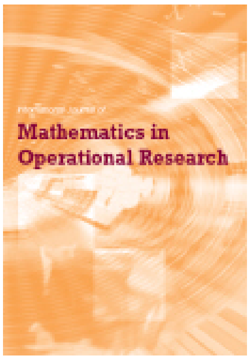

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A single server Markovian differentiated working vacation queue with server breakdown

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A single server Markovian differentiated working vacation queue with server breakdown

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Abstract: A single server Markovian queue with server breakdown has been considered here. In addition, the server goes on two types of vacation namely type-I and type-II. Type-I vacation is taken after the busy period of non-zero duration and type-II vacation occurs, at the completion of type-I vacation. When there are no customers in the system, both type-I and type-II vacations are working vacations. Also, on type-I vacation, the server may breakdown, and immediately the repair takes place. We analysed it as a quasi-birth-and-death (QBD) process, and using the matrix geometric method, the steady-state probability vector of the number of people in the queue and the stability conditions were produced. There are some system performance measures found. The effects of various parameters on the system performance measures are illustrated numerically.

Keywords: single server; differentiated vacation; breakdown; repair; QBD process.

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1 Introduction

We can observe in real-time queueing scenarios that servers operate during rest period if necessary. Because it results in a new class of queueing systems, these situations are known as queueing systems with server working vacations. Server working vacation models is mostly used in various types of sections like telecommunication systems and manufacturing systems, industrial organisations, production system and many others. Queueing system with server vacations has been analysed by many researchers and the survey paper from Doshi (1986), and the monograph from Takagi (1991), should be cited by readers.

Netus (1994) explore the matrix geometric method for the $M/M/1$ model under repair. Laxmi and Kassahun (2020), Suganya and Sivakumar (2019) and Jeyakumar and Rameshkumar (2019) are studied about the multiple vacations queing system. Agrawal et al. (2021) discussed about $M/M/1$ queueing model with working vacation and two type of server breakdown. Recently, Tian et al. (2021) analyse about a markovian queues with single working vacations and Bernoulli interruptions. Vijayashree et al. (2021) and Sampath and Jicheng (2018) talk about $M/M/1$ queueing model with differentiated vacation and interruption. Kalyanaraman and Sundaramoorthy (2019) analyse the working vacation queue with a server state dependent arrival rate and partial breakdown. In the working vacation queueing system, the server which works at a different rate, in particular with the lower service rate instead of a complete shut down during vacation. An $M/M/1$ queue with multiple working vacations, where the vacation times are exponentially distributed, was initially suggested by Servi and Finn (2002). The work to $M/G/1$ queue is expanded in Wu and Takagi (2006), although Joshi et al. (2021) discussed the $M/G/1$ queue length distribution with working vacations.

The stochastic decomposition structure of the queue length and waiting time in an $M/M/1$ working vacation queue is examined in Kim et al. (2003). The transformation of the working vacation queue from $M/M/1$ to $M^{[X]}/M/1$ in Xu et al. (2009) and Li et al. (2009) used the matrix analytic method to analyse an $M/G/1$ queue with exponential working vacation under a specific assumption. At the same time a multi-server queue with single working vacation consider in Lin and Ke (2009). But Jain and Jain (2008) examined a single working vacation model with the server breakdowns. Recently, Dasa et al. (2022) analyse about the Markovian queueing system with server failures and also Choudhary and Sharma (2022) studied about queueing model with unreliable service station. In addition to these ideas in Ke et al. (2010), we have a brief analysis of recent developments in vacation queueing models. A single-server queueing model with heterogeneous arrival and service rate was described in Yechiali and Naor (1971). The identical model was examined in Fond and Ross (1997) with the notion that any arrival would result in the server queuing, with the arrival rate being depending on server state (Shogan, 1979).

We take into account a $M/M/1$ queue with differentiated vacation and server breakdown in this object. In these sense we have seen the service rate is different but arrival rate is same for all the states. The model has been examined using the matrix geometric model.

Our proposed model has several real-life time examples. For example, if we are considering a bank with a single cashier, handling the customers is his primary job. If there are no customers in the system, the cashier can go to his secondary job. That is, he bundled the rupee notes into 100 numbers. After completing bundling the money again if the system founds to be empty cashier go to another secondary job. That is to take and proceed with the cheques from the cheque box. After completing cheque process, if the system deducts a customer then the cashier is return back to his/her primary job. That is handling the customers.

The remaining section of the paper is collected as follow. We give a model description and explain a quasi-birth-death process in Section 2. In Section 3 using the matrix geometric method, the solution of steady state is presented. In Sections 4 and 5, we give some system performance measures and a numerical analysis. A conclusion has been offered.

2 Model description

Here we are dealing $M/M/1$ queuing model with a differentiated working vacation and server breakdown. The elementary assumptions are described as following:

1 Customers arrive according to a Poisson process with rate λ.

2 Service times in the regular busy period follows an exponentially distributed with parameter μ and the service is provided by a single server.

3 We assume that there are two types of working vacations namely type-I and type-II respectively.

4 Type-I vacation taken after a busy period of non-zero duration, and type-II vacation is taken when no customers are waiting in the system when the server it returns from the vacation. Otherwise, the server switches to regular busy period.

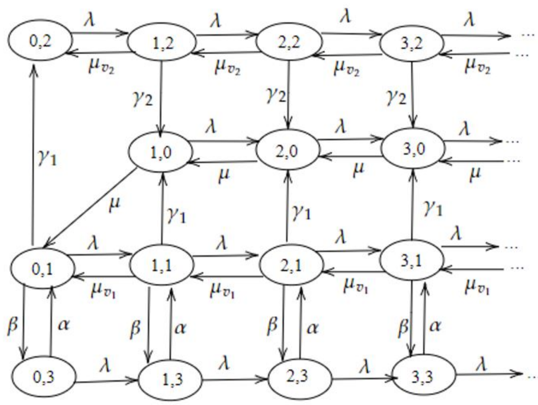
5 During type-I (II) vacations, an arriving customers are served at a rates $\mu_{v_1}(\mu_{v_2})$.

6 Assume that the durations of type-I (II) vacations are assumed to exponentially distributed with parameters γ_1 and γ_2.

7 The server may getting breakdown during type-I vacation with rate β and α is assumed to be a repair rate.

8 The model is presented schematically in Figure 1.

Figure 1 Transition diagram



2.1 The quasi-birth-and-death process

At time t, the number of customers in the systems is consider as $H(t)$ and let $I(t)$ be the server state at time t where $I(t)$ is defined as follows

$$I(t) = \begin{cases} 0, & \text{if the server is in busy period} \\ 1, & \text{if the server is in type-I vacation} \\ 2, & \text{if the server is in type-II vacation} \\ 3, & \text{if the server is in breakdown} \end{cases}$$

Let $Z(t) = \{H(t), I(t)\}$, then $\{Z(t) : t \geq 0\}$ is a continuous time Markov chain (CTMC) with state space $\Omega = \{(i, n) : i \geq 0; n = 0, 1, 2, 3\}$, where $(i, 0), i \geq 1$ denotes the system is in regular busy state and $(i, n), i \geq 0; n = 1, 2, 3$ denotes the system is in type-I vacation (type-II vacation, breakdown) states.

Using lexicographical order for the states, the infinitesimal generator of MC Q is given by,

$$Q = \begin{bmatrix} B_{00} & B_{01} & \dots & & & \\ B_{10} & B_{11} & A_0 & \dots & & \\ \vdots & A_2 & A_1 & A_0 & \dots & \\ & \vdots & A_2 & A_1 & A_0 & \dots \\ & & \vdots & \vdots & \vdots & \end{bmatrix}$$

where

$$B_{00} = \begin{bmatrix} -(\lambda+\beta+\gamma_1) & \gamma_1 & \beta \\ 0 & -\lambda & 0 \\ \alpha & 0 & -(\alpha+\lambda) \end{bmatrix},$$

$$B_{01} = \begin{bmatrix} 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}, \quad B_{10} = \begin{bmatrix} \mu & 0 & 0 \\ \mu_{v_1} & 0 & 0 \\ 0 & \mu_{v_2} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B_{11} = \begin{bmatrix} -(\lambda+\mu) & 0 & 0 & 0 \\ \gamma_1 & -(\mu_{v_1}+\gamma_1+\beta+\lambda) & 0 & \beta \\ \gamma_2 & 0 & -(\lambda+\gamma_2+\mu_{v_2}) & 0 \\ 0 & \alpha & 0 & -(\alpha+\lambda) \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -(\lambda+\mu) & 0 & 0 & 0 \\ \gamma_1 & -(\mu_{v_1}+\gamma_1+\lambda+\beta) & 0 & \beta \\ \gamma_2 & 0 & -(\gamma_2+\lambda) & 0 \\ 0 & \alpha & 0 & -(\alpha+\lambda) \end{bmatrix},$$

$$A_2 = \begin{bmatrix} \mu & 0 & 0 & 0 \\ 0 & \mu_{v_1} & 0 & 0 \\ 0 & 0 & \mu_{v_2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_0 = \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$$

3 The steady state solution

In order for $PQ = 0$ and $Pe = 1$, where e is a column vector of 1's of the proper size, let $P = (p_0, p_1, p_2, ...)$ be the stationary probability vector associated with Q.

Let $p_0 = (p_{01}, p_{02}, p_{03})$ and if $i \geq 1$ then $p_i = (p_{i0}, p_{i1}, p_{i2}, p_{i3})$. The following equations produce the sub vectors p_i if the steady state requirement is met

$$p_0 B_{00} + p_1 B_{10} = 0 \tag{1}$$

$$p_0 B_{01} + p_1 B_{11} + p_2 A_2 = 0 \tag{2}$$

$$p_i A_0 + p_{i+1} A_1 + p_{i+2} A_2 = 0 \qquad \text{for } i \geq 1 \tag{3}$$

$$p_i = p_1 R^{(i-1)} \qquad \text{for } i \geq 2 \tag{4}$$

where R is the rate matrix is the minimal non-negative solution of the matrix quadratic equation (see Netus, 1994).

$$R^2A_2 + RA_1 + A_0 = 0 \tag{5}$$

Substituting the (4) in (2) we have

$$p_0B_{01} + p_1(B_{11} + RA_2) = 0 \tag{6}$$

and the normalising condition is

$$p_0e + p_1(I - R)^{-1}e = 1 \tag{7}$$

Theorem 1: The above section is stable iff $\rho < 1$ where $\rho = \dfrac{\lambda}{\mu}$. *Proof:* Let

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \gamma_1 & -(\gamma_1 + \beta) & 0 & \beta \\ \gamma_2 & 0 & -\gamma_2 & 0 \\ 0 & \alpha & 0 & -\alpha \end{bmatrix}$$

Consider the row vector $\pi = (\pi_1, \pi_2, \pi_3, \pi_4)$ which satisfies the conditions $\pi A = 0$, $\pi e = 1$.

Following Netus (1994), the system is stable if and only if $\pi A_0 e < \pi A_2 e$. That is the system is stable if and only if $\rho < 1$. □

Lemma 2:

1 $\mu r_1^2 - (\lambda + \mu)r_1 + \lambda = 0$

The above quadratic equation has 2 real roots r_1^*, r_1 with conditions $r_1^* = 1$, $r_1 = \rho$, where $\rho = \dfrac{\lambda}{\mu}$

2 $\mu_{v_1}(r_3^2 + r_4r_7) - r_3(\mu_{v_1} + \gamma_1 + \lambda + \beta) + \alpha r_4 + \lambda = 0$

The above quadratic equation has 2 real roots $r_3 < r_3^*$ with conditions $0 < r_3 < 1$, $r_3^* > 1$, where

$$r_3^*, r_3 = \frac{(\mu_v + \gamma_1 + \lambda + \beta) \pm \sqrt{(\mu_v + \gamma_1 + \lambda + \beta)^2 - 4\lambda\mu_{v_1}}}{2\mu_{v_1}}$$

3 $\mu_{v_2}r_6^2 - (\gamma_2 + \lambda + \mu_{v_2})r_6 + \lambda = 0$

The above quadratic equation has two real roots $r_6 < r_6^*$ with conditions $0 < r_6 < 1$, $r_6^* > 1$, where

$$r_6^*, r_6 = \frac{(\lambda + \gamma_2 + \mu_{v_2}) \pm \sqrt{(\lambda + \gamma_2 + \mu_{v_2})^2 - 4\lambda\mu_{v_1}}}{2\mu_{v_1}}$$

Theorem 3: If $\rho < 1$, the matrix equation (5) has the minimal non-negative solution as follows:

$$R = \begin{bmatrix} \rho & 0 & 0 & 0 \\ r_2 & r_3 & 0 & r_4 \\ r_5 & 0 & r_6 & 0 \\ 0 & r_7 & 0 & r_8 \end{bmatrix}$$

where

$$r_2 = \frac{-r_3\gamma_1}{\mu r_1 + \mu r_3 - (\lambda + \mu)}$$

$$r_3 = \frac{(\mu_v + \gamma_1 + \lambda + \beta) - \sqrt{(\mu_v + \gamma_1 + \lambda + \beta)^2 - 4\lambda\mu_{v_1}}}{2\mu_{v_1}}$$

$$r_4 = \frac{r_3\beta}{\alpha + \lambda}$$

$$r_5 = \frac{-r_6\gamma_2}{(\mu r_1 + \mu r_6 - \lambda + \mu)}$$

$$r_6 = \frac{(\lambda + \gamma_2 + \mu_{v_2}) - \sqrt{(\lambda + \gamma_2 + \mu_{v_2})^2 - 4\lambda\mu_{v_1}}}{2\mu_{v_1}}$$

$$r_7 = 0$$

$$r_8 = \frac{\lambda}{\alpha + \lambda}$$

Proof: Let

$$R = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ r_2 & r_3 & 0 & r_4 \\ r_5 & 0 & r_6 & 0 \\ 0 & r_7 & 0 & r_8 \end{bmatrix} \tag{8}$$

$$R^2 A_2 = \begin{bmatrix} \mu r_1^2 & 0 & 0 & 0 \\ \mu r_2(r_1 + r_3) & \mu_{v_1}(r_3^2 + r_4 r_7) & 0 & 0 \\ \mu r_5(r_1 + r_6) & 0 & \mu_{v_2} r_6^2 & 0 \\ \mu r_7 r_7 & \mu_{v_1}(r_3 r_7 + r_7 r_8) & 0 & 0 \end{bmatrix} \tag{9}$$

$$RA_1 = \begin{bmatrix} -(\lambda + \mu)\gamma_1 & 0 & 0 & 0 \\ V_0 & -r_3 V_1 + \alpha r_4 & 0 & r_3\beta - r_4(\alpha + \lambda) \\ V_2 & 0 & -r_6(\lambda + \gamma_2 + \mu_{v_2}) & 0 \\ r_7\gamma_1 & -(\mu_{v_1} + \gamma_1 + \lambda + \beta) r_7 + \alpha r_8 & 0 & -r_7\beta r_8(\alpha + \lambda) \end{bmatrix} \tag{10}$$

where $V_0 = -(\lambda + \mu)r_2 + r_3\gamma_1$, $V_1 = \mu_v + \gamma_1 + \lambda + \beta$ and $V_2 = -(\lambda + \mu)r_5 + r_6\gamma_2$.

Substituting (9), (10) and A_0 into (5) gives the following set of equations

$$\mu r_1^2 - (\lambda + \mu)r_1 + \lambda = 0 \tag{11}$$

$$\mu r_2(r_1 + r_3) - (\lambda + \mu)r_2 + r_3\gamma_1 = 0 \tag{12}$$

$$\mu_{v_1}(r_3^2 + r_4 r_7) - r_3(\mu_{v_1} + \gamma_1 + \lambda + \beta) + \alpha r_4 + \lambda = 0 \tag{13}$$

$$r_3\beta - (\alpha + \lambda)r_4 = 0 \tag{14}$$

$$\mu r_5(r_1 + r_6) - (\lambda + \mu)r_5 + r_6\gamma_2 = 0 \tag{15}$$

$$\mu_{v_2} r_6^2 - (\gamma_2 + \lambda + \mu_{v_2})r_6 + \lambda = 0 \tag{16}$$

$$\mu(r_2 r_7) + r_7\gamma_2 = 0 \tag{17}$$

$$\mu_{v_1}(r_3 r_7 + r_7 r_8) - (\mu_{v_1} + \gamma_1 + \lambda + \beta)r_7 + \alpha r_8 = 0 \tag{18}$$

$$r_7\beta - r_8(\alpha + \lambda) + \lambda = 0 \tag{19}$$

From equations (11)–(19) we get the results of Theorem 3.

The non-negative solution must be minimal, since equation (5) has unique non-negative solution. □

Theorem 4: If $\rho < 1$, the stationary probability vectors $p_0 = (p_{01}, p_{02}, p_{03})$ and $p_i = (p_{0i}, p_{i1}, p_{i2}, p_{i3})$ are

$$p_{01} = \frac{(\alpha + \lambda)}{\beta} p_{03}$$

$$p_{02} = \frac{t_1\gamma_1(\alpha + \lambda)}{\lambda\beta t_1} p_{03}$$

$$p_{10} = \frac{t_3}{t_4} p_{03}$$

$$p_{11} = \frac{t_2 t_4 \mu_v - \mu t_3}{t_4\mu_v} p_{03}$$

$$p_{12} = \frac{\gamma_1(\alpha + \lambda)}{t_1\beta} p_{03}$$

$$p_{13} = \frac{t_6\mu_v p_{03} - t_5\mu p_{10}}{\mu_v\alpha}$$

$$\begin{aligned} p_{03} = \Bigg[& 1 + \frac{(\alpha + \lambda)}{\beta} + \frac{t_1\gamma_1(\alpha + \lambda)}{\lambda t_1\beta} + \frac{t_3}{t_4(1 - r_1)} \\ & + \frac{(t_2 t_4 - (\mu/\mu_v)t_3)}{t_4(1 - r_3)} \left(1 + \frac{r_2}{1 - r_1} + \frac{r_4}{1 - r_8}\right) \\ & + \left[\frac{(\alpha + \lambda)\gamma_1}{\beta t_1(1 - r_6)}\right] \left[1 + \frac{r_5}{1 - r_1}\right] + \frac{t_4 t_6 - t_5(\mu/\mu_v)t_3}{\alpha t_4(1 - r_8)} \Bigg]^{-1} \end{aligned}$$

where

$$t_1 = \lambda + \gamma_2 + r_6\mu_{v_2}$$

$$t_2 = \frac{(\alpha+\lambda)(\lambda+\beta+\gamma_1)-\alpha\beta}{\mu_v\beta}$$

$$t_3 = (\gamma_1+\mu r_2)t_2 + [\gamma_2+\mu r_5]\left[\frac{\gamma_1(\alpha+\lambda)}{\beta t_1}\right]$$

$$t_4 = \lambda+\mu-\mu r_1+(\mu/\mu_v)(\gamma_1+\mu r_2)$$

$$t_5 = \mu_{v_1}+\gamma_1+\beta+\lambda-r_3\mu_v$$

$$t_6 = t_2 t_5 - \frac{\lambda(\alpha+\lambda)}{\beta}$$

Proof: $p_{01}, p_{02}, p_{03}, p_{10}, p_{11}, p_{12}, p_{13}$ follows from equations (1), (6) and (7). □

4 Performance measures

By using the normal calculations, performance measures have been calculated as follows:

1. Mean system length when the server is an
   - busy: $E(L) = \sum_{i=1}^{\infty} ip_{i0}$
   - type-I vacation: $E(L_{v_1}) = \sum_{i=0}^{\infty} ip_{i1}$
   - type-II vacation: $E(L_{v_2}) = \sum_{i=0}^{\infty} ip_{i2}$
   - breakdown: $E(L_b) = \sum_{i=0}^{\infty} ip_{i3}$.
2. The probability that the server is in
   - regular busy: $P_b = \sum_{i=1}^{\infty} p_{i0}$
   - type-I vacation: $P_{v_1} = \sum_{i=0}^{\infty} p_{i1}$
   - type-II vacation: $P_{v_2} = \sum_{i=0}^{\infty} p_{i2}$
   - breakdown state: $P_{br} = \sum_{i=0}^{\infty} p_{i3}$.

5 Numerical analysis

In this section, we have presented some numerical illustrations in order to validate our analytical results by graphs and tables. In table 1, arrival rate $\lambda = 0.18$. The other input parameters are taken as $\alpha = 0.3$, $\beta = 0.2$, $\mu = 2.4$, $\mu_{v_1} = 1$, $\mu_{v_2} = 0.8$, $\gamma_1 = 0.4$, $\gamma_2 = 0.5$. The description of Table 1 is given as follows:

- first column represents the number of customers in the system
- second column represents the probability that there are n customers in the system and the server is in busy
- third column represents the probability that there are n customers in the system and the server is on type-I vacation
- fourth column represents the probability that there are n customers in the system and the server is on type-I vacation
- fifth column represents the probability that there are n customers in the system and the server is in breakdown.

Table 1 Performance measures

i	p_{i0}	p_{i1}	p_{i2}	p_{i3}
0	-	0.11630368639	0.61785439510	0.04845986933
1	0.02320863691	0.02047818600	0.08086539570	0.04437467832
2	0.00468929624	0.00220410402	0.01058374315	0.00091837668
3	0.00072804853	0.00023723168	0.00138521080	0.00009884654
4	0.00010283076	0.00002553367	0.00018129776	0.00001063903
5	0.00001391345	0.00000274824	0.00002372843	0.00000114510
6	0.00000184319	0.00000029580	0.00000310560	0.00000012325
7	0.00000024162	0.00000003184	0.00000040646	0.00000001327
8	0.00000003151	0.00000000343	0.00000005320	0.00000000143
9	0.00000000410	0.00000000037	0.00000000696	0.00000000015
10	0.00000000053	0.00000000004	0.00000000091	0.00000000002
11	0.00000000007	0.00000000000	0.00000000012	0.00000000000
12	0.00000000001	0.00000000000	0.00000000002	0.00000000000

Table 2 Performance measures

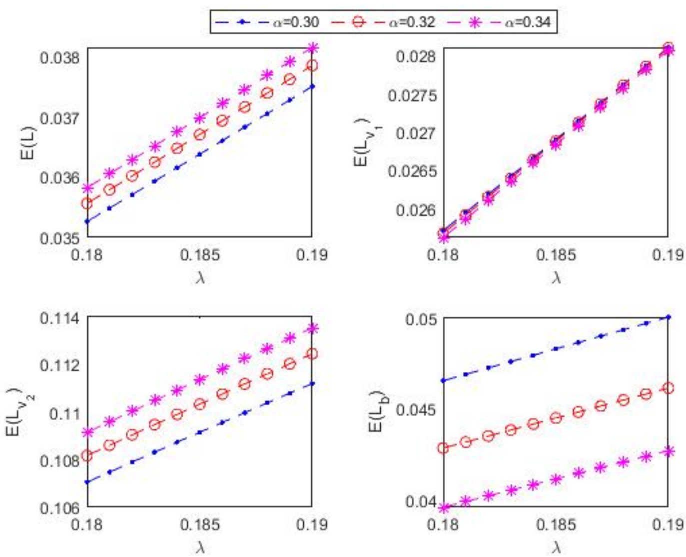
λ	α	P_b	P_{v_1}	P_{v_2}	P_{br}
0.18	0.30	0.035265311	0.139251821	0.710897344	0.093863693
	0.32	0.035567889	0.140435178	0.718331206	0.088758552
	0.34	0.035826501	0.141445609	0.724799246	0.084149944
0.185	0.30	0.036387627	0.142050353	0.703626605	0.095790606
	0.32	0.036713827	0.143311789	0.711260653	0.090616177
	0.34	0.036992756	0.144389391	0.717903421	0.085939806
0.19	0.30	0.037515667	0.144790575	0.696403949	0.097679888
	0.32	0.037866593	0.146132424	0.704235885	0.092440118
	0.34	0.038166810	0.147279277	0.711051609	0.087699311

From Table 1, we conclude if the number of customers increases then the probability values are decrease. From Table 2, we observed that if the repair rate (α) increases then the steady state probabilities increases all the states except breakdown state. And Table 3 tells us if the breakdown rate (β) increases then the steady state probabilities decreases all the states except breakdown state.

Table 3 Performance measures

λ	α	P_b	P_{v_1}	P_{v_2}	P_{br}
0.18	0.30	0.035265311	0.139251821	0.710897344	0.093863693
	0.32	0.035567889	0.140435178	0.718331206	0.088758552
	0.34	0.035826501	0.141445609	0.724799246	0.084149944
0.185	0.30	0.036387627	0.142050353	0.703626605	0.095790606
	0.32	0.036713827	0.143311789	0.711260653	0.090616177
	0.34	0.036992756	0.144389391	0.717903421	0.085939806
0.19	0.30	0.037515667	0.144790575	0.696403949	0.097679888
	0.32	0.037866593	0.146132424	0.704235885	0.092440118
	0.34	0.038166810	0.147279277	0.711051609	0.087699311

Figure 2 λ versus mean number of customers for different α (see online version for colours)



The effect of arrival rate (λ) on mean queue length for different values of α is shown in Figure 2. That is if λ increases then the mean queue length are increases. And also we can notice that if α increase then the mean queue length is increase in regular busy state and type-II vacation state. If α increases then the mean queue length is decreases in type-I vacation state and breakdown state. The effect of arrival rate (λ) on mean queue length for different values of β is shown in Figure 3. That is if λ increases then the mean queue length are increases. And also we can notice that if β rate increase then the mean queue length is decreases in regular busy state and type-II vacation state. If β rate increases the mean queue length is increases in type-I vacation state and breakdown state.

In Figure 4, we shows the effect of the service rate in type-I vacation (μ_{v_1}) on mean queue length for different values of α. We observed that if μ_{v_1} increases then the mean queue length is increases in regular busy state and type-II vacation state, and decreases in type-I vacation state and breakdown state. Also we notice that if α increase then the mean queue length is also increase in regular busy state and type-II vacation state, and mean queue length is decrease in type-I vacation state and breakdown state. In Figure 5, we shows the effect of the service rate in type-I vacation (μ_{v_1}) on mean queue length for different values of β. We observed that if μ_{v_1} increase then the mean queue length is increase in regular busy state and type-II vacation state, and decrease in type-I vacation state and breakdown state. Also we notice that if β increase then the mean queue length is also decreases in regular busy state and type-II vacation state, and mean queue length is increases in type-I vacation state and breakdown state.

Figure 3 λ versus mean number of customers for different β (see online version for colours)

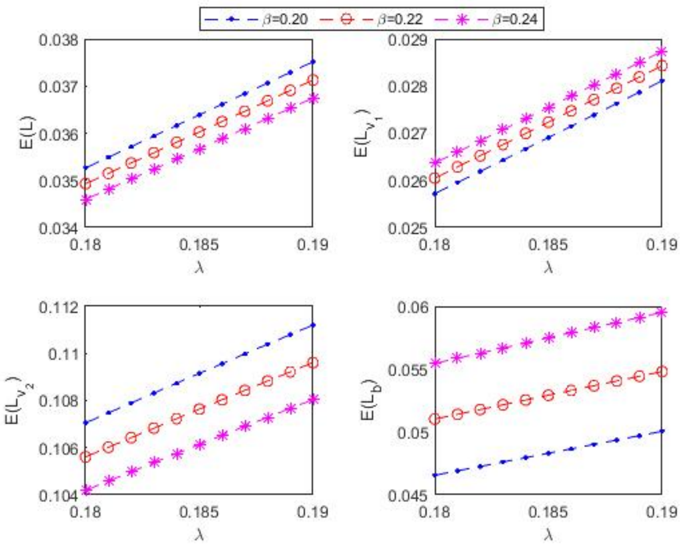


Figure 4 μ_{v_1} versus mean number of customers for different α (see online version for colours)

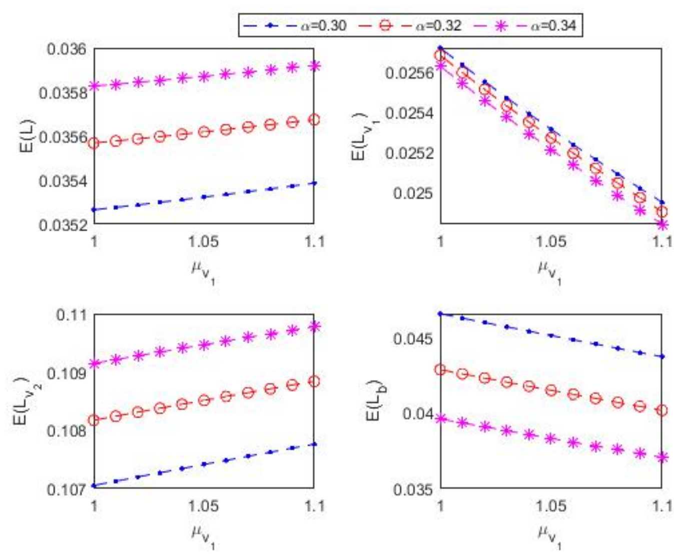


Figure 5 μ_{v1} versus mean number of customers for different β (see online version for colours)

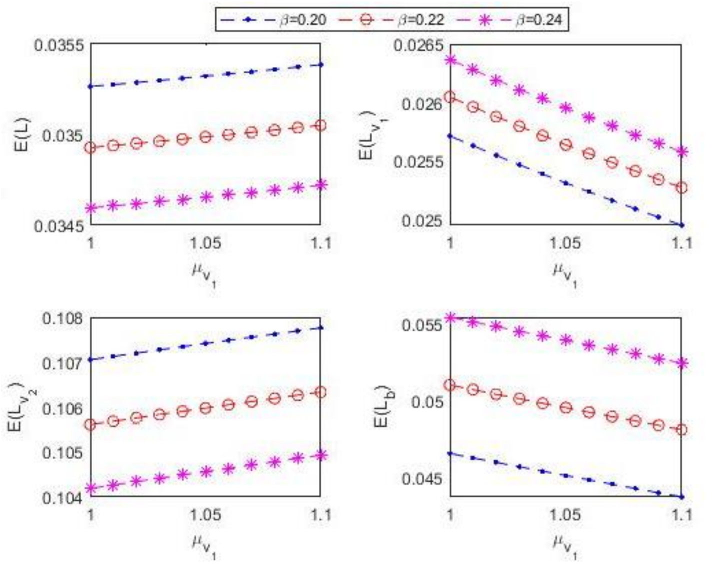
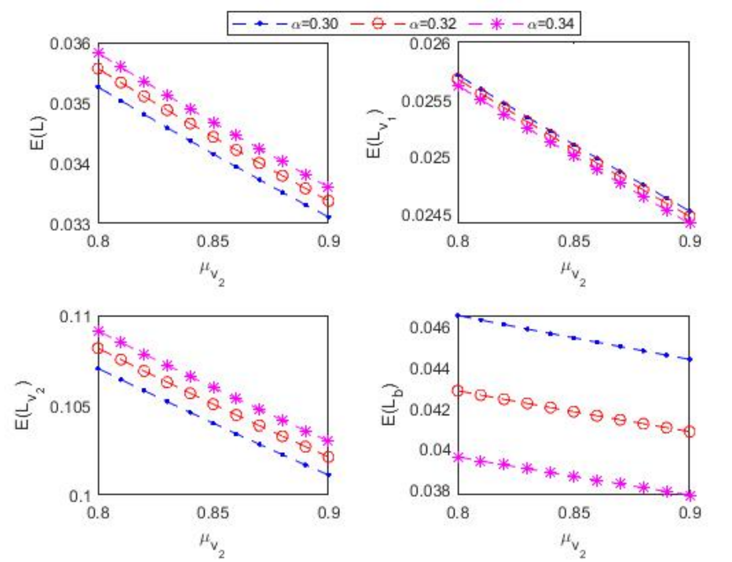
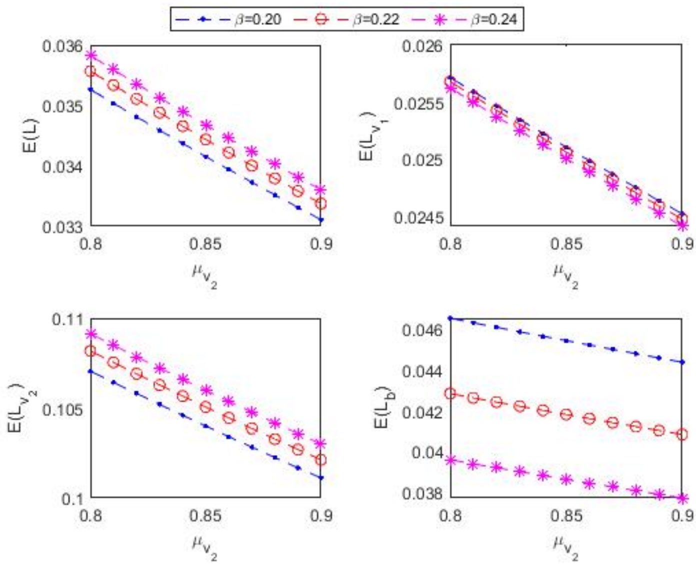


Figure 6 μ_{v2} versus mean number of customers for different α (see online version for colours)



In Figure 6, we shows the effect of the service rate in type-II vacation (μ_{v_2}) on mean queue length for different values of α. We observed that if μ_{v_2} increases then the mean queue length is decreases in all the states. Also we notice that if α increases then the mean queue length is also increases in regular busy state and type-II vacation state, and mean queue length is decrease in type-I vacation state and breakdown state. In Figure 7, we shows the effect of the service rate in type-II vacation (μ_{v_2}) on mean queue length for different values of β. We observed that if μ_{v_2} increases then the mean queue length is decreases in all the states. Also we notice that if β increases then the mean queue length is also increase in regular busy state and type-II vacation state, and mean queue length is decrease in type-I vacation state and breakdown state.

Figure 7 μ_{v_2} versus mean number of customers for different β (see online version for colours)



6 Conclusions

In this work we investigated a single server Markovian queue with server breakdown. Also we considered two types vacations, in addition both of these vacation are a working vacation. The performance measures of mean for the four states of the system are presented through the graphs and the steady state probabilities are presented through the table. In the future, the work could progress to a multi-server, differentiated working vacation queueing system with server breakdown.

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