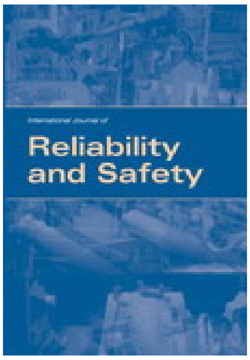

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Performance analysis of client-server distributed system using Gumbel-Hougaard family Copula

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Abstract: The current paper compares the performance of two distinct client-server distributed system architectures using the capabilities of the Gumbel-Hougaard Family Copula. Using the Gumbel-Hougaard Family Copula, this paper presents a comprehensive comparative analysis of the performance of two distinct client-server distributed system architectures. The first architecture consists of three identical clients, a load balancer and three identical servers, whereas the second employs a three-component warm standby system with imperfect switching. The failure and repair patterns in both systems are exponential. The analysis entails using supplementary variable technique and Laplace transforms to solve first-order differential equations derived from transition diagrams for each system. While failures are distributed exponentially, repair times are represented by the versatile Gumbel-Hougaard Family Copula and a general distribution. The study considers various parametric values to evaluate various reliability metrics such as system availability, system reliability, Mean Time To Failure (MTTF), MTTF sensitivity and cost function. The findings are presented in the form of tables and figures, which provide a clear visual representation of the obtained insights.

Keywords: architecture; client-server; distributed system; Gumbel-Hougaard; performance; reliability.

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1 Introduction

The client-server model is essential for streamlining data transfer and promoting economic expansion. Clients submit requests to one or more servers in this system, which then patiently await them and may work with additional servers to address the client's needs. Clients and servers frequently work on different machines connected by networks and are typically organised within a multi-tiered system architecture.

The limitations of the conventional client-host model, where a single hardware entity serves a large number of dumb terminals for distributed cloud access, are addressed by distributed systems, which are built on the client-server model. The client-server model's centralised approach, which enables information security through access controls implemented by security protocols, is a notable benefit. This framework requires little upkeep and is essential for managing distributed transactions for clients. Additional advantages include its ease of management and quick data delivery, and the system's highly centralised data storage improves data security.

A client-server distributed system's effectiveness and efficiency depend on each of its component parts. A distributed system's overall dependability, availability, reliability, mean time to failure and revenue generation are significantly impacted by the frequency and type of component failures. The most reliable key components must therefore be identified and evaluated in terms of dependability, mean time to failure, mean time between failures, availability, generated revenue and reliability.

The majority of computer systems experience network performance problems like lag, data lag and sluggish data transmission because of things like packet collisions, congestion and slowdowns. Designing electronic communication networks requires the use of mathematical modelling and performance evaluation of computer networks. Models are tools that developers use to evaluate a system before it is implemented. By evaluating the performance of computer network designs, designers have the freedom to change different network parameters during the planning stage rather than when the network is actually in use. With the aim of strategically enhancing manufacturing capacity and industry competitiveness, ongoing device performability assessment is a popular trend for assessing system performance in terms of reliability, availability, maintainability and more.

In the fields of system engineering, enhancement, and configuration, performance is crucial. Not only must systems be assumed to function correctly, but they must also do so efficiently. Numerous examples from a wide range of fields, including computer networks (including client-server configurations), telecommunications systems and industrial manufacturing systems, have shown how improving system performance can avert disasters, save time, lower costs, lower labour costs, lower risks and even save lives. Investigations into performance are conducted to assess already-in-use or upcoming systems, investigating various configurations in an effort to pinpoint the ideal design configuration.

Numerous studies in the field of reliability engineering have shown that effective performance analysis can avert disasters and save money and time. Here are a few instances: In the performance assessment framework, Yemane and Colledani (2019) presented an approach to evaluate the efficiency of uncertain manufacturing systems in unreliable machines. Zhao et al. (2021) investigated and optimise the economic performance of a cold standby system susceptible to shocks and imperfect repairs, proposing geometric process models to quantify the lifetime and repair time. A method for improving the performance of safety instrumented systems against incidents and reduction of cascading failures was studied by Xie et al. (2021) when addressing the prevention of cascading failures, Safety Instrumented Systems (SIS) reliability and durability are taken into consideration. By means of Monte Carlo simulations, the method is validated by iterative combinations using the reliability block diagram. By creating models to look into the effects of cascading failures in railway signalling systems, Xie et al. (2019) attempt to determine the average frequency of critical failures in high- or continuous-demand mode systems that are susceptible to cascading failures.

Using Generalised Stochastic Petri Nets (GSPN), Zhang et al. (2017) developed a model for evaluating the productivity of multi-stage serial manufacturing systems with rework loops and productive polymorphism. When experienced by several competing failures, Ye et al. (2020) proposed a method for predicting the performance and evaluating the probabilities of different states in serial AMSs. Models were developed by Malik and Tewari (2018) to analyse the effectiveness of the water flow system. By means of the use of the Markovian method, the performance models are created by resolving the differential difference equations that were obtained from the system's schematic diagram. Singh et al. (2016) delved into performance analysis of a system made up of two separate subsystems connected in series. They obtain and numerically analyse the performance measures associated with the system's efficiency using the copula approach. In a study on the reliability and performance evaluation of a parallel system incorporating a standby unit, four different types of failures were taken into account by Kumar et al. (2018). They developed reliability models using the Markovian method, Laplace, and supplementary variable techniques, which were then applied to gauge the system's effectiveness. To explore and enhance the operational effectiveness of a safety system exposed to aging, destructive testing and unforeseen shocks, Zhang et al. (2022) created models to assess the efficiency and efficacy of the system. Zhao et al. (2021) proposed an optimal replacement policy for two cold standby components system subject to d-shock in which repair times of the components follows geometric process. The entire components are replaced with new ones whenever the number of repair of the first component reached a threshold value.

Copula technique is a powerful technique for describing interdependence among variables that has gotten a lot of interest in a lot of domains. The joint lifetime distribution can be generated by modelling component dependence using a Copula function, making it more convenient and adaptable in applications (Nelson, 2006).

Several researchers have investigated repairable systems and proposed ways to improve reliability by contributing to making complicated systems more dependable using copula. We can mention Lado et al. (2018) who explored cost estimation based on system performance for a configuration involving two consecutive subsystems. They employed supplementary variables and gave precedence to repairing the initially failing unit. The Copula repair technique was used by Yusuf et al. (2020a, 2020b) to demonstrate the efficient functioning of a multi-computer system made up of three sequential subsystems. In order to analyse a system made up of two sequentially arranged subsystems that experienced various types of failures, Lado and Singh (2019) used the Copula technique. A repairable linear sequential 2-out-4 system's reliability assessment was created by Yusuf et al. (2019) while taking both online and offline preventive maintenance strategies into account. Additionally, Gahlot et al. (2020) studied the performance characteristics of repairable systems operating in a series configuration, considering multiple failure types and two distinct repair approaches, utilising the Copula technique. Niwas and Garg (2018) introduced an approach centred around a warranty policy without costs, which aimed to assess both the reliability and profitability of an industrial system. Meanwhile, Poonia (2021) conducted a study utilising supplementary techniques and Laplace transforms on a multi-state computer network featuring a series arrangement of three database servers and five web servers. This network was also considered for potential Copula-based repairs. Poonia (2021) delved into investigation of some performance measures of distributed computer network composing of load balancers, web servers and database servers for replication under the redundancy scheme of k-out-of-n. Chen et al. (2022) analysed the reliability index of CNC heavy duty machine based on probability importance and reliability redistribution method. El-Moumen et al. (2022) delved into reliability analysis of discrete system through Markovian and stochastic petri nets to obtain the system's performance indicator.

2 Related works and contributions of the current research

The client-server concept/client-server architecture has recently been receiving significant attention from both academia and industry due to its importance in sustaining and maintaining economy. Here, the client-server model outlines how a load balancer distributes resources and services to one or more clients. This architecture is widely used in distributed computing and can be used in a wide range of applications. The client-server concept/model is now a significant component of increasingly complicated systems, which identify a server and a client process that interacts through a load balancer to accomplish some of its functionality. The model presented in Figure 1 introduces a symmetric architecture in which all the clients and servers play the same role. Therfore, this paradigm is well suited to highly decentralised architecture, which scales better in the number of peers dimension.

For a distributed system to function optimally, it needs to possess a certain degree of reliability and availability. These two metrics of performance are commonly employed in devising maintenance strategies to enhance the system's effectiveness. To achieve this goal, Zhang (2019) conducted an investigation into the performance of a computer network by leveraging intelligent cloud computing technology and reliability theory. Meanwhile, Yusuf et al. (2021) showcased the effectiveness of a multi-computer system consisting of three sequential subsystems, utilising the Copula repair method. The reliability attributes of a computer network system, which included load balancers, distributed database servers and a centralised server configured in a series-parallel arrangement with three subsystems, are discussed in Yusuf et al. (2018). Potapov et al. (2019) delved into the assessment of reliability in a client-server information system. In a recent study, Singh et al. (2021) examined the probabilistic evaluation of a CBT network system featuring four distinct subsystems arranged in series, employing the Copula repair policy. Rahman (2018) explored the stationary availability factor for computer networks with a two-level structure and arbitrary topology. Singh et al. (2020) presented reliability metrics for three computer laboratories connected to a server in a 2-out-of-3: G configuration. More recently, Sanusi and Yusuf (2021) employed the RAMD methodology to analyse the performance of a Computer-Based Test (CBT) at the component level.

Exploring the outcomes resulting from Copula repair, Yusuf et al. (2021) investigated a configuration featuring five clients and two servers as subsystems 1 and 2, utilising the k-out-of-n: G arrangement. In a more recent effort, John et al. (2022) conducted a study that focused on analysing the reliability of a multi-hardware-software system considering interactions among failures. Addressing the issue of heterogeneity in server-client systems, Garg (2019) elaborated on a method involving Remote Procedure Call (RPC). Zhu and Pham (2019) introduced a fresh approach to modelling system reliability that accounts for hardware, software and the interactions between the two. Chopra and Ram (2019) investigated the availability and reliability of a parallel network system composed of two distinct units, utilising the copula approach. Zeng et al. (2019) explored an empirical method for analysing the reliability of hardware-software co-designed devices. Sanusi and Yusuf (2022) discussed the resilience of a distributed data centre network topology consisting of three units. Through the integration of solar panels and advancements in computer technology, Osemwengie et al. (2022) established an economical network for internet operation. This was achieved by incorporating more access points and utilising computer systems with reduced operating costs. Zhu and Pham (2018) provided a model of software reliability integrating the martingale method with environmental factors distributed by gamma. Kumar (2019) provided a review on client-server-based applications and research opportunity. A multi-computer system with n clients and the k-outof-n was explored by Rawal et al. (2022) for reliability assessment.

Nailwal and Singh (2012), Singh et al. (2016, 2022), Ram and Singh (2008) and Chantola and Singh (2020) examined system performances via supplementary variable and Laplace transform with implications of Gumbel-Hougaard copula and concluded that copula repair is better than general repair.

Table 1 Some related research on reliability and performance measure of client-server distributed systems

Reference	*System*	*Failure distribution*	*General repair distribution*	*Gumbel-Hougaard repair distribution*	*Time dependent reliability metrics*	*Standby*	*Switching failure*
Jindal et al. (2019)	Server	Exponential	N/A	N/A	N/A	N/A	N/A
Lawan et al. (2018)	Hardware-software distributed system	Exponential	N/A	N/A	N/A	Parallel	N/A
Yusuf et al. (2021)	Multi client-server network	Exponential	N/A	N/A	N/A	Parallel	N/A
Singh and Gahlot (2021)	Multi-client under k-out-of-n: G scheme	Exponential	yes	yes	yes	Parallel	N/A
Potapov et al. (2019)	Client-server architecture	Exponential	N/A	N/A	N/A	N/A	N/A
Wei (2021)	Distributed network system	N/A	N/A	N/A	N/A	N/A	N/A
Current Study	Client-server distributed system	Exponential	yes	yes	yes	Warm	yes

Researchers such as those mentioned above have made significant contributions to improving the efficiency and performance of various systems. There has been a thorough examination of the numerous factors that influence system reliability. However, more research in this field is required as there have been few studies on the performance evaluation of distributed systems. As a result, the characteristics of the Gumbel-Hougaard Family Copula were used in this study to evaluate the performance of two typical client-server distributed systems that had not previously been captured by any researcher. The contributions of this paper are as follow:

- We have formulated novel models of performance analysis of client client-server distributed system considering models; parallel and warm standby units. The switching mechanism is introduced in the second model with warm standby units. Warm standby client-server distributed systems reduce energy use and recovery period because a standby unit is partly energised and subjected to maximum stress while the primary unit is up and running and completely powered and functional after the primary unit stops working.
- Warm standby client-server model with switching failure is better than parallel client-server without switching failure.

- Warm standby client-server distributed systems possess time-dependent failure behavioural patterns; they have distinct failure rates before and after replacing the online defective units. Warm standby client-server distributed configuration is utilised in a multitude of settings, including adaptive databases, where the server retains the standby database as a duplicate of the active database; when the active database fails, the standby database resumes client applications to work with minimal service interruption.
- To develop the explicit expressions for the availability, reliability, mean time to failure, sensitivity and profit function for each model.
- To perform numerical comparison between the systems and rank the systems in terms of their availability and profit function.
- To identify the most critical/sensitive component in each system.
- To determine how failure and repair influence reliability metrics using Copula and General repairs, and to determine which repairs optimise system performance.

This paper is organised as follows, with Section 1 containing the introduction. Section 2 is devoted to a review of the literature, while Section 3 includes notations, assumptions and a description of the system. Section 4 covers Model I formulation and solution, while Section 5 covers Model I analysis, which takes into account particular cases. Section 6 presents Model II formulation and solution, Section 7 covers Model II analysis, Section 8 discussed the results and Section 9 concluded the paper with references.

3 Notations, assumptions and system description

3.1 Notations

t: Time variable on a time scale.

s: Laplace transform variable for all expressions.

ϑ_0 : Failure rate of the first client and first server.

ϑ_1 : Failure rate of the clients.

ϑ_2 : Failure rate of the servers.

ϑ_3 : Failure rate of the load balancer.

ϑ_4 : Switch Failure rate.

δ_1 : Warm standby Client failure rate.

δ_2 : Warm standby server failure rate.

$h(x)$: Repair rate of the clients.

$h(y)$: Repair rate of the servers.

$\epsilon_0(x)$: Repair rate for complete failed state of the clients.

$\epsilon_0(y)$: Repair rate for complete failed state of the servers.

$\epsilon_0(z)$: Repair rate for complete failed state of the load balancer.

$p_i(t)$: The probability that the system is in S_i state at instants for $i = 0$ to 9.

$\overline{P}(s)$: Laplace transformation of state transition probability $p(t)$.

$P_i(y,t)$: The probability that a system is in state S_i for $i = 1,...,9$, the system under repair and elapse repair time is given by (x,t) with repair variable *x* and time variable *t*.

$P_i(x,t)$: The probability that a system is in state S_i for $i = 1,...,9$, the system under repair and elapse repair time is denoted by (y,t) with repair variable *y* and time variable *t*.

$P_i(z,t)$: The probability that a system is in state S_i for $i = 1,...,9$, the system under repair and elapse repair time is denoted by (*y*, *t*) with repair variable *z* and time variable *t*.

$E_p(t)$: Expected profit function during the time interval [0, *t*).

J_1, J_2: Revenue and service cost per unit time, respective.

$\mu_0(x)$: According to the Gumbel-Hougaard family copula definition, joint probability is expressed as: $c_\theta(u_1(x), u_2(x)) = \exp\left(x^\theta + \left\{\log\phi(x)^\theta\right\}^{\frac{1}{\theta}}\right)$, $1 \le \theta \le \infty$. Where $\mu_1 = \phi(x)$ and $u_2 = e^x$.

3.2 Assumptions

1. All of the clients, load balancer and servers are in great working oer at the start.
2. Any unit failure may results in insufficient or inadequate system performance.
3. All partial failures of either client or server are restorable.
4. The required system unit should function as if it were brand new, and the repair procedure should cause no harm.
5. At initial stage, the system is operating in full capacity.
6. Failure of a particular subsystem (all clients or load balance oall servers) halt the operations of the other subsystems that have not failed.

3.3 Description of model

The client-server distributed system presented in this paper introduces a symmetric architecture, consists of three clients, one load balancer and three servers. In this system, the load balancer will deliver the request to the servers on behalf of the clients. The load balancer will choose the server with the least amount of traffic and send back the response to the client(s). When two clients and two servers die, the system continues to function, but any additional failures will result in a total or complete failure. A load balancer failure might also cause the system to crash. This system is susceptible to two types of failure: partial and total/complete failure. In the event of a total or complete failure, the system is repaired using Copula, while partially failed states are restored using General distribution. The system has nine states, six of which are active and three are in failed states. The states are described in full further down.

S_0: Represents the ideal condition, in which all three clients, the load balancer and the three servers are in perfect functioning order.

S_1: In this state, one client has failed, but the load balancer and three servers are still operational. The system is functional.

S_2: State 2 occurs when two clients fail, but the load balancer and three servers continue to function. The system is up and running.

S_3: One server is down in this state, but the load balancer and three clients are still up and running. The system is working.

S_4: Two servers have failed, but the load balancer and three clients are still operational in this state. The system appears to be functional.

S_5: Here, one server had already failed, and one client had failed unexpectedly. The load balancer, the remaining two servers and the remaining two clients are all operational. The system is up and running.

S_6: One of the clients had previously failed, and one of the servers had also failed suddenly. The load balancer, the remaining two clients and the remaining two servers are all functional. The system is working as expected.

S_7: In this state, the system is in complete failure status due to the breakdown of the load balancer.

S_8: The breakdown of the three servers has resulted in state 8, which is a complete failure.

S_9: The failure of the three servers has resulted in state 9, which is also a complete failure.

Figure 1 Client-server distributed system (Model I)

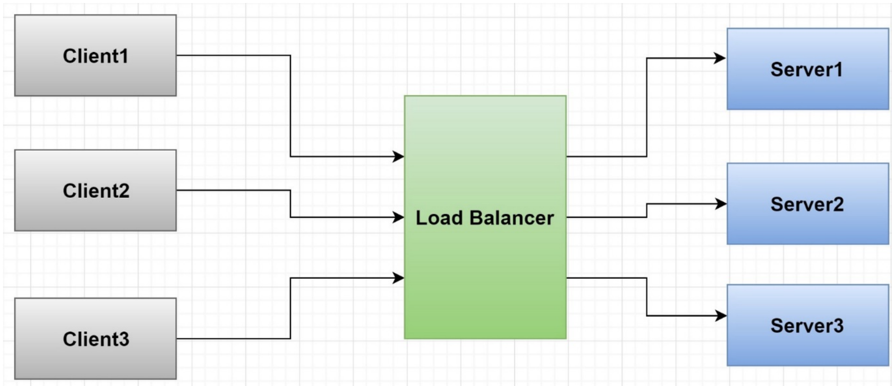
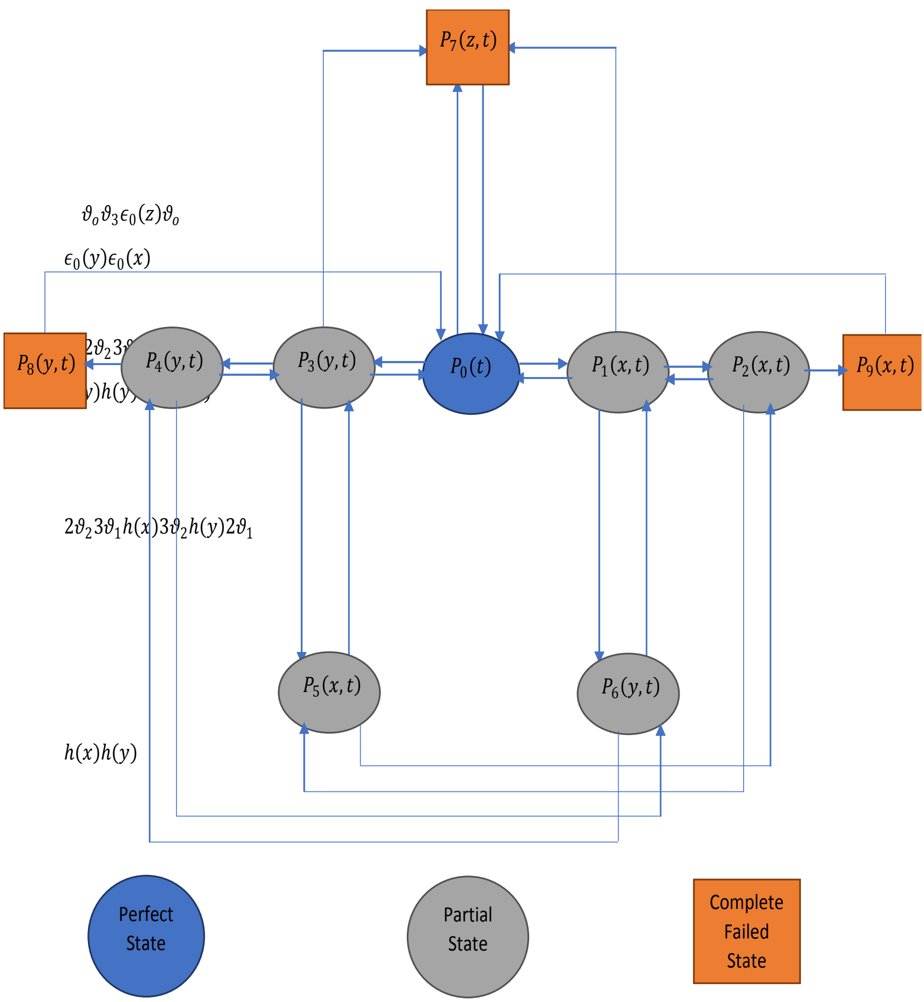


Figure 2 Transition diagram of the Model I



4 Formulation and solution of Model I

4.1 Formulation of Model I

For system modelling and analysis, reliability models were created using the supplementary variable technique and Laplace transforms. The differential equations were generated from the transition diagram using a probabilistic approach. These equations were then solved using initial and boundary conditions to obtain the steady state probabilities that serve as the foundation for the formulation of performance models.

The steps in getting the solutions of the state probabilities $P_k(s)$ for the formulation of the models involve:

a) Derivation of the partial differential equations from Figure 2.

b) Derivation of the boundary conditions of the states other than initial state.

c) Taking the Laplace transformation of (a) and (b) above.

d) Solving (c) to obtain the state probabilities $P_k(s)$.

The following partial differential equations are obtained via Figure 2 using the method used by Nelson (2006), Gulati et al. (2016), Singh and Ayagi (2017), and Gahlot et al. (2020).

$$\left[\frac{\partial}{\partial x}+3\vartheta_1+3\vartheta_2+\vartheta_3\right]P_o(t)=\int_0^\infty h(x)P_1(x,t)\,dx+\int_0^\infty h(y)P_3(y,t)\,dy+\int_0^\infty \in_0(z)P_7(z,t)\,dz+\int_0^\infty \in_0(y)P_8(y,t)\,dy+\int_0^\infty \in_0(x)P_9(x,t)\,dx \quad (1)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\vartheta_0+2\vartheta_1+3\vartheta_2+h(x)\right]P_1(x,t)=0 \quad (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\vartheta_1+2h(x)\right]P_2(x,t)=0 \quad (3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\vartheta_0+3\vartheta_1+2\vartheta_2+h(y)\right]P_3(y,t)=0 \quad (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\vartheta_2+2h(y)\right]P_4(y,t)=0 \quad (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\vartheta_1+h(x)\right]P_5(x,t)=0 \quad (6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+2\vartheta_2+h(y)\right]P_6(y,t)=0 \quad (7)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\vartheta_1+\epsilon_0\left(z\right)\right]P_7\left(z,t\right)=0 \tag{8}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\epsilon_0\left(y\right)\right]P_8\left(y,t\right)=0 \tag{9}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\epsilon_0\left(x\right)\right]P_9\left(x,t\right)=0 \tag{10}$$

With the following boundary conditions:

$$P_1\left(0,t=3\vartheta_1P_0\left(t\right)\right) \tag{11}$$

$$P_2\left(0,t\right)=6\vartheta_1\left[\vartheta_2+3\vartheta_1\vartheta_3\right]P_0\left(t\right) \tag{12}$$

$$P_3\left(0,t\right)=9\vartheta_1\vartheta_3P_0(t) \tag{13}$$

$$P_4\left(0,t\right)=6\vartheta_2\left[\vartheta_3+3\vartheta_1\vartheta_2\right]P_0\left(t\right) \tag{14}$$

$$P_5\left(0,t\right)=9\vartheta_1\vartheta_3P_0\left(t\right) \tag{15}$$

$$P_6\left(0,t\right)=9\vartheta_1\vartheta_2P_0\left(t\right) \tag{16}$$

$$P_7\left(0,t\right)=\left[\vartheta_3+3\vartheta_0\vartheta_1+3\vartheta_0\vartheta_3\right]P_0\left(t\right) \tag{17}$$

$$P_8\left(0,t\right)=6\vartheta_2^2\left[\vartheta_3+3\vartheta_1\vartheta_2\right]P_0\left(t\right) \tag{18}$$

$$P_9\left(0,t\right)=6\vartheta_1^2\left[\vartheta_2+3\vartheta_1\vartheta_3\right]P_0\left(t\right) \tag{19}$$

4.2 Model I solution

With the initial condition $P\left(0\right)=1$, we can generate the Laplace transforms of equations (1) to (10) as follow:

$$\begin{aligned}\left[S+3\vartheta_1+3\vartheta_2+\vartheta_3\right]\overline{P_0}\left(s\right)=1&+\int_0^\infty h\left(x\right)\overline{P}_1\left(x,s\right)dx+\int_0^\infty h\left(y\right)\overline{P}_3\left(y,s\right)dy\\&+\int_0^\infty \in_0\left(z\right)\overline{P}_7\left(z,s\right)dz+\int_0^\infty \in_0\left(y\right)\overline{P}_8\left(y,s\right)dy\\&+\int_0^\infty \in_0\left(x\right)\overline{P}_9\left(x,s\right)dx\end{aligned} \tag{20}$$

$$\left[s+\frac{\partial}{\partial x}+\vartheta_0+2\vartheta_1+3\vartheta_2+h\left(x\right)\right]\overline{P}_1\left(x,s\right)=0 \tag{21}$$

$$\left[s+\frac{\partial}{\partial x}+\vartheta_1+2h\left(x\right)\right]\overline{P}_2\left(x,s\right)=0 \tag{22}$$

$$\left[s+\frac{\partial}{\partial y}+\vartheta_0+3\vartheta_1+2\vartheta_2+h\left(y\right)\right]\overline{P}_3\left(y,s\right)=0 \tag{23}$$

$$\left[s+\frac{\partial}{\partial y}+\vartheta_2+2h(y)\right]\overline{P}_4(y,s)=0 \tag{24}$$

$$\left[s+\frac{\partial}{\partial x}+2\vartheta_1+h(x)\right]\overline{P}_5(x,s)=0 \tag{25}$$

$$\left[s+\frac{\partial}{\partial y}+2\vartheta_2+h(y)\right]\overline{P}_6(y,s)=0 \tag{26}$$

$$\left[s+\frac{\partial}{\partial z}+\epsilon_0(z)\right]\overline{P}_7(z,s)=0 \tag{27}$$

$$\left[s+\frac{\partial}{\partial y}+\epsilon_0(y)\right]\overline{P}_8(y,s)=0 \tag{28}$$

$$\left[s+\frac{\partial}{\partial y}+\epsilon_0(x)\right]\overline{P}_9(x,s)=0 \tag{29}$$

Also, the Laplace transforms of the boundary conditions are generated as follow:

$$\overline{P}_1(0,s)=3\vartheta_1\overline{P}_0(s) \tag{30}$$

$$\overline{P}_2(0,s)=6\vartheta_1[\vartheta_2+3\vartheta_1\vartheta_3]\overline{P}_0(s) \tag{31}$$

$$\overline{P}_3(0,s)=3\vartheta_3\overline{P}_0(s) \tag{32}$$

$$\overline{P}_4(0,s)=6\vartheta_2[\vartheta_3+3\vartheta_1\vartheta_2]\overline{P}_0(s) \tag{33}$$

$$\overline{P}_5(0,s)=9\vartheta_1\vartheta_3\overline{P}_0(s) \tag{34}$$

$$\overline{P}_6(0,s)=9\vartheta_1\vartheta_2\overline{P}_0(s) \tag{35}$$

$$\overline{P}_7(0,s)=[\vartheta_3+3\vartheta_0\vartheta_1+3\vartheta_0\vartheta_3]\overline{P}_0(s) \tag{36}$$

$$\overline{P}_8(0,s)=6\vartheta_3^2[\vartheta_3+3\vartheta_1\vartheta_2]\overline{P}_0(s) \tag{37}$$

$$\overline{P}_9(0,s)=6\vartheta_1^2[\vartheta_2+3\vartheta_1\vartheta_3]\overline{P}_0(s) \tag{38}$$

Using equations (30) to (38), i.e., the above boundary conditions, equations (20) to (29) generate:

$$\overline{P}_1(s)=3\vartheta_1\left\{\frac{1-\overline{S}_h(s+\vartheta_0+2\vartheta_1+3\vartheta_2)}{s+\vartheta_0+2\vartheta_1+3\vartheta_2}\right\}\overline{P}_0(s) \tag{39}$$

$$\overline{P}_2(s)==6\vartheta_1[\vartheta_2+3\vartheta_1\vartheta_3]\left\{\frac{1-\overline{S}_{2h}(S+\vartheta_1)}{S+\vartheta_1}\right\}\overline{P}_0(s) \tag{40}$$

$$\bar{P}_3(s) = 3\vartheta_3 \left\{ \frac{1-\bar{S}_h\left(s+\vartheta_0+3\vartheta_1+2\vartheta_2\right)}{\left(s+\vartheta_0+3\vartheta_1+2\vartheta_2\right)} \right\} \bar{P}_0(\mathrm{s}) \tag{41}$$

$$\bar{P}_4(S) = 6\vartheta_2\left[\vartheta_3+3\vartheta_1\vartheta_2\right] \left\{ \frac{1-\bar{S}_{2h}\left(s+\vartheta_2\right)}{s+\vartheta_2} \right\} \bar{P}_0(\mathrm{s}) \tag{42}$$

$$\bar{P}_5(S) = 9\vartheta_1\vartheta_3 \left\{ \frac{1-\bar{S}_h\left(s+2\vartheta_1\right)}{Ss+2\vartheta_1} \right\} \bar{P}_0(\mathrm{s}) \tag{43}$$

$$\bar{P}_6(S) = 9\vartheta_1\vartheta_2 \left\{ \frac{1-\bar{S}_h\left(s+2\vartheta_2\right)}{s+2\vartheta_2} \right\} \bar{P}_0(\mathrm{s}) \tag{44}$$

$$\bar{P}_7(S) = \left[\vartheta_3+3\vartheta_0\vartheta_1+3\vartheta_0\vartheta_3\right] \left\{ \frac{1-\bar{S}_{\in_0}(S)}{S} \right\} \bar{P}_0(\mathrm{s}) \tag{45}$$

$$\bar{P}_8(S) = 6\vartheta_2^2\left[\vartheta_3+3\vartheta_1\vartheta_2\right] \left\{ \frac{1-\bar{S}_{\in_0}(S)}{S} \right\} \bar{P}_0(\mathrm{s}) \tag{46}$$

$$\bar{P}_9(S) = 6\vartheta_1^2\left[\vartheta_2+3\vartheta_1\vartheta_3\right] \left\{ \frac{1-\bar{S}_{\in_0}(S)}{S} \right\} \bar{P}_0(\mathrm{s}) \tag{47}$$

Summing up all the probabilities that the system is working, i.e., $\bar{P}_{up}(s) = \bar{P}_0(s)+\bar{P}_1(s)++\bar{P}_2(s)+\bar{P}_3(s)+\bar{P}_4(s)+\bar{P}_5(s)+\bar{P}_6(s)$. We obtain:

$$\begin{aligned} \bar{P}_{up}(s) = \frac{1}{R(s)}\Bigg[& 1+3\vartheta_1\left\{ \frac{1-\bar{S}_h\left(s+\vartheta_0+2\vartheta_1+3\vartheta_2\right)}{s+\vartheta_0+2\vartheta_1+3\vartheta_2} \right\} \\ & +6\vartheta_1\left[\vartheta_2+3\vartheta_1\vartheta_3\right]\left\{ \frac{1-\bar{S}_{2h}\left(s+\vartheta_1\right)}{s+\vartheta_1} \right\} + 3\vartheta_3\left\{ \frac{1-\bar{S}_h\left(s+\vartheta_0+3\vartheta_1+2\vartheta_2\right)}{\left(s+\vartheta_0+3\vartheta_1+2\vartheta_2\right)} \right\} \\ & +6\vartheta_2\left[\vartheta_3+3\vartheta_1\vartheta_2\right]\left\{ \frac{1-\bar{S}_{2h}\left(s+\vartheta_2\right)}{s+\vartheta_2} \right\} \\ & +9\vartheta_1\vartheta_3\left\{ \frac{1-\bar{S}_h\left(s+2\vartheta_1\right)}{Ss+2\vartheta_1} \right\} + 9\vartheta_1\vartheta_2\left\{ \frac{1-\bar{S}_h\left(s+2\vartheta_2\right)}{s+2\vartheta_2} \right\} \Bigg] \end{aligned} \tag{48}$$

where

$$R(s) = \left\{ \begin{aligned} & \left(s+3\vartheta_1+3\vartheta_2+\vartheta_3\right) - 3\vartheta_1\bar{S}_h\left(s+\vartheta_0+2\vartheta_1+3\vartheta_2\right) \\ & -3\vartheta_3\bar{S}_h\left(s+\vartheta_0+3\vartheta_1+2\vartheta_2\right) - \left(\vartheta_3+3\vartheta_0\vartheta_1+3\vartheta_0\vartheta_3\right)\bar{S}_{\in_0}(s) \\ & -6\vartheta_2^2\left(\vartheta_3+3\vartheta_1\vartheta_2\right)\bar{S}_{\in_0}(s) - 6\vartheta_1^2\left(\vartheta_2+3\vartheta_1\vartheta_3\right)\bar{S}_{\in_0}(s) \end{aligned} \right\}$$

5 Particular cases

5.1 Availability analysis of Model I

When repairs are provided to the system, the performance of the system is referred to as availability. Here, we obtain the system availability in two different forms: when the repair follows Copula distribution and when the repair follows General distribution.

1) *System availability when the repair follows Copula distribution:* In this case, we set $S_{\epsilon_0}(s) = \overline{S}_{\exp\left[x^\theta+\{\log\varphi(x)\}^\theta\right]^{1/\theta}}(s) = \dfrac{\exp\left[x^\theta + \{\log\varphi(x)\}^\theta\right]^{1/\theta}}{+\exp\left[x^\theta + 1\{\text{og}\,\varphi(x)\}^\theta\right]^{1/\theta}}$, $\overline{S}_h(s) = \dfrac{h}{s+h}$, the failure rates are set at different values, such as $\vartheta_0 = 0.050, \vartheta_1 = 0.055, \vartheta_2 = 0.060, \vartheta_3 = 0.065, \varphi = h = x = y = z = 1$ and all the repair rates are set to 1, i.e., $h(x) = h(y) = \epsilon_0(x) = \epsilon_0(y) = \epsilon_0(z) = 1$ in equation (48). Now using the inverse Laplace transform, we can generate the availability equation for Copula repair as follows:

$$\begin{aligned} P_{up}(s) = {} & 0.03067597048e^{-2.825715957t} - 0.06282702488e^{-1.595634487t} \\ & -0.000005265514432e^{-1.33768940t} \\ & +1.090723881e^{-0.04426015551t}\, 0.01324485000e^{-1.110000000t} \\ & -0.01181819909e^{-1.120000000t} \\ & -0.01789750748e^{-2.060000000t} - 0.01560700472e^{-2.055000000t} \end{aligned} \tag{49}$$

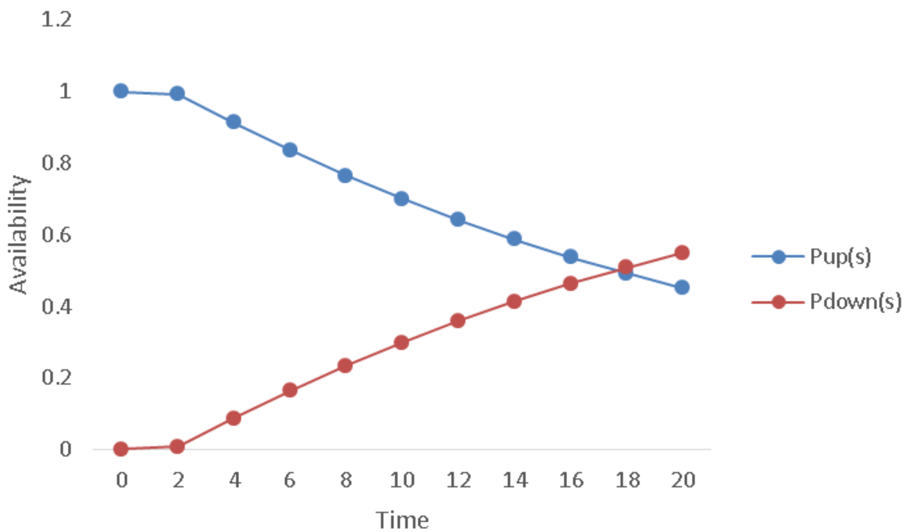
$$\begin{aligned} P_{down}(t) = 1 & - 0.03067597048e^{-2.825715957t} + 0.06282702488e^{-1.595634487t} \\ & + 0.000005265514432e^{-1.33768940t} - 1.090723881e^{-0.04426015551t} \\ & + 0.01324485000e^{-1.110000000t} + 0.01181819909e^{-1.1120000000t} \\ & + 0.01789750748e^{-2.060000000t} + 0.01560700472e^{-2.055000000t} \end{aligned}$$

Taking time $= 0, 2, 4, 6, \ldots$ and so on, we derive Table 2 and Figure 3 for system availability when the repair is done through Copula distribution.

Table 2 System availability against time for Copula repair

Time (t)	*0*	*2*	*4*	*6*	*8*	*10*	*12*	*14*	*16*	*18*	*20*
$P_{up}(s)$.	1.0000	0.9926	0.9133	0.8363	0.7655	0.7006	0.6413	0.5870	0.5372	0.4917	0.4501
$P_{down}(s)$	0.0000	0.0074	0.0867	0.1637	0.2345	0.2994	0.3587	0.4130	0.4628	0.5083	0.5499

Figure 3 System availability against time for Copula repair (see online version for colours)



2) *System availability when the repair follows General distribution:* Setting $\bar{S}_h(s) = \frac{h}{s+h}$ in equation (48) and differentiating the parameters by assigning different values as $\vartheta_0 = 0.050, \vartheta_1 = 0.055, \vartheta_2 = 0.060, \vartheta_3 = 0.065, \varphi = h = 1$, then taking inverse Laplace transform, we generate the availability equation for General repair as:

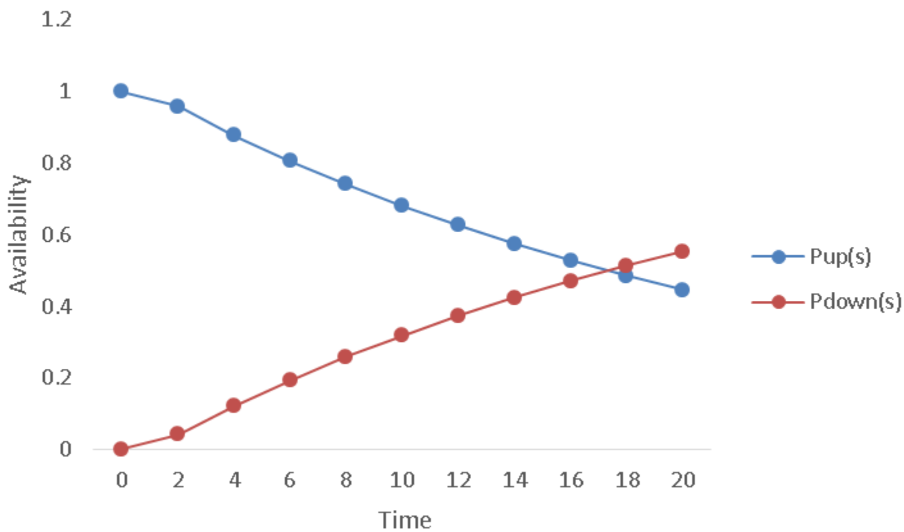
$$\begin{aligned} P_{up}(s) = {} & 0.000002051372863e^{-1.337696716t} + 0.06928702458e^{-1.045901121t} \\ & +1.039988551e^{-0.042307709374t} - 0.02140525937e^{-1.110000000t} \\ & -0.01798767113e^{-1.120000000t} - 0.02518049885e^{-2.060000000t} \\ & -90.02197722853e^{-2.055000000t} - 0.02272696895e^{-1.659095069t} \end{aligned} \tag{50}$$

Assuming time $t = 0, 2, 4, 6, \ldots$ and so on, we obtain Table 3 and Figure 4 for system availability when the repair is done via general distribution.

Table 3 System availability against time for general repair

Time (t)	*0*	*2*	*4*	*6*	*8*	*10*	*12*	*14*	*16*	*18*	*20*
$P_{up}(s)$.	1.0000	0.9583	0.8786	0.8069	0.7414	0.6812	0.6260	0.5752	0.5285	0.4856	0.4462
$P_{down}(s)$	0.0000	0.0417	0.1214	0.1931	0.2586	0.3188	0.3740	0.4248	0.4715	0.5144	0.5538

Figure 4 System availability against time for general repair (see online version for colours)



5.2 Reliability analysis of Model I

Reliability is the inability to repair the system. On the basis of this, the repair rates are all set to zero and the failure rates are set as $\vartheta_0 = 0.050, \vartheta_1 = 0.055, \vartheta_2 = 0.060, \vartheta_3 = 0.065$ in equation (48). Taking inverse Laplace transform, we obtain the expression for system reliability as given below:

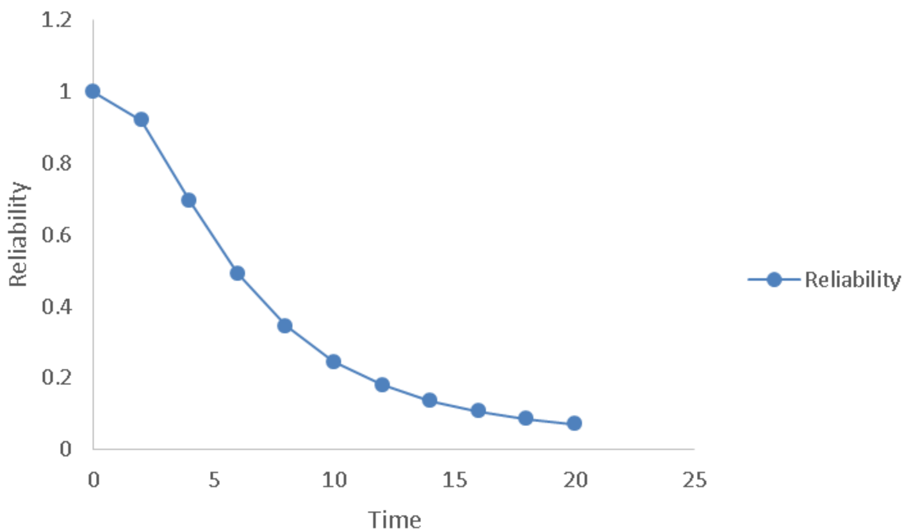
$$\begin{aligned} R(t) = {} & 2.357142857e^{-0.3400000000t} + 0.06574436620e^{-0.055000000000t} \\ & +2.600000000e^{-0.3350000000t} - 4.309591016e^{-0.4100000000t} \\ & +0.1024137931e^{-0.1200000000t} + 0.07704000000e^{-0.06000000000t} \\ & +0.1072500000e^{-0.1100000000t} \end{aligned} \tag{51}$$

Letting time $t = 0, 2, 4, 6, \ldots$ the reliability of the system is obtained and presented in Table 4 and Figure 5.

Table 4 System reliability against time

Time	*0*	*2*	*4*	*6*	*8*	*10*	*12*	*14*	*16*	*18*	*20*
$R(t)$	1.0000	0.9204	0.6956	0.4930	0.3451	0.2452	0.1795	0.1360	0.1066	0.0860	0.0709

Figure 5 System reliability against time



5.3 *Mean time to failure (MTTF) analysis of Model I*

Setting all the repairs to zero in equation (48) and limiting as s approaches zero, the expression for MTTF can be generated as:

$$MTTF = \lim_{s\to 0} \bar{P}_{up}(s) = \frac{1}{R(s)} \left\{ \begin{array}{l} 1 + \left(\dfrac{3\vartheta_1}{\vartheta_0 + 3\vartheta_1 + 3\vartheta_2}\right) + 6(\vartheta_2 + 3\vartheta_1\vartheta_3) + \left(\dfrac{3\vartheta_3}{\vartheta_0 + 3\vartheta_1 + 2\vartheta_2}\right) \\ +6(\vartheta_3 + 3\vartheta_1\vartheta_2) + \dfrac{9}{2}\vartheta_3 + \dfrac{9}{2}\vartheta_1 \end{array} \right\}$$

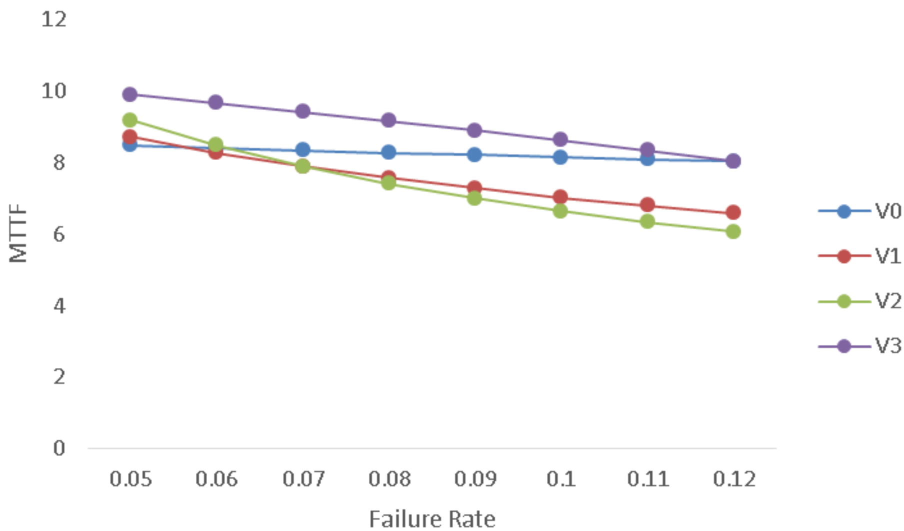
where $R(s) = 3\vartheta_1 + 3\vartheta_2 + \vartheta_3$

Setting $\vartheta_0 = 0.05, \vartheta_1 = 0.055, \vartheta_2 = 0.060, \vartheta_3 = 0.065$ and changing $\vartheta_0, \vartheta_1, \vartheta_2$ and ϑ_3 one as $0.050, 0.060, 0.070, 0.080, 0.090, 0.100, 0.110, 0.120$, respectively, in equation (48), the variation of MTTF in terms of failure rates are shown in Table 5 and Figure 6.

Table 5 MTTF against failure rate

Failure rate	*MTTF* ϑ_0	*MTTF* ϑ_1	*MTTF* ϑ_2	*MTTF* ϑ_3
0.050	8.4906	8.7188	9.1978	9.9046
0.060	8.4156	8.2798	8.4906	9.6729
0.070	8.3448	7.9027	7.9066	9.4308
0.080	8.2779	7.5742	7.4171	9.1776
0.090	8.2145	7.2849	7.0012	8.9124
0.100	8.1544	7.0276	6.6451	8.6346
0.110	8.0974	6.7971	6.3362	8.3430
0.120	8.0431	6.5891	6.0662	8.0366

Figure 6 MTTF against failure rate (see online version for colours)



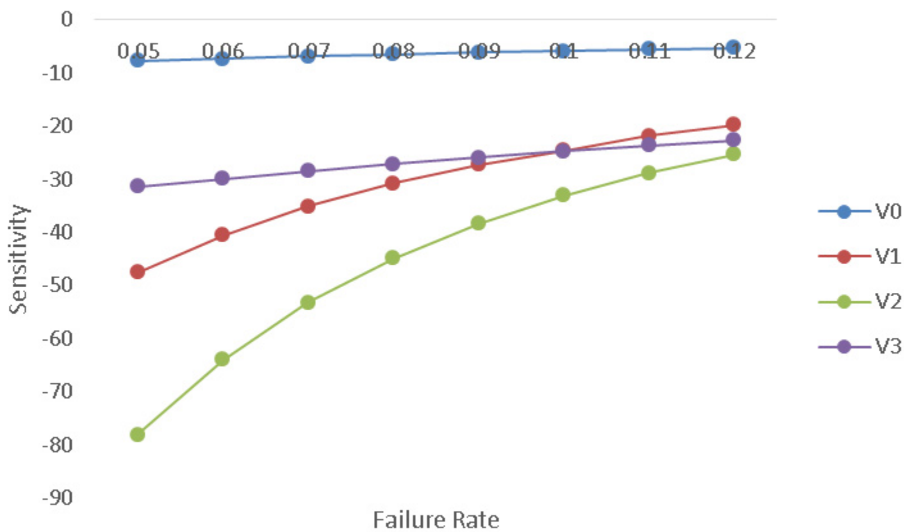
5.4 *Sensitivity analysis of Model I*

The system sensitivity can be obtained by partially differentiating MTFF with respect to failure rate parameters. By applying $\vartheta_0 = 0.050, \vartheta_1 = 0.055, \vartheta_2 = 0.060, \vartheta_3 = 0.065$ in the partial differentiation of MTFF, sensitivity of the system is obtained and presented in Table 6 and the corresponding graph in Figure 7.

Table 6 Sensitivity against failure rate

Failure rate	$\frac{\partial}{\partial \vartheta_0}(MTTF)$	$\frac{\partial}{\partial \vartheta_1}(MTTF)$	$\frac{\partial}{\partial \vartheta_2}(MTTF)$	$\frac{\partial}{\partial \vartheta_3}(MTTF)$
0.050	–7.7193	–47.5523	–78.1176	–31.4142
0.060	–7.2811	–40.5467	–63.9973	–29.8821
0.070	–6.8792	–35.0970	–53.2730	–28.4593
0.080	–6.5096	–30.7551	–44.9542	–27.1358
0.090	–6.1691	–27.2265	–38.3835	–25.9025
0.100	–5.8546	–24.5762	–33.1117	–24.7515
0.110	–5.5635	–21.8676	–28.8236	–23.6754
0.120	–5.2937	–19.7954	–25.2931	–22.6681

Figure 7 Sensitivity against failure rate (see online version for colours)



5.5 *Cost/benefit function of Model I*

The formula presented below can be used to evaluate the anticipated profit for the interval [0, t) if the service facility is always open/available.

$$E_p(t) = J_1 \int_0^t P_{up}(t)\,dt - J_2 t \tag{52}$$

where J_1 and J_2 in the interval [0, t) are the revenue generated and service cost per unit time. As in system availability, the expected profit generated from the system is also presented in two ways: when the repair follows Copula distribution and when the repair follows general distribution.

1) *Expected profit for Copula repair:* Setting $\vartheta_0 = 0.050, \vartheta_1 = 0.055, \vartheta_2 = 0.060, \vartheta_3 = 0.065$ and substituting equation (48) in equation (52), one can obtain equation (53), representing expected profit for Copula repair as given below:

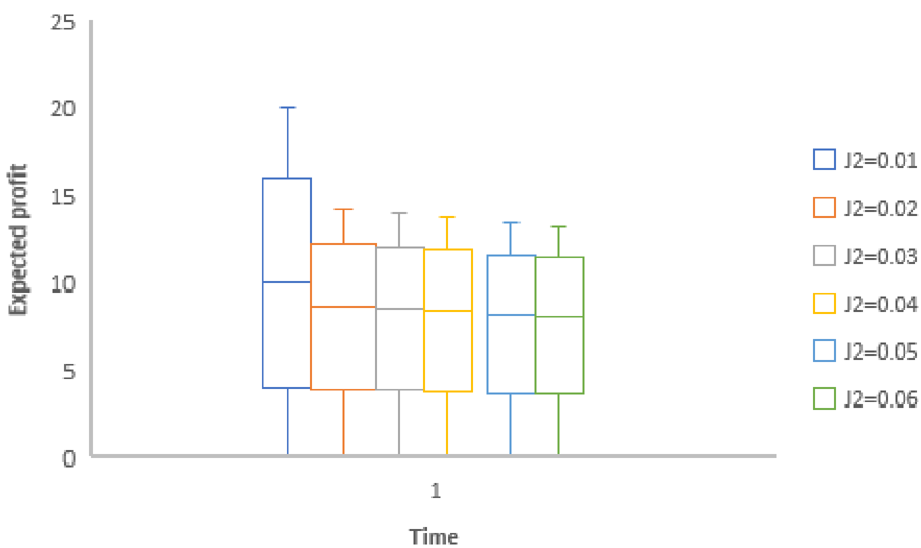
$$\begin{aligned} E_p(x) = J_1 \{ & 0.007594649499e^{-2.055000000t} + 0.008688110427e^{-2.060000000t} \\ & -0.01085599931e^{-2.825715957t} + 0.03937432124^{-1.595634487t} \\ & +0.000003936275811e^{-1.337689401t} - 24.64347150e^{-0.04426015551t} \\ & +0.01193229730e^{-1.110000000t} + 0.01055196347e^{-1.120000000t} + 24.57618222 \} \\ & -J_2 t \end{aligned} \tag{53}$$

With $J_1 = 1$, $J_2 = 0.01, 0.02, 0.03, 0.04, 0.05, 0.06,$ and $t = 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20$ and by applying Laplace transform on equation (48), we derive Table 7 and Figure 8.

Table 7 Expected profit against time for Copula repair

Time	$E_p(x)$ $J_2 = 0.01$	$E_p(x)$ $J_2 = 0.02$.	$E_p(x)$ $J_2 = 0.03$	$E_p(x)$ $J_2 = 0.04$.	$E_p(x)$ $J_2 = 0.0$.	$E_p(x)$ $J_2 = 0.06$
0	0	0	0	0	0	0
2	2.0047	1.9847	1.9647	1.9447	1.9247	1.9047
4	3.8915	3.8515	3.8115	3.7715	3.7315	3.6915
6	5.6202	5.5602	5.5002	5.4402	5.3802	5.3202
8	7.2010	7.1210	7.0410	6.9610	6.8810	6.8010
10	8.6461	8.5461	8.4461	8.3461	8.2461	8.1461
12	9.9672	9.8472	9.7272	9.6072	9.4872	9.3672
14	11.1746	11.0346	10.8946	10.7546	10.6146	10.4746
16	12.2781	12.1181	11.9581	11.7981	11.6381	11.4781
18	13.2864	13.1064	12.9264	12.7464	12.5664	12.3864
20	14.2075	14.0075	13.8075	13.6075	13.4075	13.2075

Figure 8 Expected profit against time for Copula repair (see online version for colours)



2) *Expected profit for general repair:* Using parameters of equation (53), the expression for profit function for general repair is given by equation (54) below.

$$\begin{aligned} E_p(x) = \{ & 0.01069451510e^{-2.055000000t} + 0.01222354313e^{-2.060000000t} \\ & +0.01369841269e^{-1.659095069t} - 0.000001533511175e^{-1.337696716t} \\ & -0.06624624756e^{-1.045901121t} - 24.58189535e^{-0.04230709374t} \\ & +0.01928401745e^{-1.1210000000t} + 0.01606042065e^{-1.120000000t} \\ & +24.57618222\} - J_2 t \end{aligned} \quad (54)$$

Using the values of J_1, J_2 and t above in equation (54), we get Table 8 and Figure 9 for expected profit for general repair.

Table 8 Expected profit against time for general repair

	$E_p(x)$ $J_2 = 0.01$	$E_p(x)$ $J_2 = 0.02$	$E_p(x)$ $J_2 = 0.03$	$E_p(x)$ $J_2 = 0.04$	$E_p(x)$ $J_2 = 0.0$	$E_p(x)$ $J_2 = 0.06$
0	0	0	0	0	0	0
2	1.9652	1.9452	1.9252	1.9052	1.8852	1.8652
4	5.7807	3.7407	3.7007	6607	3.6207	3.5807
6	5.4451	5.3851	5.3251	5.2651	5.2051	5.1451
8	6.9725	6.8925	6.8125	6.7325	6.6525	6.5725
10	8.3742	8.2742	8.1742	8.0742	7.9742	7.8742
12	9.6607	9.5407	9.4207	9.3007	9.1807	9.0607
14	10.8411	10.7011	10.5611	10.4211	10.2811	10.1411
16	11.9241	11.7641	11.6041	11.4441	11.2841	11.1241
18	12.9176	12.7376	12.5576	12.3776	12.1976	12.0176
20	13.8289	13.6289	13.4289	13.2289	13.0289	13.8289

Figure 9 Expected profit against time for general repair (see online version for colours)

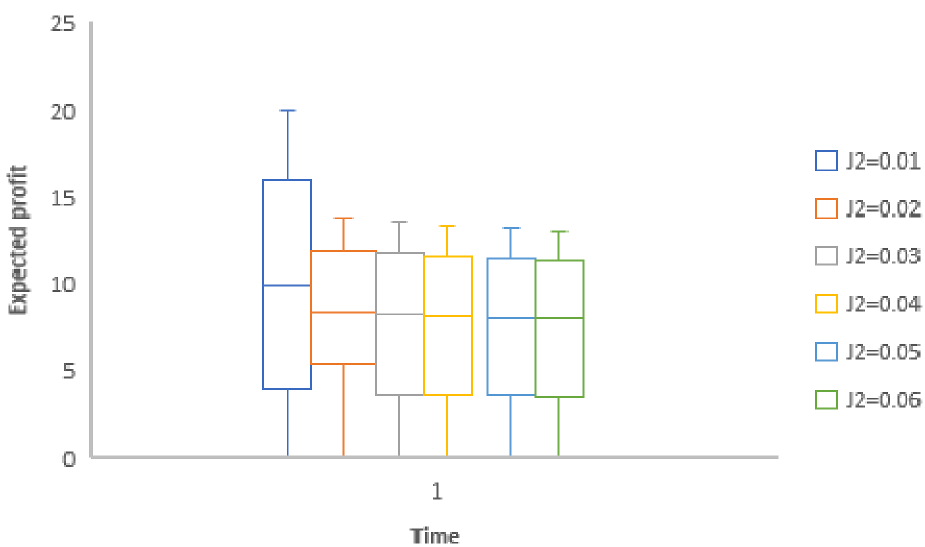
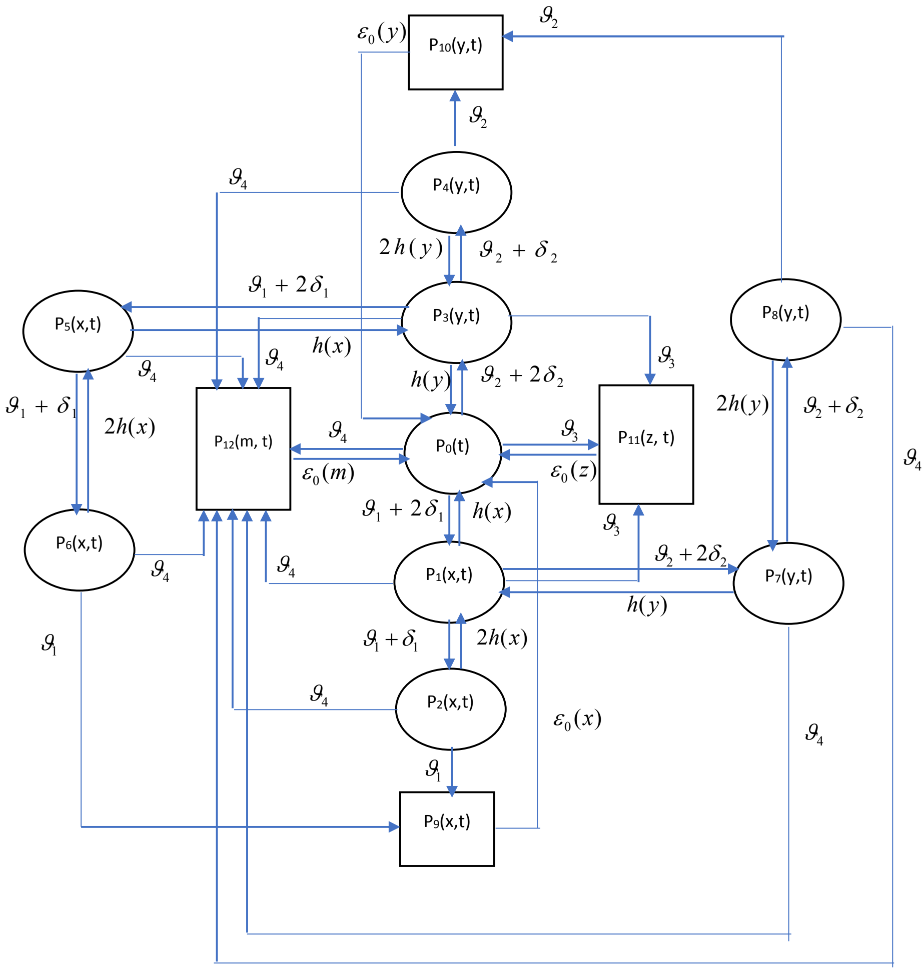


Figure 10 Transition diagram of the Model II



6 Formulation and solution of Model II

In this scenario, a warm standby client and servers are introduced to see their implication in enhancing the performance of the system. Warm standby client–server distributed systems reduce energy use and recovery period because a standby unit is partly energised and subjected to maximum stress while the primary unit is up and running and completely powered and functional after the primary unit stops working. Warm standby client-server distributed systems possess time-dependent failure behavioural patterns; they have distinct failure rates before and after replacing the online defective units. Warm standby client-server distributed configuration is utilised in a multitude of settings, including adaptive databases, where the server retains the standby database as a duplicate of the active database; when the active database fails, the standby database resumes client applications to work with minimal service interruption. Warm standby client-server distributed sensor redundancy is also used in wireless sensor networks to maintain a balance between energy consumption and the time required to activate backup sensors.

We acquire the solution for Model II by following the same steps used to formulate and solve Model I above.

$$
\begin{aligned}
P_{up}(t) = \frac{1}{J(s)}\Bigg[& 1+(\vartheta_1+2\delta_1)\left\{\frac{1-\overline{s_h}\left(s+\vartheta_3+\vartheta_4+(\vartheta_1+\delta_1)+(\vartheta_2+2\delta_2)\right)}{s+\vartheta_3+\vartheta_4+(\vartheta_1+\delta_1)+(\vartheta_2+2\delta_2)}\right\} \\
& +(\vartheta_1+\delta_1)(\vartheta_1+2\delta_1)\left\{\frac{1-\overline{s_{2h}}\left(s+\vartheta_1+\vartheta_4\right)}{s+\vartheta_1+\vartheta_4}\right\} \\
& +(\vartheta_2+2\delta_2)\left\{\frac{1-\overline{s_h}\left(s+\vartheta_3+\vartheta_4+(\vartheta_2+\delta_2)+(\vartheta_1+2\delta_1)\right)}{s+\vartheta_3+\vartheta_4+(\vartheta_2+\delta_2)+(\vartheta_1+2\delta_1)}\right\} \\
& +(\vartheta_2+\delta_2)(\vartheta_2+2\delta_2)\left\{\frac{1-\overline{s_{2h}}\left(s+\vartheta_2+\vartheta_4\right)}{s+\vartheta_2+\vartheta_4}\right\} \\
& +(\vartheta_1+2\delta_1)(\vartheta_2+2\delta_2)\left\{\frac{1-\overline{s_h}\left(s+(\vartheta_1+\delta_1)+\vartheta_4\right)}{s+(\vartheta_1+\delta_1)+\vartheta_4}\right\} \\
& +(\vartheta_1+\delta_1)(\vartheta_1+2\delta_1)(\vartheta_2+2\delta_2)\left\{\frac{1-\overline{s_{2h}}\left(s+\vartheta_1+\vartheta_4\right)}{s+\vartheta_1+\vartheta_4}\right\} \\
& +(\vartheta_1+2\delta_1)(\vartheta_2+2\delta_2)\left\{\frac{1-\overline{s_h}\left(s+(\vartheta_2+\delta_2)+\vartheta_4\right)}{s+(\vartheta_2+\delta_2)+\vartheta_4}\right\} \\
& (\vartheta_2+\delta_2)(\vartheta_1+2\delta_1)(\vartheta_2+2\delta_2)\left\{\frac{1-\overline{s_{2h}}\left(s+\vartheta_2+\vartheta_4\right)}{s+\vartheta_2+\vartheta_4}\right\}\Bigg]
\end{aligned}
\tag{55}
$$

where

$$
\begin{aligned}
J(s) = & \left(s+\vartheta_3+\vartheta_4+(\vartheta_1+2\delta_1)+(\vartheta_2+2\delta_2)\right) \\
& -\Big[(\vartheta_1+2\delta_1)\left\{1-\overline{s_h}\left(s+\vartheta_3+\vartheta_4+(\vartheta_1+\delta_1)+(\vartheta_2+2\delta_2)\right)\right\} \\
& +(\vartheta_2+2\delta_2)\left\{1-\overline{s_h}\left(s+\vartheta_3+\vartheta_4+(\vartheta_2+\delta_2)+(\vartheta_1+2\delta_1)\right)\right\} \\
& +\vartheta_1(\vartheta_1+\delta_1)(\vartheta_1+2\delta_1)\left(1+(\vartheta_2+2\delta_2)\right) \\
& \overline{s}_{\in_0}(s)+\vartheta_2(\vartheta_2+\delta_2)(\vartheta_2+2\delta_2)\left(1+(\vartheta_1+2\delta_1)\right)\overline{s}_{\in_0}(s) \\
& +\vartheta_3\left\{1+(\vartheta_1+2\delta_1)+(\vartheta_2+2\delta_2)\right\}\overline{s}_{\in_0}(s) \\
& +\vartheta_4\begin{cases}1+(\vartheta_1+2\delta_1)+(\vartheta_1+\delta_1)(\vartheta_1+2\delta_1)+(\vartheta_2+2\delta_2) \\ +(\vartheta_2+\delta_2)(\vartheta_2+2\delta_2)+2(\vartheta_1+2\delta_1)(\vartheta_2+2\delta_2)\end{cases} \\
& +(\vartheta_1+\delta_1)(\vartheta_1+2\delta_1)(\vartheta_2+2\delta_2)+(\vartheta_2+\delta_2)(\vartheta_2+2\delta_2)(\vartheta_1+2\delta_1)\Big\}\overline{s}_{\in_0}(s)\Big]
\end{aligned}
$$

7 Particular cases

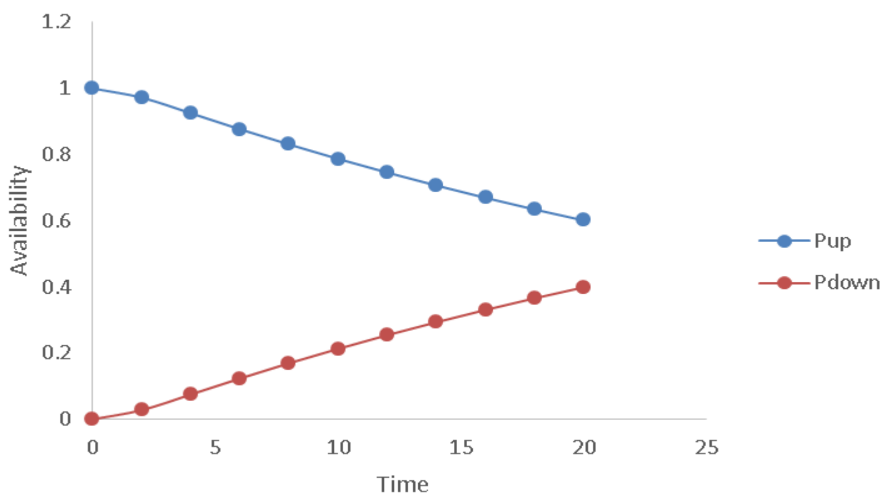
7.1 Availability analysis of Model II

(a) *System availability when repair follows Copula distribution:* We obtain Table 9 and Figure 11 for system availability when the repair is done using Copula repair distribution by substituting $\vartheta_1 = 0.050, \vartheta_2 = 0.055, \vartheta_3 = 0.060, \vartheta_4 = 0.065,$ $\delta_1 = 0.040, \delta_2 = 0.045$. in equation (55) and taking Laplace transform with $t = 0, 2, 4, 6, 8, 10,$ and so on as in case of Model I above.

Table 9 System availability against time for Copula repair

Time (t)	*0*	*2*	*4*	*6*	*8*	*10*	*12*	*14*	*16*	*18*	*20*
$P_{up}(s)$	1.0000	0.9723	0.9249	0.8766	0.8305	0.7869	0.7456	0.7064	0.6693	0.6341	0.6008
$P_{down}(s)$	0.0000	0.0277	0.0751	0.1234	0.1695	0.2131	0.2544	0.2936	0.3307	0.3659	0.3992

Figure 11 System availability against time for Copula repair (see online version for colours)

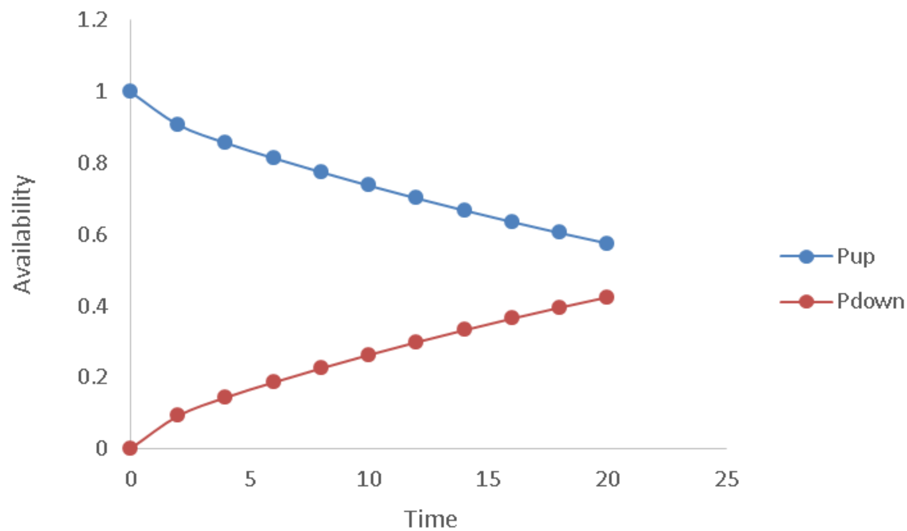


(b) *System availability when repair follows a general repair:* We obtain Table 10 and Figure 12 for system availability via general repair for Model II by following the same steps as for Model I, inserting $\vartheta_1 = 0.050, \vartheta_2 = 0.055, \vartheta_3 = 0.060, \vartheta_4 =$ $0.065, \delta_1 = 0.040, \delta_2 = 0.045$ in equation (55) and taking Laplace transform with $t = 0, 2, 4, 6, 8, 10,$ and so on.

Table 10 System availability against time for general repair

Time (t)	*0*	*2*	*4*	*6*	*8*	*10*	*12*	*14*	*16*	*18*	*20*
$P_{up}(s)$	1.0000	0.9080	0.8565	0.8142	0.7747	0.7372	0.7015	0.6676	0.6353	0.6045	0.5753
$P_{down}(s)$	0.0000	0.092	0.1435	0.1858	0.2253	0.2628	0.2985	0.3324	0.3647	0.3955	0.4247

Figure 12 System availability against time for general repair (see online version for colours)



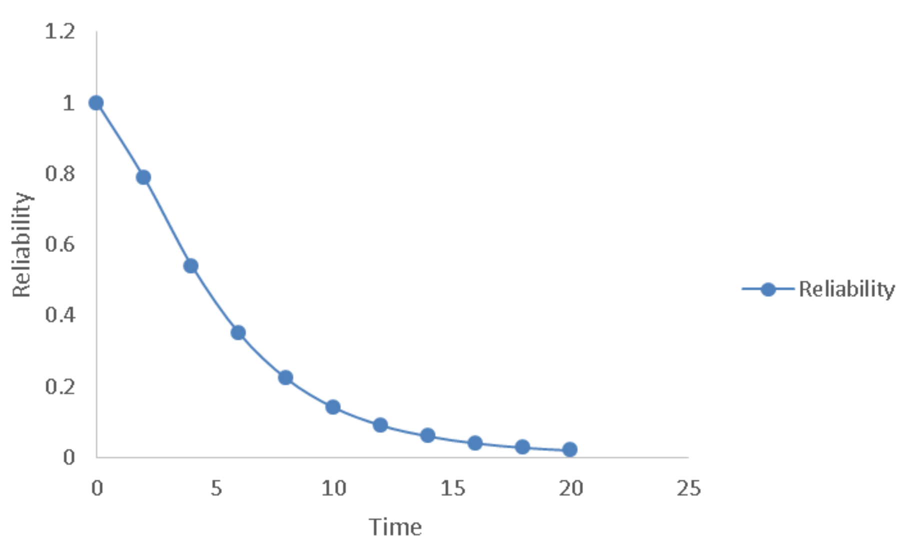
7.2 Reliability analysis of Model II

Model II reliability is presented in Table 11 and Figure 13, much like in Model I.

Table 11 System reliability against time

Time	*0*	*2*	*4*	*6*	*8*	*10*	*12*	*14*	*16*	*18*	*20*
$R(t)$	1.0000	0.7876	0.5407	0.3505	0.2224	0.1410	0.0906	0.0595	0.0401	0.0277	0.0197

Figure 12 System reliability against time



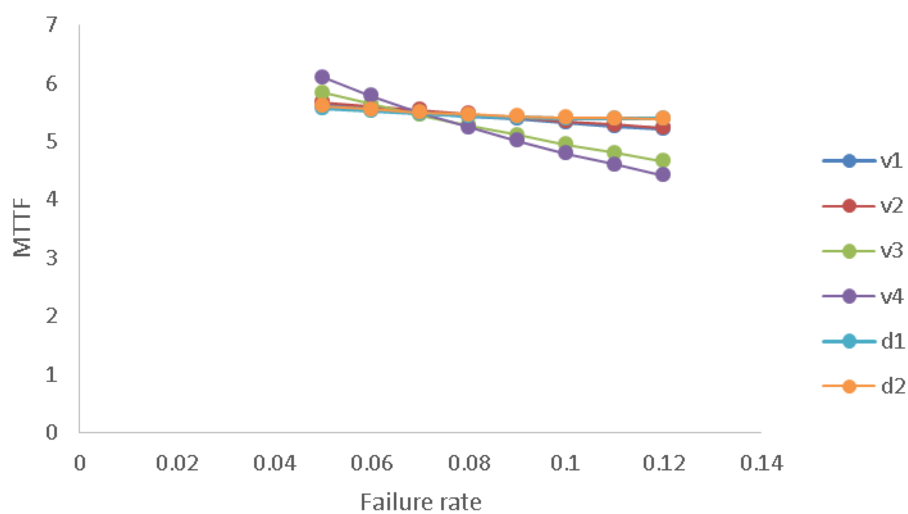
7.3 Mean time to failure (MTTF) analysis of Model II

The variation of MTTF in terms of failure rates are shown in Table 12 and Figure 14 as it was in Model I.

Table 12 MTTF against failure rates

Failure rate	*MTTF* ϑ_1	*MTTF* ϑ_2	*MTTF* ϑ_3	*MTTF* ϑ_4	*MTTF* δ_1	*MTFF* δ_2
0.050	5.646	5.6811	5.8477	6.1133	5810	5.6126
0.060	5.5797	5.6117	5.6461	5.7930	5.5279	5.5547
0.070	5.5153	5.5443	5.4573	5.5070	5.4854	5.5075
0.080	5.4527	5.4790	5.2801	5.2496	5.4522	5.4694
0.090	5.3921	5.4157	5.1135	5.0161	5.4274	5.4398
0.100	5.3333	5.3544	4.9568	4.8032	5.4100	5.4176
0.110	5.2764	5.2950	4.8089	4.6079	5.3994	5.4022
0.120	5.2213	5.2375	4.6692	4.4281	5.3949	5.3928

Figure 14 MTTF against failure rates (see online version for colours)



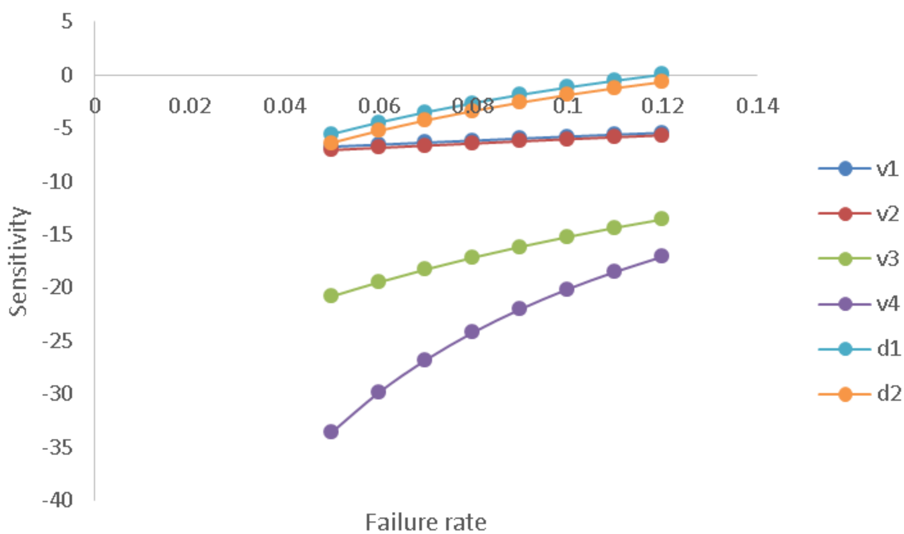
7.4 Sensitivity analysis of Model II

Sensitivity of the system of Model II is obtained and presented in Table 13 and the corresponding graph in Figure 15 just as in Model I.

Table 13 Sensitivity against failure rate

Failure rate	$\frac{\partial}{\partial \vartheta_1}(MTTF)$	$\frac{\partial}{\partial \vartheta_2}(MTTF)$	$\frac{\partial}{\partial \vartheta_3}(MTTF)$	$\frac{\partial}{\partial \vartheta_4}(MTTF)$	$\frac{\partial}{\partial \delta_1}(MTTF)$	$\frac{\partial}{\partial \delta_2}(MTTF)$
0.050	–6.735	–7.0472	–20.8389	–33.6586	–5.5578	–6.3777
0.060	–6.5434	–6.8408	–19.4994	–29.8935	–4.4584	–5.2437
0.070	–6.3501	–6.6339	–18.2812	–26.8035	–3.4872	–4.2470
0.080	–6.1578	–6.4291	–17.1703	–24.2218	–2.6232	–3.3639
0.090	–5.9684	–6.2282	–16.1547	–22.0327	–1.8498	–2.5760
0.100	–5.7833	–6.0321	–15.2241	–20.1535	–1.1538	–1.8684
0.110	–5.6031	–5.8417	–14.3694	–18.5236	–0.5242	–1.2295
0.120	–5.42839	–5.6574	–13.5827	–17.0972	0.0479	–0.6497

Figure 15 Sensitivity against failure rate (see online version for colours)



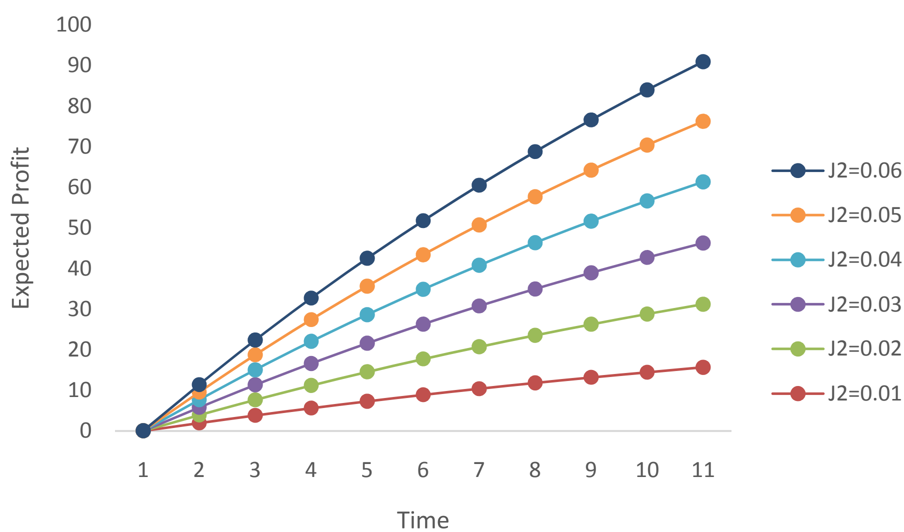
7.5 *Cost-benefit analysis of Model II*

i) *Expected profit for general repair:* Equations (52) and (55) are combined with $\vartheta_1 = 0.050, \vartheta_2 = 0.055, \vartheta_3 = 0.060, \vartheta_4 = 0.065, \delta_1 = 0.040, \delta_2 = 0.045$ and then Laplace transform is performed using $t = 0, 2, 4, 6, 8, 10,$ and so on. For Copula repair, the expected profit from Model II is shown in Table 14 and Figure 16.

Table 14 Expected profit against time for Copula repair

Time	$E_p(x)$ $J_2 = 0.01$	$E_p(x)$ $J_2 = 0.02$	$E_p(x)$ $J_2 = 0.03$	$E_p(x)$ $J_2 = 0.04$	$E_p(x)$ $J_2 = 0.0$	$E_p(x)$ $J_2 = 0.06$
0	0	0	0	0	0	0
2	1.9515	1.9315	1.8915	1.8915	1.8715	1.8515
4	3.8299	3.7899	3.7099	3.7099	3.6699	3.6299
6	5.6111	5.5511	5.43107	5.4311	5.3711	5.3111
8	7.2978	7.2178	7.0578	7.0578	6.9778	6.8978
10	8.8948	8.7948	8.5948	8.5948	8.4948	8.3948
12	10.4069	10.2869	10.0469	10.0469	9.9269	9.8069
14	11.8385	11.6985	11.4185	11.4185	11.2785	11.1385
16	13.1938	13.0338	12.7138	12.7138	12.5538	12.3938
18	14.4769	14.2969	13.9369	13.9369	13.7569	13.5769
20	15.6915	15.4915	15.0915	15.0915	14.8915	14.6915

Figure 16 Expected profit against time for Copula repair (see online version for colours)

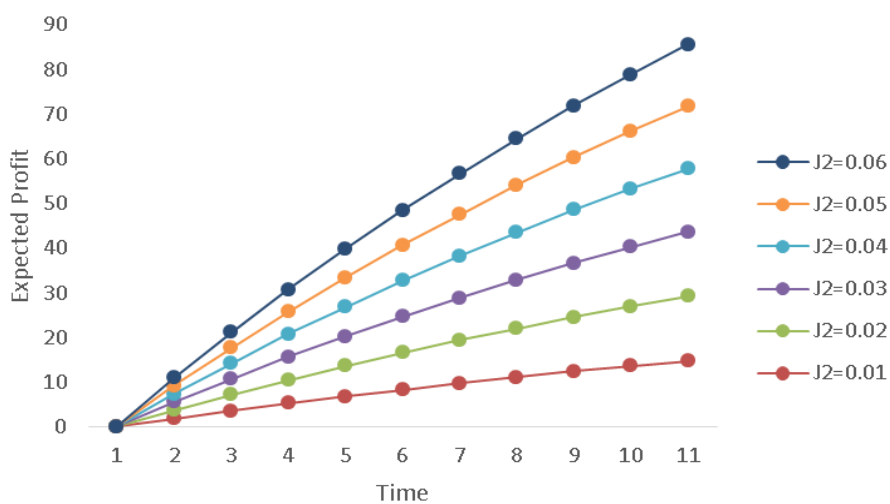


Expected profit for general repair: Adding equations (52) and (55) and replacing $\vartheta_1 = 0.050, \vartheta_2 = 0.055, \vartheta_3 = 0.060, \vartheta_4 = 0.065, \delta_1 = 0.040, \delta_2 = 0.045$ and then performing Laplace transform with $t = 0, 2, 4, 6, 8, 10$ and so on. Table 15 and Figure 17 show the estimated profit function from Model II for general repair.

Table 15 Expected profit against time for Copula repair

Time	$E_p(x)$ $J_2 = 0.01$	$E_p(x)$ $J_2 = 0.02$	$E_p(x)$ $J_2 = 0.03$	$E_p(x)$ $J_2 = 0.04$	$E_p(x)$ $J_2 = 0.05$	$E_p(x)$ $J_2 = 0.06$
0	0	0	0	0	0.	0
2	1.8777	1.8577	1.8377	1.8177	1.7977	1.777
4	3.6192	3.5792	3.5392	3.4992	3.4592	3.4192
6	5.2692	5.2092	5.1492	5.0892	5.0292	4.9692
8	6.8377	6.7577	6.6777	6.5977	6.5177	6.4377
10	8.3292	8.2292	8.1292	8.0292	7.9292	7.8292
12	9.7476	9.6276	9.5076	9.3876	9.2676	9.1476
14	11.0963	10.9563	10.8163	10.6763	10.5363	10.3963
16	12.3789	12.2189	12.0589	11.8989	11.7389	11.5789
18	13.5984	13.4184	13.2384	13.0584	12.8784	12.6984
20	14.7579	14.5579	14.3579	14.1579	13.9579	13.7579

Figure 17 Expected profit against time for general repair (see online version for colours)



8 Discussion of results

In order to have inevitable guide to this study, we give a discussion of numerical simulations with reference to system availability, reliability, Mean Time to Failure (MTTF), sensitivity and profit function of both models in this section. Here, we consider two settings: For Model I, we set the failure rates $\vartheta_0 = 0.05$, $\vartheta_1 = 0.055$, $\vartheta_2 = 0.060$, $\vartheta_3 = 0.065$ and for Model II, we set $\vartheta_1 = 0.050, \vartheta_2 = 0.055, \vartheta_3 = 0.060, \vartheta_4 = 0.065,$ $\delta_1 = 0.040, \delta_2 = 0.045$ for illustration purposes.

Table 2, Figure 3, Table 9, and Figure 11 show the system availability of Models I and II respectively, in relation to time t when the repair is performed using Copula. The availability of both models reduces as time passes for the parameters studied for both models, as can be seen in these tables and figures. This shows that the performances of both models may be accurately anticipated based on their graphical representations. Same scenarior can be observed from Table 3, Figure 4, Table 10, and Figure 12 when the repairs of Model I and Model II follow the general distribution, respectively. In comparison, the availability of both models when the repairs follow the Copula distribution appears to be better than the availability of both models when the repairs follow the General distribution. Model II is more readily available when repairs are made in accordance with both Copula and General repair procedures, as opposed to Model I, which is more readily available in such circumstances. This is actually true since adding enough spare units with adequate or inadequate switchover mechanisms can increase system availability. This alysis indicates that Copula repair is required for both models to operate for an extended period of time.

Table 4 and Figure 5 provide the information on system reliability with respect to time t for Model I while Table 11 and Figure 13 give the information on system reliability with respect to time t for Model II. We can see from these tables and their corresponding graphs that the reliability of both models decreases over time. This goes a long way towards justifying the fact that the lower the number of repairs, the worse the reliability. In comparison to availability of both models, it can be seen that the reliability of both models' plummets. This is due to lack of system repairs. This illustrates what failure to manage the structure/system entails. The reliability of Model I is higher than the reliability of Model II. This is because introducing spare units to a system without repairs is equivalent to decreasing the reliability of the system.

The MTTF for both models are shown in Table 5, Figure 6 and Table 12, Figure 14, respectively. When the failure rates of both models grow, their MTTFs lower. The MTTF of Model I is higher than that of Model II, which is similar the situation for reliability.

The information on system sensitivity studied for both Models I and II in this research are provided in Table 5, Figure 6 and Table 13 and Figure 15, respectively. The value of the sensitivity analysis illustrates the significance of each component; the more vital a component is, the greater its sensitivity value. Intriguingly, Table 4 and Figure 4 for Model I show that the sensitivity to failure rate, ϑ_3 of the load balancer has the highest value. This shows how vulnerable the system is to a load balancer failing. While for Model II, Table 13 and Figure 15 show that the sensitivity to the switch failure rate, ϑ_4 has the highest value. This has also demonstrates how susceptible the system is to a switch failure. For these reasons, system designers and maintenance engineers must develop a strategy for the upkeep of these components. The performance of both models can be optimised by limiting the occurrence of load balancer and switch failure, respectively.

A benefit-cost analysis, also known as a cost-benefit analysis, is a methodical procedure that enables businesses to assess decisions and systems as well as estimate the worth of intangible assests. Cost-benefit analysis is a general technique that is frequently used in engineering. Making the most of idea and option is crucial in many businessess today. To this end, a lot of businesses, from big ones to small startups, use cost-benefit analysis to help them make crucial and pertinent decisions. Many sectors benefit from the use of cost-benefit analysis to determine the maximum and projected value of a design/system. In general, an industrial manager will often work to increase the industry's profit, since profits are determined by both growing/rising revenue and cutting operational costs. Owing to its apparent importance in terms of boosting profits, managers will usually select this approach. When the generated revenue, J_1 is set to 1 and the service cost, J_2 is set to $0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.08, 0.09$. Table 6 and its related Figure 8 show the expected profit from Model I when the repair follows Copula distribution while Table 8 and its associated Figure 9 show the expected profit from Model I when the repair follows the General distribution. Table 14, Figure 16 and Table 15, Figure 17 show the expected profit from Model II when repairs are made using Copula repair and General repair, respectively, using the same values of revenue generated and service cost from model I. When the service cost, J_2 is decreasing, the expected profits from both models improve with time, when the repairs follow Copula repair. In general, when the service cost is low ($J_2 = 0.01$) the expected profit is higher, for both models, i.e., low-service costs generate maximum profit, whereas high service costs ($J_2 = 0.06$) yield lowest profit. Same outcome can be observed from both models when the repair follows a General distribution. On the other hand, the anticipated cost of Copula repair is much more than *t* expected cost of General repair. From this sensitivity analysis, Copula repair appears to be more advantageous than General repair. The idea that copula repair enhances system availability more than general maintenance has been validated by this analysis.

9 Conclusions

In this study, we have examined the performance of two different typical client-server distributed systems using the features of Gumbel-Hougaard family Copula. The expressions for the system characteristics such as availability, reliability, mean time to failure, MTTF sensitivity and predicted profit for both models were obtained and validated through numerical experiments. The impact of the different parameters governing each system was examined. The results are presented in tables and figures. On the basis of these results, the following conclusions were drawn:

1 It was discovered that system availability and predicted profit for both systems rise when Copula repair is employed. Therefore, this repair technique is more effective in raising predicted profit and availability. This has given engineers a new reason to accept Copula-based multi-dimension repair.

2 In addition, it was shown that a load balancer failure could significantly affect how well Model I performs.

3 It was also revealed that a switch failure could affect how well Model II performed. As a result, system designers and maintenance engineers must devise a maintenance strategy for these components.

4 Furthermore, it was found that low-service costs resulted in higher-than-expected profits for both models. Maintenance mangers and system engineers should select the ideal service fee based on the expected profit level.

5 Finally, it is obvious that Mdel II is the ideal configuration. This is corroborated by availability and revenue figures, which show that Model II is most profitable and available when Copula and General are done.

This study provides a foundation for maintenance managers and system engineers to identify the optimal sort of repairs and system configuration that will increase system performance and revenue generation. Furthermore, the models/results described in this work, if modified, will allow management to prevent wrong reliability assessments and erroneous decision-making, thus reducing unnecessary expenditures. The current research can extend to address the system with non-identical clients and servers. This topic will be explored more in our future work.

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