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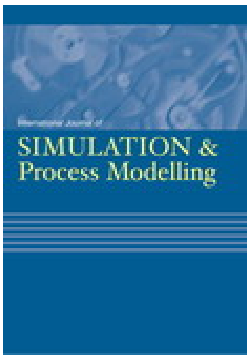

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Comparative analysis of performability and maintenance decision matrix for a repairable industrial system

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Abstract: This paper reveals the comparative analysis of performability and maintenance decision matrix (MDM) for a repairable industrial system. Wrapper forming machine, eye rolling machine, lap cutting machine, and eye forming rolling machine are the four subsystems that comprise this system. The modelling and performability analysis among various subsystems has been done using both stochastic Petri nets (SPN) approach and Markov method. The present study used the SPN as an advanced performance modelling tool and validates their results using Markov method. In addition, a MDM has been purposed which indicates the maintenance priority among various subsystems. This matrix facilitates decision making for various maintenance activities. The results of this study can be useful to maintenance management in developing a maintenance plan and procedures that achieve the ultimate goal of economical maintenance and high level of system availability.

Keywords: performability; availability; maintenance decision matrix; Markov method; stochastic Petri nets; SPN.

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1 Introduction

Modern automation requires the utilisation of state-of-the-art equipment to achieve the desired production goals of process industries. Performability and maintainability became more crucial to preserving the market's equilibrium between demand and supply. Reliability, availability, maintainability and safety (RAMS) plays an essential role to improve the system performability. Failure of these complex systems cannot be completely avoided but it can be reduced by appropriate preventative maintenance and regular inspections. This helps to reduce the plant overall operating costs (Dahiya et al., 2019; Gupta et al., 2008). The manufacturing industry faces formidable challenges in maintaining profitability and a competitive edge due to market competition and globalisation. With time and use of the machinery, the industrial system will deteriorate and wear down. The deterioration and wear might raise the possibility of machine failure and lower the output of the system (Lu et al., 2016). On the other hand, poor maintenance procedures can lead to expensive repairs and extend the system outages, which reduce the system availability and results in large production losses (Su and Liu, 2020). There are many tools and techniques for performability evaluation, but simulation-based Petri nets and Markov method are mostly used by the researchers. The novelty of this research work to analyses the system performability by using the Petri nets as an advance performance modelling tool and validates their results using Markov method.

India's automobile industry contributes significantly to the growth of the national economy. The sector's contribution to the national GDP increased from 2.77 to 7.1% between 1992 and 2021. It directly and indirectly employs around 19 million people (Economic Survey of India, 2022-23). A leaf spring is a crucial component of a vehicle assembly. This eye rolling line system consists of four subsystems such as: wrapper forming machine, eye rolling machine, lap cutting machine, and eye forming rolling machine. These subsystems are placed in combinations of both series and parallel arrangements. Performability analysis in terms of availability among various subsystems has been done by using simulation modelling, i.e., stochastic Petri nets (SPN) and a probabilistic approach, i.e., Markov method. In this research work, performability analysis in terms of availability was done. Further, a MDM has been developed which facilitates the decision making to various maintenance activities.

1.1 Markov method

Failure rate, repair rate and probability restored to an available state are the factors of interest in the reliability and availability study. State-space (or state-time) analysis can be used to determine the probabilities that a system having two states, i.e., failed or non-failed. The probabilities connected to these states can be defined either discretely or continuously. The most popular approach to state-space analysis is Markov analysis. Under the major restrictions listed below, the Markov approach can be used:

1. The process must be homogenous and the probability of transitioning from one condition to another must be constant. As a result, the technique can only be applied when the rate of repair and failure is constant.
2. Systems will independent of all past and future states except the most recent one. When evaluating repairable systems, this constraint is essential since it shows that a repair restores the system to its original state.

This method is reliable to use for availability analysis of complex systems as compare to reliability block diagram (RBD) and fault tree analysis (FTA).

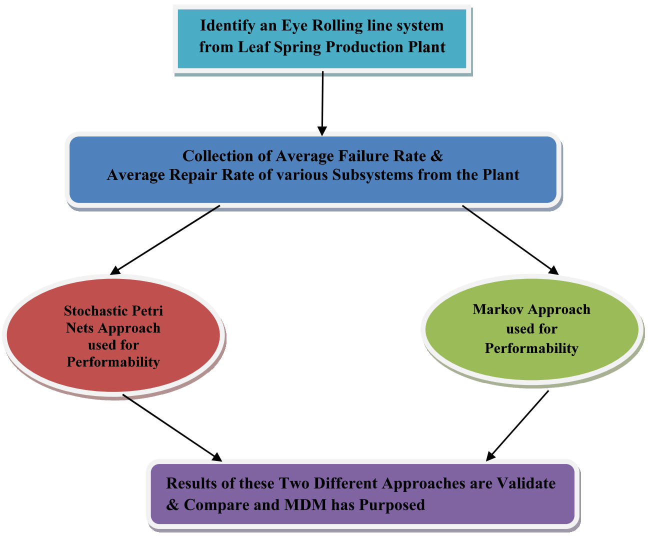
1.2 Petri nets

Carl Adam Petri developed PN in 1962 as an advancement of state-space analysis methods. In this method, mathematical and pictorial tools are used to define relationships between different conditions. Since time was not considered in the original PN, so enabled transitions fires immediately. A variant known as stochastic Petri nets (SPN) or time Petri nets (PN) was introduced in the late 1970s. This method has been improved upon numerous times with additional modelling and analysis features. Generalised stochastic petri nets (GSPN) were explored as an extension of SPN that explained transitions of PN in two types, immediate and timed transitions (Marsan et al., 1984). Extended stochastic petri net (ESPN), a kind of SPN that was expanded in 1984 by Dugan et al. was discussed (Dugan et al., 1984). A further development of the GSPN is the deterministic stochastic Petri net (DSPN) and coloured Petri net (CPN) which was introduced by Jensen (1981).

2 Literature review

Garg et al. (2010) discussed the model for blackboard manufacturing plant to assess the maintenance priorities and availability using the Runge-Kutta method. Kleyner and Volovoi (2010) developed the model using SPN for the availability analysis of electronic airbag controllers. Yang et al. (2011) proposed PN model for diagnosing faults in wind turbines. Gupta and Tewari (2010) performed probabilistic modelling and analysis for steam generator system of a power plant. Adhikary et al. (2012) have done RAM analysis for determine preventive maintenance programs of thermal power plant to improve the availability. Thangamani (2012) deals with availability analysis of lubricant systems used in combined cycle power plants using SPN. Latorre and Jiménez (2013) discussed the decision and optimisation problems using PN modelling. Kumar (2014) performed an availability analysis of air circulation systems using Markov models. Jolly and Singh (2014) demonstrated an approach to reduce downtime for repairable systems. Bosse et al. (2016) developed a PN simulation to evaluate the availability of IT systems. Wan et al. (2016) synthesised stochastic models using Markov processes in electronic systems (ES). Jiménez-Macías et al. (2017) used coloured Petri nets (CPN) for modelling of discrete event systems. Alizadeh and Sriramula (2017) used Markov analysis to develop a model for security systems. Don and Khan (2019) discussed a methodology for checking errors using hidden Markov methods. Kumar et al. (2019, 2020) attempted to use his PN approach for milk processing plant and plywood production plant to enhance system performability. Zhang (2021) developed a Markov model to determine the reliability of heat exchangers. Kumar et al. (2021) used the PN approach to perform performance analysis of complex manufacturing systems. Modgil (2022) presented simulation modelling and analysis of the container manufacturing industry using (PN). Malik et al. (2022) and Malik and Tewari (2023) improved the performance of thermal power plants for steam generation system and coal ash handling plant based on the Markov model. Optimisation of the results has been done using the particle swarm optimisation (PSO) technique. The methodology of this research work clearly shown in Figure 1.

Figure 1 Graphical representation of methodology used in this research work



3 System description

In the present work, a leaf spring producing plant situated in north India has been chosen as the case study. Eye rolling line (as shown in Figure 2) is considered as a system which comprises the following subsystems:

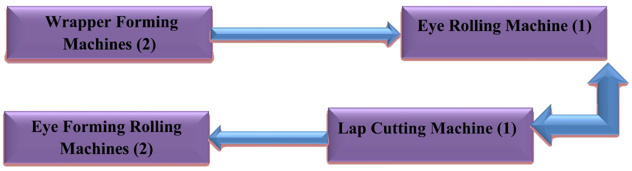
1 *Wrapper forming machine:* there are two wrapper forming machine arranged in the parallel in this subsystem. If one of the machines fails, the system goes into the reduced capacity.

2 *Eye rolling machine:* this subsystem has a single machine; failure of this machine also leads to failure of the complete system.

3 *Lap cutting machine:* this is a single-machine subsystem. A machine failure causes the entire system to fail because no redundancy is provided.

4 *Eye forming rolling machine:* this subsystem has two eye forming rolling machines in parallel arrangement. Failure of either machine reduces system capacity.

Figure 2 Eye rolling line system of leaf spring plant (see online version for colours)



4 Performance modelling of the eye rolling line system using SPN

SPN modelling of the eye rolling line of leaf spring manufacturing plant has been discussed in this section. SPN are used to show interactions among different subsystems. In the present modelling, two persons are available at repair facility.

Following assumptions are used to simulate the performance modelling of the system:

- the system may operate at reduced capacity
- standby system have similar status to active system
- repair services can be started without any delay
- failure and repair patterns follow an exponential distribution
- restored subsystem is performing as good as new subsystem.

Places:

- *SYSA:* indicates the availability of the entire system in upstate
- *SYSWFC:* indicates working of the system in full capacity
- *SYSWRC:* indicates working of the system in reduced capacity.
- *SYSUA:* indicates the unavailability of the entire system in downstate
- *RFA:* indicates the availability of repair facility
- *WFM up, ERM up, LCM Up, EFRM Up:* represents the upstate of wrapper forming machine, eye rolling machine, lap cutting machine and eye forming rolling machine
- *WFM down, ERM down, LCM Down, EFRM Down:* represents the down state (non-working state) of wrapper forming machine, eye rolling machine, lap cutting machine and eye forming rolling machine
- *WFM Rep, ERM Rep, LCM Rep, EFRM Rep:* represents repaired states of wrapper forming machine, eye rolling machine, lap cutting machine and eye forming rolling machine.

Transitions:

- *SYS Ok:* represents system is works with full capacity
- *SYS Rec:* represents system recovered to full capacity
- *SYS Fail:* represents the downstate of system
- *SYS Red:* represents the system working, but in reduced capacity
- *WFM Fail, ERM Fail, LCM Fail, EFRM Fail:* represents timid transitions associated with failure pattern of wrapper forming machine, eye rolling machine, lap cutting machine and eye forming rolling machine
- *WFM Ok, ERM Ok, LCM Ok, EFRM Ok:* represents timid transitions associated with failure pattern of wrapper forming machine, eye rolling machine, lap cutting machine and eye forming rolling machine
- *WFM Rep, ERM Rep, LCM Rep, EFRM Rep:* represents immediate transitions associated with availability of repair facility for wrapper forming machine, eye rolling machine, lap cutting machine and eye forming rolling machine.

Guard functions:

The associated guard functions for different transitions are given below:

[G1] := (#2 > 0 and #13 > 0) enables the transition *Rep Available WFM*.

[G2] := (#5 > 0 and #13 > 0) enables the transition *Rep Available ERM*.

[G3] := (#8 > 0 and #13 > 0) enables the transition *Rep Available LCM*.

[G4] := (#11 > 0 and #13 > 0) enables the transition *Rep Available EFRM*.

[G5] := (#1 < 1 or #4 < 1 or #7 < 1 or #10 < 1) enables the transition *SYS Fail*.

[G6] := (#1 > 0 and #4 > 0 and #7 > 0 and #10 > 0) disables the transition *SYS Ok*.

[G7] := (#1 < 2 and #1 > 0 or #10 < 2 and #10 > 0) enables the transition *SYS Red*.

[G8] := (#1 > 1 and #10 > 1) disables the transition *SYS Rec*.

Figure 3 Modelling of eye rolling line system of leaf spring plant using stochastic Petri nets (see online version for colours)

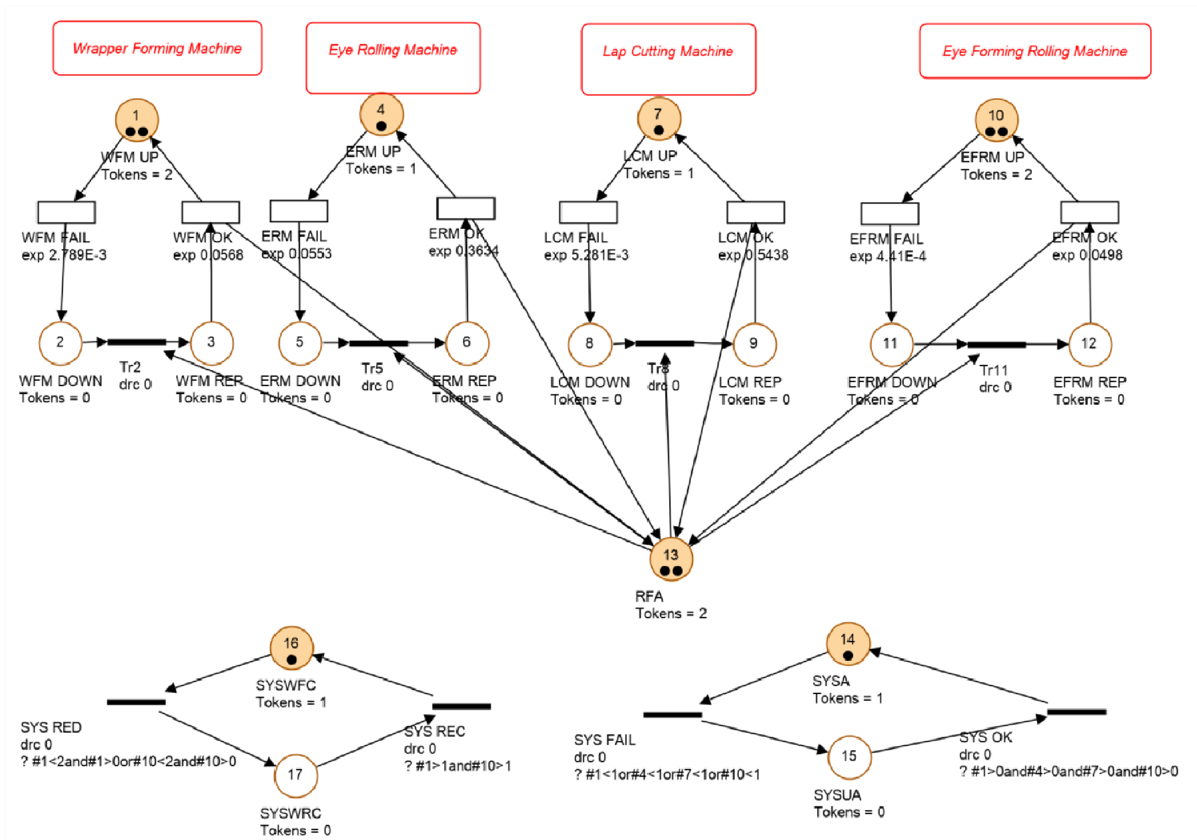


Table 1 Repair and failure data of eye rolling line system

SUBSYSTEM	*Exponentially distributed*	
	Average failure rate (θ)	*Average repair rate (φ)*
Wrapper forming	0.002789	0.05681
Eye rolling	0.055310	0.36340
Lap cutting	0.005281	0.54380
Eye forming rolling	0.000441	0.04980

The modelling of eye rolling line system of leaf spring plant using stochastic Petri nets software is shown in Figure 3. The repair and failure data was obtained from maintenance log book of the plant with the help of supervisors and maintenance personnel. It is presented in Table 1.

Licensed GRIF predicates Petri module 2022 was used to obtain the performability matrix for the various subsystems, and the results are shown in Tables 2 to 5. MATLAB software produces accurate and 3D-quality visualisations, which are illustrated in Figures 4 to 7.

Table 2 Performability matrix for wrapper forming machine

φ_1 / θ_1	*0.01681*	*0.03681*	*0.05681*	*0.07681*	*0.09681*	*Constant parameters*
0.000989	0.8564	0.8586	0.8589	0.8590	0.8592	$\theta_2 = 0.055310$, $\varphi_2 = 0.036340$
0.001889	0.8489	0.8569	0.8583	0.8588	0.8590	$\theta_3 = 0.005281$, $\varphi_3 = 0.54380$
0.002789	0.8385	0.8545	0.8572	0.8581	0.8586	$\theta_4 = 0.000441$, $\varphi_4 = 0.04980$
0.003689	0.8093	0.8462	0.8534	0.8559	0.8571	
0.004589	0.7883	0.8265	0.8334	0.8457	0.8476	

Figure 4 Variation of repair and failure rates of wrapper forming machine on system performability (see online version for colours)

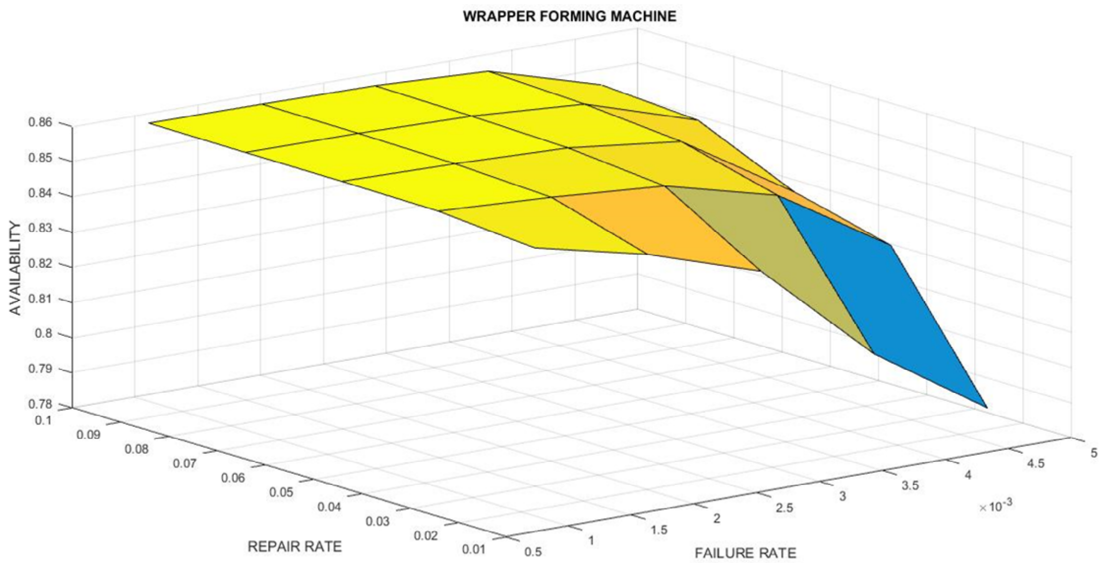


Table 3 Performability matrix for eye rolling machine

φ_2 / θ_2	*0.16340*	*0.36340*	*0.56340*	*0.76340*	*0.96340*	*Constant parameters*
0.015310	0.9029	0.9478	0.9618	0.9685	0.9725	$\theta_1 = 0.002789$, $\varphi_1 = 0.05681$
0.035310	0.8120	0.9003	0.9296	0.9443	0.9531	$\theta_3 = 0.005281$, $\varphi_3 = 0.54380$
0.055310	0.7380	0.8572	0.8995	0.9211	0.9343	$\theta_4 = 0.000441$, $\varphi_4 = 0.04980$
0.075310	0.6759	0.8161	0.8735	0.8988	0.9170	
0.095310	0.6237	0.7817	0.8449	0.8768	0.8976	

Figure 5 Variation of repair and failure rates of eye rolling machine on system performability (see online version for colours)

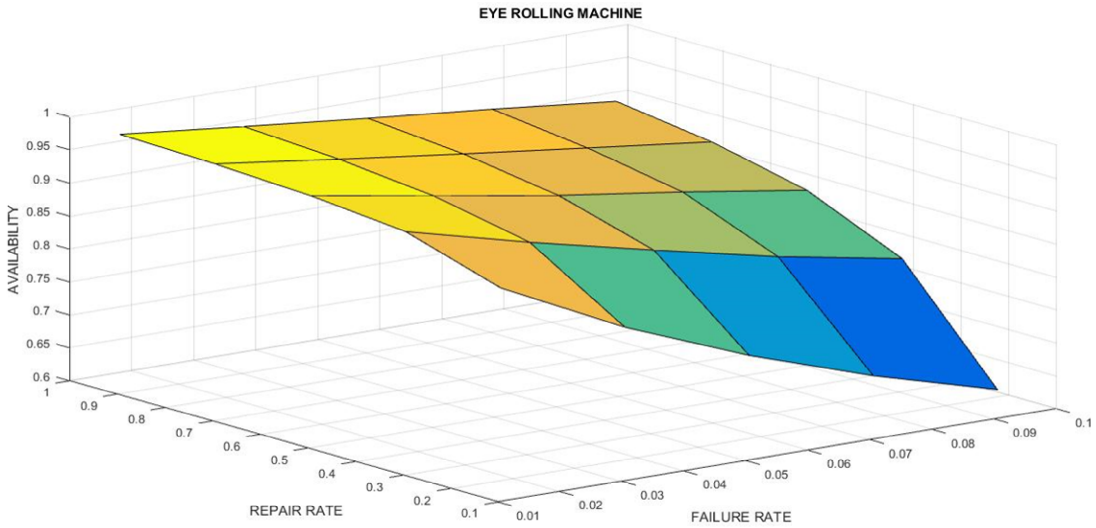


Table 4 Performability matrix for lap cutting machine

θ_3 \ φ_3	*0.01438*	*0.03438*	*0.05438*	*0.07438*	*0.09438*	*Constant parameters*
0.001281	0.8579	0.8624	0.8636	0.8642	0.8644	$\theta_1 = 0.002789$, $\varphi_1 = 0.05681$
0.003281	0.8466	0.8579	0.8607	0.8621	0.8630	$\theta_2 = 0.055310$, $\varphi_2 = 0.036340$
0.005281	0.8352	0.8527	0.8572	0.8594	0.8608	$\theta_4 = 0.000441$, $\varphi_4 = 0.04980$
0.007281	0.8251	0.8477	0.8541	0.8568	0.8586	
0.009281	0.8133	0.8432	0.8508	0.8542	0.8568	

Figure 6 Variation of repair and failure rates of lap cutting machine on system performability (see online version for colours)

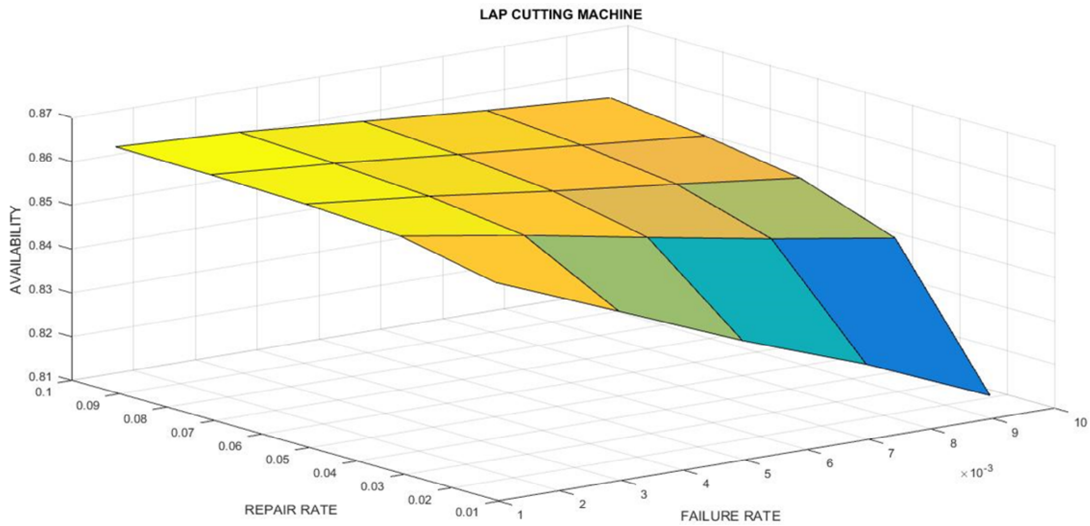


Table 5 Performability matrix for eye forming rolling machine

θ_4 \ φ_4	*0.00980*	*0.02980*	*0.04980*	*0.06980*	*0.08980*	*Constant parameters*
0.000041	0.8621	0.8623	0.8627	0.8631	0.8638	$\theta_1 = 0.002789$, $\varphi_1 = 0.05681$
0.000241	0.8588	0.8595	0.8597	0.8609	0.8613	$\theta_2 = 0.055310$, $\varphi_2 = 0.036340$
0.000441	0.8550	0.8570	0.8572	0.8576	0.8578	$\theta_3 = 0.005281$, $\varphi_3 = 0.54380$
0.000641	0.8516	0.8522	0.8528	0.8542	0.8545	
0.000841	0.8487	0.8491	0.8498	0.8508	0.8512	

Figure 7 Variation of repair and failure rates of eye forming rolling machine on system performability (see online version for colours)

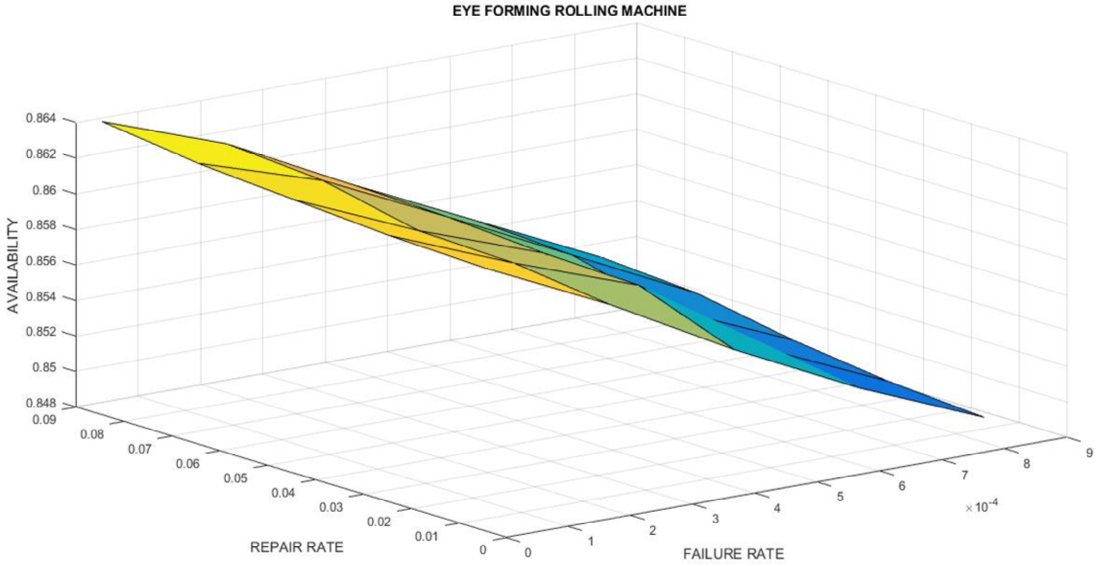
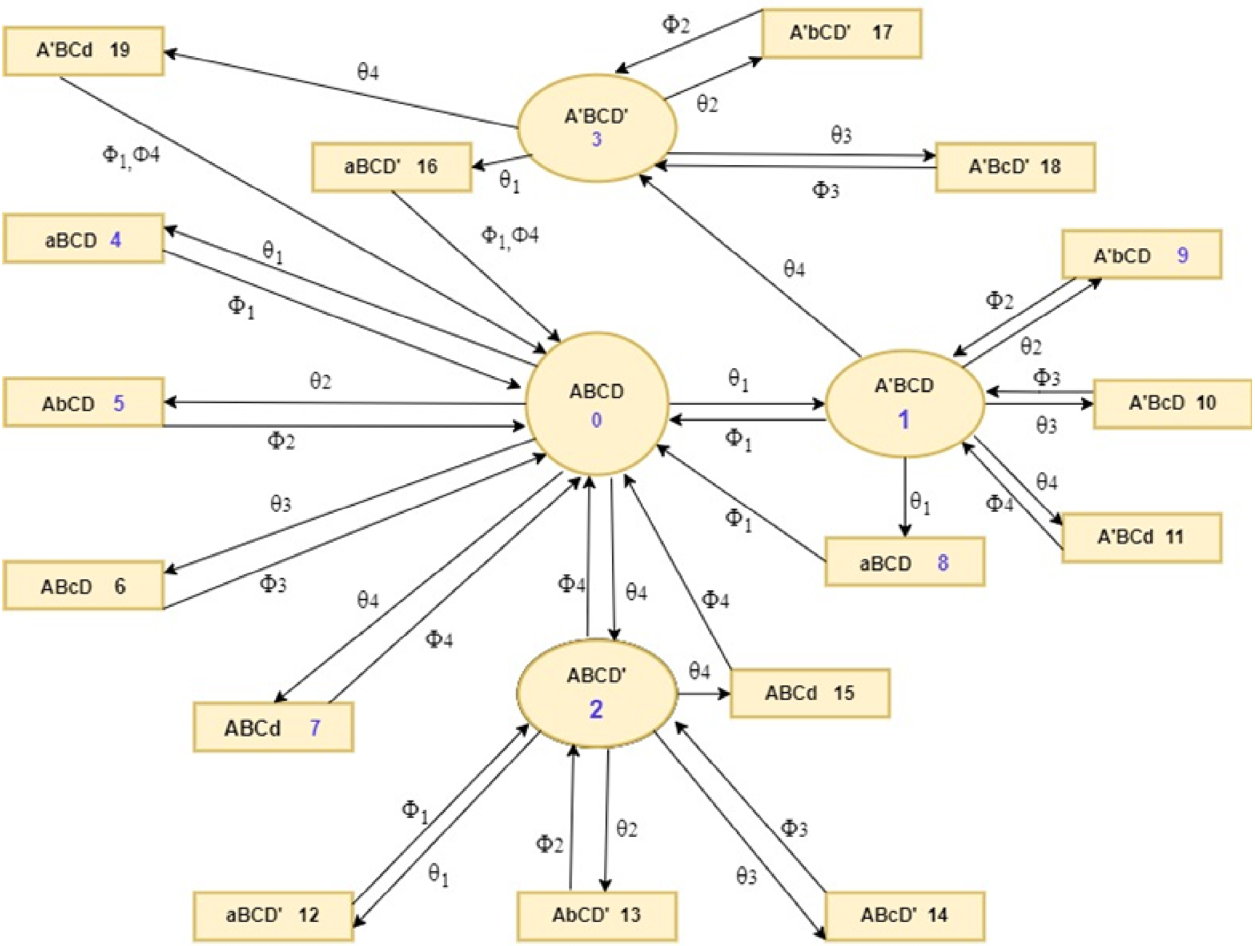


Figure 8 Transitions diagram of eye rolling line system for leaf spring plant (see online version for colours)



5 Performance modelling of the eye rolling line system using Markov method

5.1 Mathematical formulation of the system

A mathematical formulation applying mnemonic rules has been used to establish the viability of an eye rolling line system of a leaf spring manufacturing facility for all of the subsystems. The transition diagram in Figure 8 has been used to create a set of differential equations. The notations which are used in this diagram are discussed below:

- A, B, C and D: indicates subsystems are working in full capacity
- A′ and D′: indicates subsystems are working in reduce capacity
- a ,b, c and d: indicates subsystem are in failed state
- state 0: represents system working in full capacity
- states 1, 2 and 3: represents system working in reduced capacity
- states 4, 5, …, 19: represent system in failed state.
- θ_1, θ_2, θ_3, θ_4: represents the mean failure rate of different subsystems.
- φ_1, φ_2, φ_3, φ_4: represents the repair rate of different subsystems.

5.1.1 Transient state

$$\begin{aligned}&A_0(t+\Delta t)-A_0(t)\\&=[-\theta_1\Delta t-\theta_1\Delta t-\theta_2\Delta t-\theta_3\Delta t-\theta_4\Delta t-\theta_4\Delta t]A_0(t)\\&+A_1(t)\varphi_1\Delta t+A_2(t)\varphi_4\Delta t+A_4(t)\varphi_1\Delta t+A_5(t)\varphi_2\Delta t\\&+A_6(t)\varphi_3\Delta t+A_7(t)\varphi_4\Delta t+A_8(t)\varphi_1\Delta t+A_{15}(t)\varphi_4\Delta t\\&+A_{16}(t)\varphi_1\Delta t+A_{16}(t)\varphi_4\Delta t+A_{19}(t)\varphi_1\Delta t+A_{19}(t)\varphi_4\Delta t\end{aligned}$$

Dividing both sides by Δt, we get:

$$\begin{aligned}&[A_0(t+\Delta t)-A_0(t)]/\Delta t\\&=[-2\theta_1-\theta_2-\theta_3-2\theta_4]A_0(t)+A_1(t)\varphi_1+A_2(t)\varphi_4\\&+A_4(t)\varphi_1+A_5(t)\varphi_2+A_6(t)\varphi_3+A_7(t)\varphi_4+A_8(t)\varphi_1\\&+A_{15}(t)\varphi_4+A_{16}(t)\varphi_1+A_{16}(t)\varphi_4+A_{19}(t)\varphi_1+A_{19}(t)\varphi_4\end{aligned}$$

On taking limit as $\Delta t \to 0$, this yield:

$$\begin{aligned}A_0'(t)=&-K_0A_0(t)+A_1(t)\varphi_1+A_2(t)\varphi_4+A_4(t)\varphi_1\\&+A_5(t)\varphi_2+A_6(t)\varphi_3+A_7(t)\varphi_4+A_8(t)\varphi_1\\&+A_{15}(t)\varphi_4+A_{16}(t)\varphi_1+A_{16}(t)\varphi_4+A_{19}(t)\varphi_1\\&+A_{19}(t)\varphi_4\end{aligned}$$

$$\begin{aligned}A_0'(t)+K_0A_0(t)=&\varphi_1A_1+\varphi_4A_2+\varphi_1A_4+\varphi_2A_5\\&+\varphi_3A_6+\varphi_4A_7+\varphi_1A_8+\varphi_4A_{15}\\&+\varphi_1A_{16}+\varphi_4A_{16}+\varphi_1A_{19}+\varphi_4A_{19}\end{aligned} \quad (1)$$

Similarly,

$$\begin{aligned}A_1'(t)+K_1A_1(t)=&\theta_1A_0(t)+\varphi_2A_9(t)+\varphi_3A_{10}(t)\\&+\varphi_4A_{11}(t)\end{aligned} \quad (2)$$

$$\begin{aligned}A_2'(t)+K_2A_2(t)=&\theta_4A_0(t)+\varphi_1A_{12}(t)+\varphi_2A_{13}(t)\\&+\varphi_3A_{14}(t)\end{aligned} \quad (3)$$

$$A_3'(t)+K_3A_3(t)=\theta_4A_1(t)+\varphi_2A_{17}(t)+\varphi_3A_{18}(t) \quad (4)$$

Here,

$$K_0=2\theta_1+\theta_2+\theta_3+2\theta_4$$

$$K_1=\theta_1+\theta_2+\theta_3+2\theta_4+\varphi_1$$

$$K_2=\theta_1+\theta_2+\theta_3+\theta_4+\varphi_4$$

$$K_3=\theta_1+\theta_2+\theta_3+\theta_4$$

$$A_i'(t)+\varphi_jA_i(t)=\theta_jA_0(t), \quad (5)$$

where i = 4, 5, 6, 7; j = 1, 2, 3, 4.

$$A_8'(t)+\varphi_1A_8(t)=\theta_1A_0(t), \quad (6)$$

$$A_i'(t)+\varphi_jA_i(t)=\theta_jA_1(t), \quad (7)$$

where i = 9, 10, 11; j = 2, 3, 4

$$A_i'(t)+\varphi_jA_i(t)=\theta_jA_2(t), \quad (8)$$

where i = 12, 13, 14, 15; j = 1, 2, 3, 4

$$A_{16}'(t)+\varphi_1A_{16}(t)+\varphi_4A_{16}(t)=\theta_1A_3(t) \quad (9)$$

$$A_i'(t)+\varphi_jA_i(t)=\theta_jA_3(t), \quad (10)$$

where i = 17, 18; j = 2, 3

$$A_{19}'(t)+\varphi_1A_{16}(t)+\varphi_4A_{16}(t)=\theta_1A_3(t) \quad (11)$$

5.1.2 Steady state

Steady state probabilities of the system can be obtained by imposing the condition that as $t \to \infty$, $d/dt \to 0$.

In this state, equations (1) to (11) reduce to the following system of equations.

$$\begin{aligned}K_0A_0(t)=&\varphi_1A_1+\varphi_4A_2+\varphi_1A_4+\varphi_2A_5+\varphi_3A_6\\&+\varphi_4A_7+\varphi_1A_8+\varphi_4A_{15}+\varphi_1A_{16}+\varphi_4A_{16}\\&+\varphi_1A_{19}+\varphi_4A_{19}\end{aligned} \quad (12)$$

Similarly,

$$K_1A_1(t)=\theta_1A_0(t)+\varphi_2A_9(t)+\varphi_3A_{10}(t)+\varphi_4A_{11}(t) \quad (13)$$

$$K_2A_2(t)=\theta_4A_0(t)+\varphi_1A_{12}(t)+\varphi_2A_{13}(t)+\varphi_3A_{14}(t) \quad (14)$$

$$K_3A_3(t)=\theta_4A_1(t)+\varphi_2A_{17}(t)+\varphi_3A_{18}(t) \quad (15)$$

$$A_i'(t)+\varphi_jA_i(t)=\theta_jA_0(t), \quad (16)$$

where i = 4, 5, 6, 7; j = 1, 2, 3, 4

$$A_8'(t)+\varphi_1A_8(t)=\theta_1A_0(t),$$

$$A_i'(t)+\varphi_jA_i(t)=\theta_jA_1(t), \quad (18)$$

where i = 9, 10, 11; j = 2, 3, 4

$$A_i'(t)+\varphi_jA_i(t)=\theta_jA_2(t), \quad (19)$$

where i = 12, 13, 14, 15; j = 1, 2, 3, 4

$$A'_{16}(t)+\varphi_1 A_{16}(t)+\varphi_4 A_{16}(t)=\theta_1 A_3(t) \quad (20)$$

$$A'_i(t)+\varphi_j A_i(t)=\theta_j A_3(t), \quad (21)$$

where i = 17, 18; j = 2, 3

$$A'_{19}(t)+\varphi_1 A_{19}(t)+\varphi_4 A_{19}(t)=\theta_4 A_3(t) \quad (22)$$

Solving these equations recursively and we get:

$$A_1=\left[\theta_1/(\varphi_1+2\theta_1+\theta_4)\right]A_0;$$

$$A_2=\left[\theta_4/(\varphi_4+\theta_4)\right]A_0;$$

$$A_3=\left[(\theta_1\theta_4)/(\theta_4+\theta_1)(\varphi_1+2\theta_1+\theta_4)\right]A_0;$$

$$A_4=[\theta_1/\varphi_1]A_0;\ A_5=[\theta_2/\varphi_2]A_0;\ A_6=[\theta_3/\varphi_3]A_0;$$

$$A_7=[\theta_4/\varphi_4]A_0;\ A_8=[\theta_1/\varphi_1]A_0;$$

$$A_9=[\theta_2/\varphi_2]\ \left[\theta_1/(\varphi_1+\theta_1+\theta_4)\right]A_0;$$

$$A_{10}=[\theta_3/\varphi_3]\left[\theta_1/(\varphi_1+2\theta_1+\theta_4)\right]A_0;$$

$$A_{11}=[\theta_4/\varphi_4]\left[\theta_1/(\varphi_1+2\theta_1+\theta_4)\right]A_0;$$

$$A_{12}=[\theta_1/\varphi_1]\left[\theta_4/(\varphi_4+\theta_4)\right]A_0;$$

$$A_{13}=[\theta_2/\varphi_2]\left[\theta_4/(\varphi_4+\theta_4)\right]A_0;$$

$$A_{14}=[\theta_3/\varphi_3]\left[\theta_4/(\varphi_4+\theta_4)\right]A_0;$$

$$A_{15}=[\theta_4/\varphi_4]\left[\theta_4/(\varphi_4+\theta_4)\right]A_0;$$

$$A_{16}=\left[\theta_1/(\varphi_1+2\theta_1+\theta_4)\right]\left[\theta_4/(\theta_1+\theta_4)\right]\left[\theta_1/(\varphi_1+\varphi_4)\right]A_0;$$

$$A_{17}=[\theta_2/\varphi_2]\left[\theta_1/(\varphi_1+2\theta_1+\theta_4)\right]\left[\theta_4/(\theta_1+\theta_4)\right]A_0;$$

$$A_{18}=[\theta_3/\varphi_3]\left[\theta_1/(\varphi_1+2\theta_1+\theta_4)\right]\left[\theta_4/(\theta_1+\theta_4)\right]A_0;$$

$$A_{19}=\left[\theta_1/(\varphi_1+2\theta_1+\theta_4)\right]\left[\theta_4/(\theta_1+\theta_4)\right]\left[\theta_4/(\varphi_1+\varphi_4)\right]A_0.$$

Under normalising condition the sum of all the probabilities has to be equal to one, i.e.,

$$\sum A_i=1,\ \text{i.e.,}\ A_0+A_1+A_2+\ldots+A_{19}=1 \quad (23)$$

On re-arranging it as:

$$\begin{aligned}&\left[A_0+(A_1+A_2+A_3)+(A_4+A_5+A_6+A_7)\right.\\&+(A_8+A_9+A_{10}+A_{11})+(A_{12}+A_{13}+A_{14}+A_{15})\\&\left.+(A_{16}+A_{17}+A_{18}+A_{19})\right]=1\end{aligned}$$

This yields,

$$\begin{aligned}&A_0\left[1+\{\theta_1/(\varphi_1+2\theta_1+\theta_4)\}\right.\\&+\{\theta_4/(\varphi_4+\theta_4)\}+\{(\theta_1\theta_4)/(\theta_4+\theta_1)(\varphi_1+2\theta_1+\theta_4)\}\\&+\{\theta_1/\varphi_1\}+\{\theta_2/\varphi_2\}+\{\theta_3/\varphi_3\}+\{\theta_4/\varphi_4\}+\{\theta_1/\varphi_1\}\\&+\{(\theta_2/\varphi_2)(\theta_1/(\varphi_1+2\theta_1+\theta_4))\}\\&+\{(\theta_3/\varphi_3)(\theta_1/(\varphi_1+2\theta_1+\theta_4))\}\\&+\{(\theta_4/\varphi_4)(\theta_1/(\varphi_1+2\theta_1+\theta_4))\}\\&+\{(\theta_1/\varphi_1)(\theta_4/(\varphi_4+\theta_4))\}+\{(\theta_2/\varphi_2)(\theta_4/(\varphi_4+\theta_4))\}\\&+\{(\theta_3/\varphi_3)(\theta_4/(\varphi_4+\theta_4))\}+\{(\theta_4/\varphi_4)(\theta_4/(\varphi_4+\theta_4))\}\\&+\{(\theta_1/(\varphi_1+2\theta_1+\theta_4))(\theta_4/(\theta_1+\theta_4))(\theta_1/(\varphi_1+\varphi_4))\}\\&+\{(\theta_2/\varphi_2)(\theta_1/(\varphi_1+2\theta_1+\theta_4))(\theta_4/(\theta_1+\theta_4))\}\\&+\{(\theta_3/\varphi_3)(\theta_1/(\varphi_1+2\theta_1+\theta_4))(\theta_4/(\theta_1+\theta_4))\}\\&\left.+\{(\theta_1/(\varphi_1+2\theta_1+\theta_4))(\theta_4/(\theta_1+\theta_4))(\theta_4/(\varphi_1+\varphi_4))\}\right]\\&=1\end{aligned}$$

or

$$\begin{aligned}A_0=&\left[1+\{\theta_1/(\varphi_1+2\theta_1+\theta_4)\}+\{\theta_4/(\varphi_4+\theta_4)\}\right.\\&+\{(\theta_1\theta_4)/(\theta_4+\theta_1)(\varphi_1+2\theta_1+\theta_4)\}+\{\theta_1/\varphi_1\}\\&+\{\theta_2/\varphi_2\}+\{\theta_3/\varphi_3\}+\{\theta_4/\varphi_4\}+\{\theta_1/\varphi_1\}\\&+\{(\theta_2/\varphi_2)(\theta_1/(\varphi_1+2\theta_1+\theta_4))\}\\&+\{(\theta_3/\varphi_3)(\theta_1/(\varphi_1+2\theta_1+\theta_4))\}\\&+\{(\theta_4/\varphi_4)(\theta_1/(\varphi_1+2\theta_1+\theta_4))\}\\&+\{(\theta_1/\varphi_1)(\theta_4/(\varphi_4+\theta_4))\}\\&+\{(\theta_2/\varphi_2)(\theta_4/(\varphi_4+\theta_4))\}\\&+\{(\theta_3/\varphi_3)(\theta_4/(\varphi_4+\theta_4))\}\\&+\{(\theta_4/\varphi_4)(\theta_4/(\varphi_4+\theta_4))\}\\&+\{(\theta_1/(\varphi_1+2\theta_1+\theta_4))\\&\times(\theta_4/(\theta_1+\theta_4))(\theta_1/(\varphi_1+\varphi_4))\}\\&+\{(\theta_2/\varphi_2)(\theta_1/(\varphi_1+2\theta_1+\theta_4))(\theta_4/(\theta_1+\theta_4))\}\\&+\{(\theta_3/\varphi_3)(\theta_1/(\varphi_1+2\theta_1+\theta_4))(\theta_4/(\theta_1+\theta_4))\}\\&+\{(\theta_1/(\varphi_1+2\theta_1+\theta_4))\\&\left.\times(\theta_4/(\theta_1+\theta_4))(\theta_4/(\varphi_1+\varphi_4))\}\right]^{-1}\end{aligned} \quad (24)$$

The long-run availability (performability) of the system $P(\infty)$ can now be calculated using:

$$\begin{aligned}P(\infty)&=A_0+A_1+A_2+A_3\\&=\left[1+\{\theta_1/(\varphi_1+2\theta_1+\theta_4)\}+\{\theta_4/(\varphi_4+\theta_4)\}\right.\\&\left.+\{(\theta_1\theta_4)/(\theta_4+\theta_1)(\varphi_1+2\theta_1+\theta_4)\}\right]A_0\\&=(1+X)A_0\end{aligned} \quad (25)$$

where $X = 1 + \{\theta_1/(\varphi_1 + 2\theta_1 + \theta_4)\} + \{\theta_4/(\varphi_4 + \theta_4)\} + \{(\theta_1\theta_4)/(\theta_4 + \theta_1)(\varphi_1 + 2\theta_1 + \theta 4)\}$.

In steady state the long run availabilities for various permissible combinations of repair and failure rates of eye rolling line system can now be obtained using equation (25). These values are presented in Tables 6 to 9. Accurate and 3D quality plots are generated using MATLAB software which is illustrative in Figure 9 to 12.

Table 6 Performability matrix for wrapper forming machine

θ_1 \ φ_1	*0.01681*	*0.03681*	*0.05681*	*0.07681*	*0.09681*	*Constant parameters*
0.000989	0.7810	0.8179	0.8301	0.8361	0.8397	$\theta_2 = 0.055310$, $\varphi_2 = 0.036340$
0.001889	0.7290	0.7889	0.8099	0.8207	0.8273	$\theta_3 = 0.005281$, $\varphi_3 = 0.54380$
0.002789	0.6863	0.7628	0.7913	0.8062	0.8154	$\theta_4 = 0.000441$, $\varphi_4 = 0.04980$
0.003689	0.6503	0.7393	0.7739	0.7924	0.8040	
0.004589	0.6192	0.7178	0.7576	0.7794	0.7931	

Figure 9 Variation of repair and failure rates of wrapper forming machine on system performability (see online version for colours)

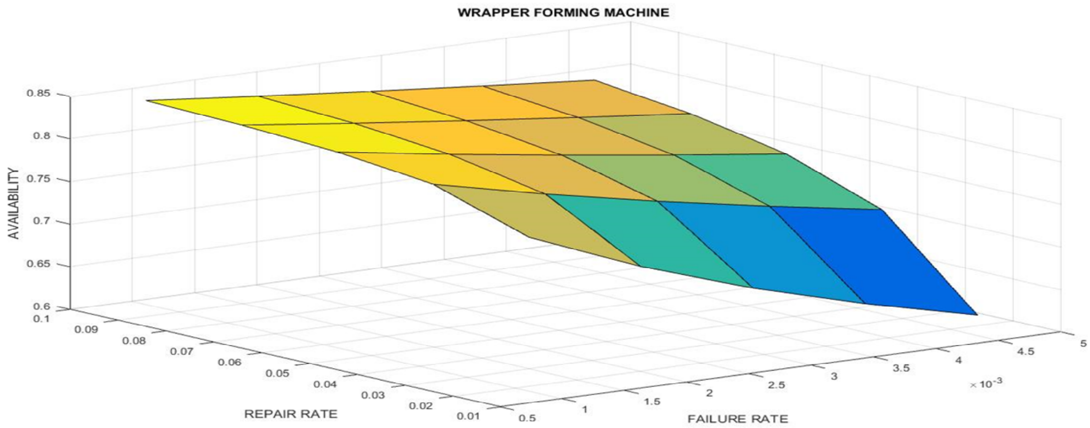


Table 7 Performability matrix for eye rolling machine

θ_2 \ φ_2	*0.16340*	*0.36340*	*0.56340*	*0.76340*	*0.96340*	*Constant parameters*
0.015310	0.8297	0.8668	0.8781	0.8837	0.8869	$\theta_1 = 0.002789$, $\varphi_1 = 0.05681$
0.035310	0.7532	0.8273	0.8516	0.8637	0.8709	$\theta_3 = 0.005281$, $\varphi_3 = 0.54380$
0.055310	0.6896	0.7913	0.8266	0.8446	0.8554	$\theta_4 = 0.000441$, $\varphi_4 = 0.04980$
0.075310	0.6359	0.7582	0.8030	0.8263	0.8405	
0.095310	0.5900	0.7279	0.7808	0.8088	0.8261	

Figure 10 Variation of repair and failure rates of eye rolling machine on system performability (see online version for colours)

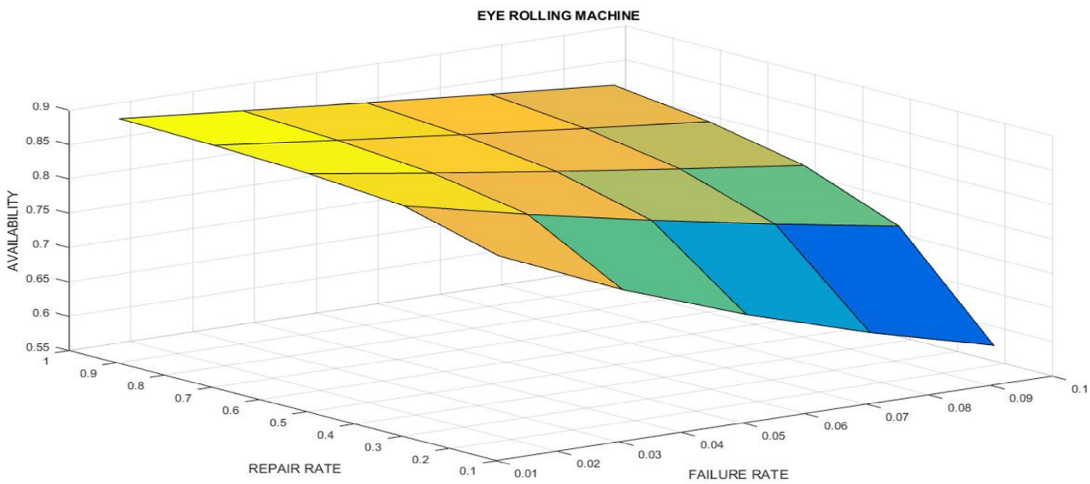


Table 8 Performability matrix for lap cutting machine

θ_3 \ φ_3	*0.1438*	*0.3438*	*0.5438*	*0.7438*	*0.9438*	*Constant parameters*
0.001281	0.7918	0.7950	0.7959	0.7963	0.7965	$\theta_1 = 0.002789$, $\varphi_1 = 0.05681$
0.003281	0.7831	0.7914	0.7936	0.7946	0.7952	$\theta_2 = 0.055310$, $\varphi_2 = 0.036340$
0.005281	0.7747	0.7877	0.7913	0.7929	0.7939	$\theta_4 = 0.000441$, $\varphi_4 = 0.04980$
0.007281	0.7665	0.7842	0.7890	0.7912	0.7925	
0.009281	0.7584	0.7806	0.7867	0.7895	0.7914	

Figure 11 Variation of repair and failure rates of lap cutting machine on system performability (see online version for colours)

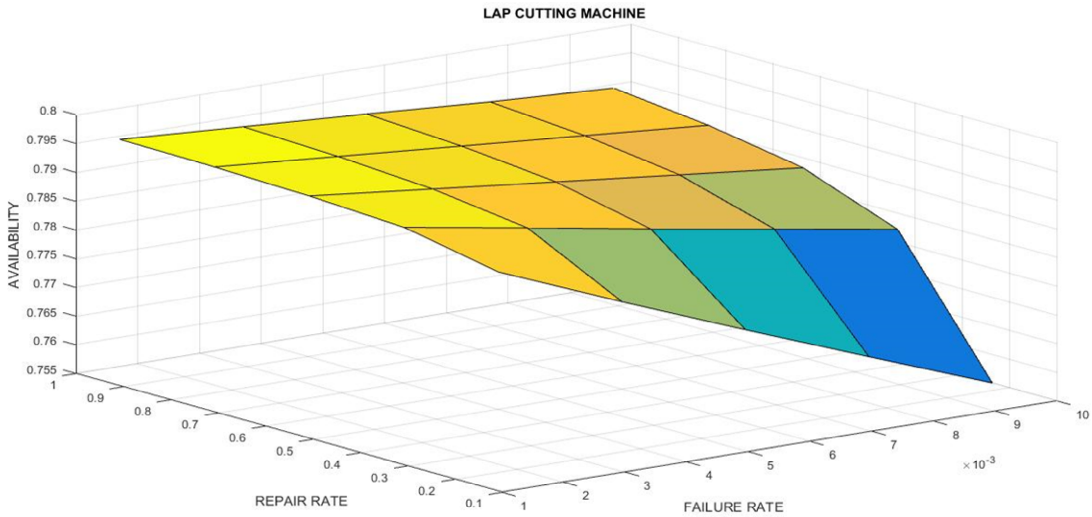


Table 9 Performability matrix for eye forming rolling machine

θ_4 \ φ_4	*0.00980*	*0.02980*	*0.04980*	*0.06980*	*0.08980*	*Constant parameters*
0.000041	0.7938	0.7956	0.7959	0.7960	0.7961	$\theta_1 = 0.002789$, $\varphi_1 = 0.05681$
0.000241	0.7820	0.7916	0.7936	0.7944	0.7949	$\theta_2 = 0.055310$, $\varphi_2 = 0.036340$
0.000441	0.7816	0.7877	0.7913	0.7928	0.7937	$\theta_3 = 0.005281$, $\varphi_3 = 0.54380$
0.000641	0.7811	0.7838	0.7890	0.7912	0.7924	
0.000841	0.7809	0.7810	0.7867	0.7895	0.7910	

Figure 12 Variation of repair and failure rates of eye forming rolling machine on system performability (see online version for colours)

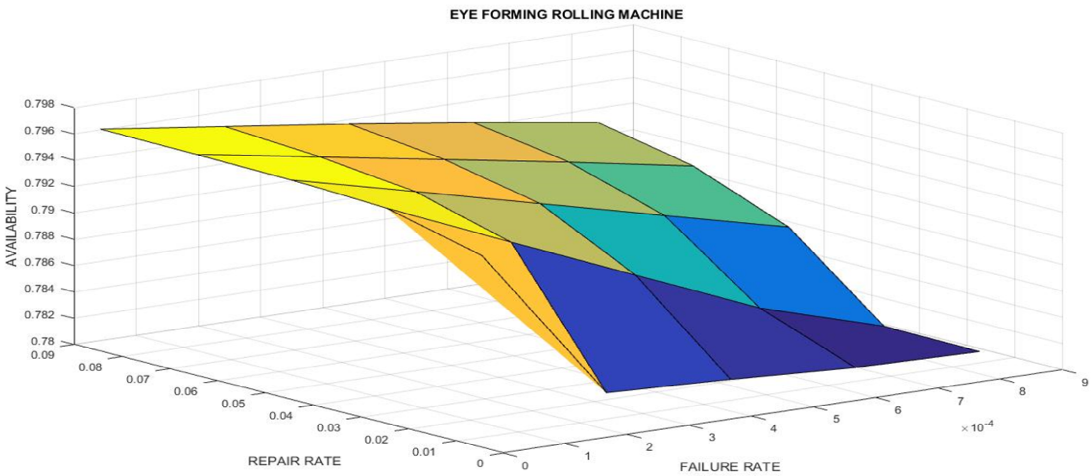


Table 10 Maintenance decision matrix (MDM)

Subsystem	*Variation in repair rates φ (failure rates θ)*	*System performability (%) using SPN*	*System performability (%) using Markov method*	*Recommend maintenance priority*
Wrapper forming machine	0.01681–0.09681 (0.000989–0.004589)	0.8592–0.7883 (7.09%)	0.8397–0.6192 (22.05%)	II
Eye rolling machine	0.16340–0.96340 (0.015310–0.095310)	0.9725–0.6237 (34.88%)	0.8869–0.5900 (29.69%)	I
Lap cutting machine	0.014380.09438 (0.001281–0.009281)	0.8644–0.8133 (5.11%)	0.7965–0.7584 (3.81%)	III
Eye forming rolling machine	0.00980–0.08980 (0.000041–0.000841)	0.8638–0.8487 (1.51%)	0.7961–0.7809 (1.52%)	IV

6 Results discussion and MDM

In the present work, the performability matrix of various subsystems for the system concerned is presented in Tables 2 to 5 using SPN and Tables 6 to 9 using Markov method respectively. The various combinations of repair and failure rates for wrapper forming machine subsystem decreases the system performability from 0.8592 to 0.7883 using SPN whereas for Markov method it decreases from 0.8397 to 0.6192 when the failure rate increases and repair rate decreases as presented in Tables 2 and 6, respectively. In the similar way, for eye rolling machine the system performability varies 0.9725–0.6237 using SPN whereas 0.8869–0.5900 for Markov method respectively as depicted in Tables 3 and 7, respectively. This subsystem has the highest impact on system performability. Lap cutting machine subsystem varies the system performability from 0.8644 to 0.8133 using SPN and from 0.7965 to 0.7584 for Markov method. For the eye forming rolling machine subsystem, the system performability varies 0.8638–0.8487 using SPN and 0.7961–0.7809 using Markov method respectively. This subsystem has the least impact on system performability. The accurate 3-D graphs of these performability matrices are clearly illustrated in Figures 4 to 7 for SPN modelling and Figures 8 to 11 for Markov method modelling respectively for different combination of repair and failure rates.

A permissible variations in failure and repair rates of various subsystems revealed that eye rolling machine subsystem has significant impact on the system performability by 34.88% using SPN whereas 29.69% while using Markov method. This subsystem needs to be placed on the top of maintenance priorities. A moderate effect has been done by wrapper forming machine having a contribution of 7.09% for SPN while 22.05% for Markov method. The variation of failure and repair rates of lap cutting machine and eye forming rolling machine shows the least effect. Based on the above discussion a MDM is proposed which clearly shows the comparative results of SPN and Markov method in Table 10. This MDM helps the practitioners to adopt suitable maintenance strategies.

7 Conclusions

A comparative study has been carried out which clearly highlighted that eye rolling machine subsystem needs the utmost attention during maintenance planning. It can be observed that the Markov method needs lots of computational effort as compared to SPN for determining the performability in terms of availability of the system. Moreover, SPN follows the dynamic behaviour of the system concerned. Therefore, for complex systems SPN is better than the Markov method to determine the performability. The proposed MDM might be quiet helpful in making decisions regarding maintenance policies and priorities which further enhance the system performability. This reduces the unplanned downtime and maintenance costs also. In addition to these benefits, system performability analysis can also be used for cost benefit analysis and spare parts inventory management purposes. In future, some advanced techniques such as spider monkey optimisation (SMO), teacher learning-based optimisation (TLBO), discrete particle swarm optimisation (DPSO), ant colony algorithm (ACA), etc. may be used for optimising the results.

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