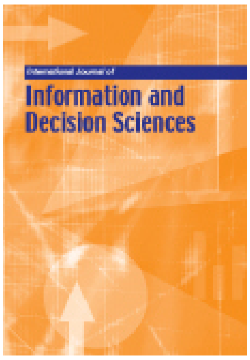

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EOQ model for time dependent demand with deterioration, inflation, shortages and trade credits

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Abstract: The inflation acts an important role for each area of life in the world. Inflation varies rapidly for high tech commodities with passing over time. This study develops an EOQ model with time sensitive demand rate for deteriorating products and shortages with inflation over a predetermined planning horizon. Mathematical formulations are prepared under two cases: 1) time for positive inventory (T_1) is greater than credit period M; 2) T_1 is less than or equal to credit period M, to gain optimal number of replenishment and cycle time. An algorithm is presented to find the most favourable cycle time so that total annual relevant profit is maximised. We then demonstrate the total profit is concave with respect to number of replenishments. Numerical examples are offered to display the model. Sensitivity investigation for variation of a number of key parameters is also discussed. Mathematica 7.0 software is used to calculate numerical results and optimality conditions.

Keywords: cash flow; inflation; non-increasing demand; credit period; shortages.

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1 Introduction

In the beginning stage EOQ models start by considering that demand rate was stable. This consideration was a grim limitation because in the real world demand of a product can be in fluctuating state. In the past decades, sufficient research papers have been published by a number of researchers for controlling inventory of deteriorating items. In actual practice, deterioration of several commodities such as medicine, volatile liquids, chemicals and some other items during storage period is non- negligible. Almost all items decrease their freshness over time, so that items cannot be used for as fresh. Therefore, loss due to decay cannot be neglected, while developing inventory models. Most of the researchers assume a consistent deterioration rate over time. Covert and Philip (2003) designed an EOQ model for Weibull distribution deteriorating items and without shortages. Mohammadi et al. (2015) developed optimal manufacture phase such that predictable total cost is reduced. Roy (2008) presented a model with time linked deterioration, selling price sensitive demand rate and carrying cost is time linked.

At present trade credit plays significant role in all type of high-tech industrial transactions. Kingsman (1983) and Daellenbach (1986) studied effects of trade credit in their EOQ models. Guchhait et al. (2014) proposed a model for decaying products in which demand rate is selling price linked under two echelon allowable delay periods. Goyal (1985) addressed an EOQ model under trade credits for particular items. Some of the related research work in this track has been adopted by Hou and Lin (2008), Teng et al. (2005, 2012), Wu (1998).

In classical inventory model, demand rate is stable, while in real life it is not evermore true. Demand for any kind of commodity is always in dynamic state. Donaldson (1977) explored an EOQ model with linear tendency in demand. Sarkar et al. (2013) described a model for non-spoilage time- sensitive demand with deterioration for predetermined manufacture rate. Other researchers such as Sarkar et al. (2014), Yang (2014), Teng et al. (2011), Ouyang et al. (2015), Goyal et al. (1992), etc.

The more open financial system, there is greater significance of exchange rate in policy design. It is normally anticipated that every additional features remains identical, if inflation in a nation rises, its products become luxurious in worldwide market to condensed demand for local legal tender. Import becomes cheaper and demand for foreign legal tender amplifies. Inflation rates differ among countries, causing global trade outlines and swaps over rates to regulate consequently. If country’s inflation rates are risen demand for its money turned down as exports turn down due to their high prices. Thus, in the study of EOQ models, effect of inflation cannot be ignored. Hou and Lin (2009) proposed a model by linearly time associated demand for deteriorating objects when supplier offers trade credits. Basu and Sinha (2007) described a model by way of time induced demand, fractional backlogging and time-connected deterioration. Buzacott (1975) analysed best possible model including inflation under dissimilar nature of pricing policies. Other associated articles can be found in Misra (1979), Jaggi et al. (2006), Sarkar and Moon (2011), Abidin and Applanaidu (2020), Mouatassim et al. (2020). Montgomery et al. (1973) first recognised in EOQ and EPQ models with incomplete backlogging and misplaced sales. A lot of researchers have measured shortages and partial backlogging in their EOQ and EPQ models such as Wee et al. (2014), Bhunia et al. (2014), Ouyang and Chang (2013), Jaggi et al. (2013), Ghiami et al. (2013), Taleizadeh and Nematollahi (2014), Tripathi (2021), Hamdi et al. (2018), Salari et al. (2017), Tripathi and Mishra (2021), etc.

Table 1 Brief literature review

Author's name	*Model*	*Deterioration*	*Delay in payments*	*inflation*	*Shortages*
Basu and Sinha (2007)	EOQ	√	√	√	√
Bhunia et al. (2014)	EOQ	√	√	x	√
Buzacott (1975)	EOQ	x	x	√	x
Covert and Philip (1972)	EOQ	√	x	x	x
Daellenbach (1986)	EOQ	x	√	x	x
Donaldson (1977)	EOQ	x	x	x	x
Ghiami et al. (2013)	EOQ	√	x	x	√
Goyal (1985)	EOQ	x	√	x	x
Goyal et al.(1992)	EOQ	x	x	x	√
Guchhait et al. (2014)	EOQ	√	√	√	x
Hou and Lin (2009)	EOQ	√	√	√	x
Jaggi et al. (206)	EOQ	√	x	√	√
Jaggi et al. (2013)	EOQ	√	√	x	√
Mohammadi et al. (2015)	EPQ	√	x	x	x
Ouyang and Chang (2013)	EPQ	x	√	x	√
Ouyang et al. (2015)	EPQ	x	√	x	x
Roy (2008)	EOQ	√	x	√	√
Sarkar and Moon (2011)	EOQ	√	x	x	√
Sarkar et al. (2013)	EOQ	√	x	x	x
Sarkar et al. (2014)	EMQ	x	x	√	√
Taleizadeh and Nematollahi (2014)	EOQ	√	√	x	x
Teng et al. (2005)	EOQ	√	√	x	x
Yang (2014)	EOQ	x	x	x	√
Wee et al. (2014)	EPQ	x	x	x	√
Wu (1998)	EOQ	√	√	x	x
Tripathi (2021)	EOQ	√	√	√	√

The main aim of this study is for finding highest profit over restricted planning horizon. The non- increasing demand in EOQ models is realistic to demand because it represents the availability of remaining inventory after time t. The rest of demand decreases with time.

The rest part of the study is arranged as follows. In next section, we explain notations and assumptions used in the whole manuscript. Section 3, gives specification of mathematical model while, section 4, illustrates detail solution measure which gives with proof of concavity of total relevant profit. Section 5, provides flowchart that describes pictorial representation of solution procedure. In section 6, numerical examples have been demonstrated followed by sensitivity study of the model. Finally, paper ends at conclusion and future research.

2 Notations and assumption

2.1 Notations

A	initial cost of order (\$/ order)
h and s	carrying and shortage cost, \$ per unit time
c	unit cost of item, \$per unit
θ	deterioration rate
q_1 and q_2	maximum inventory and shortage quantity level
Q	order quantity, ($Q = q_1 + q_2$)
H	span of planning horizon
T	cycle time
n	number of replenishment $n = H/T$
T_1	period for which no shortages occurs.T_1 is a fraction of cycle time T where $T_1 = kT(0 < k < 1)$, where is fraction of T
$D(t) = a–bt$	demand rate, $a > 0$, $b > 0$
p	selling price/unit time
r	net reduction rate of inflation
I_c	interest charged/\$ in stock/unit time
I_d	interest earned/\$/unit time $I_c > I_d$
M	permissible delay in payments
$q(t)$	inventory level at moment t
SR	total sales revenue
OC, HC and MC	total ordering, holding and material cost
IP_i, $i = 1, 2$	total interest payable

IE_i	total interest earned
$Z(n)$	total annual profit during time horizon

$$Z(n)=\begin{cases} Z_1(n) & , \text{ if } \quad T_1=\dfrac{kH}{n}\geq M \\ Z_2(n) & , \text{ if } \quad T_1=\dfrac{kH}{n}< M \end{cases}$$

$Z_1^*(n)$ and $Z_2^*(n)$	optimal $Z(n)$ for case I and II
n_1^* and n_2^*	optimal n in finite planning horizon for case I and II.

2.2 Assumption

1. demand rate is diminishing function of time
2. shortages are permitted
3. lead time in insignificant
4. planning horizon H in infinite
5. 'θ' is invariable and $0 \leq \theta < 1$
6. $D(t)$ for positive and negative inventory both are same and time dependent
7. instantaneous cash flow is considered during product transaction
8. selling price is larger than purchase cost.

3 Mathematical model

Let us consider the planning horizon H is separated into n identical sub-divisions. The demand rate $D(t)$ in linearly time- sensitive and non-increasing function of time. The period for positive inventory T_1 is a portion of cycle time T. The level of inventory exhausted due to time-linked demand and partially due to deterioration during $[0, T_1]$. Shortages occur during $[T_1, T]$ and are accumulated until T_1, before they are back ordered. So, variation of $q(t)$ at some moment t is represented as

$$\frac{dq(t)}{dt}+\theta q(t)=-D(t)\,, \qquad 0 \leq t \leq T_1 = kH/n \tag{1}$$

$$\frac{dq(t)}{dt}=-D(t), \qquad T_1 \leq t \leq T \tag{2}$$

The solution of (1) and (2) with condition $q(T_1) = 0$ are

$$q(t)=\frac{1}{\theta}\left\{\left(a+\frac{b}{\theta}\right)\left(e^{\theta(T_1-t)}-1\right)-b\left(T_1e^{\theta(T_1-t)}-t\right)\right\} \tag{3}$$

$$q(t) = -\left\{a\left(t - T_1\right) - \frac{b}{2}\left(t^2 - T_1^2\right)\right\} \tag{4}$$

The maximum inventory and shortage level in first replenishment cycle are respectively

$$q_1 = \frac{1}{\theta}\left\{\left(a + \frac{b}{\theta}\right)\left(e^{\theta T_1} - 1\right) - bT_1 e^{\theta T_1}\right\} \tag{5}$$

$$q_2 = (1-k)\left(H/n\right)\left\{a - \frac{b}{2}(1+k)\left(H/n\right)\right\} \tag{6}$$

Total *SR*, *OC*, *HC*, *SC* and *MC* during *H* are:

$$SR = \sum_{i=0}^{n-1}\left(p\int_0^T D(t)e^{-rt}dt\right)e^{-irT} = \frac{p}{r}\left\{\left(a - \frac{b}{r}\right)\left(1 - e^{-rT}\right) + bTe^{-rT}\right\}\left(\frac{1 - e^{-rH}}{1 - e^{-rH/n}}\right) \tag{7}$$

$$OC = \sum_{i=0}^{n-1} Ae^{-irT} = A\left(\frac{1 - e^{-rH}}{1 - e^{-rH/n}}\right) \tag{8}$$

$$\begin{aligned} HC &= \sum_{i=0}^{n-1}\left(h\int_0^{T_1} q(t)e^{-rt}dt\right)e^{-irT} \\ &= \frac{h}{\theta}\left[\left(a + \frac{b}{\theta}\right)\left(\frac{e^{\theta T_1} - e^{-rT_1}}{\theta + r} + \frac{e^{-rT_1} - 1}{r}\right)\right. \\ &\quad \left. - b\left\{T_1\left(\frac{e^{\theta T_1} - e^{-rT_1}}{\theta + r}\right) + \frac{1}{r}\left(T_1 e^{-rT_1} + \frac{e^{-rT_1} - 1}{r}\right)\right\}\right]\left(\frac{1 - e^{-rH}}{1 - e^{-rH/n}}\right) \end{aligned} \tag{9}$$

$$\begin{aligned} SC &= \sum_{j=0}^{n-1}\left(s\int_{T_1}^{T} -q(t)e^{-rt}dt\right)e^{-jrT} \\ &= \frac{s}{r}\left[-a\left\{\left(T - T_1\right)e^{-rT} + \frac{e^{-rT} - e^{-rT_1}}{r}\right\}\right. \\ &\quad \left. + \frac{b}{2}\left\{\left(T^2 - T_1^2\right)e^{-rT} + \frac{2}{r}\left(Te^{-rT} - T_1 e^{-rT_1} + \frac{e^{-rT} - e^{-rT_1}}{r}\right)\right\}\right]\left(\frac{1 - e^{-rH}}{1 - e^{-rH/n}}\right) \end{aligned} \tag{10}$$

$$\begin{aligned} MC &= c\sum_{j=0}^{n-1}\left[q_1 + e^{-rT}\left\{a\left(T - T_1\right) - \frac{b}{2}\left(T^2 - T_1^2\right)\right\}\right]e^{-jrT} \\ &= c\left(q_1 + e^{-rT}q_2\right)\left(\frac{1 - e^{-rH}}{1 - e^{-rH/n}}\right) \end{aligned} \tag{11}$$

Following two cases may arise due to *T* and *M*

3.1 Case I: $M \le T1 \le T$

In this case, M is shorter than T, interest is payable by vendor, total interest payable IP_1 during H is

$$IP_1 = cI_c \sum_{i-0}^{n-1} \left(\int_M^{T_1} q(t)e^{-rt}dt + \int_{T_1}^{T} q(t)e^{-rt}dt \right) e^{-irT} \quad (12)$$

Since interest is paid during $[T_1, T]$ is zero. Consequently, interest paid during planning horizon H is:

$$IP_1 = cI_c \sum_{i-0}^{n-1} \left(\int_M^{T_1} q(t)e^{-rt}dt \right) e^{-irT}$$

$$= \frac{\mathrm{cI_c}}{\theta} \left[\left(a + \frac{b}{\theta} \right) \left\{ \frac{e^{\theta T_1 - (\theta + r)M} - e^{-rT_1}}{\theta + r} + \frac{e^{-rT_1} - e^{-rM}}{r} \right\} \right. \quad (13)$$

$$\left. -b \left\{ T_1 \left(\frac{e^{\theta T_1 - (\theta + r)M} - e^{-rT_1}}{\theta + r} \right) + \frac{1}{r} \left(T_1 e^{-rT_1} - Me^{-rM} + \frac{e - rT_1}{r} \right) \right\} \right] \left(\frac{1 - e^{-rH}}{1 - e - rH/n} \right)$$

The total interest earned during H is

$$IE_1 = pI_d \sum_{i=0}^{n-1} \left(\int_0^{T_1} (a - bt)te^{-rt}dt \right) e^{-irT} = \frac{pI_d}{r}$$

$$\left[-T_1 (a - bT_1) e^{-rT_1} + - \frac{1}{r} \left\{ \left(a - \frac{2b}{r} \right) (1 - e^{-rT_1}) + 2bT_1 e^{-rT_1} \right\} \right] \left(\frac{1 - e^{-rH}}{1 - e^{-rH/n}} \right) \quad (14)$$

The total annual profit during H is

$$Z_1(n) = SR - (OC + HC + SC + MC + IP_1 - IE_1) \quad (15)$$

3.2 Case II: $M > T_1$

In this case, length of allowable delay M is longer than T. Interest earned in between $[0, T_1]$ plus interest earned from invested cash during $[T_1, M]$ after inventory is ended at time T_1. The total interest earned IE_2 is:

$$IE_2 = pI_d \sum_{i-0}^{n-1} \left\{ \int_0^{T_1} (a - bt)te^{-rt}dt + (M - T_1)e^{-rT_1} \int_0^{T_1} (a - bt)dt \right\} e^{-irT}$$

$$= \frac{pI_d}{r} \left[-T_1 (a - bT_1) e^{-rT_1} + \frac{1}{r} \left\{ \left(a - \frac{2b}{r} \right) (1 - e^{-rT_1}) + 2bT_1 e^{-rT_1} \right\} + \right. \quad (16)$$

$$\left. (M - T_1) T_1 e^{-rT_1} \left(a - \frac{bT_1}{2} \right) \right] \left(\frac{1 - e^{-rH}}{1 - e^{-rH/n}} \right)$$

The total annual profit during H is:

$$Z_2(n) = SR - \left(OC + HC + SC + MC - IE_2\right) \tag{17}$$

$Z_1(n) = Z_2(n)$ at $M = T_1$, then we have

$$Z(n) = \begin{cases} Z_1(n) & \text{if } T_1 \geq M \\ Z_2(n) & \text{if } T_1 \leq M \end{cases}$$

Based on above conversation, we develop solution algorithm for finding n^*, T^*, Q^* and $Z^*(n^*)$ values:

4 Solution algorithm

Step 1 Choose integer n such that it is greater than or equal to 1

Step 2 Find $Z_1(n)$ from (15) for unlike value of n, if $T_1 \geq M$

Step 3 Find $Z_2(n)$ from (17) for dissimilar value of n, if $T_1 \leq M$

Step 4 Repeat steps 2 and 3 for unlike values of n if $T_1 \geq M$ maximum $Z_1(n)$ is obtain form equation (15) and let $n_1^* = n.$ For different n with $T_1 \leq M$ until maximum $Z_2(n)$ is obtain from equation (17) and let $n_2^* = n.$

Step 5 With $n_1^*, n_2^*, Z_1^*(n)$ and $Z_2^*(n)$ values constitute optimal solution which follows the condition given below:

$$Z_1\left(n_1^* + 1\right) - Z_1\left(n_1^*\right) < 0 \tag{18}$$

$$Z_2\left(n_2^* + 1\right) - Z_2\left(n_2^*\right) < 0 \tag{19}$$

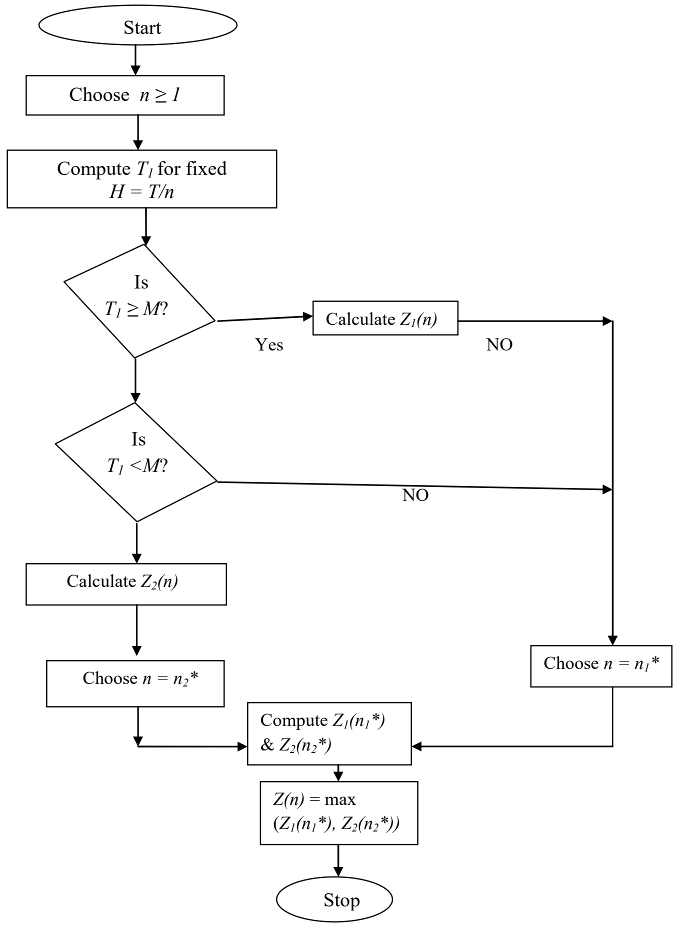
Step 6 Choose n* such that

$$Z\left(n^*\right) = \max \begin{cases} Z_1\left(n_1^*\right) & \textit{for } kH / n_1^* \geq M \\ Z_2\left(n_2^*\right) & \textit{for } kH / n_2^* \leq M \end{cases}$$

5 Flowchart

The following flow chart shows the validity of the proposed model. Number n s taken positive integer and greater than one.

Figure 1 Flow chart of the proposed model



Based on above algorithm, we present following numerical examples for finding the T^*, n^*, Q^* and $Z(n^*)$.

6 Numerical examples

- Example 1: Let us consider inventory parameters: a = 1000 units/yr, A = 80 \$/order, h = 2.5 \$/unit/yr, b = 15/unit, s = \$ 10/unit/yr, p = 50/unit/yr, I_c = 0.18/\$/year, I_p = 0.16/\$/yr, θ = 0.15, r = 0.12/\$/yr, and H = 5 years, M = 45 days = 45/360 yrs (consider year is 360 days) and c = \$ 30/unit/yr.

Using solution algorithm, numerical results are provided in the following Table 1. We find case II is the optimal solution. From Table 2, maximum total annual profit is obtained when $n(=n_2^*) = 75$, corresponding T^* = 0.06667 yrs, $T_1^* = 0.03333$ yrs, Q^* = 66.7166 units and $Z_2(n_2^*) = \$76{,}178.7$. The computational results are given in Table 2.

Table 2 The computational results (optimal solution)

Case	*n*	*T*	T_1	*Q*	*Z(n)*
I	10	0.50000	0.25000	502,975	67,212.3
	11	0.45454	0.22727	456,857	67,360.0
	12	0.41667	0.20833	418,610	67,473.0
	13	0.38462	0.19231	386,272	67,561.2
	14	0.35714	0.17857	358,571	67,627.9
	15	0.33333	0.16667	334,578	67,677.9
	16	0.31250	0.15625	313,594	67,714.3
	17	0.29412	0.14706	295,087	67,739.5
	18	0.27778	0.13889	295,087	67,755.4
	19	0.26316	0.13158	263,934	67,763.3
	20	0.25000	0.12500	250,701	67,764.4
II	21	0.23810	0.11905	238,731	68,453.7
	22	0.22727	0.11364	227,852	69,075.5
	23	0.21739	0.10870	217,922	69,638.5
	24	0.20833	0.10417	208,820	70,149.9
	25	0.20000	0.10000	200,449	70,616.0
	30	0.16667	0.08333	166,979	72,425.1
	35	0.14286	0.07143	143,086	73,830.8
	40	0.12500	0.06250	125,176	74,477.9
	45	0.11111	0.05556	111,250	75,069.2
	50	0.10000	0.05000	100,112	75,486.2
	55	0.09091	0.04545	91.0020	75,776.9
	60	0.08333	0.04167	83.4114	75,972.4
	65	0.07692	0.03846	76.9896	76,094.8
	70	0.07143	0.03571	71.4859	76,159.7
	71	0.07042	0.03521	70.4783	76,166.9
	72	0.06944	0.03472	69.4987	76,172.3
	73	0.06849	0.03425	68.5459	76,176.0
	74	0.06757	0.03378	67.6189	76,178.1
	75*	0.06667*	0.03333*	66.7166*	76,178.7*
	76	0.06579	0.03290	65.8381	76,177.8
	77	0.06494	0.03247	64.9825	76,175.4
	78	0.06410	0.03205	64.1488	76,171.7
	79	0.06329	0.03165	63.3362	76,166.6
	80	0.06250	0.03125	62.5439	76,160.3
	85	0.05882	0.02941	58.8624	76,111.1
	90	0.05556	0.02278	55.5903	76,036.4

Note: *Optimal solution.

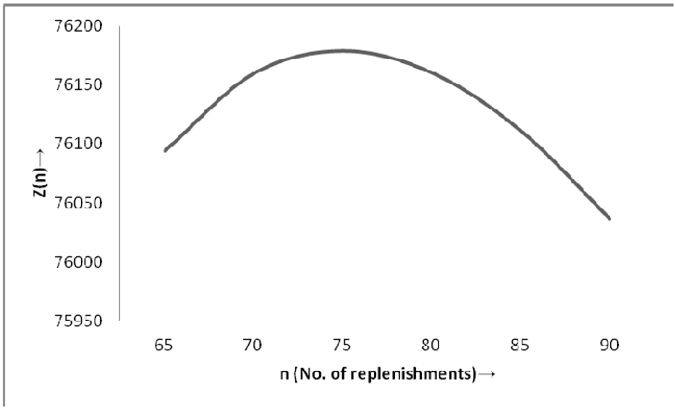
Table 3 The computation result (optimal solution)

Case	*n*	*T*	*T1*	*Q*	*Z(n)*
I	10	0.50000	0.25000	502.975	14,378.1
	11	0.45454	0.22727	456.857	14,556.7
	12	0.41667	0.20833	418.610	14,696.1
	13	0.38462	0.19231	386.272	14,805.2
	14	0.35714	0.17857	358.571	14,890.0
	15	0.33333	0.16667	334.578	14,956.4
	16	0.31250	0.15625	313.594	15,006.9
	17	0.29412	0.14706	295.087	15,044.4
	18	0.27778	0.13889	295.087	15,071.2
	19	0.26316	0.13158	263.934	15,088.9
	20	0.25000	0.12500	250.701	15,098.9
II	21	0.23810	0.11905	238.731	15,587.9
	22	0.22727	0.11364	227.852	16,027.7
	23	0.21739	0.10870	217.922	16,424.4
	24	0.20833	0.10417	208.820	16,783.5
	25	0.20000	0.10000	200.449	17,109.4
	30	0.16667	0.08333	166.979	18,357.6
	35	0.14286	0.07143	143.086	19,169.7
	40	0.12500	0.06250	125.176	19,709.0
	45	0.11111	0.05556	111.250	20,066.4
	50	0.10000	0.05000	100.112	20,296.5
	55	0.09091	0.04545	91.0020	20,433.9
	60	0.08333	0.04167	83.4114	20,501.8
	61	0.08197	0.04098	82.0427	20,508.5
	62	0.08064	0.04032	80.7183	20,513.2
	63	0.07936	0.03968	79.4359	20,516.0
	64*	0.07813	0.03906*	78.1936*	20,517.8*
	65	0.07692	0.03846	76.9896	20,516.7
	66	0.07576	0.03788	75.8221	20,513.7
	67	0.07463	0.03731	74.6895	20,509.6
	68	0.07353	0.03676	73.5902	20,504.0
	69	0.07246	0.03623	72.5228	20,497.0
	70	0.07143	0.03571	71.4859	20,488.5
	75	0.06667	0.03333	66.7166	20,427.3

Note: *Optimal solution.

The figure for previous discussion is given in Figure 2.

Figure 2 Number of replenishments vs. total annual profit ($Z(n)$) for example 1

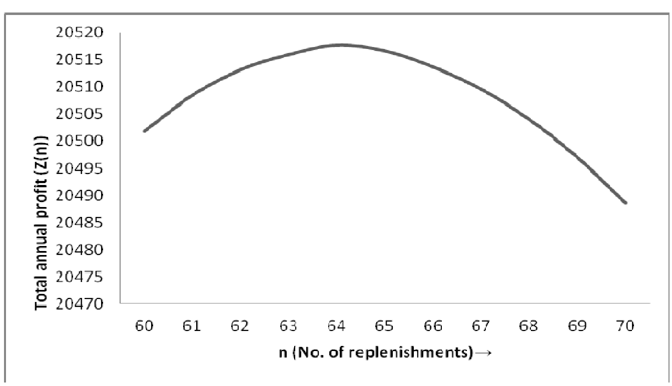


- Example 2: Let us take the inventory values a = 1,000 unit/yr, A = \$ 80/order, h = 2.5 \$/unit/yr, b = 15 units/yr, shortage cost s = 10 \$/ unit/yr, p = 35 \$/unit/yr, I_c = 0.18 \$/yr, I_p = 0.16 \$/yr θ = 0.15, r = 0.12 \$/year, and H = 5 yrs, M = 45 days = 45/360 yrs and c = \$ 30/unit/yr. Using the solution algorithm, numerical results shown in Table 3.

We obtain case II is optimal solution. From Table 3, maximum total annual profit is obtained when $n(=n_2^*)=64$ and corresponding optimal cycle time T^* = 0.078125 yr, time for positive inventory $T_1^* = 0.039062$ yrs, Q^* = 78.1936 units and $Z_2(n_2^*) = \$20,517.8$. The computational result is as follows.

The figure for above discussion is given in Figure 3.

Figure 3 Number of replenishments vs. total annual profit ($Z(n)$) for example 2



From Tables 2 and 3, we observe that total annual profit $Z(n)$ is higher if selling price (p) is higher. Also optimal number of replenishments (n) is higher for higher selling price (p).

Table 4 Change of $Z(n)$ with variation of M (in days), p, A, h, c, r, k, θ, and a

Case I					*Case II*				
M→	*50*	*55*	*60*	*65*	*M*→	*50*	*55*	*60*	*65*
n↓					*n*↓				
5	65,299.0	65,313.0	65,328.0	65,341.8	75	77,798.7	79,418.6	81,038.6	82,658.6
6	65,969.0	65,983.0	65,995.9	66,008.0	76	77,797.7	79,417.7	81,037.7	82,657.7
7	66,437.8	66,450.0	66,461.4	66,472.0	77	77,795.4	79,415.4	81,035.4	82,655.4
8	66,777.5	66,788.4	66,798.5	66,807.6	78	77,791.7	79,411.7	81,031.7	82,651.6
9	67,030.4	67,040.1	67,048.8	67,056.5	79	77,786.6	79,406.6	81,026.6	82,646.6
p→	55	60	65	70	p→	55	60	65	70
n↓					n↓				
5	83,010.0	100,738.0	118,465	136,193.0	78	94,769.9	113,368.0	131,966	150,564.0
6	83,643.5	101,332.0	119,020	136,708.0	79	94,769.4	113,372.0	131,975	150,578.0
7	84,085.3	101,746.0	119,407	137,067.0	80	94,767.5	113,375.0	131,982	150,589.0
8	84,405.6	102,046.0	119,685	137,325.0	81	94,764.3	113,376.0	131,987	150,599.0
9	84,643.7	102,268.0	119,892	137,516.0	82	94,759.8	113,376.0	131,991	150,607.0
A→	82	84	86	88	A→	82	84	86	88
n↓					n↓				
5	65,276.1	65,268.7	65,261.3	65,253.8	75	76,073.3	75,967.8	75,862.4	75,757.0
6	65,946.6	65,937.7	65,928.9	65,920.0	76	76,070.9	75,964.1	75,857.3	75,750.5
7	66,414.6	66,404.3	66,394.1	66,383.8	77	76,067.2	75,959.0	75,850.7	75,742.5
8	66,754.0	66,742.4	66,730.7	66,719.1	78	76,062.1	75,952.4	75,842.8	75,733.2
	67,006.6	66,993.6	66,980.6	66,967.5	79	76,055.6	75,944.6	75,833.6	75,722.5
h→	*3.0*	*3.5*	*4.0*	*4.5*	*h*→	*3.0*	*3.5*	*4.0*	*4.5*
n↓					*n*↓				
5	65,052.2	64,820.7	64,589.3	64,357.9	75	76,164.1	76,149.4	76,134.8	76,120.1
6	65,764.4	65,573.4	65,382.4	65,191.4	76	76,163.3	76,148.9	76,134.4	76,120.0
7	66,262.2	66,099.6	65,937.0	65,774.4	77	76,161.2	76,146.9	76,132.6	76,118.4
8	66,624.1	66,482.5	66,341.0	66,199.4	78	76,157.6	76,143.5	76,129.5	76,115.4
9	66,894.3	66,769.0	66,643.7	66,518.3	79	76,152.7	76,138.8	76,124.9	76,111.1
c→	*32*	*34*	*36*	*38*	*c*→	*32*	*34*	*36*	*38*
n↓					*n*↓				
5	58,155.7	51,027.7	43,899.8	36,771.9	75	69,173.3	62,167.9	55,162.5	48,157.1
6	58,855.5	51,755.6	44,655.7	37,555.8	76	69,172.4	62,167.1	55,161.8	48,156.5
7	59,343.5	52,262.2	45,180.9	38,099.6	77	69,170.1	62,164.9	55,159.6	48,154.4
8	59,697.5	52,629.3	45,561.2	38,493.9	78	69,166.5	62,161.3	55,156.1	48,150.9
9	59,961.3	52,902.8	45,844.4	38,786.0	79	69,161.5	62,156.4	55,151.3	48,146.1

Table 4 Change of $Z(n)$ with variation of M (in days), p, A, h, c, r, k, θ, and a (continued)

$r\rightarrow$	*0.13*	*0.15*	*0017*	*0.20*	$r\rightarrow$	*0.13*	*0.15*	*0017*	*0.20*
$n\downarrow$					$n\downarrow$				
5	62,939.9	58,239.8	53,526.4	46,441.1	75	72,663.5	66,103.5	59,986.1	51,349.8
6	63,591.0	58,853.7	54,107.4	46,978.2	76	72,658.9	66,093.5	59,973.0	51,334.5
7	64,045.6	59,281.2	54,510.4	47,347.5	77	72,652.9	66,082.5	59,959.0	51,318.6
8	64,375.6	59,591.1	54,801.6	47,612.7	78	72,645.7	66,070.5	59,944.2	51,302.1
9	64,621.5	59,821.5	55,018.0	47,809.0	79	72,637.3	66,057.6	59,928.6	51,284.9
$k\rightarrow$	*0.52*	*0.54*	*0.56*	*0.58*	$h\rightarrow$	*0.52*	*0.54*	*0.56*	*0.58*
$n\downarrow$					$n\downarrow$				
5	65,343.6	65,387.4	65,414.9	65,426.2	75	76,451.0	76,710.0	76,955.6	77,187.8
6	66,013.1	66,057.7	66,089.1	66,107.2	76	76,454.2	76,717.4	76,967.4	77,204.2
7	66,479.2	66,522.6	66,554.9	66,576.1	77	76,455.8	76,723.2	76,977.5	77,218.8
8	66,816.7	66,858.3	66,890.3	66,912.7	78	76,455.9	76,727.3	76,985.9	77,231.6
9	67,067.6	67,107.2	67,138.4	67,161.1	79	76,454.7	76,730.0	96,992.7	77,242.6
$\theta\rightarrow$	*0.16*	*0.17*	*0.18*	*0.19*	$\theta\rightarrow$	*0.16*	*0.17*	*0.18*	*0.19*
$n\downarrow$					$n\downarrow$				
5	65,131.3	64,979.0	64,826.7	64,674.4	75	76,169.9	76,161.1	76,152.3	76,143.6
6	65,831.3	65,706.9	65,582.6	65,458.3	76	76,169.1	76,160.4	76,151.8	76,143.1
7	66,320.1	66,215.5	66,110.8	66,006.1	77	76,166.9	76,158.3	76,149.8	76,141.2
8	66,675.4	66,585.3	66,495.1	66,404.9	78	76,163.2	76,154.8	76,146.4	76,137.9
9	66,940.6	66,940.6	66,782.4	66,703.3	79	76,158.3	76,150.0	76,141.6	76,133.3
$a\rightarrow$	*1,050*	*1,100*	*1,150*	*1,200*	$a\rightarrow$	*1,050*	*1,100*	*1,150*	*1,200*
$n\downarrow$					$n\downarrow$				
5	68,588.1	71,892.7	78,501.7	81,806.2	75	80,200.3	84,222.0	88,243.0	92,265.3
6	69,292.0	72,628.5	79,301.7	82,638.3	76	80,202.1	84,226.5	86,250.9	92,275.3
7	69,784.6	73,144.4	79,864.0	83,223.8	77	80,202.4	84,229.5	88,250.5	92,283.6
8	70,143.0	73,520.4	80,275.2	83,652.6	78	80,201.3	84,230.9	88,260.6	92,290.2
9	70,410.8	73,802.0	80,584.3	83,975.4	79	80,198.8	84,230.9	88,263.1	92,295.2

7 Sensitivity analysis

The variation may occur due to risks in any judgment. We now discuss effects of variation in the key parameters M,), p, A, h, c, r, k, θ and a on number of replenishments and present value of total annual profit. Using numerical data mentioned in Ex. 1, keeping other parameters same, sensitivity examination of different parameters has been provided. Results are concluded in Table 4.

Following observation can be made from Table 3.

1. When credit period (M), selling price (p), fraction of cycle time (k) and initial demand (a) increase, total annual profit $Z(n)$ will also increases, i.e., alter in M, p, k and a will lead positive alteration in $Z(n)$.
2. When A, h, r and θ will augment, $Z(n)$ will decrease, i.e., variation in A, h, r and θ will cause negative change in $Z(n)$.
3. We get same result as numerical examples 1 and 2 for increase in the number of replenishments in both cases.

8 Conclusions and future research directions

At present most of the countries facing problem on high inflation rate. So, it is essential to consider the presence of inflation in formulating inventory replenishment policy. In this study, an EOQ model is designed with linear time induced demand under shortages. The effects of deterioration, net discount rate of inflation and trade credit period have been also considered. The solution algorithm is presented for finding optimal number of replenishments, cycle time, time for positive inventory and order quantity to maximise total annual profit. Numerical examples are offered to exhibit results. This model is exceptionally constructive in retail industry like domestic goods, cloths, and other similar products. From administrative point of view following scrutiny can be made:

- Raise of credit period causes raise in $Z(n)$.
- Raise of selling price results raise in $Z(n)$.
- Increase of fraction of cycle time causes increase in $Z(n)$.
- Increase of initial demand causes increase in $Z(n)$.
- Enhance of cost of order, carrying cost, unit cost of item, net cut rate of inflation and deterioration rate will lead, shrink in total profit.

The model discussed in this study can be extended for quadratic time- dependent demand as well as exponential demand. We could also generalise the model for allowing time dependent deterioration with variable planning horizon.

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