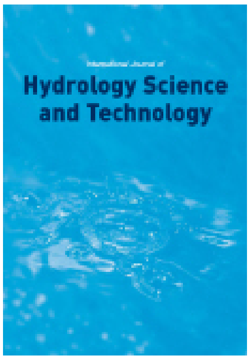

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Solute transport with decay type input source in one-dimensional heterogeneous groundwater: analytical solution

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Solute transport with decay type input source in one-dimensional heterogeneous groundwater: analytical solution

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Abstract: The process of aquifer remediation extends with the growing dependence on groundwater. Mathematical model of solute transport in porous media is important tool used to characterise the extent of approximating the shape, size and position of a contaminant. In the present study, an unsteady solute transport model advection-diffusion equation (ADE) is taken and analytical solutions were obtained by using Laplace integral transformation technique (LITT). The concentration is predicted in presence and absence of source, i.e., firstly initially medium (aquifer/air) is not supposed to be solute free, i.e., initially domain is already polluted/contaminated and secondly the medium is clean, taking decay type exponential input at origin. The dependence of velocity on space variable is of linear non-homogeneous nature due to heterogeneity of the semi-infinite horizontal dispersion medium. The dispersivity is considered square of the velocity which represents the seasonal variation of the year in tropical regions.

Keywords: advection-dispersion equation; groundwater; heterogeneity; pollution; semi-infinite medium.

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1 Introduction

Ground water contamination is an ever-increasing concern as our society's dependence on groundwater grows. As we all know, groundwater contamination in aquifer is now a day's serious problem in almost all part of the country even in the world. Mainly, the world's population increases so does the need for reliable forces of clean, non-polluted fresh water. Well known common problem of water pollution resulting from mining and due to pumping of mine water and its discharge into existing drainage system. A large amount of water is used in the coal preparation plants and beneficiation, tailing which, if discharged into the streams causes water contamination. At this stage, removable of existing contamination requires not only elimination of the contaminant source, but extraction of chemicals present in the ground water. Before physical removable of the contaminated ground water can be initiated, however, the extent and severity of the contaminant plume must be estimated.

Mathematical models of solute transport in porous media are useful tools in characterising the shape, size and position of a contaminant medium. Literatures have various references for homogeneous and heterogeneous solute movement and discussed the dispersion theory and established that dispersion is proportional to (velocity)^n, $1 \leq n \leq 2$ (Freeze and Cherry, 1979) where as other explained that the dispersion is directly proportional to velocity, i.e., $n = 1$ (Rumer, 1962). But in some works, the dispersion is squarely proportional to the velocity (i.e., $n = 2$) is considered (Taylor, 1953; Scheidegger, 1957). The analytical solution for solute movement for heterogeneous/homogeneous and layered medium followed by various litterateurs (Shamir and Harleman 1967; Serrono, 1996; Carnahan and Remer, 1984; Singh et al., 2008; Jaiswal et al., 2011; and Singh et al., 2014) were found through the transport equations with constant coefficients, prescribed with boundary and initial conditions and are solved for multi-dimensional (one, two and three) temporally/spatially domains. These solutions are being applicable to semi-infinite/infinite domain. However, for transport equations in heterogeneous media constant coefficient equation for solute movement is not suitable. The coefficients can be dependent on position and time, position/time. Relatively, some analytical results are available, especially for the case of finite media, with non-constant coefficients. The homogeneity/heterogeneity of the medium was discussed by various researchers (Lin, 1977; Valochhi, 1989; Matheron and de Marsily, 1980; van Genuchten and Alves, 1982; Yates, 1992). Further, some authors (Logan and Zlotnik, 1995; Hunt, 1998; Lotinopoulos et al., 1998; Zoppou and Knight, 1997) explained their works describing by scale-dependent dispersivity and periodic boundary conditions.

In the recent decade, a variety of problems related to hydro-environment research, the advection-diffusion equation (ADE) has been used. An analytical solution for non-homogeneous porous media has been obtained for ADE with space-dependent coefficients and formulated them (Guerrero and Skaggs, 2010). They found that, the obtained solutions for particular case of a linearly increasing dispersity are developed in detail and compared with solution from other published literature. Analytical solution for one dimensional advection diffusion equation with temporally dependent dispersion in a longitudinal semi-infinite homogenous porous media is obtained along with varying pulse-type point source through a porous medium of linear heterogeneity is taken (Singh et al., 2010; Yadav et al., 2011; Singh et al., 2012; Yadav et al., 2012). A nonlinear mathematical model to assess the effect of applied technological efforts on the conservation of forestry resources with two discrete time delays have been formulated and studied for the growth of forestry resources and achieve the sustainable development (Lata et al., 2017). Later on the, advection diffusion equation in infinite heterogeneous medium were discussed and found an analytical solution by using Green's function (Sanskrityayn and Kumar, 2016).

The impacts of landfill leachate on groundwater quality is investigated and understand the objective of groundwater quality and also explained the dynamic of trace metals in groundwater by using visual MODFLOW/MT3D along with delineates the different factors, controlling the availability and suitability of groundwater for drinking, agriculture and other purposes (Srivastava and Ramanathan, 2018). To increase fresh groundwater, pumping from production wells for beneficial use, the efficiency and viability of using artificial freshwater recharge (AFR) have been investigated. Various sets of position-temporally coefficients of the ADE have been reduced into constant coefficients to get the analytical solutions for semi-infinite and an infinite domain (Sanskrityayn and Kumar, 2018; Jaiswal and Gulrana, 2019; Thakur et al. 2019; Yadav and Kumar, 2019; Jaiswal et al., 2020; Kumar et al., 2020; Chaudhary and Singh, 2020). Gazal and Eslamian (2021) demonstrated that intensive land use in arid areas imposes tremendous pressure on groundwater. They discussed significant proportion of the case study area is hazardous to contaminants, demonstrated by the vulnerability and nitrate contamination risk maps with using remote sensing and GIS techniques. Fixed and moving boundary problems were also discussed (Rodrigo and Thamwattana, 2021) in which a unified approach have presented to solve both problems for one-dimensional heat equation. Analytical solutions have been obtained of the advection-dispersion equation with temporal coefficients and linearly moving pulse point sources using laplace integral transformation technique (LITT) (Yadav and Kumar, 2021; Jaiswal et al., 2022).

Present work deals with the heterogeneous semi-infinite medium (aquifer/air) which is not solute free initially, i.e., medium is not clean. Exponential decay source is considered at origin and concentration gradient is taken to be zero at the end of the medium. LITT is used to get an analytical solution for the problem. This work gives the prediction regarding contaminants concentration level in heterogeneous semi-infinite shallow aquifer with exponentially de-accelerating source of input concentration. The effects of various parameters are also considered in this study to get the physical insight of the problem.

2 Heterogeneous semi-infinite medium: analytical solution

The position where the contaminants meet upper most groundwater level is taken as origin. The linear advection-diffusion partial differential equation in one dimensional horizontal isotropic but heterogeneous medium in general form may be written as:

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D(x,t)\frac{\partial c}{\partial x} - u(x,t)c \right) \tag{1}$$

where, c is the solute/pollutant concentration (substances in land or water bodies like surface water, groundwater as in form of aquifers that are or potentially hazardous to the environment or human health) at a position x at time t, $D(x, t)$ represents the solute dispersion and $u(x, t)$ is velocity of the medium transporting the solute particles. At initially, the medium is not pure it may contain some contaminant. Decay type exponential source is assumed at one source and at the other end its gradient is taken to be zero. The pollutant decay and unsteady parameter is q whose dimension is inverse of the time $[T^{-1}]$. Initial and boundary conditions for considered problem are:

$$c(x,t) = c_i; t = 0, x \geq 0 \tag{2}$$

$$c(x,t) = c_0 \exp(-qt); x = 0, t \geq 0 \tag{3}$$

$$\frac{\partial c}{\partial x} = 0; x \to \infty, t \geq 0 \tag{4}$$

In present problem, seepage velocity is taken as both space dependent as well as temporally dependent due to heterogeneous medium. Being heterogeneous medium the source is taken as decay type exponential function because in practical case initially may be there is some contaminants that depend on some function of time. Dispersion is temporally dependent as well as spatially dependent where as the source is constant and discussed their results analytically in starting of decade (Jaiswal et al., 2009; Kumar et al., 2010).

2.1 Unsteady flow in heterogeneous medium:

The expression for velocity and dispersion in degenerate form are considered (Singh et al., 2012) as:

$$u(x,t) = u_0 f_1(mt)(1 + ax) \text{ and } D(x,t) = D_0 f_1^2(mt)(1 + ax)^2 \tag{5}$$

where a is the parameter of heterogeneity with dimension inverse of space variable, unsteadiness parameter is m and dimension of m is inverse of the time variable t. D_0 and u_0 referred as initial dispersion and velocity whose dimensions are $[L^2\ T^{-1}]$ and $[LT^{-1}]$. Using the expressions of dispersion coefficient and seepage velocity from equation (5), equation (1) will become

$$\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(D_0 f_1^2(mt)(1+ax)^2\frac{\partial c}{\partial x}-u_0 f_1(mt)(1+ax)c\right) \tag{6}$$

Or

$$\frac{1}{f_1(mt)}\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(D_0 f_1(mt)(1+ax)^2\frac{\partial c}{\partial x}-u_0(1+ax)c\right) \tag{7}$$

A new time variable, T (Crank, 1975) with decay type exponential function is taken for $f_1(mt) = \exp(-mt)$

$$T=\int_0^t f_1(mt)dt \tag{8}$$

The dimension of T is same as t. The value of $T = 0$ for $t = 0$ by taking a suitable form of $f_1(mt)$ so that the conditions are unaffected by new variable T. (Jaiswal et al., 2009; Kumar et al, 2010) introduced a space variable as,

$$X=\frac{1}{a}\log(1+ax) \tag{9}$$

In new frame of space and time, the problem and their conditions are

$$\frac{\partial c}{\partial T}=D_0 f_1(mt)\frac{\partial^2 c}{\partial X^2}-u_0 f_2(mt)\frac{\partial c}{\partial X}-au_0 c \tag{10}$$

$$c(X,T)=c_i; T=0, X\geq 0, \tag{11}$$

$$c(X,T)=c_0(1-qT); X=0, T\geq 0, \tag{12}$$

$$\frac{\partial c}{\partial X}=0; X\to\infty, T\geq 0, \tag{13}$$

where, $f_2(mt) = 1-\lambda f_1(mt)$ is another time dependent expression in non-dimensional variable mt and $\lambda = (aD_0/u_0)$ is non-dimensional coefficient. q and m are both unsteady parameters and have same dimension but notations are different according to place. Therefore q not replaced by m in equation (12) and have equal values for discussion in next section. To remove the decay term from (10), choose a transformation

$$c = Cexp(-au_0T) \tag{14}$$

With the help of above transformation equation (13), equation (10) will become

$$\frac{\partial C}{\partial T}=D_0 f_1(mt)\frac{\partial^2 C}{\partial X^2}-u_0 f_2(mt)\frac{\partial C}{\partial X} \tag{15}$$

Another frame of space and time may introduce by the transformation:

$$Z_1 = \frac{f_2(mt)}{f_1(mt)} X \tag{16}$$

and

$$\zeta_1 = \int_0^t f_2^2(mt)dt \tag{17}$$

or

$$\zeta_1 = (1/m)\left[-\log\left(1-mT\right) + (\lambda^2/2)\left\{1-(1-mT)^2\right\} - 2\lambda\{1-(1-mT)\}\right] \tag{18}$$

The time frame T has to be expressed in new time frame ζ_1 by using equations (8) and (17) as:

$$T = \gamma_1 \zeta_1 \tag{19}$$

where $\gamma_1 = (1-\lambda)^{-2}$.

Thus the advection–diffusion equation (15) and their conditions (11–13), in another space as well as time frame may be rewritten as:

$$\frac{\partial C}{\partial \zeta_1} = D_0 \frac{\partial^2 C}{\partial Z_1^2} - u_0 \frac{\partial C}{\partial Z_1} \tag{20}$$

$$C\left(Z_1, \zeta_1\right) = c_i ; \zeta_1 = 0, Z_1 \geq 0, \tag{21}$$

$$C\left(Z_1, \zeta_1\right) = c_0\left(1 - q\gamma_1\zeta_1\right) exp\left(A\zeta_1\right); Z_1 = 0; \zeta_1 \geq 0,\ A = au_0\gamma_1 \tag{22}$$

$$\frac{\partial C}{\partial Z_1} = 0; Z_1 \to \infty, \zeta_1 \geq 0. \tag{23}$$

To solve equation (20) with initial and boundary conditions equations (21–23), Laplace integral transform technique (LITT) is used. The analytical solution of ADE with prescribed initial and boundary conditions is

$$\begin{aligned} c(x,t) = c_i - \frac{c_i}{2}(C_1' + D_1') + \frac{c_0}{2}(C_1 + D_1) \\ - \frac{c_0 q \gamma_1}{4\beta}\left\{\left(2\beta\zeta_1 - Z_1\sqrt{\frac{1}{D_0}}\right)C_1 + \left(2\beta\zeta_1 + Z_1\sqrt{\frac{1}{D_0}}\right)D_1\right\} \end{aligned} \tag{24}$$

where

$$C_1 = \exp\left(\frac{u_0}{2D_0}Z_1 - \beta\sqrt{\frac{1}{D_0}}Z_1\right) erfc\left(\frac{Z_1}{2}\sqrt{\frac{1}{D_0\zeta_1}} - \beta\sqrt{\zeta_1}\right)$$

$$D_1 = \exp\left(\frac{u_0}{2D_0}Z_1 + \beta\sqrt{\frac{1}{D_0}}Z_1\right) erfc\left(\frac{Z_1}{2}\sqrt{\frac{1}{D_0\zeta_1}} + \beta\sqrt{\zeta_1}\right)$$

$$C_1' = \exp\left(\frac{u_0}{2D_0}Z_1 - \alpha\sqrt{\frac{1}{D_0}}Z_1\right) erfc\left(\frac{Z_1}{2}\sqrt{\frac{1}{D_0\zeta_1}} - \alpha\sqrt{\zeta_1}\right)$$

$$D_1' = \exp\left(\frac{u_0}{2D_0} Z_1 + \alpha\sqrt{\frac{1}{D_0}} Z_1\right) erfc\left(\frac{Z_1}{2}\sqrt{\frac{1}{D_0\zeta_1}} + \alpha\sqrt{\zeta_1}\right)$$

$$\beta^2 = A + \frac{u_0^2}{4D_0}; A = au_0\gamma_1; \alpha^2 = u_0^2 / 4D_0$$

$$Z_1 = \frac{f_2(mt)}{f_1(mt)} X; X = \frac{1}{a}\log(1+ax); f_2(mt) = 1 - \lambda f_1(mt); \zeta_1 = T / \gamma_1; \gamma_1 = (1-\lambda)^{-2}$$

$$\lambda = \frac{aD_0}{u_0}; T = \int_0^t f_1(mt)dt; f_1(mt) = \exp(-mt).$$

For exponentially decay type input condition solute transport model is solved and its solution is expressed by (24).

2.2 *An Analytical solution for the heterogeneous medium if initially it is solute free*

The solution of the above problem, i.e., if the medium is solute free initially ($c_i = 0$), is as

$$c(x,t) = \frac{c_0}{2}(C_1 + D_1) - \left(\frac{c_0 q\gamma_1}{4\beta}\right)\left\{\left(2\beta\zeta_1 - Z_1\sqrt{\frac{1}{D_0}}\right)C_1 + \left(2\beta\zeta_1 + Z_1\sqrt{\frac{1}{D_0}}\right)D_1\right\} \quad (25)$$

3 Result and discussion

Analytical solutions are obtained for one dimensional ADE in presence of initial source and in absence of initial source by equation (24) and equation (25). The concentration values (c) for equation (24) are evaluated with the input values: reference concentration (c_0) = 1, initial velocity (u_0) = 0.61 (km/year), initial dispersivity (D_0) = 0.71 (km^2/year), heterogeneity parameter (a) = 0.15 (km^{-1}), unsteady parameter (m) = 0.1 (km^{-1}), and q = 0.1 (km^{-1}). Initial source concentration (c_i) = 0.5. Concentration attenuation with time and space for x = 0 to 1 (*km*) are discussed, at t = 2.5, 3.0 and 3.5 where t taken in years. The domain's length is considered $\ell =$ 1.0 km, then the Peclet number, $P_e = (u\ell / D) = 0.859$. Distribution of concentration is illustrated from Figure 1 with solid line are drawn for decelerating flow $f_1(mt) = \exp(-mt)$ and dashed lines are drawn for sinusoidal flow field as $f = (1-\sin(mt)$. A sinusoidal function is of periodic and oscillate between higher and lower value concentration. This function shows real situation in distribution of solute concentrations in tropical reasons. It is predicted that, the sinusoidal source is disperse faster than exponentially source concentration which can be show in tropical region. It is also observed from Figure 2, the concentration (c) at origin diminishes with respect to time. Figure 2 is drawn for sinusoidal and exponentially accelerating source concentration. In both the Figure 1 and Figure 2, the diffusion of sinusoidal source concentration with increasing position and time is faster than that of exponential source.

Figure 1 Distribution of solute concentration for sinusoidal and exponentially decay function for heterogeneity parameter (a) = 0.15 (km^{-1}) at different time (year)

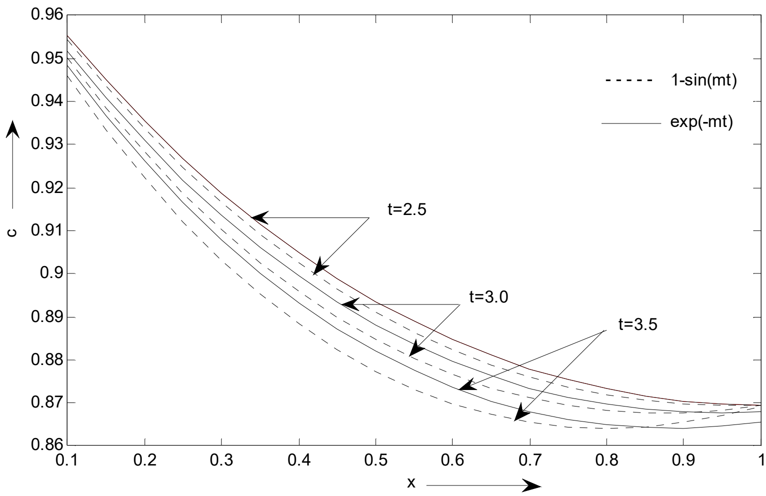
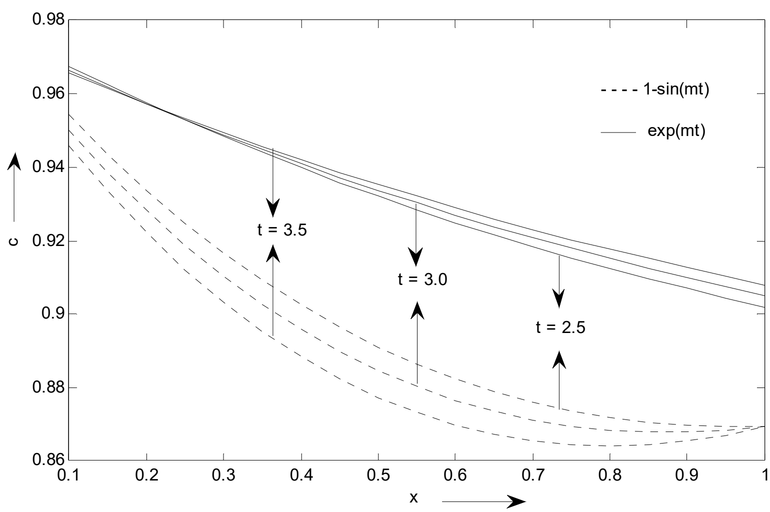


Figure 2 Distributions of solute concentration for sinusoidal and accelerating exponentially function for heterogeneity parameter (a) = 0.15 (km^{-1}) at different time (year)



In Figure 3, is plotted for accelerating and de accelerating function of concentration and the nature of the input concentration almost like as the Figure 2. Figure 4 shows the effect of heterogeneity. The concentration values of the solution (24) are calculated for different values of t and a at $t = 2.5$ and $a = 0.2, 0.25, 0.3$, for two sources, i.e., for sinusoidal and the exponential de-accelerating flow field. It is observed that the nature of the both source functions is not change with heterogeneity. It is clear that the sinusoidal source is much more suitable for tropical region where as for non-tropical region accelerating/de-accelerating exponential source, linear and constant input sources are good. For analytical solution (25), the natures of input sources are unaltered. We conclude the if there is no source, i.e., if medium (aquifer/air) is solute free then its concentration decreases much faster than in presence of the source due to diffusion and velocity which are of variable coefficient. It is very clear from the solutions (24) and (25) that, in the presence of source, the rehabilitation is slower than in absence of source, i.e., with source its concentration decrease slowly in comparison to absence of source. The concentration pattern with various time are predicted in three figures and show distribution of contaminants with increasing space position.

Figure 3 Distributions of solute concentration for decay and accelerating exponentially function for heterogeneity parameter (a) = 0.15 (km^{-1}) at different time (year)

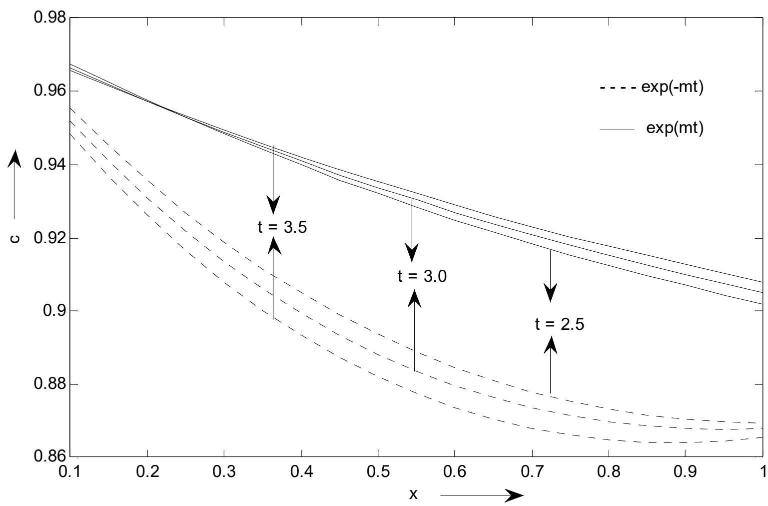
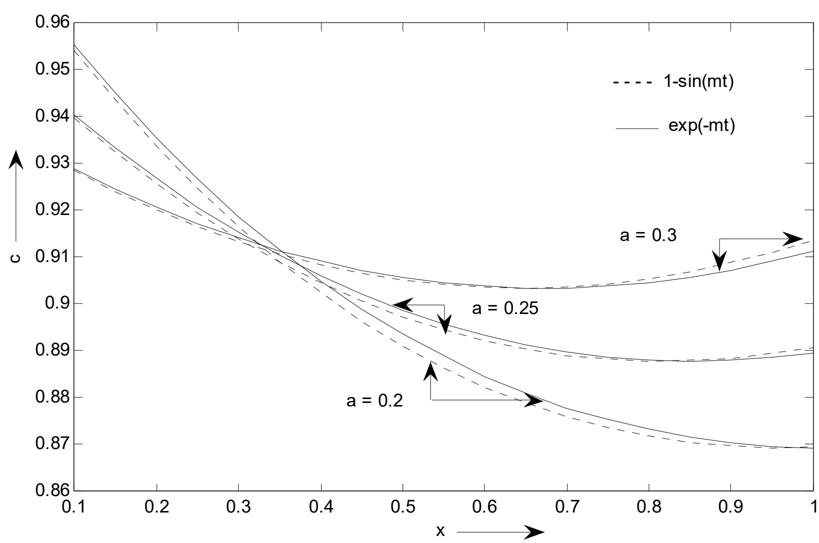


Figure 4 Distributions of solute concentration for sinusoidal and decay exponentially function for different heterogeneity parameter (a) at one time t = 2.5 year



4 Conclusions

An analytical solution of ADE for one dimensional in heterogeneous medium is obtained with the help of LITT. The concentration at origin is uniform source of exponentially de-accelerating type. Such type of situations appears at municipal sewage treatment plant area, garbage disposal site of factory, etc. where concentration of contaminant is of decreasing nature. The seepage velocity and diffusion are both taken into account in degenerate form, reflected in equation (5). By the use of appropriate new position and time variable transformation, variable form of ADE is converted into constant form. Velocity change reflects the impact of time dependency as the heterogeneous effect which will also vary by taking different values in account for both parameters. In this paper we observed that, the profile of concentration decreases and its rehabilitation process take time due to presence of initial concentration of contaminants. Also we observed that, the contaminant concentration decreases gradually in sinusoidal form function in compared of exponential function. It means rehabilitation processes is faster in sinusoidal form function than exponential function. Impacts of inhomogeneous parameter a are considered and their effect are depicted in the graph. Analytical solutions validate the numerical solutions. The numerical codes like MATLAB pdepe and others like FEFLOW, MODFLOW, MT3DMS, are found suitable to get the numerical solution of the proposed problem because each such code is developed with fixed ends of the space domain.

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