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Robust supplier selection under uncertain costs and delivery delay times

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Abstract: We address the supplier selection problem under uncertainty, motivated by the current economic situation of global trade. The intense search among organisations for responsiveness in meeting market demands has directed efforts toward supply chain optimisation. Consequently, the decision regarding the best supplier choice has become vital for the success of organisations, requiring a high level of accuracy and assertiveness under complex and uncertain environments. To support decision-making in global sourcing environments, we propose a robust optimisation model that incorporates cost and time uncertainties that commonly arise in the context of worldwide raw materials supply. The model includes raw materials inventory management, preventing stockouts and violations of physical storage constraints, while considering deviations of the uncertain parameters. We analyse the behaviour of the proposed model using 324,000 scenarios generated by Monte Carlo simulations. The results show that the proposed model increases the level of robustness without significantly increasing the value of the objective function when uncertain costs and times attain their worst-case scenarios (highest deviation). On average, the objective function values increased only 3.58% in the worst case, considering 20 products, 40 periods, 60 suppliers, and an uncertainty level of 50%. [Received: 1 June 2022; Accepted: 10 May 2023]

Keywords: supplier selection; uncertainty; inventory; VUCA; robust optimisation.

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1 Introduction

The characteristics of the global competitive market environment, where product lifecycles are increasingly shorter and dynamic, require organisations to have high production efficiency and low operating costs, leading to the search for innovation in ways to conduct new business (Pan and Nagi, 2010). The current market behaviour is well characterised as a VUCA environment, which stands for volatility, uncertainty, complexity, and ambiguity. Consequently, decision-makers face complicated situations, with multiple options, unpredictable consequences for scenarios and a large amount of input data (Abel et al., 2020; Giones et al., 2019; Du and Chen, 2018).

Inside the VUCA framework's global scenario, each organisation is forced to optimise its global supply chain, known as global sourcing (GS), to manage its resources better. Supply Chain Management is currently focusing deeply on Global Supply and its uncertainties. For example, Park et al. (2018) proposed a decision support system for operational risk management of global supply chains. Managing supplier relationships becomes a crucial competitive factor for the performance of the entire chain (Park et al., 2010). One of the key links of GS is the supply of raw materials (RM). The decision to select the best suppliers is challenging and complex because it impacts the performance of the entire GS. A delay in the supply of RM can cause interruptions in the company's processes, delay in the delivery of products, loss of sales, contracts and customers, and affect the company's image in the market (Amorim et al., 2016).

This whole context is demanding attention for recent research on supplier selection related to quantitative decision modelling. For example, Ekici et al. (2021) deal with supplier selection and order quantity allocation for a single retailer that orders from multiple suppliers. However, this literature is still scarce, especially when considering global supply, which commonly involves many costs and uncertainties (Wetzstein et al., 2019). In this type of environment where intercontinental transactions are carried out, it is necessary to schedule orders and close RM purchasing contracts many planning horizons ahead (Suri, 2010). This peculiarity indicates that many uncertainties and risks are involved, such as price and delivery time (Abel et al., 2020).

Within this context, we propose a mathematical model to support decision-making in supplier selection in an environment subject to uncertainty, using the robust optimisation (RO) approach. RO has been applied in the supply chain context in recent years, but there is a gap in the literature specifically related to supplier selection in a GS environment (Kisomi et al., 2016).

This study contributes to practical managers by presenting a mathematical model to help supply chain decision-makers to optimise supplier selection problem (SSP) decisions under costs and lead time uncertainty. The main advantages of our approach are:

1 our model is based on widely used static RO strategies, which are adapted to SSP decision-making

2 it can be used to solve large-scale instances on top of general-purpose optimisation software without requiring any implementation of other algorithms

3 it is the first to incorporate the influence of uncertainties in most of the three cost classes (static, dynamic, and hidden costs) indicated by Holweg et al. (2011) through RO, in addition to considering the control of RM inventory and preventing from stockouts and violations of physical storage constraints.

Additionally, this study contributes to link the literature on GS and supplier selection through a quantitative approach in a global transaction environment subject to uncertainties by modelling the various costs involved in GS that are only superficially addressed by the literature on the subject. For example, Suri (2010) argues that it is necessary to assess the true cost of a supplier with a long lead time to reduce the flow time through the chain. Holweg et al. (2011) discuss how GS operations have a complex context of tied costs that can be divided into three classes, static, dynamic, and hidden costs, which must be known before structuring a GS operation.

The remainder of this paper is organised as follows. In Section 2, we present the definition of the addressed problem and the relevant literature. In Section 3, we propose the RO model for supplier selection under uncertainty, after introducing a deterministic counterpart model. The results of the computational experiments are described and analysed in Section 4, and the conclusions are presented in Section 5.

2 Problem definition and related literature

In this section, we first define the SSP and its context. Subsequently, we present studies that used deterministic models, stochastic models, and other approaches to propose solutions to the problem.

2.1 The supplier selection problem

According to Aouadni et al. (2019), the SSP is a procurement decision-making problem that addresses the definition of methods and models to analyse and measure the performance of a set of suppliers to improve customer's competitiveness. This decision is complex because of the diversity of quantitative and qualitative criteria in the evaluation and decision-making process. Therefore, given a set of suppliers available in different geographic regions and their respective costs, the SSP consists of determining which of them to contract and when, as well as the amount of each RM to be purchased from each supplier, to satisfy a demand for RMs in a finite time horizon in a multiperiod and multi-item environment (Cunha et al., 2018).

The continuous search for optimal purchase orders among various suppliers with different prices and delivery times promotes a trade-off involving deciding to search for more expensive products with shorter delivery times and *vice versa*. This type of decision is exemplified by Figure 1, which presents a hypothetical situation in which a manufacturer needs 3,000 units of a given item and has several approved suppliers scattered around the world. Nearby suppliers have a lower delivery lead time and higher costs. As the distance increases, supplier prices decline, but delivery times increase (Suri, 2010). The SSP seeks a trade-off that aims to divide this demand among several suppliers to minimise all costs involved in the operation and fulfil all deadlines agreed upon through contracts with the customers of the buying company.

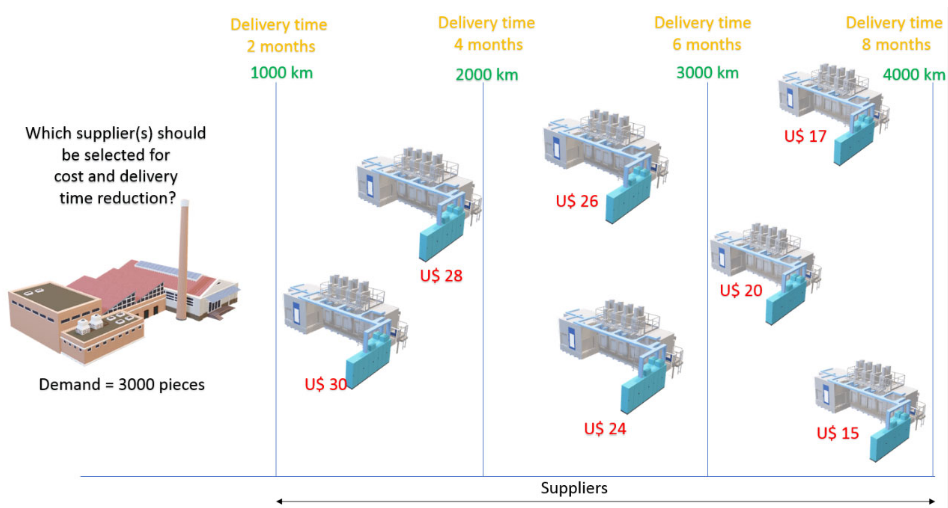
2.2 Related literature

Several studies have proposed the use of optimisation models to address the SSP under a deterministic bias, using mixed-integer linear programming (MIP) and mixed-integer nonlinear programming (MINLP). Aouadni and Rebai (2013) present an MIP model that incorporates safety stock in its formulation and decides which order lot size to allocate to the chosen supplier in each scenario. Bohner and Minner (2017) proposed a MIP that considers suppliers that offer quantity and business volume discounts, and they are both subject to failure in a supply chain problem involving SSP.

Ware et al. (2014) addressed the SSP through an MINLP model, in a dynamic environment where a commercial organisation wants to optimise costs in its RM purchase decisions, maximising its operational outcome. In Choudhary and Shankar (2013), the SSP is addressed in a combined MIP model, which integrates three classes of problems studied in the literature: lot sizing, transportation model and inventory control. Cunha et al. (2018) presented a combined MIP involving SSP decisions, lot sizing, and production sequencing in the context of the chemical industry, in a plant located in Brazil. Last, Son and Van Hop (2021) proposed a MIP to the SSP in recyclables materials in two steel factories in Vietnam.

Some studies have addressed the uncertainty observed in practice through stochastic programming. Coronado (2007) developed a nonlinear stochastic programming model that addresses the market of fossil fuel distributors for automotive vehicles. Purohit et al. (2016) proposed a combined lot sizing and SSP model with dynamic and stochastic demand. This model also considers decisions on economies of scale, inventory level, service levels, and successive selections of the same supplier within a planning horizon. Sawik (2014) proposed a coordinated SSP model that addresses portfolio order delivery scheduling in the presence of disruption risks in the supply chain. Curcio (2017) presented a combined two-stage stochastic model involving lot sizing decisions, production scheduling and the SSP in a food products industry. Last, Chintapalli (2021) presented a stochastic programming model with recourse and solves it using sample average approximation (SAA).

Figure 1 The intercontinental supplier sourcing problem (see online version for colours)



Some studies have addressed simulation techniques to support decisions in SSP. Wu and Olson (2008) and Azadeh and Alem (2010) used data envelopment analysis (DEA) to evaluate risk within supply chain. Ding et al. (2003, 2005) presented a simulation-optimisation approach using a genetic algorithm (GA). Firouz et al. (2017) presented a simulation model that evaluates the value of the objective function of a heuristic algorithm based on decomposition. Franco and Alfolson-Lizarazo (2020) presented a simulation-optimisation approach based on the stochastic counterpart of a sample path method for optimising tactical and operative decisions in the pharmaceutical supply chain.

Other methods often employed to address the studied problem are heuristic techniques. The main goal of heuristics is the relatively fast provision of good and feasible solutions, not always optimal, to optimisation problems. This set of techniques is used by Rabbani et al. (2014), Aouadni and Rebai (2013) and Cao et al. (2012) through GA to support decision methods for the SSP. Another well-known heuristic technique used in this context is Tabu Search, found in studies by Guo et al. (2016) and Wang and Zhong (2010).

Dweiri et al. (2016), Yadav and Sharma (2015, 2016) and Kaya and Yet (2019) used the multi-criteria decision making (MCDM) approaches analytical hierarchy process (AHP), decision making trial and evaluation laboratory (DEMATEL) and Bayesian networks (BN) for supplier selection in automotive supply chains. The Pareto method is used by Konur et al. (2017) for the selection of second-order suppliers from a retailer. Shadkam and Bijari (2017) proposed a novel multi-objective simulation optimisation method for the SSP in a stochastic environment. This method consists of four basic modules: cuckoo optimisation algorithm (COA), discrete event simulation (DES), supply chain model (SCM) and generalised data envelopment analysis (GDEA). Mirzaee et al. (2018) proposed a preemptive fuzzy goal programming approach. Hasan et al. (2020) propose a decision support system using a fuzzy-based technique for order preference by similarity to ideal solution (TOPSIS) method and considering resilient suppliers in the Logistic 4.0 environment.

Other methods often employed to address the SSP are combined hybrid approaches. Xu et al. (2021) used fuzzy multi-objective mixed integer programming (MOMIP) for automotive sensors. Cárdenas-Barrón et al. (2021) proposed a new MIP heuristic based on new valid inequalities. Kaur and Prakash Singh (2021) proposed a MIP multi-stage hybrid model where the suppliers are evaluated using DEA and prioritised using fuzzy analytical hierarchical process and technique for order of preference by similarity to ideal solution (FAHP-TOPSIS) in an automotive company. Liu et al. (2022) proposed a stochastic bi-objective MIP that uses conditional value-at-risk (CVaR) as a risk measure and apply the heuristic NSGA-II to find solutions for this model. Foroozesh et al. (2021) proposed an interval-valued fuzzy-stochastic group decision model, using fuzzy logic in a stochastic environment simulated by Monte Carlo.

Some studies have addressed the uncertainty observed in practice through RO. Fu et al. (2014) proposed a RO model for the SSP under uncertain demand. Niroomand et al. (2018) proposed a multi-objective RO model for SSP and production decisions in cardboard box manufacturing, considering uncertain costs and demands. Isaloo and Paydar (2020) proposed a bi-objective RO model considering uncertainty costs throughout a plastic injection industry chain. Mazahir and Ardestani-Jaafari (2020) proposed a framework based on a two-stage RO where the SSP and respective suppliers' capacity commitments constitute the first-stage decisions, and the allocation of products from each supplier to a market is the second-stage decision. To solve the two-stage RO, the authors used column-and-constraint generation (C&CG) algorithm and affinely adjustable robust counterpart (AARC). Solgi et al. (2021) proposed bi-objective RO model to provide resilient SSP and order allocation for satellites and their subsystems in response to uncertainty and disruption risks. Thevenin et al. (2022) considered delivery lead-time uncertainty in a single-item variant of SSP.

The innovation of our work in comparison to the research of Thevenin et al. (2022) consists of:

1 we consider more types of relevant costs in SSP decisions in the GS context in the objective function

2 we consider a problem with multiple items

3 we manage to solve large instances resorting to a general-purpose MIP software.

To the best of our knowledge, no other study addresses precisely the same characteristics.

There is also a gap in practice, as the costs involved in the SSP decision are superficially addressed in Holweg et al. (2011) and Suri (2010) making it challenging to build exact mathematical models to support the decision. The developed model proposes to reduce this gap, by modelling static, dynamic, and hidden costs, providing solutions for decision-makers such as managers, supervisors, coordinators, and analysts, among other professionals involved in GS management.

A summarised overview of the 37 papers and two doctoral theses discussed in this section is presented in Table 1, including a brief description of their main purposes.

Table 1 Summary of all the revised papers dealing with SSP

Authors	*Main purpose*
Ding et al. (2003)	Proposes an approach that uses DES to evaluate the performance of the SSP and GA to identify the optimal portfolio based on performance indexes estimated by the simulation.
Ding et al. (2005)	Presents a simulation optimisation methodology to support decisions on the SSP. The methodology is composed of three basic modules: a genetic GA optimiser, a DES and a supply chain modelling framework.
Coronado (2007)	A nonlinear stochastic programming model is developed that addresses the market of fossil fuel distributors for automotive vehicles.
Wu and Olson (2008)	Presents three approaches of risk evaluation models within in the SSP: chance constrained programming (CCP), DEA, and MOPMIP model.
Wang and Zhong (2010)	Proposes a Tabu Search to the SSP considering multi-period price rebate and logistics costs.
Azadeh and Alem (2010)	Presents three SSP models with a decision-making scheme for choosing appropriate method for the SSP under certainty, uncertainty, and probabilistic conditions. These models are, DEA, fuzzy data envelopment analysis (FDEA), and chance constraint data envelopment analysis (CCDEA).
Cao et al. (2012)	A one-step product family optimisation model integrating SSP decision is proposed based on multinomial logit consumer choice rule.
Choudhary and Shankar (2013)	Proposes a MIP to simultaneously determine the timings of procurement, lot sizes, suppliers, and carriers to be chosen to incur the least total cost over the planning horizon.
Aouadni and Rebai (2013)	Proposes a MIP for a mono-item multi period inventory lot-sizing problem with the SSP including safety stock.
Rabbani et al. (2014)	Proposes a bi-objective model for supplier selection portfolio problem that can control delayed, disrupted, and defected supplies via scenario analysis.
Sawik (2014)	Proposes a coordinated SSP model that addresses portfolio order delivery scheduling in the presence of disruption risks in the supply chain.
Ware et al. (2014)	Proposes a MINLP to address the SSP demonstrating the dynamics involved in this type of decision.
Fu et al. (2014)	Proposes a stochastic programming model for the SSP under an uncertain demand. To solve this model, the authors developed a RO approach to create a protection against the worst-case scenario possible.
Yadav and Sharma (2015)	Uses DEA and AHP methodologies to address the SSP.

Table 1 Summary of all the revised papers dealing with SSP (continued)

Authors	*Main purpose*
Guo et al. (2016)	Proposes a Tabu Search for supply chain node configuration, with SSP decisions, analysing the influence of the level of uncertainty on robust result.
Yadav and Sharma (2016)	Used AHP approach to address the SSP in an automobile company.
Dweiri et al. (2016)	Used AHP approach to address the SSP in an automobile company in Pakistan.
Purohit et al. (2016)	Proposes a combined lot sizing and the SSP model with dynamic and stochastic demand.
Curcio (2017)	Presents a combined two-stage stochastic model involving lot sizing decisions, production scheduling and the SSP in a food products industry.
Bohner and Minner (2017)	Proposes a MIP that considers suppliers that offer quantity and business volume discounts, and they are both subject to failure, in a supply chain problem involving SSP.
Firouz et al. (2017)	Proposes a decomposition-based heuristic algorithm combined with simulation. While the decomposition-based heuristic determines a solution with supplier selection and inventory decisions, the simulation model evaluates the objective function value corresponding to each generated solution
Shadkam and Bijari (2017)	Proposes a multi-objective simulation optimisation method to the SSP in a stochastic environment. This method consists of four basic modules: COA, DES, SCM and GDEA.
Konur et al. (2017)	Proposes a bi-objective continuous review inventory control model with order splitting among multiple suppliers, where both expected costs and carbon emissions per unit time are minimised.
Mirzaee et al. (2018)	Proposes a preemptive fuzzy goal programming approach to solve a SSP considering a multi-period, multi-product, multi-supplier, multi-objective cases as well as quantity discount subject to budget and capacity limitations for both buyers and suppliers.
Cunha et al. (2018)	Proposes a combined MIP involving the SSP decisions, lot sizing, and production sequencing in the context of the chemical industry, in a plant located in Brazil.
Niroomand et al. (2018)	Proposes a multi-objective RO model for the SSP and production decisions in cardboard box manufacturing, considering uncertain costs and demands. To solve the RO model, a weighted global criterion approach was applied to find Pareto optimal solutions.
Kaya and Yet (2019)	Proposes a combined DEMATEL in BN construction to address the SSP decisions.
Mazahir and Ardestani-Jaafari (2020)	Proposes a framework based on a two-stage RO where the SSP and respective suppliers' capacity commitments constitute our first-stage decisions, and the allocation of products from each supplier to a market is the second-stage decision. To solve the two-stage RO the authors used two approaches: C&CG algorithm and AARC framework.
Franco and Alfolson-Lizarazo (2020)	Presents a simulation-optimisation approach based on the stochastic counterpart or sample path method is used for optimising tactical and operative decisions in the pharmaceutical supply chain.

Table 1 Summary of all the revised papers dealing with SSP (continued)

Authors	*Main purpose*
Hasan et al. (2020)	Proposes a decision support system using a fuzzy-based TOPSIS method and considering resilient suppliers in the Logistic 4.0 environment
Isaloo and Paydar (2020)	Proposes a bi-objective RO model, considering uncertainty costs throughout a plastic injection industry chain. To solve the proposed model, some of the multi-objective exact methods such as the weighted sum, weighted goal programming, and LP-metric method was used.
Solgi et al. (2021)	Proposes bi-objective RO model to provide resilient solutions to the SSP and order allocation for satellites and their subsystems in response to uncertainty and disruption risks. To solve the proposed bi-objective model, the augmented -constraint method is proposed, which ensures strong Pareto solutions.
Son and Van Hop (2021)	Proposes a MIP to the SSP in recyclables materials in two steel factories in Vietnam. The model was solved by the hybrid meta-heuristic of particles swarm optimisation and grey wolf optimisation (PSO-GWO).
Xu et al. (2021)	Used MOMIP for automotive sensors with uncertain objective weights. An extended interactive algorithm is developed to solve the model.
Cárdenas-Barrón et al. (2021)	Proposes a new MIP heuristic through new valid inequalities.
Kaur and Prakash Singh (2021)	Propose a MIP multi-stage hybrid model where the suppliers are then evaluated using DEA and prioritised using FAHP-TOPSIS in an automotive company, considering positive events such as Industry 4.0 and negative events such as natural and man-made disasters.
Foroozesh et al. (2021)	Proposes an interval-valued fuzzy-stochastic group decision model, using fuzzy logic applied in a stochastic environment simulated by Monte Carlo to support SSP decisions in a green loop automotive supply chain.
Chintapalli (2021)	Presents a stochastic programming model with recurse solved by SAA.
Liu et al. (2022)	Proposes a stochastic bi-objective MIP where CVaR is used to measure the risk. A heuristic NSGA-II is applied to solve the model that integrates product family and SSP decisions, considering: global pandemic, political unrest, and natural disaster risks.
Thevenin et al. (2022)	Proposes a RO model considering delivery lead-time uncertainty in a single-item variant of the SSP. The RO model was solved using a row and column generation algorithm and three heuristic approaches: a fix-and-optimise, a genetic algorithm, and a hybrid robust counterpart.

3 Proposed models

In this section, we first present a deterministic MIP model, based on a proposed extension of the MINLP model published by Ware et al. (2014). We chose this model because its objective function incorporates service level, delivery time and other relevant costs related to SSP and GS. Then, we propose a counterpart RO model with uncertain costs and delivery time.

3.1 Deterministic supplier selection with inventory control

To model the problem described in Section 2.1, we propose the supplier selection with inventory control (SSIC) model, which is defined in a multi-supplier, multi-item, and multi-period environment. The objective function consists of acquisition costs, transportation costs, operating costs, estimated average delivery delay time of suppliers (based on history), inventory carrying costs and inventory backlog costs. At first, we assume that all these costs are known in advance; hence, the SSIC model is deterministic.

The main differences between the model published by Ware et al. (2014) and the SSIC model are as follows:

1. The model proposed in this study is linear and can be implemented in general-purpose MIP software, such as GAMS/CPLEX, a characteristic of great importance in practice given that this type of software has become increasingly more efficient in solving a large class of problems.
2. We incorporated inventory control of products obtained from suppliers (RM), allowing the demand of a given period to be met with products obtained in previous periods.
3. We created constraints that limit the inventory backlog level and satisfy the physical warehousing storage.

To define this model, consider the following sets, parameters, and decision variables:

- Sets

  T set of time periods

  S set of suppliers

  P set of product types.

- Parameters

  cc_{tsp} unit purchase cost of product p from supplier s in period t

  ct_{ts} transportation cost of a product coming from supplier s in period t

  C_{tsp} ability of supplier s to supply product p in period t

  d_{tp} demand of product p in period t

  q_{tsp} operational unit cost of supplier s for product p in period t

  l_{tsp} delay cost to supplier s for product p in period t

  dl_{tsp} estimated delivery delay time from supplier s for product p in period t

  M sufficiently large number, defined by the cumulative demand

  θ service level agreement (SLA) required by the company

  Φ maximum delay level allowed for meeting the demand

  h^{+}_{tp} cost of keeping one unit of product p in stock in period t

h_{tp}^{-} delivery delay cost of one unit of product p in period t

w_{tp} storage capacity for product p in period t.

- Variables

X_{tsp} amount of product p obtained from supplier s in period t

Y_{ts} binary variable equal to 1 if, and only if, supplier s is used in period t

I_{tp}^{+} amount of product p in stock in period t

I_{tp}^{-} amount of product p delayed in period t.

From these definitions, the SSIC model is given by:

$$\begin{aligned} \min \sum_{\substack{t\in T\\ s\in S\\ p\in P}} cc_{tsp}X_{tsp} + \sum_{\substack{t\in T\\ s\in S}} ct_{ts}Y_{ts} + \sum_{\substack{t\in T\\ s\in S\\ p\in P}} (1-\theta)q_{tsp}X_{tsp} \\ + \sum_{\substack{t\in T\\ s\in S\\ p\in P}} l_{tsp}dl_{tsp}X_{tsp} + \sum_{\substack{t\in T\\ p\in P}} h_{tp}^{+}I_{tp}^{+} + \sum_{\substack{t\in T\\ p\in P}} h_{tp}^{-}I_{tp}^{-} \end{aligned} \tag{2.1}$$

$$\text{s.t.} \quad \sum_{s\in S} X_{tsp} + I_{t-1,p}^{+} - I_{tp}^{+} + I_{tp}^{-} - I_{t-1,p}^{-} = d_{tp} \qquad \forall t\in T, \forall p\in P \tag{2.2}$$

$$X_{tsp} \le C_{tsp} \qquad \forall t\in T, \forall s\in S, \forall p\in P \tag{2.3}$$

$$\sum_{p\in P} X_{tsp} \le MY_{ts} \qquad \forall t\in T, \forall s\in S \tag{2.4}$$

$$I_{tp}^{-} \le \Phi d_{tp} \qquad \forall t\in T, \forall p\in P \tag{2.5}$$

$$I_{tp}^{+} \le w_{tp} \qquad \forall t\in T, \forall p\in P \tag{2.6}$$

$$Y_{ts} \in \{0,1\} \qquad \forall t\in T, \forall s\in S \tag{2.7}$$

$$X_{tsp} \ge 0 \qquad \forall t\in T, \forall s\in S, \forall p\in P \tag{2.8}$$

$$I_{tp}^{-} \ge 0 \qquad \forall t\in T, \forall p\in P \tag{2.9}$$

$$I_{tp}^{+} \ge 0 \qquad \forall t\in T, \forall p\in P. \tag{2.10}$$

The objective function (2.1) consists of minimising the total cost of the system, given by the costs of purchasing the products, shipping costs for all suppliers, operating costs of making a purchase from any supplier according to an imposed service level, late product delivery costs, inventory carrying costs and inventory backlog costs. Parameter q_{tsp} in the third term consists of hidden costs such as currency exchange rate variations, increase in fuels involved in the chain (oil, gas and carbon emission tax), costs of unplanned trips for prospecting and after-sales, risk of loss of intellectual capital and the static costs of maintaining local offices and agencies in supplier markets in order to keep contact and negotiate with suppliers scattered around the globe. Parameter l_{tsp} in the fourth term consists of loss costs distributed across the three cost classes, appropriating the obsolescence costs due to design changes not aligned with the delivery lead time and costs of lost sales due to unmet deadlines and problems with quality. For more details on the compositions of such costs, see Holweg et al. (2011).

Because parameters q_{tsp} and l_{tsp} are volatile and highly influenced by several variables active in the logistics chain, the activities of estimating and correctly appropriating these values into products or services are complex. Thus, the third term of the objective function represents the operating costs for that operation, restricted to a service level required by the buying company. The fourth term of the objective function represents the costs that the buying company will bear if any supplier delays delivery of RMs within the agreed deadline, under the risk of loss of sales because of this delay. Finally, it is worth remarking that even though the fourth and sixth terms are related to late delivery costs, there are some clear differences between them. Indeed, the cost h_{tp}^{-} becomes active when the buying company chooses to delay some order(s) to obtain an economic advantage with shipping, customs clearance, and duplicates, among others; while cost l_{tsp} escapes the decision power of the buying company because it becomes active when the supplier(s) delays the delivery of the products for some reason, such as machine breakage, lack of planning, and lack of RMs, among others (even though it has agreed to meet the delivery date of the customer).

Constraints (2.2) balance the inventory of products, while constraints (2.3) enforce each supplier's capacity for each product. Constraints (2.4) ensure that there may be a purchase from a given supplier s only if that supplier is selected in period t. In fact, if supplier s is selected in period t, then Y_{ts} assumes the value of 1; otherwise, it assumes the value of 0. Note that if a supplier is selected, it must deliver all product itself, i.e., without using any subcontractors for this purpose.

Constraints (2.5) limit the inventory backlog using parameter Φ, which is defined by the buying company. These constraints are important to guarantee that the inventory does not compromise the service level required by the organisation. If in a given period t the demand value for some product p is null, the backlog cannot be carried. This premise is adopted following the approach of the quick response manufacturing (QRM) philosophy (Suri, 1998) and is treated carefully in the computational tests in Section 4 to prevent periods with no demand for a product. Constraints (2.6) limit the size of the stock in hand according to the physical area in the warehouse destined for the storage of product p in period t. Finally, constraints (2.7) to (2.10) impose the domain of the decision variables.

3.2 Robust supplier selection with inventory control

In the context discussed here, it is relevant to consider that the costs and delivery times considered in objective function (2.1) are subject to uncertainties. Thus, in this section, the SSIC model is extended to consider costs and time uncertainties through RO.

RO is a paradigm to model optimisation problems in which input parameters are subject to uncertainty. The purpose is that during planning, even without knowing the values to be assumed by these parameters, it is possible to determine solutions that remain feasible in practice. For this, it is assumed that the parameters are random variables that can take on values in a constrained and bounded set, known as the *uncertainty set* (Soyster, 1973). Many Industrial Engineering problems are being solved using RO, such as layout problems (Lashgari et al., 2021). We chose RO to model uncertainty in this work because it has several advantages concerning other approaches, summarised by Alem and Morabito (2015) as follows:

1 It is not necessary to infer probability distributions to the uncertain parameters since the robust counterparts only require the random variables to be symmetrical in a predefined range. For the decision maker, it can be natural to infer a minimum and maximum bound for the variation of uncertain parameters based on historical data or even in her/his experience, instead of precisely determining the value of the variable in each scenario and its probability of occurrence.

2 The RO model allows the decision maker to simply incorporate his attitude towards the risk. Hence, she/he knows precisely what the theoretical worst case is, different from the stochastic programming model, which must be solved a priori to know the worst case.

3 The RO model is tractable computationally because the robust counterpart maintains computational complexity of the deterministic problem. However, it is worth noting that non-polyhedral sets of intense (ellipsoidal, for example) can make the computationally robust equivalent intractable.

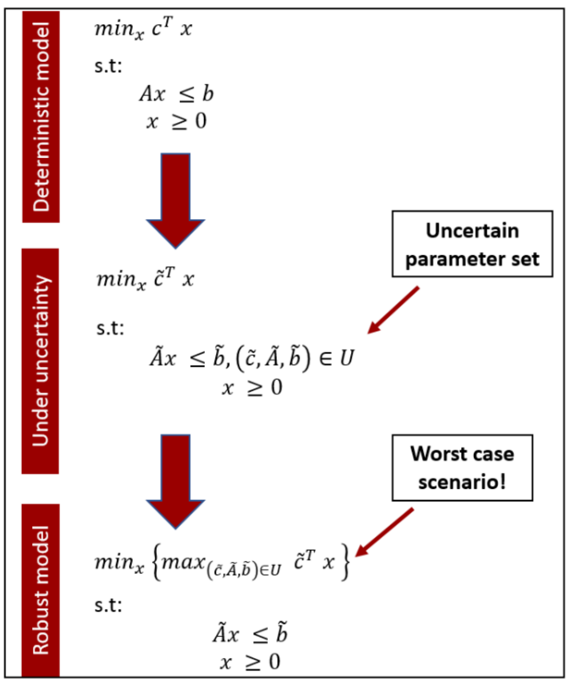
Different types of uncertainty sets with their respective formulations have been proposed in the literature (Ben-Tal and Nemirovski, 2000; Bertsimas and Sim, 2003, 2004). We adopt the successful approach of Bertsimas and Sim (2003, 2004), based on a polyhedral representation of the uncertainty set, which considers a maximum number of random variables that attain their worst case in each constraint i of the problem, through a parameter Γ_i called the uncertainty budget. Thus, it is possible to control the trade-off between the probability of a constraint violation and the additional costs in the objective function value to protect against that violation. This approach ensures that the solution remains immune to risks in any realisation of the uncertain data, to the detriment of the objective function (Bertsimas and Sim, 2004).

When the uncertainties are in the coefficients of the objective function, as is the case considered in this paper, RO provides solutions that do not excessively deteriorate the value of the objective function when the uncertain parameters reach their worst case. In a minimisation problem, assuming a single uncertainty budget Γ, the solutions provided by the RO model answer the following question: What solution ensures the lowest possible cost when up to Γ components of the uncertain parameters assume their worst case? Note that this analysis involves a limited number of parameters reaching their worst case.

RO based on polyhedral sets with an uncertainty budget has been successfully applied in several contexts, showing significant advantages in supporting decision-making, as addressed by Bertsimas and Thiele (2006), Alem and Morabito (2012), Paiva and Morabito (2014), Munhoz and Morabito (2014), Righetto et al. (2016), Rocco and Morabito (2016), De La Vega et al. (2019, 2020), Martins de Sá et al. (2018) and Munari et al. (2019). In addition to the advantages provided in practice, its success is due to the ease of incorporating uncertainties to models with continuous or discrete domains because the robust model belongs to the same complexity class as the nominal model. The abovementioned studies are recommended for more details on the theory and application of RO.

To better understand the RO overall architecture, consider the generic parameters A, b, c; the uncertainty set U; and a non-negative variable x. Figure 2 presents a flowchart that illustrate all the steps of applying RO to a problem. The first step is the construction of the deterministic optimisation model. Then, the uncertain parameters are considered as random variables that can take on values in a restricted and bounded uncertainty set (step 2 in Figure 2). The worst-case scenario that can occur is that the uncertain parameters reach the maximum deviation. In this context, the RO approach aims to minimise the worst-case scenario (step 3 in Figure 2).

Figure 2 An illustration of the RO overall architecture (see online version for colours)



In the RO model proposed in this section, uncertainty is considered in the five cost classes of objective function (2.1) (unit costs, transportation costs, operational costs of the purchase order, on-hand inventory costs, and inventory backlog costs) as well as in the delivery delay times that appear in the fourth term of this function. Consider the sets J^{φ}, where all the indices of the corresponding uncertain parameters are contained, with $\varphi = [cc, ct, q, dl, h^{+}, h^{-}]$; the nominal cost values cc_{tsp}, ct_{tsp}, q_{tsp}, dl_{tsp}, h^{+}_{tp}, h^{-}_{tp}; their corresponding maximum deviations $\widehat{cc}_{tsp}, \widehat{ct}_{tsp}, \hat{q}_{tsp}, \widehat{dl}_{tsp}, \hat{h}^{+}_{tp}, \hat{h}^{-}_{tp}$. Then, the following uncertainty set U is defined:

$$U=\left\{\begin{array}{c}\widetilde{cc}_{tsp},\widetilde{ct}_{tsp},\tilde{q}_{tsp},\widetilde{dl}_{tsp},\tilde{h}^{+}_{tp},\tilde{h}^{-}_{tp}\in R^{+}\Big|\\ \widetilde{cc}_{tsp}=cc_{tsp}+\widehat{cc}_{tsp}\zeta^{cc}_{tsp},\forall(t,s,p)\in J^{cc};\\ \widetilde{ct}_{ts}=ct_{ts}+\widehat{ct}_{ts}\zeta^{ct}_{ts},\forall(t,s)\in J^{ct};\\ \tilde{q}_{tsp}=q_{tsp}+\hat{q}_{tsp}\zeta^{q}_{tsp},\forall(t,s,p)\in J^{q};\\ \widetilde{dl}_{tsp}=dl_{tsp}+\widehat{dl}_{tsp}\zeta^{sl}_{tsp},\forall(t,s,p)\in J^{dl};\\ \tilde{h}^{+}_{tp}=h^{+}_{tp}+\hat{h}^{+}_{tp}\zeta^{h+}_{tp},\forall(t,p)\in J^{h+};\\ \tilde{h}^{-}_{tp}=h^{-}_{tp}+\hat{h}^{-}_{tp}\zeta^{h-}_{tp},\forall(t,p)\in J^{h-};\\ \sum\limits_{(t,s,p)\in J^{cc}}\zeta^{cc}_{tsp}\le\Gamma^{cc};0\le\zeta^{cc}_{tsp}\le 1,\forall(t,s,p)\in J^{cc};\\ \sum\limits_{(t,s)\in J^{ct}}\zeta^{ct}_{ts}\le\Gamma^{ct};0\le\zeta^{ct}_{ts}\le 1,\forall(t,s)\in J^{ct};\\ \sum\limits_{(t,s,p)\in J^{q}}\zeta^{q}_{tsp}\le\Gamma^{q};0\le\zeta^{q}_{tsp}\le 1,\forall(t,s,p)\in J^{q};\\ \sum\limits_{(t,s,p)\in J^{dl}}\zeta^{dl}_{tsp}\le\Gamma^{dl};0\le\zeta^{dl}_{tsp}\le 1,\forall(t,s,p)\in J^{dl};\\ \sum\limits_{(t,p)\in J^{h+}}\zeta^{h+}_{tp}\le\Gamma^{h+};0\le\zeta^{h+}_{tp}\le 1,\forall(t,p)\in J^{h+};\\ \sum\limits_{(t,p)\in J^{h-}}\zeta^{h-}_{tp}\le\Gamma^{h-};0\le\zeta^{h-}_{tp}\le 1,\forall(t,p)\in J^{h-};\end{array}\right\} \tag{3.1}$$

The uncertainty budget vector (Γ^{cc}, Γ^{ct}, Γ^{q}, Γ^{dl}, Γ^{h+}, Γ^{h-}) used in (3.1) indicates how the random variables are modeled in this formulation, considering the independence of the random variables. The analysis of the worst case for this set considers that all uncertain parameters reach their maximum deviation. For this reason, and following Bertsimas and Sim (2003), we consider only positive deviations of the uncertain parameters (i.e., only the positive half-interval of variations). Note that a negative deviation can never lead to the worst-case performance when the uncertain parameters are in the objective function. Hence, this modelling assumption is without any loss of generality, even if we may observe negative deviations in practice.

The robust counterpart of formulation (2.1)–(2.10), in which all uncertainties are allocated in the objective function, is obtained by solving a min-max problem, presented in (3.2), where the aim is to minimise the detriment of the objective function value when the uncertain parameters attain their worst case.

$$\begin{aligned}&\min_{X,Y,I^{+},I^{-}}\left\{\max_{\widetilde{cc}_{tsp},\widetilde{ct}_{ts},\tilde{q}_{tsp},\widetilde{dl}_{tsp},\tilde{h}^{+}_{tp},\tilde{h}^{-}_{tp}}\left(\sum_{(t,s,p)\in J^{cc}}\widetilde{cc}_{tsp}X_{tsp}+\sum_{(t,s)\in J^{ct}}\widetilde{ct}_{ts}Y_{ts}\right.\right.\\&+\sum_{(t,s,p)\in J^{q}}\tilde{q}_{tsp}(1-\theta)X_{tsp}+\sum_{(t,s,p)\in J^{dl}}\widetilde{dl}_{tsp}l_{tsp}X_{tsp}\\&\left.\left.+\sum_{(t,p)\in J^{h+}}\tilde{h}^{+}_{tp}I^{+}_{tp}+\sum_{(t,p)\in J^{h-}}\tilde{h}^{-}_{tp}I^{-}_{tp}\right)\right\}\end{aligned} \tag{3.2}$$

Given a feasible solution X^*, Y^*, I^{+*}, I^{-*} of problem (3.2) in terms of the outer minimisation variables, we can rewrite the inner maximisation problem of (3.2) as follows:

$$\max_{\zeta_{tsp}^{cc},\zeta_{ts}^{ct},\zeta_{tsp}^{q},\zeta_{tsp}^{dl},\zeta_{tp}^{h+},\zeta_{tp}^{h-}} \sum_{(t,s,p)\in J^{cc}} \left(\widehat{cc}_{tsp}\zeta_{tsp}^{cc}\right) X_{tsp}^* + \sum_{(t,s)\in J^{ct}} \left(\widehat{ct}_{ts}\zeta_{ts}^{ct}\right) Y_{ts}^*$$
$$+ \sum_{(t,s,p)\in J^{q}} \left(\hat{q}_{tsp}\zeta_{tsp}^{q}\right)(1-\theta) X_{tsp}^* + \sum_{(t,s,p)\in J^{dl}} \left(d\hat{l}_{tsp}\zeta_{tsp}^{dl}\right)\left(l_{tsp}\right) X_{tsp}^* \tag{3.3}$$
$$+ \sum_{(t,p)\in J^{h+}} \left(\hat{h}_{tp}^{+}\zeta_{tp}^{h+}\right) I_{tsp}^{+*} + \sum_{(t,p)\in J^{h-}} \left(\hat{h}_{tp}^{-}\zeta_{tp}^{h-}\right) I_{tsp}^{-*}$$

s.t.

$$\sum_{(t,s,p)\in J^{cc}} \zeta_{tsp}^{cc} \le \Gamma^{cc} \tag{3.4}$$

$$\sum_{(t,s)\in J^{cc}} \zeta_{ts}^{ct} \le \Gamma^{ct} \tag{3.5}$$

$$\sum_{(t,s,p)\in J^{q}} \zeta_{tsp}^{q} \le \Gamma^{q} \tag{3.6}$$

$$\sum_{(t,s,p)\in J^{dl}} \zeta_{tsp}^{dl} \le \Gamma^{dl} \tag{3.7}$$

$$\sum_{(t,p)\in J^{h+}} \zeta_{tsp}^{h+} \le \Gamma^{h+} \tag{3.8}$$

$$\sum_{(t,p)\in J^{h-}} \zeta_{tsp}^{h-} \le \Gamma^{h-} \tag{3.9}$$

$$0 \le \zeta_{tsp}^{cc} \le 1, \quad \forall (t,s,p) \in J^{cc} \tag{3.10}$$

$$0 \le \zeta_{ts}^{ct} \le 1, \forall (t, s) \in J^{ct} \tag{3.11}$$

$$0 \le \zeta_{tsp}^{q} \le 1, \forall (t, s, p) \in J^{q} \tag{3.12}$$

$$0 \le \zeta_{tsp}^{dl} \le 1, \forall (t, s, p) \in J^{dl} \tag{3.13}$$

$$0 \le \zeta_{tp}^{h+} \le 1, \forall (t, p) \in J^{h+} \tag{3.14}$$

$$0 \le \zeta_{tp}^{h-} \le 1, \forall (t, p) \in J^{h-} \tag{3.15}$$

where the terms in the objective function related only to the nominal values cc_{tsp}, ct_{tsp}, q_{tsp}, dl_{tsp}, h_{tp}^{+}, h_{tp}^{-} are omitted as they are not relevant in the maximisation problem. Since the resulting problem (3.3)–(3.15) is feasible and bounded, through the concept of duality, the following equivalent minimisation problem can be obtained, which can be replaced in (3.2) without loss of generality:

$$\min_{\left(\begin{smallmatrix}\lambda^{cc},\, \lambda^{cc},\, \lambda^{q},\, \lambda^{dl},\, \lambda^{h+},\, \lambda^{h-} \\ \mu^{cc}_{tsp},\, \mu^{cc}_{tp},\, \mu^{q}_{tsp},\, \mu^{dl}_{tsp},\, \mu^{h+}_{tsp},\, \mu^{h-}_{tp}\end{smallmatrix}\right)} \Gamma^{cc}\lambda^{cc} + \Gamma^{ct}\lambda^{ct} + \Gamma^{q}\lambda^{q} + \Gamma^{dl}\lambda^{dl} + \Gamma^{h+}\lambda^{h+} + \Gamma^{h-}\lambda^{h-}$$
$$+ \sum_{(t,s,p)\in J^{cc}} \mu^{cc}_{tsp} + \sum_{(t,s)\in J^{ct}} \mu^{ct}_{ts} ++ \sum_{(t,s,p)\in Jq} \mu^{q}_{tsp} + \sum_{(t,s,p)\in J^{dl}} \mu^{dl}_{tsp} + \sum_{(t,p)\in J^{h+}} \mu^{h+}_{tsp} \tag{3.16}$$
$$+ \sum_{(t,p)\in J^{h-}} \mu^{h-}_{tsp}$$

s.t.

$$s.t. \quad \lambda^{cc} + \mu^{cc}_{tsp} \geq \widehat{cc}_{tsp} X_{tsp}, \qquad \forall (t,s,p)\in J^{cc} \tag{3.17}$$

$$\lambda^{ct} + \mu^{ct}_{ts} \geq \widehat{ct}_{ts} Y_{ts}, \qquad \forall (t,s)\in J^{ct} \tag{3.18}$$

$$\lambda^{q} + \mu^{q}_{tsp} \geq \hat{q}_{tsp}(1-\theta) X_{tsp}, \qquad \forall (t,s,p)\in J^{q} \tag{3.19}$$

$$\lambda^{dl} + \mu^{dl}_{tsp} \geq \widehat{dl}_{tsp} l_{tsp} X_{tsp}, \qquad \forall (t,s,p)\in J^{dl} \tag{3.20}$$

$$\lambda^{h+} + \mu^{h+}_{tp} \geq \hat{h}^{+}_{tp} I^{+}_{tp}, \qquad \forall (t,p)\in J^{h+} \tag{3.21}$$

$$\lambda^{h-} + \mu^{h-}_{tp} \geq \hat{h}^{-}_{tp} I^{-}_{tp}, \qquad \forall (t,p)\in J^{h-} \tag{3.22}$$

$$\lambda^{cc} \geq 0 \tag{3.23}$$

$$\lambda^{ct} \geq 0 \tag{3.24}$$

$$\lambda^{q} \geq 0 \tag{3.25}$$

$$\lambda^{dl} \geq 0 \tag{3.26}$$

$$\lambda^{h+}_{tp} \geq 0 \tag{3.27}$$

$$\lambda^{h-}_{tp} \geq 0 \tag{3.28}$$

$$\mu^{cc}_{tsp} \geq 0, \qquad \forall (t,s,p)\in J^{cc} \tag{3.29}$$

$$\mu^{ct}_{ts} \geq 0, \qquad \forall (t,s)\in J^{ct} \tag{3.30}$$

$$\mu^{q}_{tsp} \geq 0, \qquad \forall (t,s,p)\in J^{q} \tag{3.31}$$

$$\mu^{dl}_{tsp} \geq 0, \qquad \forall (t,s,p)\in J^{dl} \tag{3.32}$$

$$\mu^{h+}_{tp} \geq 0, \qquad \forall (t,p)\in J^{h+} \tag{3.33}$$

$$\mu^{h-}_{tp} \geq 0, \qquad \forall (t,p)\in J^{h-} \tag{3.34}$$

By incorporating model (3.16)–(3.34) into (3.2) and then combining the result with the original SSIC model, we obtain the robust counterpart of SSIC with uncertain costs and delivery delay times, named hereafter as RSSIC:

$$\begin{aligned}
&\min_{\begin{pmatrix} \lambda^{cc}, \lambda^{ct}, \lambda^{q}, \lambda^{dl}, \lambda^{h-} \\ \mu^{cc}_{tsp}, \mu^{ct}_{ts}, \mu^{q}_{tsp}, \mu^{dl}_{tsp}, \mu^{h}_{tp} \\ X, Y, I^{+}, I^{-} \end{pmatrix}} \sum_{\substack{t\in T \\ s\in S \\ p\in P}} cc_{tsp} X_{tsp} + \sum_{\substack{t\in T \\ s\in S}} ct_{ts} Y_{ts} + \sum_{\substack{t\in T \\ s\in S \\ p\in P}} (1-\theta) q_{tsp} X_{tsp} \\
&+ \sum_{\substack{t\in T \\ s\in S \\ p\in P}} l_{tsp} d_{tsp} X_{tsp} + \sum_{\substack{t\in T \\ p\in P}} h^{+}_{tp} I^{+}_{tp} + \sum_{\substack{t\in T \\ p\in P}} h^{-}_{tp} I^{-}_{tp} + \Gamma^{cc} \lambda^{cc} + \Gamma^{ct} \lambda^{ct} + \Gamma^{q} \lambda^{q} \\
&+ \Gamma^{dl} \lambda^{dl} + \Gamma^{h+} \lambda^{h+} + \Gamma^{h-} \lambda^{h-} + \sum_{(t,s,p)\in J^{cc}} \mu^{cc}_{tsp} + \sum_{(t,s)\in J^{ct}} \mu^{ct}_{ts} + \sum_{(t,s,p)\in J^{q}} \mu^{q}_{tsp} \\
&+ \sum_{(t,s,p)\in J^{dl}} \mu^{dl}_{tsp} + \sum_{(t,p)\in J^{h+}} \mu^{h+}_{tsp} + \sum_{(t,p)\in J^{h-}} \mu^{h-}_{tsp}
\end{aligned} \tag{3.35}$$

s.t. constraints (2.2)–(2.10)

constraints (3.17)–(3.34)

It is worth mentioning a few advantages of the proposed model, which are:

- It considers supply capacities, limitation of physical inventory space, decisions to carry inventories for the next horizons, strategic delaying the delivery of material, and several costs inherent to the management of the supply chain in the context of the SSP. This whole context enables the decision-maker to find an optimal solution that satisfies all these requirements and brings increased efficiency in supply chain management in the context addressed.
- From a technical point of view, the model is a MIP that can be straightforwardly solved by general-purpose MIP software, including those open-source and free, and it can be solved in acceptable computational times in a practical scenario. Furthermore, it was possible to solve large-scale instances.

The model has a few disadvantages as well, which are:

- Practitioners who are not familiar with OR techniques may have difficulties in the computational implementation and use of the model, since it requires prior knowledge of a modelling language used in optimisation.
- The quality of the solutions provided as output depends on estimating the uncertainty sets. If the estimate is not accurate, the model will likely provide solutions that might present uncertainty, as expected.

4 Computational experiments

This section presents the results of computational experiments with the RSSIC model proposed in Section 3.2. All experiments were conducted using GAMS 26.0.0 software and the general-purpose MIP solver of the IBM CPLEX Optimization Studio v.12.8, on a computer with an Intel Core i7-3537U 2.00 GHz processor, 16 GB of RAM and Windows 10 operating system. The stop criterion was established as the time limit of 3,600 seconds or a 0% optimality gap.

4.1 Data description

For the creation of the instances, different classes were defined that indicate the size of the instances depending on the cardinality of their sets. Instances are made up of variations within the classes obeying the established probability distributions and the cardinalities of the sets. For this purpose, we generated four instance classes, namely 5P-10T-15S, 10P-20T-30S, 15P-30T-45S and 20P-40T-60S, where *a*P-*b*T-*c*S means *a* number of products, *b* number of periods and *c* number of suppliers. Five instances were created for each of the four classes. They were randomly generated according to the distributions presented in Table 2, which are based on the example provided by Ware et al. (2014). Recall that cc_{tsp} is the unit purchase cost, ct_{ts} is the transportation cost, C_{tsp} is the supplier ability, d_{tp} is the product demand, q_{tsp} is the operational unit cost, l_{tsp} is the delay cost, dl_{tsp} is the estimated delivery delay time, and h_{tp}^{+} is the cost of keeping one unit of product. It is worth mentioning that all randomly generated parameters are realistic and in accordance with previous practical experiences of the authors.

Table 2 Values used in the generation of instance parameters

Parameter	*Distribution*
cc_{tsp}	~Uniform [3; 7]
ct_{ts}	~Uniform [500; 2,500]
C_{tsp}	~Uniform [500; 2,500]
d_{tp}	~Uniform [1,000; 5,000]
q_{tsp}	~Uniform [3; 10]
l_{tsp}	~Uniform [8; 17]
dl_{tsp}	~Uniform [0; 4]
h_{tp}^{+}	~Uniform [3.3; 7.7]

We also adopted the following parameter settings: 95% service level agreement (θ = 0.95); physical storage limitation of 3,000 products of each type in each period (w_{tp} = 3,000), and a maximum number of late orders less than 10% of all orders in the portfolio (Φ = 0.90). Last, the initial stocks of all products were considered equal to zero, and the cost to delay delivery of an item (h_{tp}^{-}) was considered as the maximum value of parameter l_{tsp} considering all suppliers, given by $\max_s l_{tsp}$.

4.2 Model evaluation and managerial insights

We ran computational experiments to evaluate the performance of RSSIC model and the behaviour of the solutions obtained according to the uncertainties of the input data. These experiments were conducted separated by instance class. For each uncertain parameter, there could be up to five worst-case occurrences, that is, each of the budgets of uncertainty (Γ^{cc}, Γ^{ct}, Γ^{q}, Γ^{dl}, Γ^{h+}, Γ^{h-}) assuming values in the set {1,…,5}. Additionally, the uncertain parameters assumed a controlled deviation γ, using the following variation values: 10%, 25% and 50%, with $\widehat{cc}_{tsp} = \gamma cc_{tsp}$ and so on with the other costs. In this context, when the budgets of uncertainty assume null values (i.e., [Γ^{cc}, Γ^{ct}, Γ^{q}, Γ^{dl}, Γ^{h+}, Γ^{h-}] = 0), only the deterministic part of the model is active. In these cases, the final value of the objective function corresponds to a solution of the deterministic model SSIC. When at least one of the budgets assumes a positive value, the respective costs may vary according to the controlled deviation γ, thus allowing reaching their worst-case deviations. Scenarios were considered in which only one given budget assumes a positive value (the others assume null values), allowing analysis of the impact of variation in each cost and scenarios in which all budgets simultaneously assume the same positive value.

Figure 3 Results of RSSIC model – classes 5P-10T-15S and 10P-10T-30S (see online version for colours)

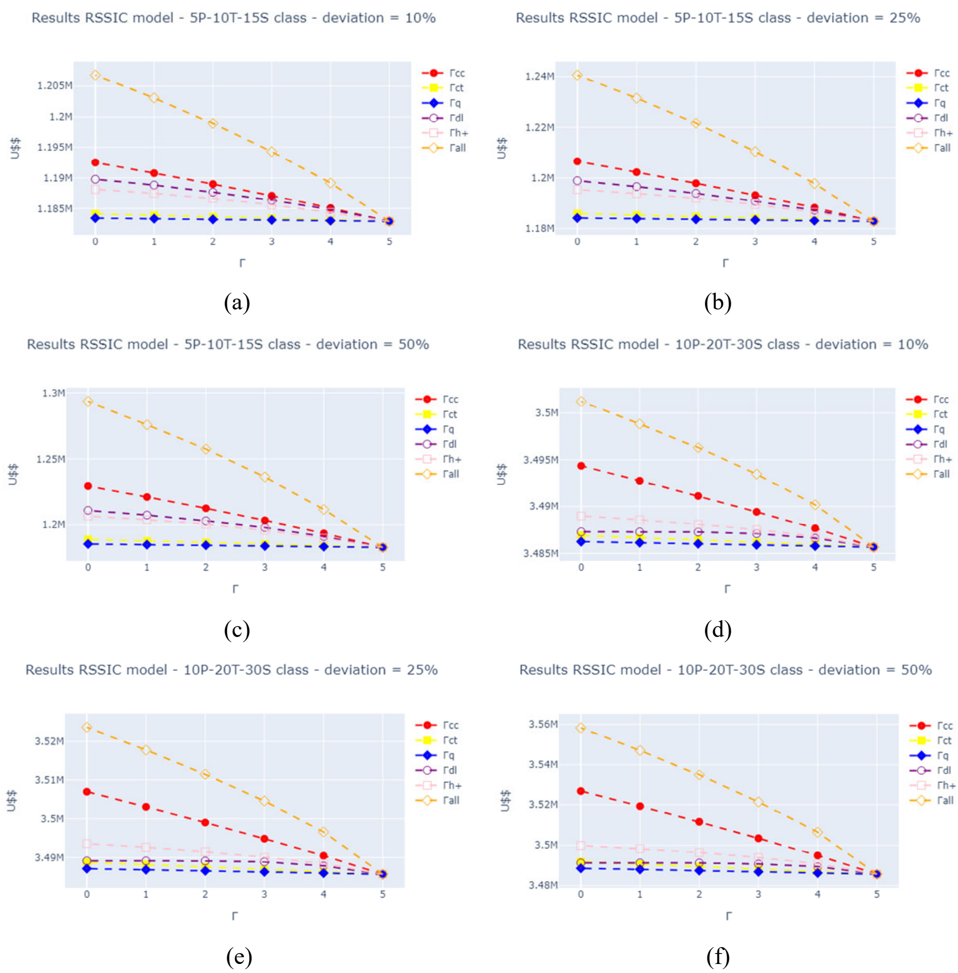
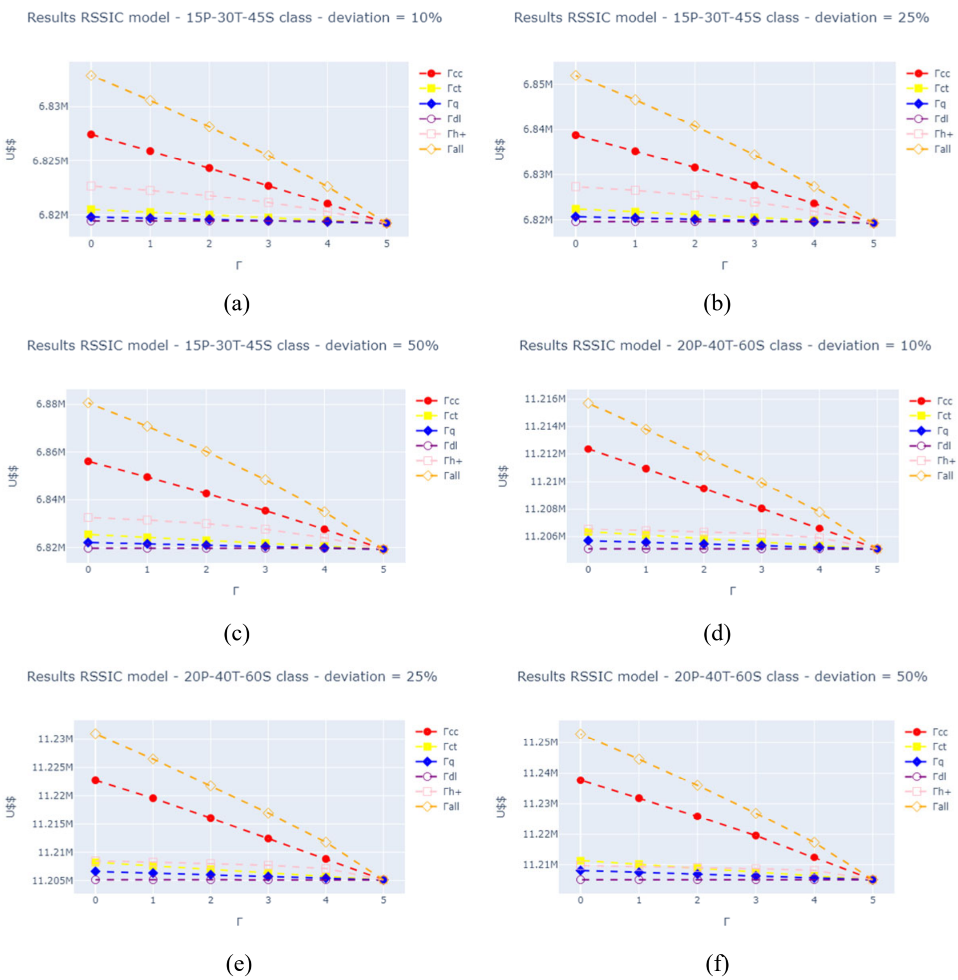


Figure 3 shows the results of the experiments with instances in classes 5P-10T-15S and 10P-20T-30S, whereas Figure 4 shows the results with instances in classes 15P-30T-45S and 20P-40T-60S, according to the different configurations of budget of uncertainty values. In all charts presented in these figures, the x-axis shows the value of the corresponding budget Γ (maximum number of uncertain parameters that can achieve their worst case), and the y-axis shows the optimal value of the objective function in currency units (United States dollars). The MIP solver proved optimality for all instances in less than 1 hour in all scenarios.

Figure 4 Results of RSSIC model – classes 15P-30T-45S and 20P-40T-60S (see online version for colours)



(a) (b) (c) (d) (e) (f)

The results in Figures 3 and 4 indicate that the increase in the optimal value of instances in all classes, in the worst case, is always less than the deviation of the uncertain parameters and the Risk measure drops significantly with few uncertain variables going to their worst case. This indicates the achievement of satisfactory solutions from a financial point of view. Performing an analysis by class, the following can be observed:

- *Class 5P-10T-15S:* when the costs are analysed individually, the one that has the greatest impact on the value of the solution is purchase costs, promoting an increase ranging from 0.19% to 0.81% when the deviation is 10%; when the deviation is increased to 25%, the increase in the solution value ranges from 0.46% to 2.00%; and when the deviation reaches its maximum value at 50%, the increase in the solution value ranges from 0.91% to 3.94%. The second-highest impact on the solution values occurs when the delivery delay times vary within the limits (that is, $\Gamma^{dl} > 0$). When the deviation is 10%, the increase in the solution value ranges from 0.16% to 0.58%; when the deviation is 25%, the encumbrance ranges from 0.37% to 1.36%; and when the deviation is 50%, the increment ranges from 0.70% to 2.37%. These results are somewhat surprising because the highest costs, namely, the transportation costs, did not yield the greatest degradation value in the objective function as expected. This occurred due to three factors: restrictions in the satisfaction of the service level agreement set at 95%; the maximum level of delay in meeting the demand fixed at 10% for all evaluated instances; and decisions to stock RMs for future horizons, avoiding shipping in all planning horizons. When all costs are analysed simultaneously at the worst case the greatest impact occurs when the deviation is 50%, as expected; the increase ranges from 2.43% to 9.36%.

- *Class 10P-20T-30S:* when the costs are analysed individually, the previous class scenario remains, and the purchase costs remain the highest impact on the objective function. Variation in this cost promotes an increase ranging from 0.06% to 0.25% when the deviation is 10%. When the deviation increases to 25%, the increase ranges from 0.14% to 0.61%. When the deviation reaches its maximum value of 50%, the increase in the solution value ranges from 0.27% to 1.18%. The second-highest impact on the solution value arose when variations in the delay time parameters ($\Gamma^{dl} > 0$) are assumed. The cost of late delivery promotes an increase ranging from 0.11% to 0.16% when the deviation is 50%. The variation in the other costs promotes small increments, not significantly changing the solution values. When all costs attain their worst case simultaneously (last five rows), the worst case occurs when the deviation is 50%, burdening the solution value by 0.59% to 2.08%.
- *Class 15P-30T-45S:* analysing costs individually, the only cost that has a significant impact on the value of the solution is the cost of purchase, promoting an increase in the magnitude of 0.06% to 0.29%, when the deviation is at 25%, and an increase in the magnitude of 0.12% to 0.54% when the deviation reaches its maximum value. When all costs are analysed simultaneously, the impact when the deviation is at 50% remains in the range of 0.23% and 0.90%, and the impact when the deviation is at 25% is 0.12% to 0.41%. These favorable results were obtained by increasing the supply of supplier's options, and the increased planning horizons, of the previous classes, allowing the model to find less costly solutions.
- *Class 20P-40T-60P:* analysing costs individually, the model can achieve satisfactory solutions since it manages to split demand among several suppliers, not significantly increasing the value of the solution. The worst case occurs when the cost of buying is perturbed. This perturbation promotes the increase from 0.07% to 0.23% when the deviation is at its maximum value. When all costs attain their worst case simultaneously for the deviation of 25%, the value of the objective function increases from 0.06% to 0.23%. When the deviation reaches its maximum value (50%), the solutions' values increase from 0.10% to 0.43%. Again, as presented in the result of the previous class, the increase in supply and planning horizons contributed to the achievement of better solutions.

Finally, we observe that two situations occur for the scenarios with $\Gamma^{h-} > 0$, in which the parameter backlog inventory cost (h_{tp}^{-}) can reach its worst case: either small variations occur in the objective function values and the value stabilises rapidly, or the value reaches the worst case in the first occurrence of its budget, thus remaining constant throughout the other scenarios. The low influence and variation in this cost were also motivated by constraints (2.5), which impose a maximum delay rate on the demand portion at the service level's expense.

4.3 PoR and risk analysis through Monte Carlo simulation

To determine the robustness level of the solutions provided by the RSSIC model, we carried out experiments using Monte Carlo simulations to assess the risk of deterioration of the solution values in a real scenario. Different simulations were performed to evaluate the possible realisations of the uncertain parameters, using three probability distributions, namely normal, triangular, and uniform, and three uncertainty levels, given by 10%, 25% and 50% deviations. Additionally, we considered seven different combinations of uncertain parameters, namely $\{\widetilde{cc}_{tsp}\}, \{\widetilde{ct}_{tsp}\}, \{\tilde{q}_{tsp}\}, \{\widetilde{dl}_{tsp}\}, \{\hat{h}_{tp}^{+}\}, \{\hat{h}_{tp}^{-}\}$, and $\{\widetilde{cc}_{tsp}, \widetilde{ct}_{tsp}, \tilde{q}_{tsp}, \widetilde{dl}_{tsp}, \tilde{h}_{tp}^{+}, \tilde{h}_{tp}^{-}\}$. We selected the first instance of each class for these experiments. For each instance, each uncertainty level, each probability distribution, and each combination of uncertain parameters we generated 1,000 samples, hence we performed sensitivity analyses using 324,000 scenarios.

The use of three probability distributions enables different scenarios by which the model could be evaluated and eliminates any bias caused by only one distribution. In the simulations, a controlled interval was selected in which the value of each random sample remained restricted. This interval assumed that the controlled deviation could have variations above or below the original deviation, where the values were truncated to not be outside the range $[\psi - \hat{\psi}, \psi + \hat{\psi}]$, where ψ represents the nominal value of the parameter in the instance and $\hat{\psi}$ represents the deviation to which the parameter is subjected $(\hat{\psi} = \gamma\psi)$. The interval corresponds to the entire sample space that the random variable may assume and is known as the full interval. This assumption was adopted because, in a real-life situation, positive and negative cost fluctuations are commonly observed (for example, due to currency exchange rates). Note that it does not conflict with the definition of the uncertainty set (3.1), defined in Section 3, as the worst-case of the uncertain parameters in the RSSIC model are always attained with positive deviations only. Including negative deviations in the simulation is essential to represent a real-life context, while in the model we can omit them without loss of generality. As the uncertain parameters are costs and delivery time, when positive deviations occur, costs are increasing (worst scenarios). When negative deviations occur, the reverse process occurs, and costs decrease (best case scenarios).

In the simulations in which the normal distribution was adopted, the following parameters were considered: the mean value of the parameter, given by its nominal value (ψ) and the standard deviation to which this parameter would be subjected (γ). In the simulations with the triangular distribution, the parameters were as follows: the lower value of the parameter subject to the deviation $(\psi - \hat{\psi})$, the upper value of the parameter subject to the deviation $(\psi + \hat{\psi})$ and the nominal value of the parameter (ψ). Last, in the scenarios in which the uniform probability distribution was used, the following were adopted as bounds: the lower value of the parameter subject to the deviation $(\psi - \hat{\psi})$ and the upper value of the parameter subject to the deviation $(\psi + \hat{\psi})$.

The main objective of the simulations was to measure the impact of uncertain parameters on the solution value and analyse the risk of deterioration of this value (that is, leaving the solution infeasible with respect to the goal) in various scenarios and distributions of varied probabilities. Three performance measures, detailed below, were proposed for the analysis: the probability of the solution deteriorating (Risk), the price of robustness (PoR) and a performance measure that evaluates the average impact on the solution value when the scenario had a deteriorated value (RI). The empirical performance measure Risk was adopted rather than the theoretical boundary suggested by Bertsimas and Sim (2003) because, on many occasions, the latter offers far-from-reality probabilities. The calculation of these measures is carried out as follows, for a given instance, uncertainty level and probability distribution:

- *Probability of the solution deteriorating (risk):* for a given deviation γ and a given configuration of the budgets of uncertainty, let x^*_{rob} be the optimal solution of the instance obtained with model RSSIC, with corresponding optimal value z^*_{rob}. For each randomly generated sample using the same deviation, considering realisation values for the uncertain parameters, we recompute the value of x^*_{rob} in the objective function (2.1) using these realisations instead of the nominal parameter values, which results in $\overline{z}^*_{rob}$. Then, if $\overline{z}^*_{rob}$ is greater than z^*_{rob}, a counter that accounted for deteriorated scenarios is incremented. At the end of the calculation, the final value of this counter and its proportion relative to the population of 1,000 samples is reported, thus obtaining the frequency of deterioration of the objective function value (or of goal violation).
- *PoR:* this measure shows the relative difference between the optimal values obtained by the robust (RSSIC) and deterministic (SSIC) models and was calculated as follows: $PoR = \left(\dfrac{z^*_{rob} - z^*_{\det}}{z^*_{\det}} \right) \cdot 100\%$, where z^*_{rob} and $z^*_{\det}$ represent the optimal values of the RSSIC and SSIC models for the same instance, respectively. This measure shows the impact on the solution value, which the protection function promotes, deteriorating the solution value in risk aversion.
- *Average relative increase (RI):* for the solutions with $\overline{z}^*_{rob} > z^*_{rob}$ we stored the accumulated sum of the difference between these values in a variable. At the end, the average relative increase in the sampled solution value $\overline{z}^*_{rob}$ can be obtained concerning the optimal value z^*_{rob}.

The calculation of the performance measures Risk and RI is summarised in Algorithm 1. This algorithm was applied to each instance, uncertainty level, probability distribution, combination of uncertain parameters, and configuration of budgets of uncertainty used in the experiments. We used seven combinations of uncertain parameters, namely $\{\widetilde{cc}_{tsp}\}$, $\{\widetilde{ct}_{tsp}\}$, $\{\tilde{q}_{tsp}\}$, $\{\widetilde{dl}_{tsp}\}$, $\{\hat{h}^+_{tp}\}$, $\{\hat{h}^-_{tp}\}$, and $\{\widetilde{cc}_{tsp}, \widetilde{ct}_{tsp}, \tilde{q}_{tsp}, \widetilde{dl}_{tsp}, \tilde{h}^+_{tp}, \tilde{h}^-_{tp}\}$. For example, in the first combination, only $\widetilde{cc}_{tsp}$ was considered as an uncertain parameter and hence we generated random values (realisations) for this parameter only – all the other parameters were defined in the sample using their nominal values. In the last combination of uncertain parameters, we generated realisations for all parameters originally defined as uncertain in the RSSIC model. These combinations allow us to analyse the impact of uncertainty in each parameter individually and then, eventually, the impact of uncertainties in all parameters simultaneously.

Algorithm 1: Calculation of performance measures Risk and RI using Monte Carlo simulation.

Input: Instance; uncertain parameters; configuration of budgets of uncertainty [Γ^{cc}, Γ^{ct}, Γ^{q}, Γ^{dl}, Γ^{h+}, Γ^{h-}] and the corresponding optimal solution x^*_{rob} with value z^*_{rob}; uncertainty level γ; probability distribution.

Generate 1,000 samples considering the uncertainty level γ and the specified uncertain parameters;

    For each sample **do**

    Calculate $\bar{z}^*_{rob}$, the objective function value of x^*_{rob} using (2.1) and the sampled values as the values of the uncertain parameters;

    If $\bar{z}^*_{rob} > z^*_{rob}$ **then**

      count_RISK=*count_RISK*+1;

      sum_RI = *sum_RI* + $(\bar{z}^*_{rob} > z^*_{rob})$;

    End-If

End-For

Risk=*count_RISK*/1000;

RI=*sum_RI*/1000;

Figure 5 Results of Monte Carlo simulation – class 5P-10T-15S (see online version for colours)

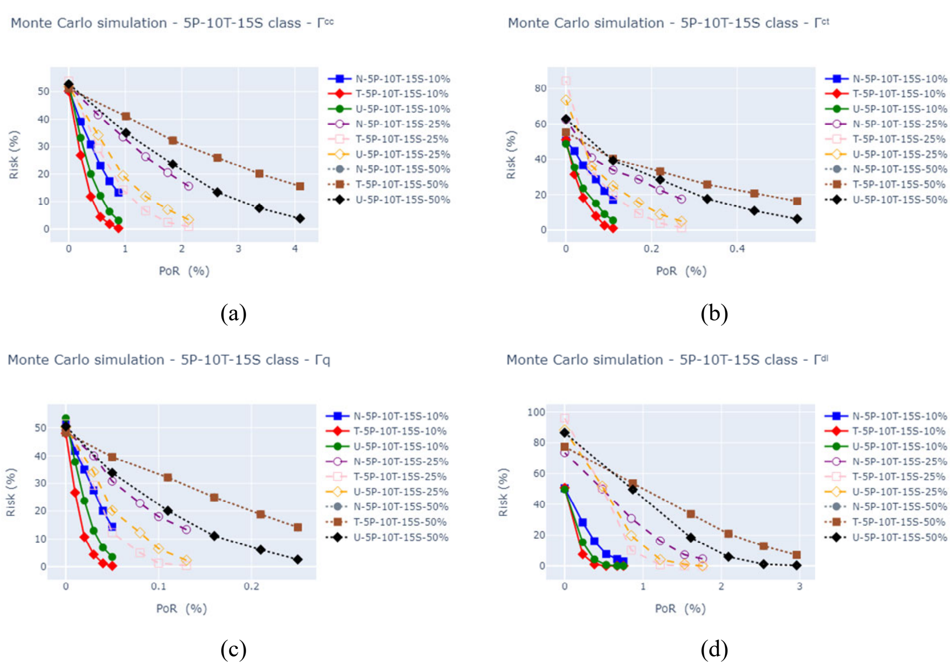
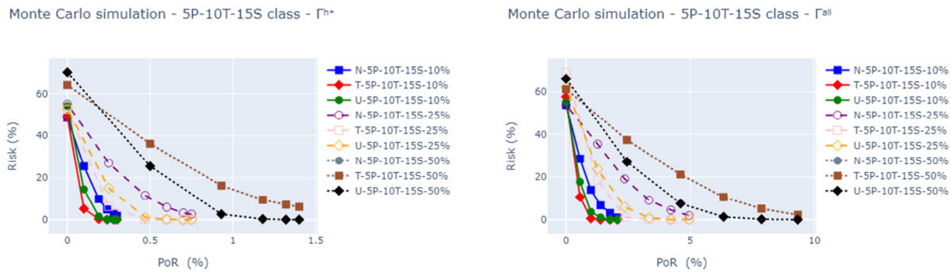
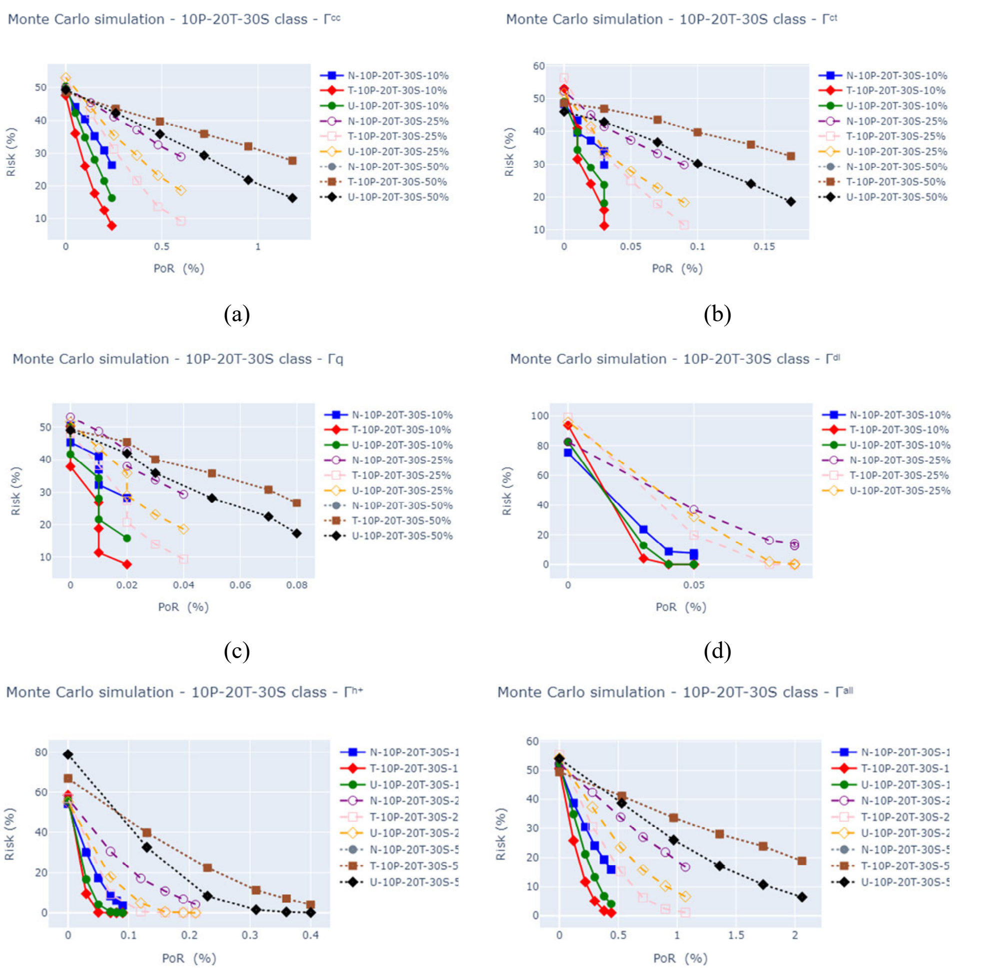


Figure 5 Results of Monte Carlo simulation – class 5P-10T-15S (continued) (see online version for colours)



(e) (f)

Figure 6 Results of Monte Carlo simulation – class 10P-20T-30S (see online version for colours)



(a) (b)

(c) (d)

(e) (f)

The results of the experiments with the seven combinations of uncertain parameters and the three probability distributions are presented in Figures 5–8, in an aggregate form. We aggregate the instances using the form: probability distribution – class – deviation. Hence, the combination N-15P-30T-45S-50%, indicates that we consider all instances in class 15P-30T-45S, a normal probability distribution and a deviation equal to 50%. All charts in the figures show the risk versus PoR analysis for all the respective values in percentages. The values in the points of each curve correspond to the results of 1,000 samples, calculated as described above. When all results of uncertain variables during simulation are zero, the subfigures or curves are not displayed.

In the analysis of the 324,000 scenarios, we observed that the values of measure performance RI for instances with 40 periods (largest horizon planning) only increased by 3.58% (maximum value during all the simulations) when the deviation was considered of 50% (highest deviation value). Since this performance measure is considered quite satisfactory, we do not plot their respective results.

Figure 7 Results of Monte Carlo simulation – class 15P-30T-45S (see online version for colours)

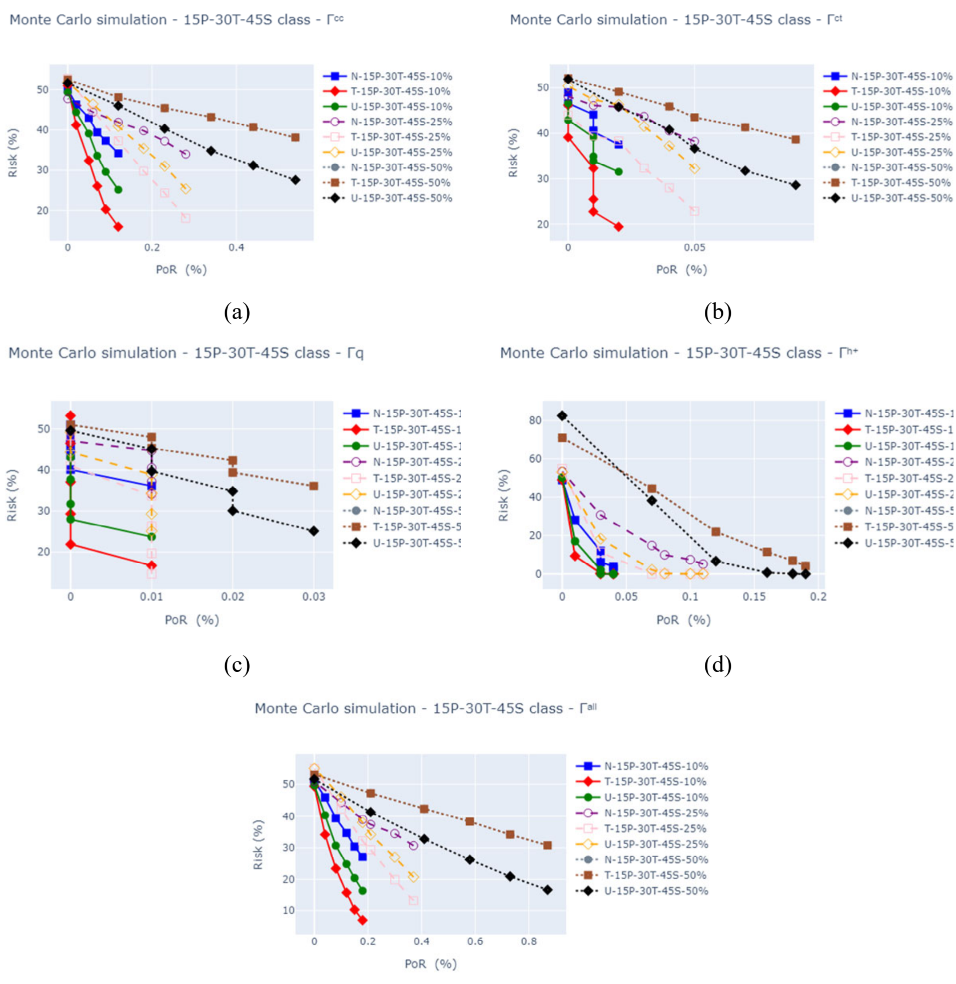
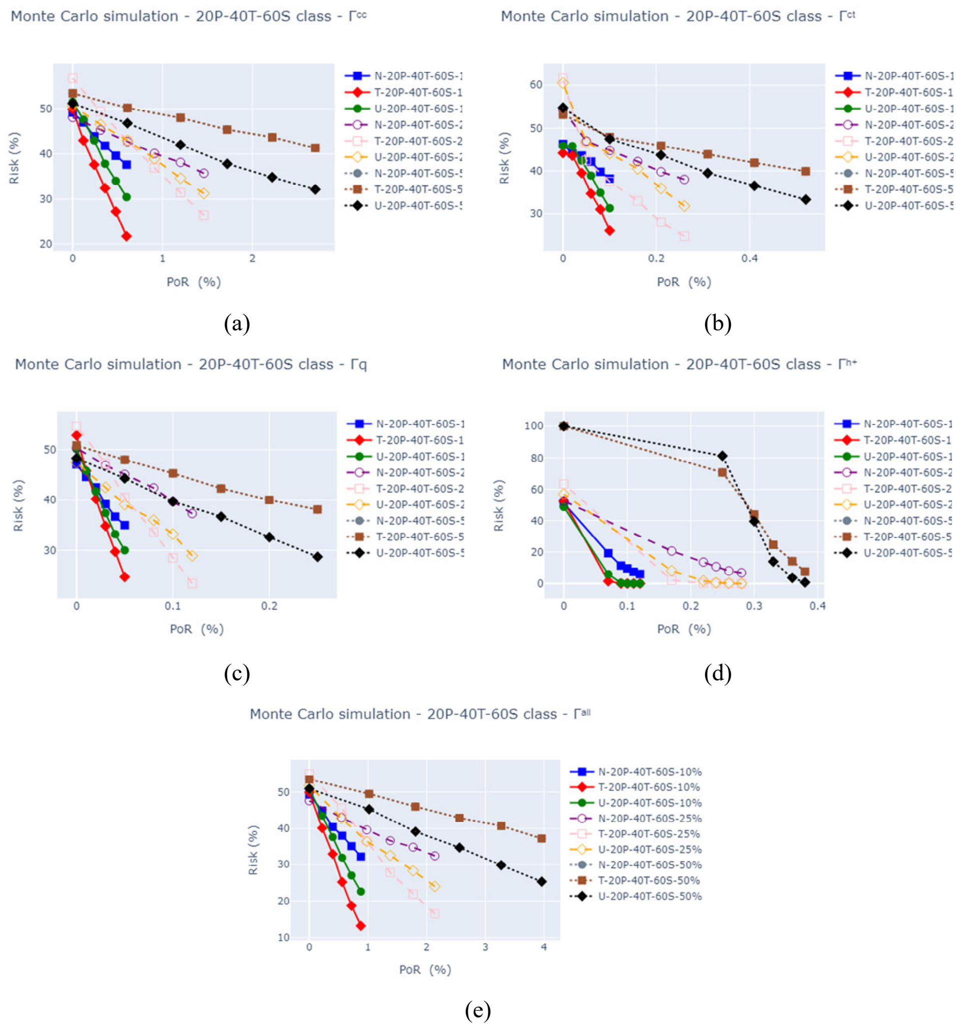


Figure 8 Results of Monte Carlo simulation – class 20P-40T-60S (see online version for colours)



The results plotted in Figures 5–8 show that the increments in the PoR performance measure are significantly smaller than the deviation of 10% in the uncertain parameters. Additionally, the Risk measure declines as more variables attain their worst case, reaching relatively low values. This trade-off between these performance measures (PoR x Risk) highlights the advantages of using the RO approach to support decision-making in the addressed context. RO yielded very satisfactory solutions, not significantly burdening the solution values, when the parameters were perturbed inside the considered uncertainty levels. Note that although the random variables have different behaviours according to each probability distribution, the approach considered in the RSSIC model remains valid because there are no drastic changes in behaviour in the solutions and the conclusions obtained remain. Performing an analysis by class, the following can be observed:

- *Class 5P-10T-15S:* when the costs are analysed individually and the deviation is 10%, the highest PoR is given by purchase costs, ranging from 0.21% to 0.81% across all three distributions. The second-highest price is given by late delivery costs, ranging from 0.23% to 0.75% across all three distributions. This occurred because the parameter is linked to the maintenance of the service level agreement, which is set at 95% for all computational tests. When the deviation increases to 25%, the situation remains, but the level of encumbrance in the PoR ranges from 0.52% to 2.12% when the purchase cost is perturbed and 0.48% to 1.76% when the late delivery time is perturbed. When the deviation reaches its maximum value (50%), the solutions are more affected, ranging from 1.01% to 4.09% when the purchase cost is perturbed and 0.87% to 2.96% when the late delivery time is disturbed. We carry out an additional analysis for the samples in which all parameters are considered uncertain. This analysis consists of determining if the sum of the prices of robustness of all the individual parameters is exactly equal to the PoR when all costs are analysed simultaneously. This can highlight which parameter most affects the objective function value. To exemplify this situation, consider $\Gamma = 1$ for all cases when the tests are performed with the parameters going to the worst case separately (combinations 1 to 6) and the deviation is considered equal to 10%; the prices of robustness are 0.21%, 0.02%, 0.01%, 0.10% and 0.0%. The sum of all these individual results is equal to 0.57%, which exceeds by 0.1% (0.56%) the value obtained by the PR performance measure when all costs are analysed simultaneously. The situation remains when $\Gamma > 1$ so that the solutions improve successively, becoming less costly with each component going to the worst case, given that when $\Gamma = 2$, 3 and 4, the solution improves by 0.03%, and when $\Gamma = 5$, the solution improves by 0.02%. When the deviation is 25%, the situation remains; for $\Gamma = 1$, the solution improves by 0.06%; for $\Gamma = 2$, 3 and 4, the solution improves by 0.09%; and for $\Gamma = 5$, the solution improves by 0.08%. When the deviation reaches 50% and $\Gamma = 1$, 2, the solution improves by 0.09%; when $\Gamma = 3$, the solution improves by 0.05%; when $\Gamma = 4$, the solutions are the same; and when $\Gamma = 5$, the solution improves by 0.08%.
- *Class 10P-20T-30S:* when the costs are analysed individually, the highest PoR is given by the purchase costs, ranging from 0.05% to 0.24% when the deviation is 10%; ranging from 0.13% to 0.60% when the deviation is 25%; and ranging from 0.26% to 1.18% when the deviation is 50%. This was due to the high volume of product demand for this class, noting that this cost is directly proportional to consumer demand, and because in this class the planning horizon is extended, this cost has a greater impact on this class. The second highest PoR is paid when the cost to carry inventory is uncertain, ranging from 0.03% to 0.09% when the deviation is 10%, 0.07% to 0.21% when the deviation is 25%; and 0.13% to 0.40% when the deviation is 50%. This situation occurred because in this class, the number of periods is higher than the RM delivery times, making it worthwhile to carry inventory for future demands to avoid shipping payments (the highest costs of the entire chain) in all planning horizons. A point that should be emphasised occurs when the estimated delivery time in supply delay is evaluated in the deviation of 50% [Figure 6(d)]. Despite the model not being able to lower the value of the Risk performance measure in all evaluated distributions, that is, all requests will be received late in this case, the value of the solution increased only 0.11%. This scenario only occurred in this

instance. In the other instances of the class, the solutions were the same as in the RSSIC model. When all costs are analysed simultaneously, the sum of the PoR of all individual uncertain costs, when $\Gamma = 1$ and 2, is exactly equal to the value obtained when all costs are evaluated simultaneously. When $\Gamma = 3$, the solution improves by 0.01%; when $\Gamma = 4$, the solution improves by 0.02%; and when $\Gamma = 5$, the solution improves by 0.01%. This is explained by the stabilisation in the RI performance measure value according to the increase in $\Gamma^{dl} = 2, 3, 4, 5$ according to the distribution. The stabilisation of this value according to $\Gamma^{h+} = 3, 4, 5$ also contributes to the null risk performance measure value in this class.

- *Class 15P-30T-45S:* again, when costs are analysed individually, the most significant relevance occurs when the purchase cost parameter is perturbed. The price paid for robustness remained in the ranges of 0.02% to 0.12%, 0.06% to 0.28% and 0.12% to 0.54%, respectively to deviations 10%, 25%, and 50%. When all costs are analysed simultaneously, the situation remains analogous to the previous class. For $\Gamma = 1, 2$ we obtained the same values of when all were evaluated simultaneously. For $\Gamma = 3$, the solution improves by 0.01%, and for $\Gamma = 4, 5$ the solutions are identical. In this class, the stabilisation of values begins to be influenced, when the costs to carrier stock into hand and inventory costs to delay deliveries are perturbed. Therefore, their respective stabilisations start in the sets $\Gamma^{h+} = \{3, 4, 5\}$ and $\Gamma^{h-} = \{0, 1, 2, 3, 4, 5\}$.
- *Class 20P-40T-60S:* similarly, the highest price for robustness is obtained when the purchase cost is perturbed. The price paid for robustness remained in the ranges of 0.12% to 0.60%, 0.31% to 0.46% and 0.17% to 0.31%, respectively. To deviations 10%, 25%, and 50%. The second highest price paid was reached when the cost to carrier inventory was perturbed, remaining in the range of: 0.7% to 0.12%; 0.17% to 0.28%, and 0.25% to 0.38%, respectively to deviations 10%, 25%, and 50%. When all costs are analysed simultaneously, it was identified that the sum of the price of the robustness of all individual uncertain costs was exactly equal to the value obtained when all were evaluated simultaneously in all occurrences. In this class, stabilisation begins when $\Gamma q \geq 2$, $\Gamma^{h+} > 2$, $\Gamma^{h-} \geq 0$.

For better visualisation of the results in this section, Table 3 summarises the main findings.

Table 3 Summary of the main findings of this section

Finding	*Main idea*
1	The RSSIC model can solve large-scale instances in acceptable computational time.
2	The optimal solutions of the RSSIC model indicate that the increase in the optimal value of instances in all classes, in the worst case, is always less than the deviation of the uncertain parameters and the Risk measure drops significantly with few uncertain variables going to their worst case.
3	The Monte Carlo simulation indicated the robustness of the RSSIC model, and the solution values increased only 3.58% on average, considering deviations of 50%.

5 Conclusions

We presented a solution approach for decision-makers, such as managers, supervisors, coordinators, and analysts, regarding supplier selection under uncertainties. This solution approach is based on an effective mathematical model that can be easily incorporated into expert systems like enterprise resource planning (ERP). The study focused on contributing both to GS chain management by modelling and analysing most of the three cost classes indicated by Holweg et al. (2011) and Suri (2010); and to operations research literature, by developing and analysing a RO model to formulate the supplier selection problem.

5.1 Managerial implications

The proposed model allowed us to incorporate and analyse the costs cited by Holweg et al. (2011), such as currency exchange rate variations, increases in fuels involved in the chain (oil, gas, and carbon emission tax), costs with unscheduled trips for prospecting and after-sales, and intellectual capital losses. Similarly, it is possible to address the following costs noted by Suri (2010): costs of maintaining local offices and agencies to keep contact and negotiate with suppliers scattered around the globe, obsolescence costs due to design changes not aligned with delivery lead times, and costs of lost sales due to unmet deadlines and to problems with quality.

For this purpose, these costs were considered as components of the supplier operational unit cost (q_{tsp}) and the supplier delay cost (l_{tsp}). Because these two parameters are volatile and highly influenced by several variables active in the logistics chain, estimating and correctly appropriating these values into products or services are complex. For this reason, these costs had not yet been mathematically evaluated in the literature on the subject effectively. This study sought to reduce this gap, contributing to state of the art.

5.2 Theoretical Implications

First, a deterministic model was proposed, based on the linearisation of the model published by Ware et al. (2014) and its extension by including inventory control and warehousing constraints. In addition, an RO model with uncertainties in costs and delivery times was also proposed to evaluate the performance of supplier selection in an uncertain environment. As identified in the literature review, this is the first RO model proposed for the supplier selection problem under uncertainties that can be used to solve large-scale instances resorting only to general-purpose MIP software. This model is also the first to incorporate the influence of uncertainties in most of the three cost classes (static, dynamic, and hidden costs) indicated by Holweg et al. (2011) in a supplier selection decision through RO, and to consider the control of RM inventory, preventing stockouts and violations of physical storage constraints.

Computational experiments considering uncertainty in costs and delivery delay times, presented in Section 4, demonstrate that the proposed RO approach enhances the level of robustness of solutions in risk aversion. This is supported by the encumbrance level promoted in the solution values when protection against uncertainty was used; the increase in the optimal value in the worst case is always less than the deviation of the uncertain parameters. This indicates that the model provides sufficiently robust solutions against variations in these costs, bringing advantages from a financial point of view.

Monte Carlo simulations considering different probability distributions showed no significant change in behaviour in the solutions, even though the behaviour of the uncertain parameters was different according to the probability distributions used in each scenario. For the 324,000 scenarios, we observed that all values of RI (average relative increase of objective function value) performance measure were always less than the deviation considered in all scenarios. In this analysis, the objective function values increased only 3.58% in the worst case, considering 20 products, 40 periods, 60 suppliers and an uncertainty level of 50%.

Also, it can be observed that the solution stabilised in class 10P-20T-30S when more than two random variables go to the worst case in the cost of delay. In class 15P-30T-45S we observed the same effect when some random variable going to the worst case was allowed in the backlog stock cost and when three or more random variables were allowed in the cost of loading inventory. Finally, in class 20P-40T-60S it was observed that the stabilisations were met when $\Gamma^{h-} \geq 0$, $\Gamma^{h+} > 2$ e $\Gamma^{q} > 2$ demonstrating that the model was able to achieve stability in the increasing value of the objective function when few random variables going to the worst case were allowed, proving the effectiveness of the OR approach in the application used in this paper.

5.3 Future research

As a future study, we intend to analyse the model's performance using real-life data to evaluate how the solutions and the respective decisions change. This analysis can be used to assess strategies for estimating the parameters related to hidden costs because these costs are often considered intangible. Another possibility is to develop models that consider other significant costs mentioned by Holweg et al. (2011), e.g., unplanned urgent shipments to meet rush orders. It is also relevant to include the costs noted by Suri (2010), that is, unplanned design changes, which generate obsolete stock of finished products, intermediate items, and RM.

5.4 Research limitation

During the computational experiments, we observed that another critical parameter for supplier selection problem models is demand uncertainty. Because one of the focal points of this study is intercontinental transactions, it is necessary to negotiate product purchases many planning periods ahead due to the delivery logistics of RM. Hence, another RO model can be proposed to protect solutions against this variation. Another limitation of this study is the consideration only of decisions related to the selection of suppliers. The recent literature has some models that integrate logistic and production decisions, and this is an attractive field to be explored in the supplier selection context.

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Availability of data and material

The samples used in the computational tests are available in metada.

Code availability

The programs used in the computational tests are available in metada.

Consent for publication

All authors of this work have consent for publication

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