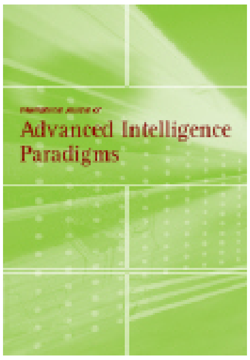

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Some results on edge irregular product vague graphs

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Abstract: Recently, vague graph is a highly growing research area as it is the generalisation of the fuzzy graphs. In this paper, we analysed the concepts of edge regular product vague graphs and its properties. The concepts of edge irregular product vague graphs, strongly edge irregular product vague graphs are analysed with properties.

Keywords: product vague graph; edge regular and irregular product vague graph; strongly edge irregular.

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1 Introduction

Graph theory has found its importance in many real-time problems. Recent applications in graph theory is quite interesting analysing any complex situations and moreover in engineering applications. It has got numerous applications on operations research, system analysis, network routing, transportation and many more. To analyse any complete information we make intensive use of graphs and its properties. For working on partial information or incomplete information or to handle the systems containing the elements of uncertainty we understand that fuzzy logic and its involvement in graph theory is applied. In 1975, Rosenfeld discussed the concept of fuzzy graphs whose ideas are implemented by Kauffman in 1973. The fuzzy relation between fuzzy sets was also considered by Rosenfeld who developed the structure of fuzzy graphs, obtaining various analogous results of several graph theoretical concepts. Bhattacharya (1987) gave some remarks of fuzzy graphs. The complement of fuzzy graphs was introduced by Mordeson and Nair (2001). Atanassov introduced the concept of intuitionistic fuzzy relation and intuitionistic fuzzy graphs (Atanassov, 1986, Atanassov et al., 2003; Shannon and Atanassov, 1994, 1995). Talebi and Rashmanlou (2013) studied the properties of isomorphism and complement of interval-valued fuzzy graphs. They defined isomorphism and some new operations on vague graphs (Talebi et al., 2013, 2016). Borzooei and Rashmalou (2017, 2015, 2016a, 2016b, 2016c) analysed new concepts of vague graphs, degree of vertices in vague graphs, more results on vague graphs, semi global domination sets in vague graphs with application and degree and total degree of edges in bipolar fuzzy graphs with application. Rashmanlou and Jun (2013) defined the complete interval-valued fuzzy graphs. Rashmanlou and Pal (2015, 2013d) studied intuitionistic fuzzy graphs with categorical properties, some properties of highly irregular interval-valued fuzzy graphs, more results on highly irregular bipolar fuzzy graphs (Rashmanlou et al., 20141), balanced interval-valued fuzzy graphs (Rashmanlou and Pal, 2013b, 2013a) and antipodal interval-valued fuzzy graphs. Samanta and Pal (2013) investigated fuzzy k-competition and p-competition graphs, and concept of fuzzy planar graphs in Pal et al. (2013a, 2013b). Also they introduced fuzzy tolerance graph (Samanta and Pal, 2011, 2012) bipolar fuzzy hypergraphs and given several properties on it. Pal and Rashmanlou (2013) defined many properties of irregular interval-valued fuzzy graphs. Ghorai and Pal (2016, 2017) analysed the properties of regular product vague graphs and product vague line graphs. In this article, we define the product vague graphs and investigate some interesting properties of regular, irregular and edge regular product vague graphs. Likewise, we analyse some concepts on product vague line graphs. For other notations and terminologies the readers are referred to Akram and Karunambigai (2011), Atanassov (1986), Atanassov et al. (2003), Bhattacharya (1987), Borzooei and Rashmanlou (2015, 2017), Karunambigai et al. (2011) and Karunambigai and Kalaivani (2011).

2 Preliminaries

In this section we give some definitions which are prerequisites applied throughout this paper.

Definition 2.1 (*Kauffman*, *1973*)*:* A graph is an ordered pair $G = (V, E)$, where V is the set of vertices of G and E is the set of all edges of G. Two vertices x and y in an undirected graph G are said to be adjacent in G if xy is an edge of G. A simple graph is an undirected graph that has no loops and not more than one edge between any two different vertices.

Definition 2.2 (*Kauffman*, *1973*)*:* A subgraph of a graph $G^* = (V, E)$ is a graph $H = (W, F)$, where $W \subseteq V$ and $F \subseteq E$.

We write $xy \in E$ to mean $(x, y) \in E$, and if $e = xy \in E$, we say x and y are adjacent. Formally, given a graph $G^* = (V, E)$, two vertices $x, y \in V$ are said to be neighbours or adjacent nodes, if $xy \in E$. The neighbourhood of a vertex v in a graph G^* is the induced subgraph of G^* consisting of all vertices adjacent to v and all edges connecting two such vertices. The neighbourhood of v is often denoted by $N(v)$. The degree $deg(v)$ of vertex v is the number of edges incident on v. The open neighbourhood for a vertex v in a graph G^* consists of all vertices adjacent to v but not including v, i.e., $N(v) = u \in V: uv \in E$. If v is included in $N(v)$, then it is called closed neighbourhood for v and is denoted by $N[v]$, i.e., $N[v] = N(v) \cup v$. A regular graph is a graph where each vertex has the same open neighbourhood degree. A complete graph is a simple graph in which every pair of distinct vertices has an edge.

Definition 2.3 (*Ramakrishna*, *2009*)*:* A vague relation B on a set V is a vague relation from V to V such that $t_B(xy) \leq \min(t_A(x), t_A(y))$, $f_B(xy) \geq \max(f_A(x), f_A(y))$ where A is a vague set on a set V and for a vague relation B on A for all $x, y \in V$.

Definition 2.4 (*Ramakrishna*, *2009*)*:* Let $G^* = (V, E)$ be a crisp graph. A pair $G = (A, B)$ is called a vague graph on a crisp graph $G^* = (V, E)$ where $A = (t_A, f_A)$ is a vague set on V and $B = (t_B, f_B)$ is a vague set on $E \subseteq V \times V$ such that $t_B(xy) \leq \min(t_A(x), t_A(y))$, $f_B(xy) \geq \max(f_A(x), f_A(y))$ for each edge $x, y \in E$. Otherwise A is the vague set on V and B is a vague relation on V.

We consider G^* as crisp graph and G as product vague graph.

Definition 2.5 (*Ghorai and Pal*, *2016*)*:* A product vague graph G of $G^* = (V, E)$ is a pair $G = (V, A, B)$ where $A = (t_A, f_A)$ is an vague set in V and $B = (t_B, f_B)$ is a vague relation on V^2 such that $t_B(xy) \leq t_A(x) \times t_A(y)$, $f_B(xy) \geq f_A(x) \times f_A(y)$ for all $x, y \in V$.

Figure 1 Example of product vague graph

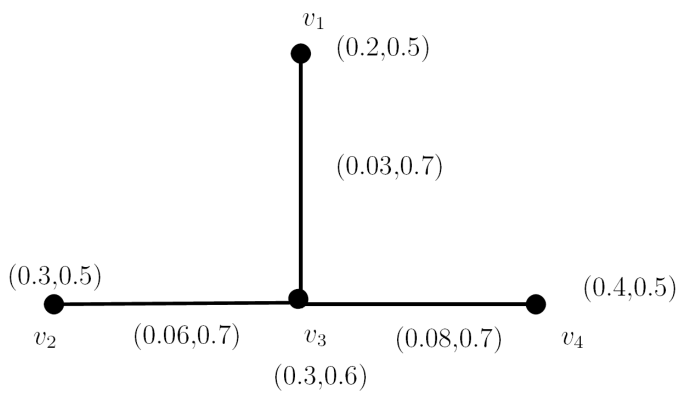
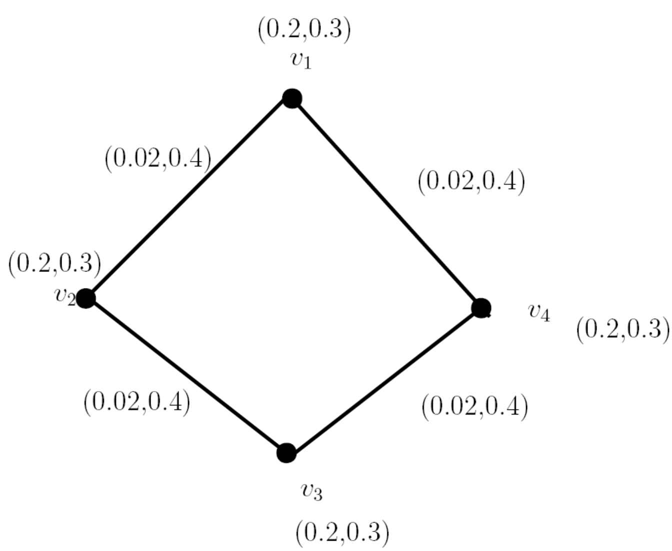


Figure 2 (0.04, 0.8)-regular and (0.24, 1.1)-totally regular product vague graph



Definition 2.6 (*Ghorai and Pal*, *2016*)*:* A product vague graph $G = (V, A, B)$ of $G^* = (V, E)$ is said to be strong if $t_B(xy) = t_A(x) \times t_A(y)$, $f_B(xy) = f_A(x) \times f_A(y)$ for all $xy \in E$.

Definition 2.7 (*Ghorai and Pal, 2016*)*:* Let $G = (A, B)$ be a product vague graph of $G^* = (V, E)$. The open neighbourhood degree of a vertex v in G is defined by $deg(v) = (deg^t(v), deg^f(v))$, where $deg^t(v) = \sum_{\substack{u \neq v \\ uv \in E}} t_B(uv)$ and $deg^f(v) = \sum_{\substack{u \neq v \\ uv \in E}} f_B(uv)$. If all the vertices of G have same open neighbourhood degree (d_1, d_2), then G is called (d_1, d_2)-regular product vague graph.

Definition 2.8 (*Ghorai and Pal, 2016*)*:* Let $G = (A, B)$ be a product vague graph of $G^* = (V, E)$. The closed neighbourhood degree of a vertex v is defined by $deg[v] = (deg^t[v], deg^f[v])$, where $deg^t[v] = deg^t(v) + t_A(v)$ and $deg^f[v] = deg^f(v) + f_A(v)$. If each vertex of G has the same closed neighbourhood degree (f_1, f_2), then G is called (f_1, f_2)-totally regular product vague graph.

3 Edge regular and irregular product vague graphs

Definition 3.1: Let $G = (V, A, B)$ be a product vague graph and let $e_{ij} = v_iv_j \in E$ be an edge of G. Then the degree of the edge e_{ij} is defined as $d_t(e_{ij}) = deg^t(v_i) + deg^t(v_j) - 2t_B(e_{ij})$ and $d_f(e_{ij}) = deg^f(v_i) + deg^f(v_j) - 2f_B(e_{ij})$. The edge degree of G is $d(e_{ij}) = (d_t(e_{ij}), d_f(e_{ij}))$.

Definition 3.2: Let $G = (V, A, B)$ be a product vague graph and let $e_{ij} = v_iv_j \in E$ be an edge of G. Then the total degree of the edge e_{ij} is defined as $d_t[e_{ij}] = d_t(e_{ij}) + t_B(e_{ij})$ and $d_f[e_{ij}] = d_f(e_{ij}) + f_B(e_{ij})$ and $d[e_{ij}] = (d_t[e_{ij}], d_f[e_{ij}])$.

Definition 3.3: Let $G = (V, A, B)$ be a product vague graph on $G^* = (V, E)$. If each edge in G has the same degree (r_1, r_2), then G is said to be an (r_1, r_2)-edge regular product vague graph.

Definition 3.4: Let $G = (V, A, B)$ be product vague graph on $G^* = (V, E)$. G is said to be an irregular product vague graph if there exists a vertex which is adjacent to vertices with distinct edge degrees.

Definition 3.5: Let $G = (V, A, B)$ be a product vague graph on $G^* = (V, E)$. G is said to be strong irregular product vague graph if every pair of vertices have distinct edge degree.

Definition 3.6: Let $G = (V, A, B)$ be a product vague graph on $G^* = (V, E)$. G is said to be strong totally irregular product vague graph if every pair of vertices have distinct total edge degree.

Definition 3.7: Let $G = (V, A, B)$ be a product vague graph on $G^* = (V, E)$. G is said to be edge irregular product vague graph if there exists an edge which is adjacent with the edges having distinct edge degree.

Definition 3.8: Let $G = (V, A, B)$ be a product vague graph on $G^* = (V, E)$. G is said to be edge totally irregular product vague graph if there exists an edge which is adjacent with the edges having distinct total edge degree.

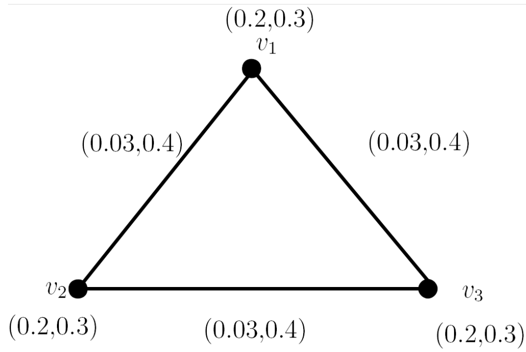
Definition 3.9: Let $G = (V, A, B)$ be a product vague graph on $G^* = (V, E)$. G is said to be highly irregular product vague graph if every vertex is adjacent to vertices with distinct edge degrees.

Proposition 3.1: Let $G = (V, A, B)$ be a product vague graph on $G^* = (V, E)$. If B is constant then G is edge regular if and only if G^* is edge regular.

Remark 3.1: In any product vague graph G, if degree of all the edges is zero, the graph G is both regular and totally regular.

Consider the product vague graph G (see Figure 3). In this graph, we see that $deg(v_1) = deg(v_2) = deg(v_3) = (0.06, 0.08)$. Hence it is (0.06, 0.08)-regular but $deg(v_1v_2) = deg(v_2v_3) = deg(v_3v_1) = (0, 0) = 0$. Hence the graph is edge regular with degree 0.

Figure 3 (0.06, 0.8)-regular and (0, 0)-dge regular product vague graph



Proposition 3.2: Let $G = (V, A, B)$ be a product vague graph from $G^* = (V, E)$ and B is a constant function. If G is strongly irregular product vague graph then G is edge irregular product vague graph.

Proof: Let $G = (V, A, B)$ be a product vague graph defined from $G^* = (V, E)$.

Let us assume that B is a constant function where $B(uv) = (t_B(uv), f_B(uv)) = (r_1, r_2)$. r_1 and r_2 are constants for all $uv \in E$.

Let us consider minimum of four vertices say u, v, w and x and the edges uv, vw and vx which are adjacent in G.

Supposing G is strongly irregular product vague graph:

- Every pair of vertices in G have distinct degrees.
- Degree of u, v, w and x are not equal.

Therefore we have,

$$\left(deg^t(u), deg^f(u)\right) \neq \left(deg^t(v), deg^f(v)\right) \neq \left(deg^t(w), deg^f(w)\right)$$

$$\Rightarrow deg^t(u) \neq deg^t(v) \text{ or}$$

$$deg^f(u) \neq deg^f(v) \text{ and}$$

$$deg^t(v) \neq deg^t(w) \text{ or}$$

$$deg^f(v) \neq deg^f(w)$$

$$\Rightarrow deg^t(u) + deg^t(v) \neq deg^t(v) + deg^t(w) \text{ or}$$

$$deg^f(u) + deg^f(v) \neq deg^f(v) + deg^N_{\mu_A}(w)$$

$$\Rightarrow deg^t(u) + deg^t(v) - 2r_1 \neq deg^t(v) + deg^t(w) - 2r_1 \text{ or}$$

$$deg^f(u) + deg^f(v) - 2r_2 \neq deg^f(v) + deg^f(w) - 2r_2$$

$$\Rightarrow deg^t(u) + deg^t(v) - 2t_B(uv) \neq deg^t(v) + deg^t(w) - 2t_B(vw) \text{ or}$$

$$deg^f(u) + deg^f(v) - 2f_B(uv) \neq deg^f(v) + deg^f(w) - 2f_B(vw)$$

$$\Rightarrow (d_t(uv), d_f(uv)) \neq (deg_t(vw), deg_f(vw))$$

$$\Rightarrow d(uv) \neq d(vw).$$

Similarly we can prove that $d(vw) \neq d(vx)$, i.e., an edge uv which is adjacent with the other edges vw and vx have distinct degrees.

Hence, G is an edge irregular product vague graph. □

Proposition 3.3: Let $G = (V, A, B)$ be a product vague graph of G^* and B is a constant function. If G is an edge irregular product vague graph, then G is an edge totally irregular product vague graph.

Proof: Let us assume that B is a constant function.

Let $B(uv) = (r_1, r_2)$ for all $uv \in E$, where r_1 and r_2 are constants.

Suppose that G is an edge irregular product vague graph. Then there exist an edge adjacent with the edges with distinct degrees.

Let uv be an edge which is adjacent with edges uw and ux which are incident at the vertex u and vy is the edge which is incident with the vertex v. Then $d(uw) \neq d(ux) \neq d(vy)$, where uw, ux and vy are adjacent with the edge uv in E.

Consider $d(uw) \neq d(ux) \neq d(vy)$

$$\Rightarrow (d_t(uw), d_f(uw)) \neq (d_t(ux), d_f(ux)) \neq (d_t(vy), d_f(vy))$$

$$\Rightarrow (d_t(uw), d_f(uw)) + (r_1, r_2) \neq (d_t(ux), d_f(ux)) + (r_1, r_2)$$
$$\neq (d_t(vy), d_f(vy)) + (r_1, r_2)$$

$$\Rightarrow d(uw) + B(uw) \neq d(ux) + B(ux) \neq d(vy) + B(vy)$$

$$\Rightarrow d[uw] \neq d[ux] \neq d[vy]$$

where uw, ux and vy are adjacent edges of uv in E.

Hence G is an edge totally irregular product vague graph. □

Proposition 3.4: Let $G = (V, A, B)$ be a product vague graph of G^* and B is a constant function. If G is an edge totally irregular product vague graph, then G is an edge irregular product vague graph.

Proof: The proof is same as proposition 3.3.

Remark 3.2: By the propositions 3.3 and 3.4, we have the result as follows. Let $G = (V, A, B)$ be a product vague graph of G^* and B is a constant function. Then G is an edge totally irregular product vague graph if and only if G is an edge irregular product vague graph.

Remark 3.3: Let $G = (V, A, B)$ be a product vague graph of G^*. If G is both an edge irregular product vague graph and an edge totally irregular product vague graph, then B need not be a constant function.

Proposition 3.5: Let $G = (V, A, B)$ be a product vague graph of G^* and B is a constant function. If G is highly irregular product vague graph, then G is an edge irregular product vague graph.

Proof: Let $G = (V, A, B)$ be a product vague graph of G^*.

Let us assume that B is a constant function. Let $B(uv) = (r_1, r_2)$ for all $uv \in E$, where r_1 and r_2 are constants. Let v be any vertex adjacent with u, w and x. Then uv, vw and vx are adjacent edges in G.

Supposing that G is highly irregular product vague graph.

- Every vertex adjacent to the vertices in G have distinct degrees.

$$\Rightarrow deg(u) \neq deg(v) \neq deg(x)$$

$$\Rightarrow \left(deg^t(u), \deg^f(u)\right) \neq \left(deg^t(v), deg^f(v)\right) \neq \left(deg^t(x), deg^f(x)\right)$$

$$\Rightarrow deg^t(u) \neq deg^t(v) \text{ (or)} deg^f(u) \neq deg^f(v)$$

In the similar way we can prove that $d(vw) \neq d(vx)$, i.e., an edge uv which is adjacent to the edges vw and vx have distinct degrees.

Hence G is an edge irregular product vague graph. □

4 Strongly edge irregular and strongly edge totally irregular product vague graph

In this section we define the strongly edge irregular and strongly edge totally irregular product vague graphs. Also we discuss some of the properties of it.

Definition 4.1: Let $G = (V, A, B)$ be a product vague graph on $G^* = (V, E)$. Then G is said to be strongly edge irregular product vague graph if every pair of edges have distinct degrees.

Definition 4.2: Let $G = (V, A, B)$ be a product vague graph on $G^* = (V, E)$. Then G is said to be strongly edge totally irregular product vague graph if every pair of edges have distinct total degrees.

Proposition 4.1: Let $G = (V, A, B)$ be a product vague graph on $G^* = (V, E)$. If G is strongly edge irregular product vague graph, then G is an edge irregular product vague graph.

Proof: Let $G = (V, A, B)$ be a product vague graph on G^*: (V, E).

Assume that G is strongly edge irregular product vague graph:

- Every pair of edges in G have distinct degrees.
- There exists an edge adjacent with the edges having distinct degrees.
- Hence G is an edge irregular product vague graph. □

Proposition 4.2: Let $G = (V, A, B)$ be a product vague graph on $G^* = (V, E)$. If G is strongly edge totally irregular product vague graph, then G is an edge totally irregular product vague graph.

Proof: Proof is same as proposition 4.1. □

Proposition 4.3: Let $G = (V, A, B)$ be a product vague graph on $G^* = (V, E)$ and B is a constant function. If G is a strongly edge irregular product vague graph, then G is a highly irregular product vague graph.

Proof: Let $G = (V, A, B)$ be an product vague graph on $G^* = (V, E)$.

Assume that B is a constant function.

Let $B(uv) = (r_1, r_2)$ for all $uv \in E$ and r_1, r_2 are constants.

Let v be any vertex adjacent with u, w and x.

Then uv, vw and vx are adjacent edges in G.

Supposing that G is a strongly edge irregular product vague graph, i.e., every pair of edges in G have distinct degrees

$$\Rightarrow d(uv) \neq d(vw) \neq d(vx)$$

$$\Rightarrow \left(d_t(uv), d_f(uv)\right) \neq \left(d_t(vw), d_f(vw)\right) \neq \left(d_t(vx), d_f(vx)\right)$$

$$\Rightarrow \left(deg^t(u) + deg^t(v) - 2t_B(uv), deg^f(u) + deg^f(v) - 2f_B(uv)\right)$$
$$\neq \left(deg^t(v) + deg^t(w) - 2t_B(vw), deg^f(v) + deg^f(w) - 2f_B(vw)\right)$$

$$\Rightarrow deg^t(u) + deg^t(v) - 2t_B(uv) \neq deg^t(v) + deg^t(w) - 2t_B(vw) \text{ (or)}$$
$$deg^f(u) + deg^f(v) - 2f_B(uv) \neq deg^f(v) + deg^f(w) - 2f_B(vw)$$

$$\Rightarrow deg^t(u) + deg^t(v) - 2r_1 \neq deg^t(v) + deg^t(w) - 2r_1 \text{ (or)}$$
$$deg^f(u) + deg^f(v) - 2r_2 \neq deg^f(v) + deg^f(w) - 2r_2$$

$$\Rightarrow deg^t(u) + deg^t(v) \neq deg^t(v) + deg^t(w) \text{ (or)}$$
$$deg^f(u) + deg^f(v) \neq deg^f(v) + deg^f(w)$$

$$\Rightarrow deg^t(u) \neq deg^t(w) \text{ (or) } deg^f(u) \neq deg^f(w)$$

$$\Rightarrow \left(deg^t(u), deg^f(u)\right) \neq \left(deg^t(w), deg^f(w)\right)$$

$$\Rightarrow deg(u) \neq deg(w)$$

Similarly by taking $(d_t(vw), d_f(vw)) \neq (d_t(vx), d_f(vx))$, we get $deg(w) \neq deg(x)$.

So, we have $deg(u) \neq deg(w) \neq deg(x)$. Thus every vertex is adjacent to vertices with distinct degrees. Hence G is highly irregular product vague graph. □

Proposition 4.4: Let $G = (V, A, B)$ be a product vague graph on $G^* = (V, E)$ and B is a constant function. If G is a strongly edge totally irregular product vague graph then G is a highly irregular product vague graph.

Proof: Proof is similar to proposition 4.3. □

5 Conclusions

The theory of graph is very much useful to solve lot of real-time applications and combinatorial problems. Many mathematical fields such as algebra, number theory, topology, operations research and optimisation techniques involve the ideas of graph theory. Many partial information leads to inconclusive results and it could be resolved using vague graphs concepts. Also the effectiveness of vague concepts rather than fuzziness is a great advantage as it measures both the lower and upper bounds of membership values in the interval [0, 1]. In this paper, we analysed more about irregularity specially edge irregularity of product vague graphs and prove some properties on it. In future we can implement the concepts of regularity and irregularity in the area of product vague hypergraphs.

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