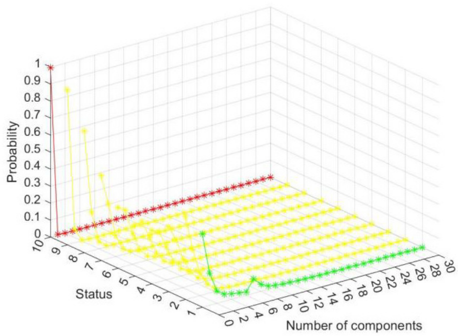


(a) $p = 2$ (b) $p = 4$

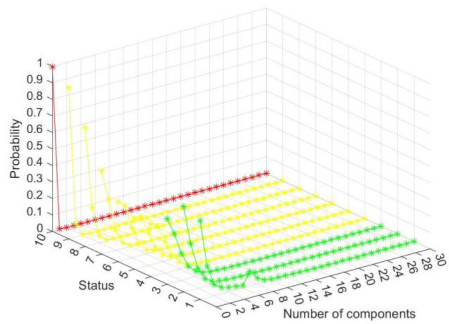
(c) $p = 6$ (d) $p = 8$

As shown in Figure 9, due to the inventory sub-system only storing repaired and ordered components, under the given condition T, q, regardless of the p variation, the number of components in the inventory sub-system that are in a failure state is always zero. As the p increases, the components that arrive at the repair sub-system are in poor condition, and their condition after repair tends to be concentrated in poorer states. Therefore, the probability that the state of components in the inventory sub-system is in the preventive maintenance zone becomes progressively larger. At the same time, the increase of p leads to an increased need for ordering new components, thereby causing the probability of component states in the inventory sub-system to gradually increase in the normal zone.

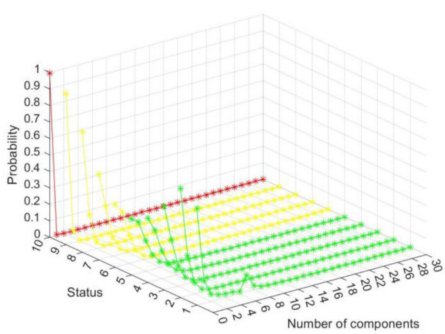
Figure 9 The influence of preventive maintenance threshold on the probability of inventory sub-system state (see online version for colours)



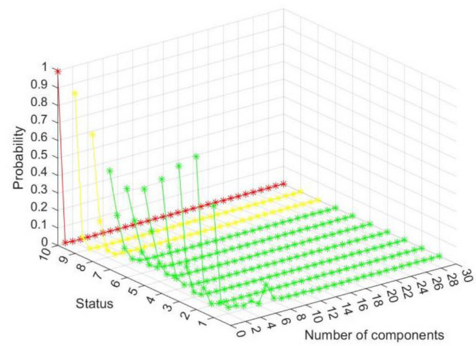
(a) $p = 2$



(b) $p = 4$



(c) $p = 6$

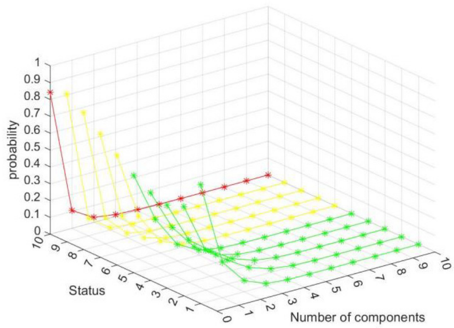


(d) $p = 8$

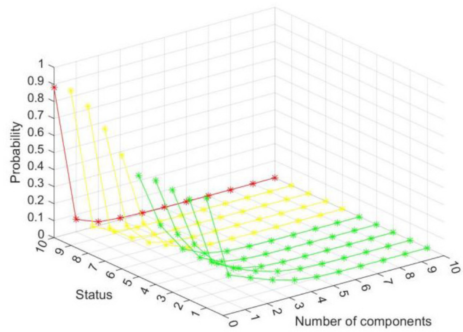
5.1.3 The influence of periodic sequences T on the probability of system states

As shown in Figures 10 to 12, under the given condition of p and q, the probability distribution of component quantities in different states in the three sub-systems varies noticeably with different inspections. This indicates the effectiveness of the dynamic decision model for variable periodic sequence. Only by optimising and obtaining the optimal periodic sequence we can ensure higher production efficiency of the production sub-system, higher maintenance effectiveness of the maintenance sub-system and higher spare parts replenishment effectiveness of inventory sub-systems.

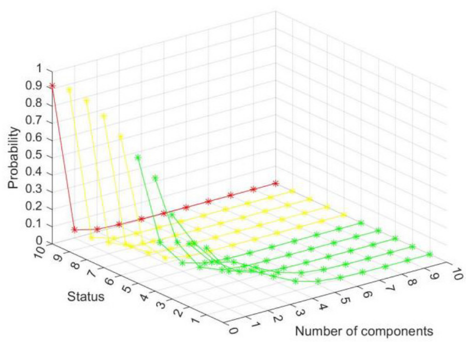
Figure 10 The influence of periodic sequence on the probability of production sub-system state (see online version for colours)



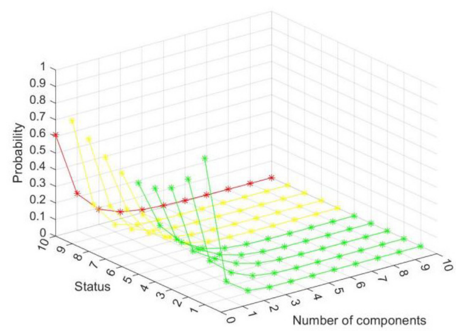
(a) $T = [30, 20, 10]$



(b) $T = [35, 15, 10]$

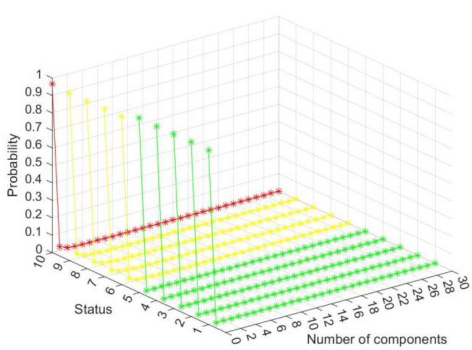


(c) $T = [30, 25, 5]$

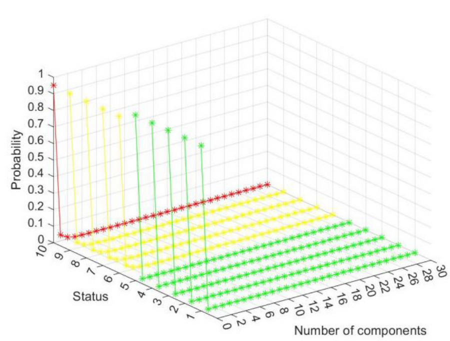


(d) $T = [25, 20, 15]$

Figure 11 The influence of periodic sequence on the probability of maintenance sub-system state (see online version for colours)

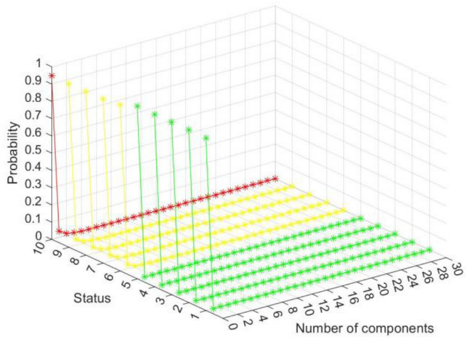


(a) $T = [30, 20, 10]$

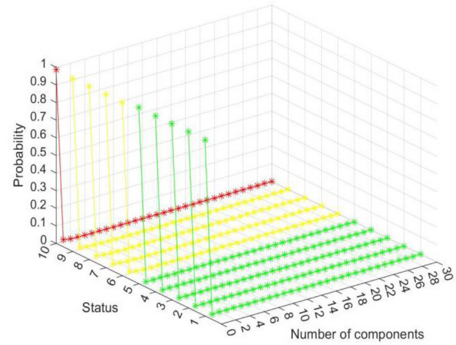


(b) $T = [35, 15, 10]$

Figure 11 The influence of periodic sequence on the probability of maintenance sub-system state (see online version for colours) (continued)

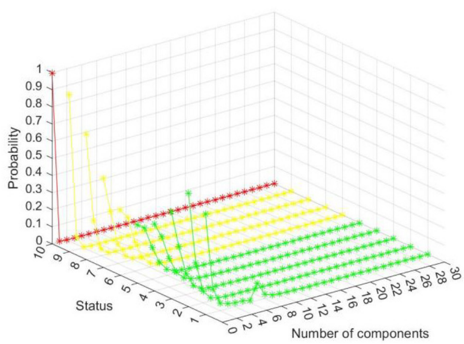


(c) $T = [30, 25, 5]$

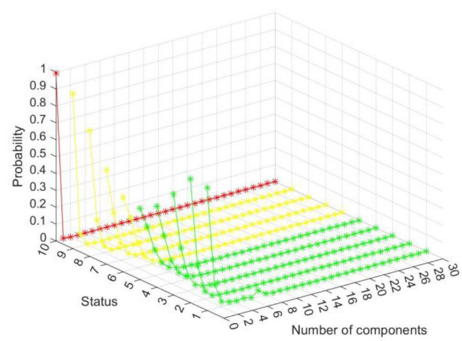


(d) $T = [25, 20, 15]$

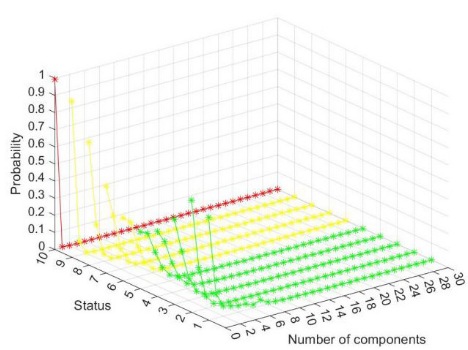
Figure 12 The influence of periodic sequence on the probability of inventory sub-system state (see online version for colours)



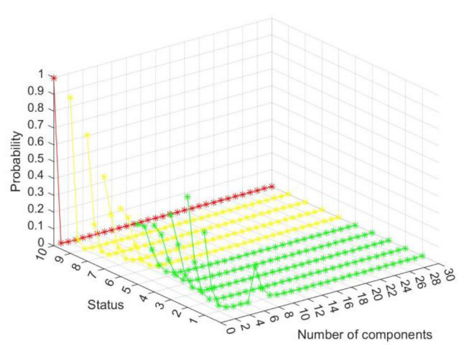
(a) $T = [30, 20, 10]$



(b) $T = [35, 15, 10]$



(c) $T = [30, 25, 5]$

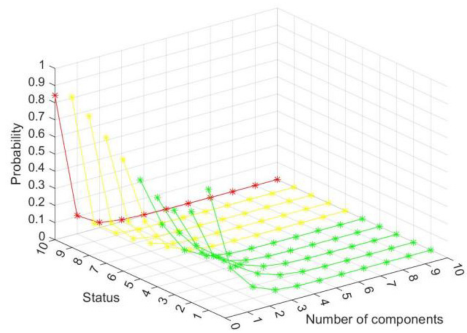


(d) $T = [25, 20, 15]$

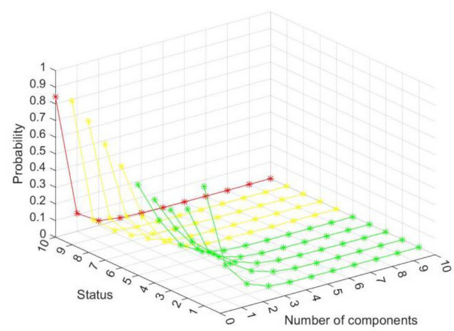
5.1.4 The influence of ordering batch sequences q on the probability of system states

As shown in Figures 13 to 15, it can be observed that under the given conditions p and T, different ordering batch sequences q result in significantly different probabilities of the quantity of components in various states in the three sub-systems. This also demonstrates the effectiveness of the dynamic decision model based on variable ordering batch sizes. By optimising the optimal ordering batch sequence, it can ensure higher production efficiency in the production sub-system, better maintenance effectiveness in the maintenance sub-system and higher spare parts replenishment effectiveness in the inventory sub-system.

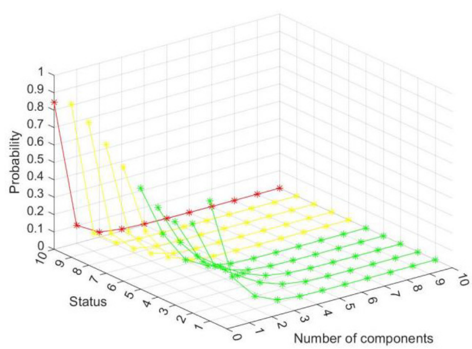
Figure 13 The influence of ordering sequence on the probability of production sub-system state (see online version for colours)



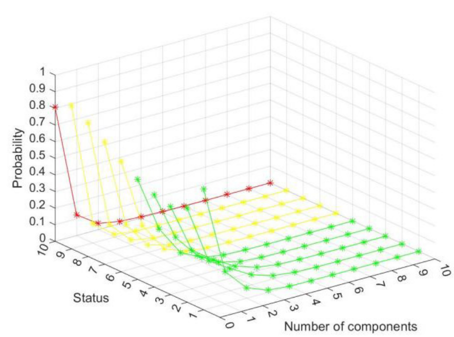
(a) $q = [3,5,7]$



(b) $q = [4,4,6]$

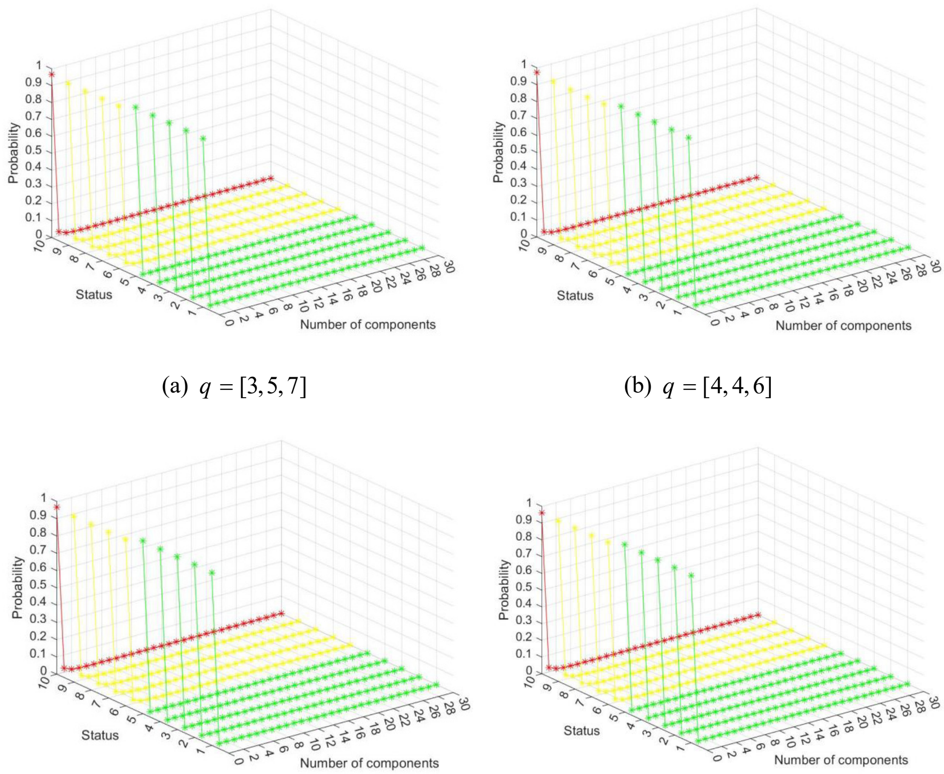


(c) $q = [3,6,6]$



(d) $q = [4,5,6]$

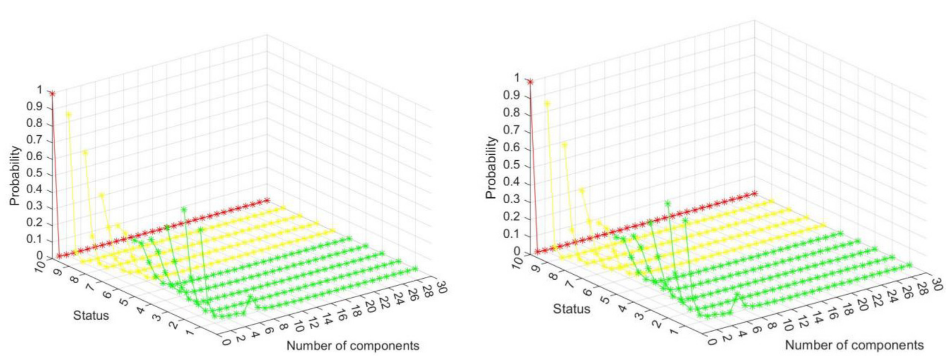
Figure 14 The influence of ordering sequence on the probability of maintenance sub-system state (see online version for colours)



(a) $q = [3, 5, 7]$ (b) $q = [4, 4, 6]$

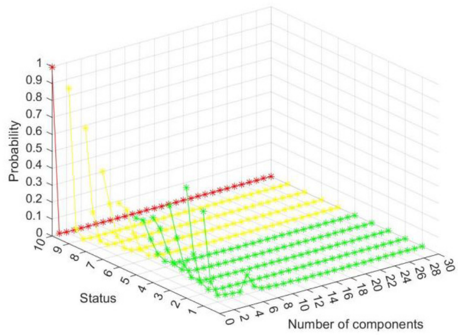
(c) $q = [3, 6, 6]$ (d) $q = [4, 5, 6]$

Figure 15 The influence of ordering sequence on the probability of inventory sub-system state (see online version for colours)

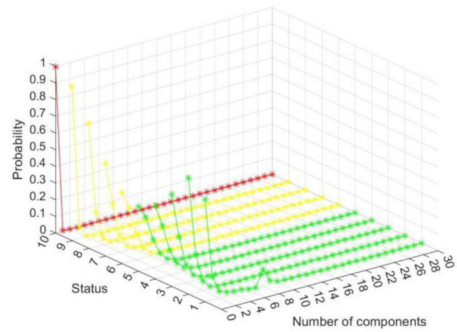


(a) $q = [3, 5, 7]$ (b) $q = [4, 4, 6]$

Figure 15 The influence of ordering sequence on the probability of inventory sub-system state (see online version for colours) (continued)



(c) $q = [3,6,6]$



(d) $q = [4,5,6]$

5.1.5 The influence of decision variables p, T, q on the total cost

As shown in Table 3, there is a significant variation in the total cost of the system with the change of decision variables.

Table 3 The impact of different decision variables on the total cost within the system

Variables	*Decision variables and their values*			$C_{\text{total}}(\times 10^4)$
	T	q	p	
p	[30, 20, 10]	[3, 5, 7]	2	3.38
	[30, 20, 10]	[3, 5, 7]	4	4.96
	[30, 20, 10]	[3, 5, 7]	6	3.12
	[30, 20, 10]	[3, 5, 7]	8	10.93
T	[30, 20, 10]	[3, 5, 7]	6	3.12
	[35, 15, 10]	[3, 5, 7]	6	3.67
	[30, 25, 5]	[3, 5, 7]	6	5.36
	[25, 20, 15]	[3, 5, 7]	6	1.61
q	[30, 20, 10]	[3, 5, 7]	6	3.12
	[30, 20, 10]	[4, 4, 6]	6	3.41
	[30, 20, 10]	[3, 6, 6]	6	5.44
	[30, 20, 10]	[4, 5, 6]	6	7.79

With the increasing preventive maintenance threshold p, there is a noticeable trend of initially decreasing and then increasing the total cost of the system. This is because within a certain range, an increase in the preventive maintenance threshold can reduce excessive maintenance and consequently lower the total cost of the system. However, once it reaches a certain level, inadequate maintenance may occur, leading to an increase in the probability of system failure and associated costs. Selecting the optimal preventive maintenance threshold can effectively reduce the total cost of the system.

By adjusting the periodic sequences T, there will be significant changes in the total cost of the system. This is because in a periodic sequence, if a cycle is too large/small, it directly affects the maintenance activities and system performance for that cycle and subsequent cycles. Taking the case of a cycle being too large as an example, the probability of component failures in the production sub-system increases during this period, leading to an increase in costs incurred within the sub-system. Meanwhile, the depreciation recovery after a failure may result in idle capacity in the maintenance sub-system, leading to a decrease in maintenance demand accumulated for the next cycle, however, it also increases the costs of inventory ordering. However, in the subsequent cycles, the increase in the number of new components leads to a decrease in the system's maintenance demand, thereby reducing the costs in the following cycles. This demonstrates the necessity of sequential decision-making.

As the ordering batch sequence q in the decision variables changes, the total cost of the system will also undergo significant variations. This is because within an ordering batch sequence, if one of the ordering batch sizes is too large or too small, it will similarly affect the system states and costs in the current and subsequent cycles. If the ordering batch size is too large in the current cycle, it will directly increase inventory costs. However, in subsequent cycles, the increased number of new components in the system will improve the initial state of the production sub-system, reducing the probability of component failures during the cycle, this in turn reduces the maintenance needs of the maintenance sub-system, resulting in lower maintenance and penalty costs.

In conclusion, combined with the analysis of experimental results, it can be observed that changes in all decision variables have an interpretation impact on the total cost of the system. Furthermore, the optimisation results of each variable also have a direct influence each other, indicating necessity of joint decision-making.

5.2 Analysis of joint decision-making results

The optimal solution was obtained using a combination of dynamic programming and genetic algorithm. The parameters of the genetic algorithm are set as: the initial population size is 50, selection pressure is 0.8, crossover probability is 0.8, mutation probability is 0.2 and the number of generations is 50. Owing to the stochastic nature of genetic algorithm optimisation, we performed 50 optimisations and selected the best result as the final satisfactory solution. The final solution yields the minimum cost for the given time $C_{total} = 1.33 \times 10^5$, representing the optimal sequence of strategy combinations $\{p, T, q\} = M, \lambda, \xi$. The total system operating time is divided into six cycles, with significant differences in the length of each detect cycle and the quantity of orders. Moreover, the quantity of orders in the preceding cycle directly affects the length and quantity of orders in subsequent cycles.

5.3 Sensitivity analysis

The system parameters and cost parameter values for the production, maintenance and inventory sub-systems are adjusted by approximately 20% increment while keeping the remaining parameters unchanged. The goal is to observe the minimum cost and changes in decision variables before and after the adjustments. This analysis will help evaluate the sensitivity of the decision model to these parameters and its adaptability to different systems.

5.3.1 Parameters of the production sub-system

According to Table 4, as the number of components M increases, the total cost of the system also increases. In this case, to maintain the minimum total cost, the sequence of cycles, order batches and preventive maintenance thresholds need to be adjusted. Overall, preventive maintenance shows an increasing trend, which can reduce the demand for maintenance and lower maintenance costs. The increase in component quantity naturally leads to an increase in demand for ordering, to satisfy the needs for more disposals and replacements. The variation of the inspection cycle will be adjusted accordingly based on the maintenance threshold and the quantity of orders. One notable pattern is that when the quantity of orders increases, the cycle for the following week significantly increases.

Table 4 The influence of production sub-system system parameter M, λ, ξ on the optimal strategy

Variables	*Values*	$C_{\text{total}}(\times 10^5)$	*Decision variables and their values*		
			T	*q*	*p*
	8	1.01	[20.94, 10.11, 44.45, 37.98, 21.63, 20.08]	[0, 2, 6, 8, 1, 0]	4
M	10	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	12	1.58	[22.65, 11.83, 10.56, 32.84, 36.17, 35.94]	[0, 4, 9, 10, 3, 0]	6
	0.15	1.34	[31.37, 15.13, 13.46, 17.10, 39.21, 33.53]	[0, 3, 6, 6, 3, 0]	7
λ	0.20	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	0.25	1.61	[18.95, 11.02, 29.95, 28.70, 35.06, 26.33]	[0, 3, 10, 2, 6, 0]	4
	0.65	1.30	[23.16, 11.51, 13.36, 26.83, 38.39, 32.93]	[0, 9, 9, 10, 1, 1]	6
ξ	0.80	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1,2]	5
	0.90	1.42	[24.03, 14.07, 12.13, 35.38, 31.80, 32.62]	[0, 4, 10, 9, 2, 0]	5

As the degradation parameter of the components λ increases, to minimise the total cost, each cycle in the sequence must be relatively small. Therefore, in the optimised results, the preventive maintenance threshold gradually decreases, reducing the maintenance costs. The order quantity may slightly increase due to the increased probability of component failures. The inspection process will also make corresponding adjustments based on the changes in the other two decision factors, aiming to ensure a lower total cost.

As the components scrap rate ξ increases, the total cost of the system increases. To ensure the lowest total cost, it is possible to achieve this by reducing some or all cycles in the cycle sequence, and increasing some or all order quantities in the order batch sequence. Reducing the cycle can lower the probability of component failures, thereby reducing the costs associated with failures and component replacements. By changing the order batch sequence, it can ensure an adequate supply of components for replacements and reduce the costs associated with component storage.

5.3.2 Cost parameters of production sub-system

According to Table 5, it can be observed that as the cost parameters of the production sub-systems increase, the total cost generated by the system also increases. To reduce costs, it can decrease the likelihood of component failures in a sub-system by reducing the size of cycles in the periodic sequence of the C_f, C_r. For C_d, if it is larger, it indicates that the component has a higher recycle value or lower usage fee. One approach is to increase the number of defective components replaced with new ones in a timely manner, which can effectively reduce the costs incurred during the operation of the sub-system. For C_{rset}, the cost incurred due to maintenance preparation can be reduced by decreasing the number of cycles in the periodic sequence within a given period.

Table 5 The influence of production sub-system cost parameter C_f, C_d, C_{rset}, C_r on the optimal strategy

Variables	*Values*	$C_{total}(\times 10^5)$	*Decision variables and their values*		
			T	*q*	*p*
C_f	800	1.29	[23.75, 15.01, 17.19, 34.80, 32.02, 26.05]	[0, 3, 8, 10, 1, 0]	5
	1000	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	1200	1.37	[24.10, 17.31, 15.37, 35.35, 31.29, 26.26]	[0, 4, 8, 10, 3, 0]	3
C_d	16	1.34	[23.89, 13.36, 11.19, 31.63, 33.27, 31.45]	[0, 3, 10, 9, 1, 1]	7
	20	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	24	1.32	[23.53, 17.31, 10.22, 35.51, 37.05, 26.38]	[0, 3, 7, 10, 1, 0]	4
C_{rset}	1200	1.28	[23.25, 17.03, 19.23, 25.01, 29.37, 31.66]	[0, 4, 9, 10, 2, 2]	4
	1500	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	1800	1.35	[25.21, 10.37, 53.42, 34.18, 14.87, 11.96]	[0, 3, 10, 1, 1, 0]	9
C_r	16	1.33	[23.68, 11.07, 13.21, 25.00, 40.90, 32.01]	[0, 4, 9, 10, 3, 1]	4
	20	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	24	1.33	[22.91, 10.33, 13.31, 29.63, 36.36, 31.30]	[0, 3, 9, 9, 1, 1]	5

5.3.3 Parameters of maintaining sub-system

As shown in Table 6, the total cost of the system decreases as the number of maintenance stations in the maintenance sub-system increases. This is because the construction cost of the repair stations has been neglected in this study. With the increase of N, the maintenance capability of the sub-system is enhanced, allowing for the early repair of more components. This can reduce or prevent overdue maintenance of components and decrease the order quantity, thus reducing costs.

As μ_m increases, the repair rate of the sub-system improves, thereby enhances the maintenance capability of the sub-system. Therefore, its impact on the optimal strategy is generally similar to the number of repair stations N.

Parameters λ_c and λ_p represent the effectiveness of corrective maintenance and preventive maintenance, respectively, and as these parameters increase, the total cost of the system decreases. This is because with an increase in the parameters, the maintenance efficiency of corrective and preventive maintenance improves, allowing the components to be restored to a better condition, thereby reducing the number of maintenance actions and related costs. It may even lead to a decrease in the order quantity, thus reducing the overall cost of the system.

Table 6 The influence of maintenance sub-system system parameter $N, \mu_m, \lambda_c, \lambda_p$ on the optimal strategy

Variables	*Values*	$C_{total}(\times 10^5)$	*Decision variables and their values*		
			T	*q*	*p*
N	4	1.34	[23.17, 15.51, 13.96, 10.62, 37.89, 18.84]	[0, 3, 7, 8, 1, 0]	7
	5	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	6	1.26	[22, 53, 10.61, 17.62, 52.35, 37.44, 9.44]	[0, 2, 6, 10, 1, 0]	4
μ_m	0.3	1.34	[24.01, 13.02, 14.25, 35.79, 36.65, 26.27]	[0, 4, 10, 8, 2, 0]	5
	0.4	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	0.5	1.31	[33.67, 11.13, 15.28, 40.05, 38.65, 11.23]	[0, 4, 7, 10, 1, 0]	5
λ_c	4	1.37	[26.67, 11.13, 15.28, 50.05, 38.65, 11.23]	[0, 4, 7, 10, 1, 0]	3
	5	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	6	1.30	[23.72, 11.79, 13.35, 38.84, 37.03, 25.27]	[0, 3, 8, 8, 1, 0]	6
λ_p	2	1.36	[24.92, 10.07, 11.13, 32.52, 35.80, 35.57]	[0, 4, 9, 9, 1, 0]	4
	3	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	4	1.31	[23.38, 13.03, 11.07, 39.11, 35.78, 27.67]	[0, 2, 6, 9, 1, 0]	6

5.3.4 Cost parameters of maintaining sub-system

According to Table 7, the total cost of the system increases with an increase in the cost parameter of the maintenance sub-system. This is because as the three cost parameters of the sub-system increase, the costs incurred due to maintenance also increase under the same conditions. To still minimise the total cost, it is necessary to reduce the maintenance of the components. However, at the same time, reducing maintenance increases the likelihood of component failures and replacements in the system. Firstly, the costs incurred due to failures will increase. Secondly, the quantity of components that need to be ordered due to replacements will increase, leading to an increase in the related ordering costs. Furthermore, the magnitude at which these two factors increase costs is greater than the magnitude at which maintenance costs decrease, resulting in an overall gradual increase in costs.

Table 7 The influence of maintenance sub-system cost parameter $C_{\mathrm{c}}, C_{\mathrm{p}}, C_{\mathrm{ms}}$ on the optimal strategy

Variables	*Values*	$C_{\text{total}}(\times 10^5)$	*Decision variables and their values*		
			T	*q*	*p*
C_{c}	200	1.24	[23.79, 14.65, 26.23, 36.55, 39.219.56]	[0, 3, 10, 3, 2, 0]	5
	250	1.33	[20.43,10.03,10.16,31.25,37.03,36.08]	[0,3,9,9,1,2]	5
	300	1.53	[24.73, 26.51, 19.24, 43.05, 36.47]	[0, 5, 10, 10, 0]	4
C_{p}	75	1.28	[25.32, 10.07, 21.05, 34.93, 30.61, 29.02]	[0, 4, 9, 10, 1, 0]	5
	90	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	105	1.34	[23.73, 11.91, 19.44, 37.14, 38.25, 19.53]	[0, 3, 7, 6, 1, 0]	6
C_{ms}	16	1.31	[24.01, 13.21, 16.00, 30.20, 33.51, 31.15]	[0, 5, 10, 9, 1, 1]	4
	20	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	24	1.36	[23.75, 10.85, 12.04, 39.63, 39.47, 24.15]	[0, 3, 8, 9, 1, 0]	6

5.3.5 Parameters of inventory sub-system

From Table 8, it can be observed that as the system parameters μ, σ of the inventory sub-system increase, the total costs also increase. As the system parameters μ increase, the lead time for ordering also increases, and as σ increases, the level of certainty decreases. In this scenario, if the inventory sub-system increases inventory availability through replenishment, it can reduce the costly consequences of untimely replacements. However, at the same time, it will increase the high costs associated with ordering and holding inventory. Therefore, there is a clear trade-off between the changes in decision variables, but there is no direct rule or pattern.

Table 8 The influence of inventory sub-system system parameter μ, σ on the optimal strategy

Variables	*Values*	$C_{\text{total}}(\times 10^5)$	*Decision variables and their values*		
			T	*q*	*p*
μ	16	1.23	[19.17, 11.29, 11.42, 38.35, 10.71, 28.93]	[0, 1, 4, 6, 1, 0]	7
	20	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	24	1.52	[23.31, 20.06, 15.80, 32.28, 37.08, 21.34]	[3, 9, 10, 1, 0]	4
σ	8	1.29	[23.13, 13.64, 15.81, 39.34, 36.30, 21.78]	[0, 3, 9, 10, 10, 1]	3
	10	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	12	1.40	[24.90, 12.75, 10.35, 39.58, 37.32, 25.10]	[0, 3, 9, 9, 1, 0]	7

5.3.6 Cost parameters of inventory sub-system

From Table 9, it is evident that the total cost of the system increases with an increase in the three cost parameters of the inventory sub-system, in order to ensure the total cost remains at its minimum level in this scenario. For C_{iset}, by increasing the maintenance threshold, the maintenance demand has been reduced. For C_{is}, since its value is relatively small, the resulting change in total cost is also comparatively small. For C_{io}, the ordering demand can be further reduced by adjusting the preventive maintenance threshold.

Table 9 The influence of inventory sub-system cost parameter $C_{iset,}C_{is}$, C_{io} on the optimal strategy

Variables	*Values*	$C_{total}(\times 10^5)$	*Decision variables and their values*		
			T	*q*	*p*
C_{iset}	400	1.23	[24.43, 11.28, 14.54, 38.13, 34.37, 23.94]	[0, 3, 8, 8, 1, 0]	4
	500	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	600	1.40	[24.92, 11.11, 13.13, 31.29, 37.09, 32.46]	[0, 3, 9, 9, 1, 0]	6
C_{is}	1.6	1.33	[23.75, 11.44, 14.85, 37.48, 36.52, 25.96]	[0, 3, 9, 9, 1, 0]	5
	2.0	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	2.5	1.34	[24.46, 13.02, 11.04, 40.03, 33.21, 28.24]	[0, 3, 8, 10, 1, 0]	5
C_{io}	800	1.27	[25.36, 17.31, 14.14, 39.18, 35.72, 18.28]	[0, 4, 9, 9, 1, 0]	6
	1000	1.33	[20.43, 10.03, 10.16, 31.25, 37.03, 36.08]	[0, 3, 9, 9, 1, 2]	5
	1200	1.36	[23.64, 10.63, 11.72, 32.17, 37.20, 31.52]	[0, 3, 9, 9, 1, 1]	5

This result indicates that, while ensuring the minimum overall system cost, adjustments to the relevant parameters will affect the three decision variables to varying degrees, thus demonstrating the necessity and effectiveness of joint decision research.

6 Conclusions

This paper investigates the multi-component repairable system utilising Markov multi-state modelling, considering the interdependencies between production, maintenance and spare parts inventory. Joint decisions regarding maintenance and spare parts inventory are made from a system perspective. A joint strategy is proposed for imperfect preventive maintenance and spare parts inventory management based on non-periodic inspections and a unified maintenance threshold. A corresponding joint sequential decision model is established, and a combined approach of global dynamic programming and local genetic algorithm is employed for joint optimisation. Numerical experiments and analysis of joint decisions are conducted using the case of wind turbine spindles. From the experimental results, adjustments to system parameters and cost parameters in the three sub-systems have significant effects on the total maintenance cost and decision variables. Specifically, the periodic sequence has a notable impact on inspection frequency and cost, order batch sequence affects replacement progress and cost and the maintenance threshold influences the demand for component maintenance. Thus, the established model effectively addresses the joint decision-making problem of maintenance and spare parts inventory in repairable multi-component systems. Sequential decisions can effectively address fluctuations in inspection and maintenance requirements caused by component failure and ordering, thereby reducing maintenance and spare parts costs.

Joint decisions ensure the dynamic optimal combination of various decision variables. Therefore, adopting such dynamic decision-making strategies can better adapt to the actual production and market environment of repairable multi-component systems, especially in systems with significant degradation fluctuations or uncertain spare parts replenishment, thus maintaining system operation and maintenance costs at a lower level.

There are still two directions for further research in this study. Firstly, the degradation process should consider actual production conditions and establish the correlation between productivity, production quality, degradation state and degradation rate. Secondly, the use of surrogate modelling approximation in the local optimisation process can further improve the optimisation solving process, effectively reducing the computational burden of the optimisation process.

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