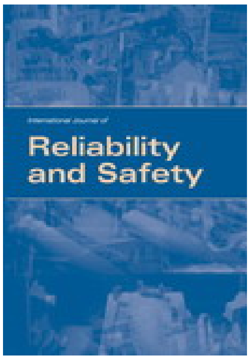

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A study on step stress partially accelerated life test under adaptive type-II progressive hybrid censoring for inverse Lomax distribution

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Abstract: Using traditional life tests in testing and reliability theory may result in extremely few failures by the completion of the experiment, leading to poorer estimations. To get the required failure as rapidly as possible for better estimation, products are exposed to higher stress levels, and this process is known as accelerated life testing. This paper develops a step stress partially accelerated life test that employs adaptive type-II progressive hybrid censoring scheme and assumes that the lifespan of test items follows a two-parameter inverted Lomax distribution. The likelihood and log-likelihood functions were created for adaptive type-II progressive hybrid censoring scheme data in order to get the point and interval of the model parameters using the maximum-likelihood estimation approach. Furthermore, using a Monte Carlo simulation analysis, the biases and mean square errors of the maximum-likelihood estimators are estimated to examine their performances in the presence of censoring introduced in this work.

Keywords: partially accelerated life test; inverse Lomax distribution; Newton Raphson; adaptive type-II progressive hybrid censoring; simulation study.

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1 Introduction

In reliability theory and life data analysis, the analyst provides a mathematical framework to fit life data from a sample of products to make forecasts about their life in real use. They will have to gather information about the appropriate probability distribution of product life data that can be used to model the product's life. Then, the predictions about the product's reliability, mean life time, and other life characteristics can be estimated based on the estimates of the distribution's parameters. However, obtaining the necessary information about the failures of extremely reliable products and materials in order to make an efficient forecast about their expected life is a difficult task. Thus, Accelerated Life Test (ALT) can be incorporated for obtaining rapid failure information in a short period of time by testing products at higher testing conditions (known as stress of acceleration) than they are supposed to operate in real life. ALT and Partially Accelerated Life Test (PALT) are two main categories that have grown in popularity in life testing for acquiring relevant data in a shorter period of time than standard life tests.

Owing to the different stress loading mechanisms, constant stress and step stress (shortened as CSALT and SSALT) are two commonly considered categories of ALTs in the research community Nelson (1990). CSALT puts the products to test through constant accelerated stress levels until all the products fail or the test is revoked due to censoring. For additional information, see Yang (1994), Watkins and John (2008), Kamal (2013), Saxena et al. (2012a), Kamal et al. (2014), Gao et al. (2016) and Han and Bai (2019). Whereas, in SSALT, products are first exposed to some high stress until a specified time, which is followed by items being switched to the next high-stress level, and so on. SSALT models have been studied by a number of researchers, including Miller and Nelson (1983), Bai et al. (1989), Saxena et al. (2012b, 2012c), Kamal et al. (2013a), Han and Bai (2020), Kamal et al. (2020) and Hakamipour (2021). The core premise in ALT is to have a mathematical relation that can be used to relate a unit's lifetime to stress. Sometimes, in scenarios, no such relation is known or assumed. Thus, in these kinds of cases, PALTs are indeed better tests for statistical modelling. The PALTs, in which both normal and accelerated conditions are used to test items, were used successfully by the experts to extrapolate the accelerated data to normal operating conditions.

Partial ALTs have been broadly classified into two types: CSPALT and SSPALT. In CSPALT, products are tested either under ordinary or increased circumstances until the program is done. Many authors, including Bai and Chung (1992), Zarrin et al. (2012), Kamal et al. (2013a, 2013b), Hassan et al. (2020) and Alam et al. (2021) investigated CSPALT using various failure data. SSPALT begins by subjecting test units to regular utilisation situations for a pre-determined amount of time. If items do not fail up to that point, they will be subjected to increased stress. The stress level is raised repetitively till all the products fail or the test is revoked due to censoring.

In testing and reliability studies, there are circumstances where components may be abandoned or removed from the experiment, or the investigator may not be able to obtain complete information on failure times. The data collected under such situations is referred to as censored data, and the framework is referred to as censoring. The two most prevalent filtering techniques are Types I and II. Several researches have been conducted to investigate the statistical inferences associated with hybrid censoring, which is really a blend of Type-I and II censor schemes. For further information see Childs et al. (2003) and Gupta and Kundu (1998). Above approaches have the drawback of the inability to remove test items from tests at any time, with the exception of the terminal point. To deal with this problem, more extensive censoring approaches such as Adaptive Type-II Progressive Hybrid Censoring Scheme (AdT-IIPH-CS) are increasingly widely used to address this issue.

Rao (1992) pointed out that SSPALT might be more advantageous in terms of saving time and money. When the test units are extremely reliable and the mathematical relationship between stress and life is unknown or cannot be inferred, it should be used for reliability evaluation. Rahman et al. (2019) addressed the SSPALT and employed a Maximum-Likelihood Estimation (MLE) technique based on T-IIPH-CS with items having a lifespan that follows an exponentiated exponential distribution. In an ALT with k increasing stress levels, Kamal (2021a, 2021b) investigated a T-IIPH-CS. Given that the failure times reflect a generalised Pareto distribution, the MLE strategy is used to make inferences. He then provided a simulation analysis to demonstrate the effectiveness of the proposed inferential methodologies. Alam and Ahmed (2020) explored an SSPALT with AdT-IIPH-CS in which the lifespan of the test units follows an exponentiated Pareto distribution and generated parameter MLEs. Kamal et al. (2021) used the MLE technique to estimate parameters in SSPALT with AdT-IIPH-CS using NH distribution for failure times. The optimal A and D test tactics are then provided by them. Alam and Ahmed (2022) investigated the maintenance policies under SSPALT for a T-IIPH-CS scheme where the lifespan of experimental units is supposed to have a generalised inverted exponential distribution. Kamal (2021a, 2021b) presented progressive hybrid censored masked data for the three-component hybrid system using SSPALT and derived inferences for linear power hazard rate distribution parameters. As a real-world application, he applied his methodology to the failure times of a plane's ac system. Alam and Ahmed (2022), Alam et al. (2022a, 2022b) and Lone et al. (2022) presented a study on ALT using step stress and constant stress and also provide the application of geometric process in ALT. Kamal et al. (2022a, 2022b) tackled with CSPALT and SSPALT with T-IIPH-CS masked data and different other censoring plans, respectively.

The proposed study is motivated by two factors. First, the inverted Lomax distribution has been shown to be extremely adaptable in terms of failure rates that are not monotonic. It also has diverse uses in random processes, economics, and actuarial sciences Kleiber and Kotz (2003). Second, no prior work based on Two-Parameter Inverted Lomax (TPIL) distribution under SSPALT that employed AdT-IIPH-CS has been undertaken. In this study, AdT-IIPH-CS is combined with SSPALT to construct a SSPALT with AdT-IIPH-CS using the TPIL distribution as a lifetime model. The rest of the work is presented as: Section 2 includes the model framework as well as the testing process. Section 3 goes through estimating points and intervals estimates. Section 4 carries a simulation example to evaluate the performance of the parameters. Section 5 concludes the paper with some comments.

2 Testing framework and procedure

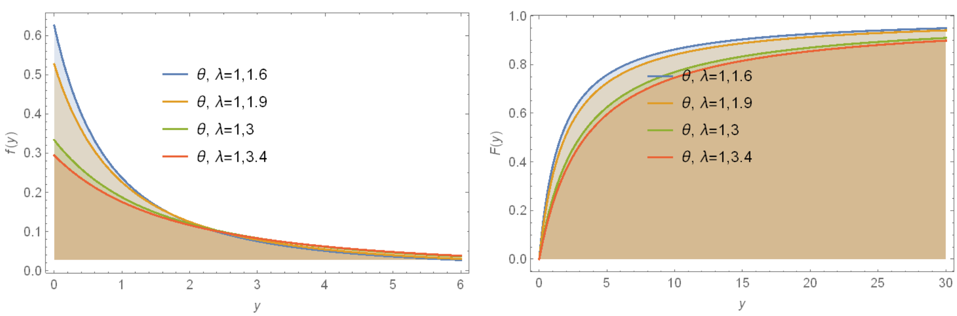
If the random variable X has a Lomax distribution, then the transform variable $Y = 1/X$ will have a TPIL distribution with θ and λ, as shape and scale parameters. The TPIL distribution's Probability Density Function (pdf) and Cumulative Distribution Function (cdf) are as follows:

$$y,\theta,\lambda) = \frac{\theta\lambda}{y^2}\left(1+\frac{\lambda}{y}\right)^{-(\theta+1)} \quad ; \quad y,\theta,\lambda \geq 0 \tag{1}$$

$$F(y,\theta,\lambda) = \left(1+\frac{\lambda}{y}\right)^{-\theta} \quad ; \quad y,\theta,\lambda \geq 0 \tag{2}$$

Figure 1 shows some shapes of pdf and cdf of TPIL with different combinations of input values of parameters.

Figure 1 pdf and cdf curves of TPIL distribution (see online version for colours)



The Reliability Function (rf) and Hazard Function (hf) of the TPIL distribution are as:

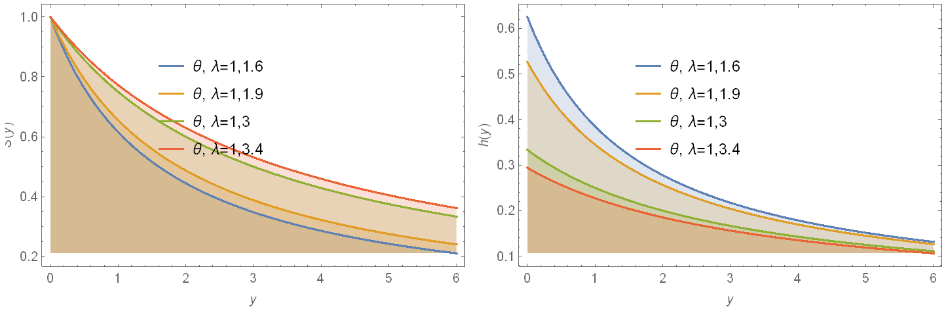
$$S(y,\theta,\lambda) = 1-\left(1+\frac{\lambda}{y}\right)^{-\theta} \quad ; \quad y,\theta,\lambda \geq 0 \tag{3}$$

$$h(y,\theta,\lambda) = \frac{\theta\lambda\left(1+\frac{\lambda}{y}\right)^{-(\theta+1)}}{y^2\left[1-\left(1+\frac{\lambda}{y}\right)^{-\theta}\right]} \quad ; \quad y,\theta,\lambda \geq 0 \tag{4}$$

Figure 2 shows some shapes of pdf and cdf of TPIL with different input values of parameters.

From the relevant curves of pdf, cdf, rf and the hf for different shape and scale parameters values are shown in Figures 1 and 2, it is observed that, at fix shape parameter $(\theta = 1)$ and different scale parameters values $(\lambda > 1)$, the nature of hazard curve is decreasing. The TPIL distribution is a specific form of the kind-2 generalised beta distribution and is an attractive substitute to commonly employed distributions such as Weibull, inverse Weibull, generalised Exponential, Gamma, etc. as well.

Figure 2 rf and hf curves of TPIL distribution (see online version for colours)



Let's suppose we have two stress levels for the SSAPALT, in which one is normal stress and the other is accelerated stress. Then, the very next equation describes the pdf for variable Y through SSPALT:

$$g(y)=\begin{cases} 0 & y\le 0 \\ f_1(y)=f(y,\theta,\lambda) & 0<y\le\tau \\ f_2(y) & y>\tau \end{cases} \tag{5}$$

where $f_1(y)$ denotes the normal stress pdf and is provided by equation (1), whereas $f_2(y)=\frac{\theta\lambda\beta}{(\tau+\beta(y-\tau))^2}\left(1+\frac{\lambda}{(\tau+\beta(y-\tau))}\right)^{-(\theta+1)}$ is used to represent the accelerated stress pdf. The variable transform proposed by DeGroot and Goel (1979) is used to produce the pdf $f_2(y)$, and the approach is presented in the following equation:

$$y=\begin{cases} T & \text{if} \quad T\le\tau \\ \tau+\beta^{-1}(T-\tau) & \text{if} \quad T>\tau \end{cases} \tag{6}$$

In the preceding equation (6), T is the items' life span under normal operation, τ is the timing upon which stress is shifted and β would be the acceleration factor.

In the T-IIP-CS environment, the reliability practitioner specifies the number of components that will fail (say m) out of the total number of components n that will be analysed. When the initial failure occurs, R_1 components from among the $n-1$ leftover (surviving) components are randomly removed from the life investigation. Similarly, when the next failure occurs, R_2 of the leftover $n-2-R_1$ assessment elements are removed from the assessment. This procedure will be repeated till a m-th failure happens. At this stage, all of the leftover $R_m=n-m-R_1-R_2-\ldots-R_m-1$ surviving assessment elements are removed from the assessment. Many writers, including Balakrishnan (2007) and Balakrishnan et al. (2000) advocated the premise of progressive censorship.

If a life examination experiment terminates at a random instant m in $(Y_{m:m:n},\kappa)$, where $1\le m\le n$ and $0<\kappa<\infty$ are specified as previous and $Y_{1:m:n}\le Y_{2:m:n}\le\ldots\le Y_{m:m:n}$ are the sorted failing timings of the elements as a result of the assessment, then

$\left(R_1, R_2, ..., R_m\right)$ is known as the PH-CS. If the m-th observation occurs before the point $\kappa(> Y_{m:m:n})$, the investigation terminates at $Y_{m:m:n}$. Alternatively, the inspection will terminate at κ with $Y_{j:m:n} < \kappa < Y_{j+1:m:n}$ and any remaining $\left(n - \sum_{i=1}^{j} R_i - j\right)$ existent unfailed elements will be censored at κ.

If a life examination experiment stops at a random moment m in $\left(Y_{m:m:n}, \kappa\right)$, where $1 \leq m \leq n$ and $0 < \kappa < \infty$ are defined to prior and $Y_{1:m:n} \leq Y_{2:m:n} \leq ... \leq Y_{m:m:n}$ are the ordered failure times consequential from the test, then $\left(R_1, R_2, ..., R_m\right)$ is called Progressive Hybrid Censoring (PHC) scheme. If the m-th progressively censored observed will occur before the point $\kappa\left(\kappa > X_{m:m:n}\right)$, then the investigation ends at the moment $Y_{m:m:n}$. Otherwise, the examination will end at the moment κ with $Y_{j:m:n} < \kappa < Y_{j+1:m:n}$, and all the leftover $\left(n - \sum_{i=1}^{j} R_i - j\right)$ existing units are censored at κ. In this case, j is a random variable representing the number of failed units up to κ. The reliability engineer has a tough time dealing with the censoring tactics discussed previously, and the practitioner may wind up with a small sample size (even it is equal to zero). As a result, using traditional suggestion methodologies to provide high-quality results is no longer a viable option. Adaptive censoring is offered as a novel censoring strategy to overcome this issue. This approach was initially introduced by Ng et al. (2009).

The observed quantity of failures m is set to antecedent in this strategy, and the investigation instant is allowed to run across instant κ. If $Y_{m:m:n} < \kappa$, the investigation will proceed using progressively censoring $\left(R_1, R_2, R_3, ..., R_m\right)$, otherwise, the ongoing components (existing units), that are not removed from the test following the $\left(j+1\right)th$ to $\left(m-1\right)$th experimental failures. If m observed failure is obtained, all survivor elements $R_m = n - m - \sum_{i=1}^{j} R_i$ are removed from the test somewhere at position $Y_{m:m:n}$, i.e., $R_{j+1} = ... = R_{m-1} = 0$. This class of censoring approach is characterised as an AdT-IIPH-SC if the practitioner is free to adjust the value *k*. This change in κ is made to take use of the benefits of reduced inspection time and a greater chance of overseeing diverse failures.

3 Estimation procedure

Let a sample of n items assigned to SSPALT for testing under normal stress level and let $y_{1:m:n} < y_{2:m:n} < ...y_{n_u:m:n}$ be observed failure times with corresponding removals $R_1, R_2, ..., R_u$ upto time τ. And then stress is changed to accelerated stress level. Suppose $y_{n_u+1:m:n} \leq \kappa < y_{J+1:m:n} < ... < y_{m:n:n}$ are the observed failures under accelerated stress as per the AdT-IIPH-SC defined in previous section. Therefore, under SSPALT with AdT-IIPH-SC, following data may be observed:

$$y_{1:m:n} < y_{2:m:n} < \ldots y_{n_u:m:n} \leq \tau < y_{n_u+1:m:n} \leq \kappa < y_{J+1:m:n} < \ldots < y_{m:n:n}$$

3.1 Point estimation

In this subsection, the MLE approach will be used to produce the point estimates of the parameters. Based on provided lifetime data of size m and the procedure outlined in Ismail (2014), the likelihood function under SSPALT for AdT-IIPH-CS from the TPIL distribution works in the following way:

$$L(\theta,\lambda,\beta) \propto \prod_{i=1}^{m} f_1(y_{i:m:n}) f_2(y_{i:m:n}) \prod_{i=1}^{J} (S_1(\tau))^{R_i} (S_2(y_{m:m:n}))^{\left(n-m-\sum_{i=1}^{J} R_i\right)} \tag{7}$$

where $S_1(\tau) = 1-\left(1+\frac{\lambda}{\tau}\right)^{-\theta}$, $S_2(y_{m:m:n}) = 1-\left(1+\frac{\lambda}{(\tau+\beta(y_{m:m:n}-\tau))}\right)^{-\theta}$, $\ln L = \ln L(\theta,\lambda,\beta)$, $\gamma_i = \tau+\beta(y_i-\tau)$, $\gamma_{m:m:n} = \tau+\beta(y_{m:m:n}-\tau)$, $J = n_u + n_a$, where n_u is the amount of components that are unsuccessful in the normal circumstance and n_a is the number of components that are unsuccessful in accelerated circumstance. Now, the log-likelihood function takes the following form:

$$\begin{aligned}\ln L &= m\ln(\beta) + 2m\ln(\lambda) + 2m\ln(\theta) + \left[\sum_{i=1}^{m} y_i^{-2} + \sum_{i=1}^{m} (\gamma_i)^{-2}\right] \\ &\quad -(\theta+1)\left[\sum_{i=1}^{m} \ln(\mathfrak{B}_i) + \sum_{i=1}^{m} \ln(\mathcal{W}_i)\right] + \sum_{i=1}^{J} R_i \ln\left[1-(\mathfrak{B}_\tau)^{-\theta}\right] \\ &\quad + \sum_{i=1}^{J}\left(n-m-\sum_{i=1}^{J} R_i\right)\ln\left[1-(\mathcal{W}_m)^{-\theta}\right]\end{aligned} \tag{8}$$

where $1+\frac{\lambda}{y_i} = \mathfrak{B}_i$; $1+\frac{\lambda}{\tau} = \mathfrak{B}_\tau$; $1+\frac{\lambda}{\gamma_i} = \mathcal{W}_i$; $1+\frac{\lambda}{\gamma_{m:m:n}} = \mathcal{W}_m$. Now, to get the model parameters' MLEs, first differentiate the previous equation with respect to θ, λ and β and equal all derived derivatives to zero as follows:

$$\begin{aligned}\frac{\partial \ln L}{\partial \theta} &= 2m\theta^{-1} - \left[\sum_{i=1}^{m} \ln(\mathfrak{B}_i) + \sum_{i=1}^{m} \ln\left(1+\frac{\lambda}{\gamma_i}\right)\right] \\ &\quad + \sum_{i=1}^{J} R_i \frac{(\mathfrak{B}_\tau)^{-\theta}\ln(\mathfrak{B}_\tau)}{1-(\mathfrak{B}_\tau)^{-\theta}} + \sum_{i=1}^{J}\left(n-m-\sum_{i=1}^{J} R_i\right)\frac{(\mathcal{W}_m)^{-\theta}\ln(\mathcal{W}_m)}{1-(\mathcal{W}_m)^{-\theta}} = 0\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{\partial \ln L}{\partial \lambda} &= 2m\lambda^{-1} - (\theta+1)\left[\sum_{i=1}^{m} y_{y_i}^{-1}(\mathfrak{B}_i)^{-1} + \sum_{i=1}^{m} (\gamma_i)^{-1}(\mathcal{W}_i)^{-1}\right] \\ &\quad + \sum_{i=1}^{J} R_i \left[\frac{(\mathfrak{B}_\tau)^{-\theta-1}}{1-(\mathfrak{B}_\tau)^{-\theta}}\right] + \sum_{i=1}^{J}\left(n-m-\sum_{i=1}^{J} R_i\right)\left[\frac{(\mathcal{W}_m)^{-\theta-1}}{1-(\mathcal{W}_m)^{-\theta}}\right] = 0\end{aligned} \tag{10}$$

$$\frac{\partial \ln L}{\partial \beta} = m\beta^{-1} - 2\sum_{i=1}^{m}\left(\gamma_i - \tau\right)\beta^{-1}(\gamma_i)^{-3} + \left(\theta+1\right)\left[\sum_{i=1}^{m}\frac{\frac{\lambda\left(\gamma_i - \tau\right)\beta^{-1}}{(\gamma_i)^2}}{\left(\mathcal{W}_i\right)}\right]$$
$$+\sum_{i=1}^{J}\left(n-m-\sum_{i=1}^{J}R_i\right)\left[\frac{\left(\mathcal{W}_m\right)^{-\theta-1}\frac{\lambda\left(\gamma_{m:m:n}-\tau\right)\beta^{-1}}{\gamma_{m:m:n}}}{1-\left(\mathcal{W}_m\right)^{-\theta}}\right] = 0 \tag{11}$$

Because equations (9) and (11) are nonlinear in nature, manually solving them is challenging. As a result, an iterative approach such as the Newton–Raphson method should be used to generate MLEs of the parameters. In this study, the *R* software's Optim() function is used to construct numerical approximations.

3.2 Interval estimation

Here, the interval estimates for the parameters using asymptotic theory of MLEs based on SSPALT for AdT-IIPH-CS from the TPIL distribution are obtained. The asymptotic distribution of MLEs of θ, λ and β take the following form presented in the form of equation (12) as follows:

$$\left(\left(\hat{\theta}-\theta\right),\left(\hat{\lambda}-\lambda\right),\left(\hat{\beta}-\beta\right)\right) \to N\left(0, I^{-1}\left(\theta,\lambda,\beta\right)\right) \tag{12}$$

The above procedure was suggested by Miller and Nelson (1983) and $I^{-1}\left(\theta,\lambda,\beta\right)$ denotes the variance-covariance matrix of θ, λ and β. The $100\left(1-\varphi\right)\%$ estimated two-sided bounds of confidence for the parameters θ, λ and β are now established as follows:

$$\hat{\theta} \pm Z_{\varphi/2}\sqrt[-1]{I_{11}^{-1}\left(\hat{\theta},\hat{\lambda},\hat{\beta}\right)};\ \hat{\lambda} \pm Z_{\varphi/2}\sqrt[-1]{I_{22}^{-1}\left(\hat{\theta},\hat{\lambda},\hat{\beta}\right)} \text{ and } \hat{\beta} \pm Z_{\varphi/2}\sqrt[-1]{I_{33}^{-1}\left(\hat{\theta},\hat{\lambda},\hat{\beta}\right)}$$

The 3×3 matrix $I^{-1}\left(\theta,\lambda,\beta\right)$ which is approximately equal to the matrix of Fisher information. The elements of $I^{-1}\left(\theta,\lambda,\beta\right); i=1,2,3, j=1,2,3$, can be approximated by $I_{ij}\left(\hat{\theta},\hat{\lambda},\hat{\beta}\right); i=1,2,3, j=1,2,3$ and the elements of $I_{ij}\left(\hat{\theta},\hat{\lambda},\hat{\beta}\right)$ are given as follows:

$$\frac{\partial^2 \ln L}{\partial \theta^2} = -2m\theta^{-2} + \sum_{i=1}^{J}R_i\frac{\left(\mathfrak{B}_\tau\right)^{-\theta}\ln\left(\mathfrak{B}_\tau\right)}{1-\left(\mathfrak{B}_\tau\right)^{-\theta}}\left[-\ln\left(\mathfrak{B}_\tau\right) - \frac{\left(\mathfrak{B}_\tau\right)^{-\theta}\ln\left(\mathfrak{B}_\tau\right)}{1-\left(\mathfrak{B}_\tau\right)^{-\theta}}\right]$$
$$+\sum_{i=1}^{J}\left(n-m-\sum_{i=1}^{J}R_i\right)\frac{\left(\mathcal{W}_m\right)^{-\theta}\ln\left(\mathcal{W}_m\right)}{1-\left(\mathcal{W}_m\right)^{-\theta}} \tag{13}$$

$$\frac{\partial^2 \ln L}{\partial \lambda^2} = -2m\lambda^{-2} - (\theta+1)\left[\sum_{i=1}^{m} y_i^{-2} (\mathfrak{B}_i)^{-2} + \sum_{i=1}^{m} (\gamma_i)^{-2} (\mathcal{W}_i)^{-2}\right] + \sum_{i=1}^{J} R_i \frac{\tau^{-1}(\mathfrak{B}_\tau)^{-\theta-1}}{1-(\mathfrak{B}_\tau)^{-\theta}}\left[-(\theta+1)(\mathfrak{B}_\tau) - \frac{\theta(\mathfrak{B}_\tau)^{-\theta-1}}{1-(\mathfrak{B}_\tau)^{-\theta}}\right] + \sum_{i=1}^{J}\left(n-m-\sum_{i=1}^{J} R_i\right)(\gamma_{m:m:n})^{-1}\frac{(\mathcal{W}_m)^{-\theta-1}}{1-(\mathcal{W}_m)^{-\theta}}\left[-(\theta+1)(\mathcal{W}_m) - \frac{\theta(\mathcal{W}_m)^{-\theta-1}}{1-(\mathcal{W}_m)^{-\theta}}\right] \tag{14}$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = -m\beta^{-2} + 6\sum_{i=1}^{m}(\gamma_i-\tau)^2 \beta^{-1}(\gamma_i)^{-4} + (\theta+1)\sum_{i=1}^{m}\frac{\dfrac{\lambda\beta^{-1}(\gamma_i-\tau)^2}{(\gamma_i)^3}}{(\mathcal{W}_i)}\left[-2+\frac{\lambda}{\mathcal{W}_i}\right] + \sum_{i=1}^{J}\left(n-m-\sum_{i=1}^{J} R_i\right)\frac{(\mathcal{W}_m)^{-\theta-1}\dfrac{\lambda\beta^{-1}(\gamma_{m:m:n}-\tau)^2}{y_{m:m:n}\gamma}}{\left[1-(\mathcal{W}_m)^{-\theta}\right](\gamma_{m:m:n})^2}\left[-1+\frac{(\theta+1)\lambda}{(\mathcal{W}_m)} - \frac{\lambda}{1-(\mathcal{W}_m)}\right] \tag{15}$$

$$\frac{\partial^2 \ln L}{\partial \theta \partial \lambda} = -\left[\sum_{i=1}^{m}(\mathfrak{B}_i)^{-1} y_i^{-1} + \sum_{i=1}^{m}(\mathcal{W}_i)^{-1}(\gamma_i)^{-1}\right] + \sum_{i=1}^{J} R_i \frac{\left[1-(\mathfrak{B}_\tau)^{-\theta}\right]\tau^{-1}(\mathfrak{B}_\tau)^{-\theta-1}\left[1-\theta\ln(\mathfrak{B}_\tau)\right] - \left[\theta\tau^{-1}(\mathfrak{B}_\tau)^{-2\theta-1}\ln(\mathfrak{B}_\tau)\right]}{\left[1-(\mathfrak{B}_\tau)^{-\theta}\right]^2} + \sum_{i=1}^{J}\left(n-m-\sum_{i=1}^{J} R_i\right)\frac{\left[1-(\mathcal{W}_m)^{-\theta}\right]\left[1-\theta\ln(\mathcal{W}_m)\right] - \left[\theta(\gamma_{m:m:n})^{-1}(\mathcal{W}_m)^{-\theta}\ln(\mathcal{W}_m)\right]}{(\mathcal{W}_m)^{\theta+1}\left[1-(\mathcal{W}_m)^{-\theta}\right]^2 \gamma_{m:m:n}} \tag{16}$$

$$\frac{\partial^2 \ln L}{\partial \beta \partial \theta} = \left[\sum_{i=1}^{m}\frac{\lambda(\gamma_i-\tau)\beta^{-1}}{(\gamma_i)^2(\mathcal{W}_i)}\right] + \sum_{i=1}^{J}\left(n-m-\sum_{i=1}^{J} R_i\right)\left(\frac{\lambda(\gamma_{m:m:n}-\tau)\beta^{-1}}{\gamma_{m:m:n}}\right) \times \frac{(\mathcal{W}_m)^{-\theta-1}}{1-(\mathcal{W}_m)^{-\theta}}\left[-\ln(\mathcal{W}_m) - \theta\frac{(\mathcal{W}_m)^{-\theta-1}}{1-(\mathcal{W}_m)^{-\theta}}\right] \tag{17}$$

$$\frac{\partial^2 \ln L}{\partial \beta \partial \lambda} = (\theta+1)\sum_{i=1}^{m}\frac{\lambda(\gamma_i-\tau)\beta^{-1}}{(\gamma_i)^2(\mathcal{W}_i)}\left[\frac{1}{\lambda} - \frac{(\gamma_i)^{-1}}{(\mathcal{W}_i)}\right] + \sum_{i=1}^{J}\left(n-m-\sum_{i=1}^{J} R_i\right) \times \frac{(\mathcal{W}_m)^{-\theta-1}\lambda\beta^{-1}(\gamma_{m:m:n}-\tau)}{\left(1-(\mathcal{W}_m)^{-\theta}\right)\gamma_{m:m:n}}\left[-(\theta+1)\frac{(\gamma_{m:m:n})^{-1}}{(\mathcal{W}_m)} + \frac{1}{\lambda} - \frac{\theta(\gamma_{m:m:n})^{-1}(\mathcal{W}_m)^{-\theta-1}}{1-(\mathcal{W}_m)^{-\theta}}\right] \tag{18}$$

4 Simulation study

Since it is theoretically not achievable to evaluate the presentation of different censorings for different values of model parameters. For this job, many software or simulation techniques are used. In this segment, the Monte-Carlo simulation procedure is applied to evaluate the performance of MLEs. This performance is recorded in the language of Mean Squared Errors (MSEs) and biases of MLEs. The following three progressive censorings are chosen for this assignment;

Scheme (I) $R_1 = R_2 = R_3 = ... = R_{m-1}$, $R_m = n - m$

Scheme (II) $R_1 = n - m$, $R_2 = R_3 = R_4 ... = 0$

Scheme (III) $R_1 = R_2 = R_3 = ... = R_{m-1}$, $R_m = n - 2m + 1$

For this task, 1000 simulation-based MSEs and biases are estimated. The steps for this procedure are;

1. The values of parameters $n, m, \tau, \kappa, \theta, \lambda$ and β are specified first.
2. After the specification of parameter values, generate a random sample from IL distribution with size n by the Inverse CDF method in both situations (regular and accelerated circumstances).
3. Generate the progressive hybrid censored sample for the parameters $n, m, \tau, \kappa, \theta, \lambda$ and β by using the technique discussed in equation (6).
4. The sample data set for the APHCT-II is;

   $$y_{1:m:n} < x_{y:m:n} < ... y_{n_u:m:n} \le \tau < y_{n_u+1:m:n} \le \kappa < y_{J+1:m:n} < ... < y_{m:n:n}$$

5. Compute the values of MSEs and biases correlated with MLEs of parameters and the computing values presented in Tables 1, 2, 3 and 4 at different values of parameters.

Table 1 MSEs and biases when $\theta = 0.89, \lambda = 1.6, \beta = 1.2, \tau = 1.7$ and $\kappa = 5$

$(n.m)$	*Schemes*	*Values of* θ		*Values of* λ		*Values of* β	
		Bias	*MSE*	*Bias*	*MSE*	*Bias*	*MSE*
(50, 12)	1	0.575	0.492	0.447	0.439	0.334	0.268
	2	0.632	0.529	0.592	0.502	0.445	0.364
	3	0.516	0.504	0.492	0.446	0.476	0.385
(70, 12)	1	0.505	0.428	0.363	0.388	0.319	0.238
	2	0.559	0.463	0.364	0.372	0.349	0.275
	3	0.436	0.449	0.428	0.344	0.435	0.352
(90, 12)	1	0.439	0.348	0.284	0.298	0.285	0.218
	2	0.484	0.334	0.292	0.420	0.233	0.239
	3	0.354	0.327	0.318	0.264	0.321	0.249

Table 1 MSEs and biases when $\theta = 0.89, \lambda = 1.6, \beta = 1.2, \tau = 1.7$ and $\kappa = 5$ (continued)

$(n.m)$	*Schemes*	*Values of* θ		*Values of* λ		*Values of* β	
		Bias	*MSE*	*Bias*	*MSE*	*Bias*	*MSE*
(50, 20)	1	0.353	0.276	0.205	0.177	0.205	0.154
	2	0.377	0.228	0.210	0.193	0.243	0.186
	3	0.392	0.247	0.286	0.205	0.256	0.188
(70, 20)	1	0.229	0.165	0.128	0.165	0.140	0.112
	2	0.248	0.189	0.229	0.159	0.176	0.127
	3	0.233	0.165	0.176	0.148	0.163	0.102
(90, 20)	1	0.168	0.126	0.107	0.094	0.093	0.015
	2	0.166	0.119	0.174	0.106	0.128	0.068
	3	0.156	0.129	0.108	0.114	0.109	0.070

Table 2 MSEs and biases when $\theta = 0.76, \lambda = 1.6, \beta = 1.7, \tau = 2.4$ and $\kappa = 9$

$(n.m)$	*Schemes*	*Values of* θ		*Values of* λ		*Values of* β	
		Bias	*MSE*	*Bias*	*MSE*	*Bias*	*Bias*
(50, 12)	1	0.384	0.290	0.387	0.299	0.308	0.218
	2	0.339	0.318	0.479	0.384	0.353	0.268
	3	0.325	0.278	0.435	0.328	0.329	0.248
(70, 12)	1	0.297	0.210	0.286	0.221	0.312	0.174
	2	0.247	0.208	0.349	0.378	0.298	0.187
	3	0.216	0.153	0.380	0.347	0.245	0.192
(90, 12)	1	0.198	0.174	0.287	0.174	0.271	0.181
	2	0.279	0.213	0.333	0.317	0.258	0.176
	3	0.228	0.216	0.310	0.289	0.224	0.181
(50, 20)	1	0.126	0.110	0.211	0.186	0.224	0.174
	2	0.197	0.188	0.265	0.190	0.234	0.208
	3	0.168	0.136	0.227	0.156	0.219	0.247
(70, 20)	1	0.117	0.105	0.132	0.118	0.143	0.093
	2	0.129	0.106	0.179	0.134	0.165	0.133
	3	0.180	0.123	0.148	0.094	0.125	0.087
(90, 20)	1	0.068	0.046	0.096	0.063	0.074	0.055
	2	0.095	0.054	0.089	0.085	0.096	0.081
	3	0.088	0.055	0.086	0.058	0.076	0.098

Table 3 MSEs and biases when $\theta = 0.65, \lambda = 1.2, \beta = 1.7, \tau = 2.8$ and $\kappa = 9$

$(n.m)$	*Schemes*	*Values of* θ		*Values of* λ		*Values of* β	
		Bias	*MSE*	*Bias*	*MSE*	*Bias*	*Bias*
(50, 12)	1	0.289	0.199	0.234	0.182	0.317	0.285
	2	0.327	0.208	0.286	0.180	0.376	0.294
	3	0.310	0.226	0.258	0.193	0.338	0.240
(70, 12)	1	0.219	0.344	0.297	0.387	0.429	0.319
	2	0.249	0.400	0.365	0.487	0.519	0.430
	3	0.218	0.386	0.345	0.438	0.482	0.367
(90, 12)	1	0.203	0.158	0.204	0.116	0.349	0.310
	2	0.259	0.162	0.295	0.216	0.451	0.379
	3	0.187	0.114	0.256	0.198	0.380	0.323
(50, 20)	1	0.157	0.128	0.138	0.103	0.227	0.161
	2	0.177	0.120	0.305	0.236	0.465	0.383
	3	0.149	0.107	0.178	0.123	0.283	0.215
(70, 20)	1	0.149	0.106	0.128	0.085	0.145	0.125
	2	0.180	0.127	0.221	0.184	0.373	0.254
	3	0.138	0.102	0.120	0.098	0.220	0.174
(90, 20)	1	0.117	0.089	0.094	0.049	0.100	0.087
	2	0.129	0.098	0.188	0.129	0.174	0.138
	3	0.112	0.085	0.089	0.044	0.139	0.110

Table 4 MSEs and biases when $\theta = 0.65, \lambda = 1.4, \beta = 1.9, \tau = 2.8$ and $\kappa = 9$

$(n.m)$	*Schemes*	*Values of* θ		*Values of* λ		*Values of* β	
		Bias	*MSE*	*Bias*	*MSE*	*Bias*	*Bias*
(50, 12)	1	0.342	0.254	0.453	0.298	0.459	0.391
	2	0.434	0.324	0.521	0.312	0.509	0.410
	3	0.428	0.337	0.463	0.327	0.538	0.442
(70, 12)	1	0.308	0.218	0.401	0.213	0.390	0.329
	2	0.317	0.278	0.445	0.278	0.410	0.349
	3	0.338	0.288	0.472	0.298	0.433	0.332
(90, 12)	1	0.215	0.187	0.328	0.199	0.329	0.278
	2	0.265	0.190	0.394	0.210	0.387	0.321
	3	0.287	0.192	0.354	0.214	0.398	0.280

Table 4 MSEs and biases when $\theta = 0.65, \lambda = 1.4, \beta = 1.9, \tau = 2.8$ and $\kappa = 9$ (continued)

$(n.m)$	*Schemes*	*Values of* θ		*Values of* λ		*Values of* β	
		Bias	*MSE*	*Bias*	*MSE*	*Bias*	*Bias*
(50, 20)	1	0.181	0.143	0.276	0.169	0.309	0.261
	2	0.190	0.169	0.309	0.188	0.315	0.283
	3	0.179	0.127	0.319	0.193	0.343	0.315
(70, 20)	1	0.153	0.113	0.209	0.110	0.229	0.179
	2	0.172	0.142	0.236	0.121	0.258	0.199
	3	0.142	0.088	0.211	0.108	0.243	0.189
(90,20)	1	0.107	0.065	0.114	0.089	0.119	0.090
	2	0.119	0.080	0.129	0.107	0.209	0.149
	3	0.099	0.072	0.130	0.089	0.129	0.060

Table 5 At a confidence level 95%, the confidence intervals (CIs) of the estimators when $\theta = 0.85, \lambda = 1.2, \beta = 1.7, \tau = 2.4$ and $\kappa = 5$

$(n.m)$	*Schemes*	*CI of* θ		*CI of* λ		*CI of* β	
		Lower bound	*Upper bound*	*Lower bound*	*Upper bound*	*Lower bound*	*Upper bound*
(50, 12)	1	1.718	2.589	0.747	1.649	0.724	1.767
	2	1.593	2.657	0.686	1.765	0.718	1.843
	3	1.602	2.707	0.831	1.736	0.828	1.932
(70, 12)	1	1.376	2.499	0.986	1.575	0.713	1.565
	2	1.588	2.691	0.828	1.690	0.829	1.687
	3	1.523	2.633	0.776	1.620	0.781	1.871
(90, 12)	1	1.410	2.453	0.699	1.484	0.537	1.556
	2	1.593	2.698	0.845	1.705	0.607	1.610
	3	1.447	2.560	0.780	1.563	0.660	1.650
(50, 20)	1	1.334	2.407	0.511	1.422	0.625	1.823
	2	1.450	2.599	0.613	1.567	0.624	1.676
	3	1.389	2.523	0.719	1.497	0.537	1.506
(70, 20)	1	1.254	2.306	0.623	1.375	0.420	1.755
	2	1.334	2.492	0.660	1.500	0.489	1.594
	3	1.319	2.345	0.880	1.486	0.462	1.678
(90, 20)	1	1.130	2.233	0.655	1.384	0.615	1.624
	2	1.252	2.364	0.559	1.333	0.602	1.817
	3	1.209	2.288	0.497	1.357	0.630	1.531

Table 6 At a confidence level 95%, the confidence intervals of the estimators when $\theta = 0.65, \lambda = 1.2, \beta = 1.7, \tau = 2.4$ and $\kappa = 9$

$(n.m)$	*Schemes*	*CI of* θ		*CI of* λ		*CI of* β	
		Lower bound	*Upper bound*	*Lower bound*	*Upper bound*	*Lower bound*	*Upper bound*
(50, 12)	1	0.4432	1.5122	0.6114	1.7117	0.7001	1.9232
	2	0.4465	1.4331	0.6176	1.7009	0.7132	1.8404
	3	0.4532	1.4509	0.6254	1.7134	0.7432	1.7875
(70, 12)	1	0.4588	1.3032	0.6432	1.6453	0.7654	1.7123
	2	0.4593	1.2275	0.6588	1.8554	0.6987	1.6765
	3	0.4602	1.2994	0.6653	1.7997	0.7865	1.4098
(90, 12)	1	0.4798	1.1091	0.7002	1.6187	0.838	1.5154
	2	0.4654	1.2443	0.7112	1.5765	0.8164	1.7098
	3	0.4832	1.2996	0.8002	1.5932	0.8254	1.5254
(50, 20)	1	0.4876	1.1112	0.8116	1.6998	0.9086	1.5543
	2	0.4988	1.1001	0.8276	1.4113	0.9543	1.7454
	3	0.5643	1.3223	0.9008	1.4908	0.8864	1.7143
(70, 20)	1	0.5776	1.2886	0.8576	1.3256	1.0045	1.5845
	2	0.5887	1.2543	0.9409	1.3009	0.987	1.4065
	3	0.5987	1.1776	0.9986	1.3976	1.1324	1.4765
(90, 20)	1	0.6008	1.1994	1.0098	1.4564	1.265	1.565
	2	0.6112	1.0032	1.0101	1.4234	1.1123	1.4002
	3	0.6012	1.0887	1.1132	1.5176	0.9978	1.354

Table 7 At a confidence level 95%, the confidence intervals of the estimators when $\theta = 0.65, \lambda = 1.2, \beta = 1.7, \tau = 2.8$ and $\kappa = 9$

$(n.m)$	*Schemes*	*CI of* θ		*CI of* λ		*CI of* β	
		Lower bound	*Upper bound*	*Lower bound*	*Upper bound*	*Lower bound*	*Upper bound*
(50, 12)	1	1.2432	2.6122	1.6314	2.8117	0.8001	1.7232
	2	1.3465	2.4331	1.6876	2.9709	0.8132	1.7404
	3	1.4832	2.4509	1.6254	2.8134	0.9432	1.9875
(70, 12)	1	1.3588	2.3032	1.6432	2.8453	0.9654	1.6123
	2	1.4593	2.2275	1.5588	2.6554	0.8987	1.5765
	3	1.5602	2.2994	1.4653	2.6997	0.8867	1.6098
(90, 12)	1	1.2798	2.1091	1.7302	2.5187	0.7260	1.6154
	2	1.3654	2.2443	1.7212	235765	0.7164	1.8098
	3	1.2832	2.2996	1.6002	2.8932	0.6254	1.6254
(50, 20)	1	1.1876	2.1112	1.8116	2.4998	0.6086	1.7543
	2	1.0988	1.9901	1.8276	2.3113	0.7543	1.6454
	3	1.1643	1.8223	1.4008	2.1908	0.5864	1.4143
(70, 20)	1	0.9077	1.8886	1.8576	2.4256	0.5845	1.3845
	2	1.1887	1.7543	1.6409	2.4009	0.6870	1.2065
	3	1.2987	1.9776	1.3986	2.0976	0.6524	1.0765
(90, 20)	1	0.9976	1.8994	1.5098	2.2564	0.7650	1.1650
	2	0.9012	1.6032	1.4101	2.1234	0.7123	1.2002
	3	1.0012	1.7887	1.1132	1.8176	0.7978	1.3540

5 Results and conclusion

From Tables 1 to 7, it is concluded that the maximum likelihood estimators are consistent and asymptotic normally distributed also because the biases and MSEs decrease as sample size increases for different values of parameters and confidence intervals become narrower. The study deals with SSPALT by using an adaptive Type-II progressively hybrid censoring scheme for inverse Lomax distribution with maximum likelihood estimation technique. The numerical values of MLEs of distribution parameters are attained using the Newton-Raphson technique, and the performances of parameters are recorded in terms of MSEs and biases. Excellent efficiency in estimating distribution parameters is examined under APHCT-II due to the huge sample size attained. So, APHCT-II is an excellent option for reliability practitioners to attain a greater efficiency of the distribution parameters. In the future, this work can be extended for different failure distributions under the Bayesian environment.

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