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A review on estimation of vehicle tyre-road friction

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Abstract: The tyre-road friction coefficient (TRFC) is not only related to pavement conditions, but also affected by factors such as tyre material, tyre pressure and ambient temperature; in addition, there are problems such as sensor measurement noise, signal transmission hysteresis, parameter uncertainty or time denaturation in the actual vehicle system. These problems make the real-time robust estimation of the friction coefficient and its stability analysis more complicated, Therefore, the identification of TRFC has always been a key topic and difficult issue in research. This paper provides a comprehensive technical review of the currently widely used TRFC estimation method. First, various filters and observers and their improved versions to solve different problems are introduced. Then the model-based estimation algorithm is comprehensively expounded. The paper summarises the research results of sensor-based and neural network-based methods, analyses the new method brought about by the structural characteristics of distributed drive electric vehicles to estimate the friction coefficient, and looks forward to the future development direction.

Keywords: vehicle state; tyre-road friction; Kalman filter; particle filter; Luenberger observer; nonlinear observer; tyre model; distributed drive; intelligent tyres; neural network.

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1 Introduction

Autonomous driving technology is a rapidly developing technology, and its research and development includes computer vision, learning, perception, planning and other fields. Thanks to its outstanding advantages such as convenience, safety, improved traffic conditions and consumer-oriented starting point, many technology companies and car manufacturers are experimenting with autonomous driving cars (Hussain and Zeadally, 2019). However, no matter what level of automation the vehicle's autonomous driving has reached, autonomous driving systems require the interaction of tyre road information, being able to process this information in a timely and accurate manner has a great impact on vehicle dynamics (Lin et al., 2014), such as vehicle speed, vehicle sideslip angle, tyre-road friction coefficient (TRFC), etc.

The TRFC is determined by road surface and tyre, The dynamic performance of an automobile is restricted not only by the driving force, but also by the adhesion conditions of road surface and ground, the friction rate of the driving wheel cannot be greater than TRFC, otherwise the driving wheel will slip. According to road adhesion conditions and accurate vehicle speed information, it can assist the vehicle's active safety system, such as electronic stability program (ESP), traction control system (TCS), etc., to adjust the control strategy to enhance driving stability (Zhao et al., 2009). Distributed drive electric vehicles are driven by electric motors integrated into the wheel rims or hubs. Compared to conventional vehicles, distributed drive electric vehicles have the following characteristics (Chen et al., 2013):

1 the torque of the motor can be directly observed, and the observation accuracy is high, which makes it easy to accurately estimate the speed

2 accurate estimation of four-wheel longitudinal force in real-time, which helps to improve the accuracy of the estimation of the peak friction coefficient of the road and the range of applicable working condition.

In harsh external environments such as slippery roads, high-speed driving, or rapid turns, or under special or even dangerous driving conditions, it is extremely important to correct the trajectory of the vehicle and prevent the wheels from being locked in dangerous situations; meanwhile, with the development of autonomous vehicles, in order to ensure the safety of vehicles, combining with the required functions to create vehicles that drive themselves without driver intervention, the Driver Assistance System was born, commonly referred to as advanced driver assistance system (ADAS) (Acosta et al., 2017). One of the ways to develop these ADAS in the future is to obtain basic information that affects the driving of the vehicle, TRFC is one of the most important parameters (Martinez and De Wit, 2007). The TRFC mainly depends on three factors:

1 tyre (model, size, pressure, wear, etc.)

2 ground (type of cover, road condition, etc.)

3 there is a third factor (snow, water, dust, etc.) at the interface between the tyre and the road.

TRFC is an important input parameter for studying vehicle dynamics control (Fu et al., 2018), real-time and accurate acquisition of TRFC helps the development of active safety control systems for vehicle. In special weather conditions, road surface conditions and potentially dangerous operations such as lane changes and overtaking, etc., it can assist drivers to reduce the occurrence of road traffic accidents and improve the stability of car driving. Many scholars have studied the identification of roads through different methods, according to the principle analysis of TRFC estimation algorithm, as shown in Figure 1, the method of obtaining the friction coefficient of vehicle's current driving pavement can be divided into two categories (Wang et al., 2014; Ding et al., 2016):

1 *Cause-Based method*. This method mainly analyses the relationship between TRFC and other related parameters, and establishes a predictive model based on the relationship between the influencing factors and TRFC, estimates TRFC based on the influencing factor values such as vehicle parameters, tyre parameters, pavement lubrication parameters and pavement parameters. Therefore, the premise of obtaining an accurate TRFC is that a large number of external factors need to be determined (Bachmann, 1998). Among them, the pavement lubrication parameters and pavement type parameters that have a key impact on TRFC require special sensors to obtain. The cause-based pavement identification method has the following three problems:

    1 needing to use additional sensors (Wang et al., 2004; Yin et al., 2007)

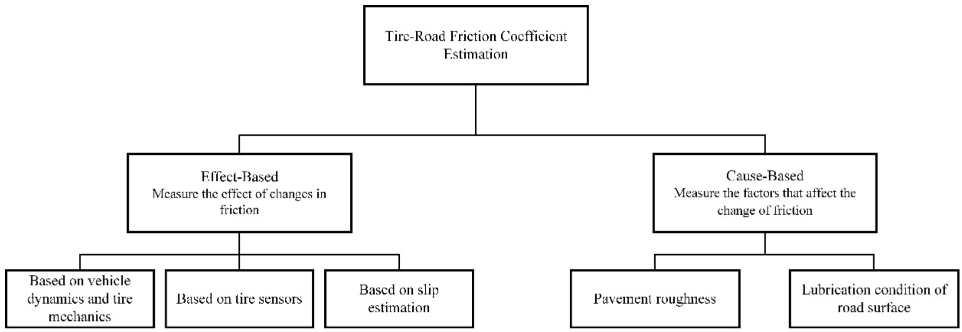
    2 requiring a lot of complex data processing processes

    3 the measurement of the sensor is inaccurate under certain special operating conditions.

2 *Effect-based method*. This method obtains measurable parameters that can represent the driving state of the car by measuring the results caused by different TRFC during the driving of the vehicle, TRFC that cannot be directly measured is obtained by estimation algorithms such as Kalman filtering algorithm, least squares algorithm, recursion algorithm, etc. (Hou, 2020).

Based on the research progress of TRFC estimation in recent years, this paper introduces the technical overview of the techniques and fusion methods commonly used in parameter estimation and briefly explains them separately, analyses the estimation method of TRFC and the research on TRFC estimation of distributed drive vehicles is summarised. Organising and evaluating the existing various friction coefficient estimation methods, pointing out the advantages and disadvantages of various estimation methods, and prospecting for its development.

Figure 1 Estimation method of TRFC



2 Filter technology

2.1 Kalman filter

2.1.1 Kalman filter

As an optimal state estimation algorithm, Kalman filter (KF) uses observations combined with the system's model to estimate the state of the system, and Kalman gain is used to correct the state prediction value; its filtering algorithm based on Bayesian principle collects observation data on the input and output of the system, and then optimally estimates the state of the system (Li et al., 2018). KF is an estimation algorithm for a recursive process, that is, as long as the estimated value of the state at the previous time and the observed value of the state at the current time are known, the estimated value of the current state can be calculated. The description of the state of the system can be represented by the following output equation and state equation:

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{1}$$

$$y_k = Cx_k + Du_k + z_k \tag{2}$$

where A, B, C, D represent the system parameter matrix, respectively; x is the system status; k is the time series; u is the system input; z, w are the measurement noise and process noise, respectively; y is the system output observation.

KF as a widely used filter design in current linear systems, its own defects directly limit its application scope and the accuracy of the estimation results. The classical KF theory is only applicable to linear systems, and most of the control objects in the engineering field cannot be expressed in linear form. Moreover, the KF needs to assume that the system noise and the measurement noise are Gaussian white noise. If the noise is coloured noise, it will weaken the algorithm's ability to suppress the noise. In addition, it is necessary to obtain the initial value of the measured noise covariance matrix through a large number of tests and online debugging.

Many new results have been developed in the literature based on the KF principle. Since KF theory was proposed, it has been continuously developed and improved, and its expression form has been changing. From the initial estimation of the state of a linear system, it has evolved to be able to predict the state of a nonlinear system, from the simplest KF algorithm to, for example, the extended Kalman filter (EKF), the unscented

Kalman filter (UKF) and the cubature Kalman filter (CKF). The application of KF algorithm in cases such as nonlinear system problems and various types of system noise characteristics is enhanced to varying degrees to improve the approximate accuracy of the results and avoid more complex operations.

2.1.2 Extended Kalman filter

The EKF provides a method for tracking the state of a nonlinear system from noise measurements and noise inputs. EKF is a generalised algorithm of KF theory for nonlinear systems. EKF implements recursive filtering by taking the nonlinear system model for a first-order approximation of the Taylor expansion near the best estimate point of its state, linearising the nonlinear function, and finally using the classical KF formulation. The Taylor series is used to expand the nonlinear function, discard the higher order derivative components, and simply linearise the nonlinear model so that the linearised model can be estimated more accurately using the KF algorithm. The formula of the EKF is as follows:

$$\tilde{X}_{k+1} = F_k \hat{X}_k + B_k u_k \tag{3}$$

$$\overline{P}_{k+1} = F_k P_k F_k^T + Q \tag{4}$$

Update

$$K_{k+1} = \overline{P}_{k+1} H^T (H\overline{P}_{k+1} H^T + R)^{-1} \tag{5}$$

$$\hat{X}_{k+1} = \tilde{X}_{k+1} + K_{k+1}(Z_{k+1} - H\tilde{X}_{k+1}) \tag{6}$$

$$P_{k+1} = (I - K_{k+1} H)\overline{P}_{k+1} \tag{7}$$

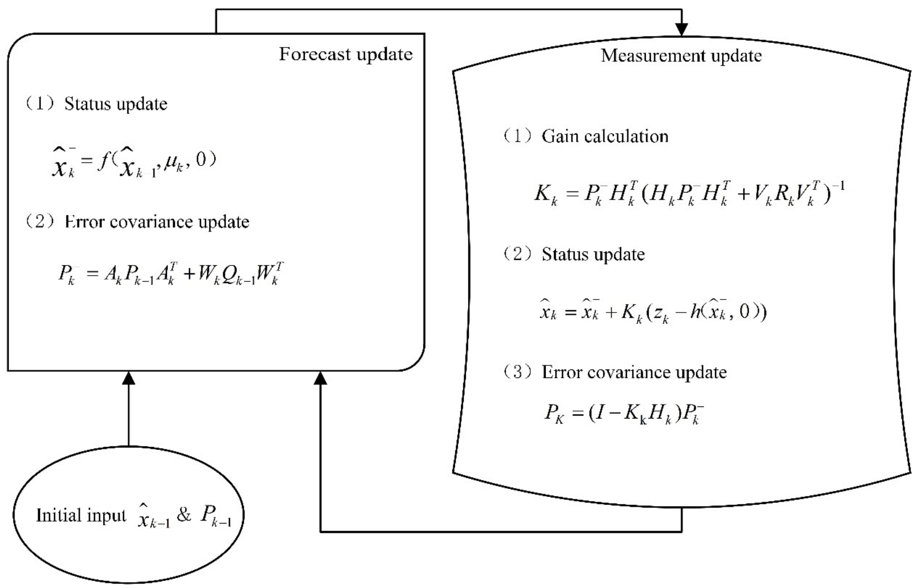
where $\hat{X}, \tilde{X}$ are the estimated and predicted values, respectly; F, H are the state transition matrix and observation matrix, respectly; P, B are the covariance matrix and control matrix; u is the control vector; K is the Kalman gain; I is the unit matrix; R, Q are the error.

The principle of EKF algorithm is shown in Figure 2, which mainly consists of two processes, the measurement update and the prediction update, that is, first perform state prediction and error covariance prediction according to the initialisation conditions, then perform gain calculation, then update the state and error covariance, loop the process, gradually converge the filtering process, and finally get the unbiased estimate of the state.

Gopinath and Das (2018) improved the performance of the EKF observer by adaptive tuning to provide better noise suppression for fast dynamic response in TRFC estimation under steady-state operation. Li et al. (2014) based on EKF theory, a three-degree-of-freedom vehicle model using the Dugoff tyre model was developed to achieve the estimation of vehicle state variables through sensor information. Li (2015) created a tyre model with combined longitudinal and lateral tyre forces and used the EKF algorithm to estimate TRFC. Kaur and Sahambi (2016) proposed a Fractional Order Gain KF (FOGKF) by adding a feedback loop to the method and using the fractional-order derivative of the previous gain as feedback in order to avoid the results of EKF divergence due to large errors. Improved Kalman filter improves root mean square error by 17% compared to the performance of the standard KF, fractional order KF and UKF.

Zong et al. (2011, 2013) used the Dual EKF (DEKF) technique to design an estimator for joint estimation of vehicle state and TRFC based on the three-degree-of-freedom vehicle model and the magic formula tyre model, and verified its accuracy. Estimation of tyre force and TRFC μ on asphalt pavement based on on-board sensor estimation and eight-degree-of-freedom vehicle model by applying Extended Kalman Bucy Filter (EKBF) and Bayesian hypothesis selection method, however, when the slip rate and slip angle of this method are very small, the estimation system cannot update the estimated value of TRFC in time (2010).

Figure 2 EKF algorithm schematic



EKF is widely used because of its simple steps. However, the EKF algorithm itself has the following drawbacks (Chen, 2012):

1 The EKF algorithm requires a local linearisation of the nonlinear system, which introduces large errors when the higher-order characteristics of the nonlinear object cannot be ignored.

2 The linearisation process of the nonlinear system involves the calculation of the Jacobi matrix, and the iterative calculation process needs to complete the replacement of the Jacobi matrix. For relatively complex nonlinear systems, the calculation of the Jacobi matrix is difficult. Especially for building real-time state estimation of complex systems, EKF is difficult to meet the real-time requirements.

For large-scale systems or complex equations, solving Jacobi matrices is very complicated. In addition, neglecting the higher-order terms of the Taylor expansion leads to an increase in truncation error, which degrades or even diverges the estimation performance of systems with strong nonlinearity. The EKF algorithm achieves nonlinear state estimation by approximating a nonlinear function, while other KF-based algorithms, such as UKF and CKF, approximate the posterior probability distribution of random variables.

2.1.3 Unscented Kalman filter

The UKF is a nonlinear filtering method based on the EKF that does not require a linearised model of the system and does not require solving the system differential (Zhao and Lin, 2010). The UKF algorithm process includes filter initialisation setting, Sigma point set calculation, time update equation and measurement update equation (Wang and Zhao, 2018), introduces unscented transformation (UT) to describe the filtering problem. The mean and variance can be approximately obtained through certain regular weights and sampling. Compared with the EKF algorithm, UKF avoids the complex calculation of the Jacobi matrix of highly nonlinear system functions, and at the same time significantly improves the estimation accuracy and convergence velocity without increasing the amount of calculation.

Zhao et al. (2016) used UKF to build a vehicle state and TRFC estimator, so that the estimation of the two was combined, and parameters such as vehicle speed and yaw angular velocity were used as input variables for friction coefficient estimation, improving the accuracy of the original input variables, thereby improving the final estimation accuracy of the algorithm. Zhang et al. (2022) proposed a state estimation method based on Enhanced Adaptive UKF (EAUKF), and compared the method with other KF algorithms, thus confirming that the proposed method has better state quantity estimation effect. However, as the number of iterations increases and the total execution time grows, it is still necessary to design the appropriate optimal decay speed parameter to avoid inaccurate state estimation in systems with rapidly changing noise. For articulated heavy vehicles, Morrison and Cebon (2016) verified that a Linear Adaptive UKF (LAUKF) using a 5-degree-of-freedom single-track vehicle model and linear adaptive tyres outperforms the linear KF at constant velocity or during emergency braking, especially under low-friction conditions, after computer simulations and vehicle test data comparison experiments. Zhang and Li (2016) proposed an Adaptive UKF (AUKF) algorithm by combining the UKF with the suboptimal Sage-Husa noise estimator, this algorithm can adapt to different road conditions and maintain high accuracy with strong robustness in the presence of vehicle parameter perturbation and model errors.

Wang et al. (2022) introduced fuzzy control theory and decaying memory filtering idea into UKF, introduced the concept of fuzzy forgetting factor, and dynamically adjusted the size of forgetting factor by fuzzy control, so that the estimation algorithm can have better convergence and tracking ability in different situations. A Fuzzy Forgetting Factor UKF (FFUKF) was designed to improve the tracking performance of the algorithm, which solves the problem that the traditional UKF cannot quickly track time-varying nonlinear systems. Fading Memory UKF (FMUKF) based on exponentially weighted fading memory, based on the traditional UKF, uses fading memory filtering to solve the problem of excessive filtering error or even scattering caused by inaccurate models, enhancing the stability of UKF, improve the estimation accuracy of the algorithm, and have a certain adaptiveness (Fu et al., 2018). Zhang et al. (2015) combined the ant colony algorithm with the UKF to reduce the estimation error and better track the virtual experimental values by relying on the optimisation-seeking effect of the ant colony algorithm, which improves the estimation accuracy and robustness of the UKF algorithm. Wielitzka et al. (2018) proposed a method for online estimation of bounded maximum friction coefficients based on serial sensors using a joint sensitivity-based UKF, introducing local sensitivity analysis to achieve robust estimation of parameter estimates without drift when excitation was insufficient. Pichlik and Zdenek (2018) proposed a new approach to locomotive wheel slip control in which the UKF was used as an estimator to estimate the relative adhesion force as an input to the PI controller in order to limit the applied traction force and to limit the wheel slip speed. This method does not require additional input signal filtering and does not require actual train speed information.

2.1.4 Cubature Kalman filter

CKF is a nonlinear Gaussian filtering method (Arasaratnam and Haykin, 2009; Arasaratnam et al., 2010), in contrast to the EKF and UKF, the CKF based on the third-order spherical radial volume criterion uses a set of volume points to approximate the state mean and covariance of the system (Sharma et al., 2017; Li et al., 2019), its accuracy of the probability distribution after approximating the nonlinear transformation is better than that of UKF, which is more adaptable, faster in calculation, more accurate and insensitive to the measurement error and system size, etc. (Liu et al., 2020; Chen, 2014).

Based on the Dual CKF theory, Li et al. (2015) designed a vehicle driving state estimator and a TRFC estimator, and interconnected the two information to form a closed-loop feedback correction to update the observed signal and achieve accurate estimation of TRFC. Zhang et al. (2022) developed a tyre model with higher accuracy than the brush model. After that, an improved square root CKF (Improved SCKF, ISCKF) algorithm based on the maximum correntropy criterion (MCC) was proposed. The estimation scheme combining the vehicle dynamics model, the tyre model and the ISCKF algorithm can both suppress the interference of anomalous measurement noise and adapt to changes in road friction conditions.

2.2 Particle filter

Regardless of EKF or UKF, the basic assumption is that the process noise and measurement noise of the system belong to Gaussian distribution, however, due to the complex driving conditions and vehicle environment, it is difficult to realise in practical applications. The application of particle filter (PF) technology has cleverly solved this problem (Gordon et al., 1993; Hu and Jing, 2005). Whether the system and noise are linear or nonlinear, Gaussian or non-Gaussian there are no excessive restrictions, can effectively solve the nonlinear non-Gaussian problem. By finding a set of propagating random samples (particles) in the state space to approximate the probability density function, the state minimum variance estimate is obtained by replacing the integration operation with the sample mean, the probability density function of the particles gradually approximates the probability density function of the state as the number of particles increases, achieving optimal Bayesian estimation. The unscented PF (UPF) algorithm combines the advantages of UKF and PF algorithms to improve the efficiency of processing nonlinear problems. Based on this algorithm to develop a nonlinear vehicle state estimator, compared with PF, UPF reduces the computing time with the same accuracy and has better tracking performance for real vehicle experimental data (Lin et al., 2014). Li et al. (2020) used the extended Kalman particle filter (EKPF) algorithm to estimate road slope with faster convergence and better tracking accuracy while meeting the real-time requirements. However, on downhill roads where engine braking was predominant, the estimator will not be activated due to the inability to obtain accurate braking power.

2.3 Fusion filter

In order to maintain the accuracy of vehicle state parameter estimation in a complex estimation environment under various driving conditions, the interacting multiple model (IMM) method is proposed and applied to the estimation of vehicle dynamic states (Jin and Yin, 2015; Jung and Choi, 2018; Zhang and Li, 2017; Wang et al., 2022), the IMM algorithm is adaptive and can effectively adjust the probability of each sub-model in real-time and keep the fusion output always tracking the sub-model output with small error according to the model transfer matrix. Tsunashima et al. (2006) proposed the IMMEKF vehicle state estimation method to construct a system model by 10 subsystem models considering tyre nonlinearity and different road friction conditions, and in which probability transformation can be performed to support high accuracy estimation of state and TRFC.

Genetic algorithm (GA) is a stochastic, parallel and adaptive search algorithm that simulates the evolution of organisms in nature, the UKF corresponds to the process noise covariance matrix $\boldsymbol{Q}$ and the measurement noise covariance matrix $\boldsymbol{R}$, which are the parameters to be optimised by GA. Both $\boldsymbol{Q}$ and $\boldsymbol{R}$ are used as optimisation parameters and the optimal parameters are obtained by adaptive processing. Zhou et al. (2019) proposed a new adaptive filtering algorithm by combining UKF and GA, established a 7-degree-of-freedom nonlinear vehicle dynamics model, combined with the 'magic formula' tyre model, and demonstrated through simulation and experiment that the estimation results of GA/UKF algorithm have higher accuracy and interference resistance.

However, the PF algorithm suffers from the particle degradation problem (Van der Merwe et al., 2000), to address this issue (Shen et al., 2014; Liu et al., 2017) proposing iterative extended Kalman filtering auxiliary particle filtering (IEKFAPF) to improve particle sampling and estimation accuracy improvement, the IEKF was applied to update the observed information to obtain the importance density function close to the real state, and the APF was resampled by the generated importance density function combined with the observed information, and the estimation performance of this algorithm was proved to be better than that of the UKF algorithm by real vehicle experiments.

Huang and Lin (2013) proposed to combine S-Correction Adaptive Kalman Filter (S-Correction AKF) and fuzzy Kalman filter (FKF) for car state estimation, where the FKF was based on fuzzy logic inference, based on the ratio of the actual variance of the measurement information obtained in real-time to the theoretical variance, the measurement noise matrix was adjusted online in real-time by the designed fuzzy system. S-Correction AKF algorithm was based on the derivation of mathematical theory to directly weight the estimation error covariance matrix, the joint algorithm has better robustness and estimation accuracy, and its estimation results can track the virtual test values well.

Ghandour et al. (2011) combined EKF, UKF and nonlinear least squares to estimate future transient tyre-road forces and maximum friction coefficients relying on a four-wheel vehicle model, and used the expected forces and maximum friction coefficients to predict two risk indicators: lateral load transfer (LTR) and lateral slip indicator (LSI).

The comparison and analysis of the various filters mentioned above is shown in Table 1.

Table 1 Features, advantages and disadvantages of each filter

Filter	*Features*	*Advantages*	*Disadvantages*
EKF	Transforming a nonlinear system model into a linear system model, linearising the nonlinear function	Reduced estimation bias of KF in nonlinear systems	Discarding higher-order derivative components in the model linearisation process with low accuracy and divergent results
DEKF	Contains two EKF that communicate with each other and correct each other's estimation results	Reduce the effects of measurement and system noise and keep accuracy within reasonable limits	Severe errors can occur under low friction road surfaces
IMM-EKF	Integrated in-wheel motor-driven vehicle in-vehicle sensors enable multiple vehicle road system models to adapt to variable driving conditions	High accuracy, low computational effort, hybrid dynamic system with switching mode	Algorithm effectiveness still needs to be verified under various complex road conditions
UKF	Finding a Gaussian distribution that approximates the true distribution for a nonlinear system model	Higher accuracy compared to EKF, avoiding the complex operation of Jacobi matrix	Large calculation volume, algorithm stability is not high, error accumulation leads to system dispersion
AUKF	Simultaneous estimation of process noise of a system using a suboptimal Sage-Husa noise estimator	Higher accuracy compared to UKF, with strong robustness, can adapt to different road surfaces	The vehicle driving conditions under complex working conditions and complex road conditions are not considered, and the experimental sample is small
EAUKF	Designing modulation factor *b* in the exponential decay function to overcome estimation inaccuracies due to rapid changes in the dependent variable	Compared with UKF has some improvement effect	When the state variable changes rapidly, the noise cannot be estimated accurately and the total execution time is long
FFUKF	Introduction of fuzzy forgetting factor, dynamic adjustment of forgetting factor size by fuzzy control	Stronger traceability and better convergence and convergence velocity compared to UKF	The algorithm is more complex, and the value of the constant forgetting factor *f* needs to be discussed categorically
FM-UKF	Introduce attenuation memory filtering and design exponential attenuation factor to ensure the estimator works in the best condition	Good estimation accuracy and stability relative to UKF, improved response speed, and some adaptiveness	Algorithm stability under different road conditions is not verified

Table 1 Features, advantages and disadvantages of each filter (continued)

Filter	*Features*	*Advantages*	*Disadvantages*
CKF	Based on the third-order spherical radial volume criterion, a set of volume points is used to approximate the state mean and covariance of the system	More adaptable, faster and more accurate calculation compared to UKF	High sensor requirements and complex algorithm structure with high computational effort
ISCKF	Adaptive adjustment of measurement noise covariance based on MCC	Effective estimation accuracy on low- and high-friction pavements with good stability, adaptiveness and robustness	The stability and accuracy of the algorithm still need to be verified in real-world experiments under different operating conditions
PF	Non-Gaussian distribution using a large number of particles	Better accuracy and simpler sampling compared to EKF	Requires a large number of sample points, which can fail during the run and limit the approximation effect
UPF	The deterministic sampling strategy is used to obtain an importance function that outperforms the ordinary PF algorithm by UT transformation	Improved particle degradation phenomenon, improved filtering accuracy, and better tracking performance	Cannot meet the requirements of high precision and high dynamic target tracking
GA/UKF	Optimisation of process noise covariance matrix and measurement noise covariance matrix using genetic algorithm	High estimation accuracy, algorithm resistant to interference, easy to measure using parameters	The test conditions are not universal, and the reliability of the algorithm needs to be proven under more conditions
IEKF-APF	Apply IEKFAPF to generate importance density functions that are closer to the true state and resample them in combination with the observed quantities	Solve the particle degeneracy problem with higher performance relative to UKF estimation	The algorithm is complex, and the estimates are biased when the tyre enters a highly nonlinear state

3 Observer technology

3.1 Luenberger observer

When the vehicle dynamical system can be defined as a deterministic system, the problem of estimating the parameters of the vehicle tyre-road interaction can be formulated as an observer design problem. Luenberger Observer (LO) is a method based on modern control theory to achieve the purpose of state estimation through the pole configuration of the system. Unlike KF, LO improves the performance of the estimation algorithm in terms of real-time, accuracy and robustness to parameters by setting different design goals and configuring different observer feedback matrices (Rabhi, 2005). Parameters such as longitudinal and lateral acceleration, steering angle, wheel angular velocity and sideslip rate are measured in the literature, and an EKF and a method based on the LO are used in the nonlinear vehicle model to estimate tyre force and road slope (Dakhlallah et al., 2008; Sebsadji et al., 2008). The advantage of LO is that it can construct states that cannot be directly measured, perform state feedback, and obtain better performance; however, at the same time, compared with sliding mode observer (SMO), robust observer, etc., the anti-disturbance performance is poor. When the system is at a low frequency, the anti-disturbance of the observer will be suppressed, and the response speed is slow. There is still a lot of room for improvement on the basis of traditional observers.

3.2 Sliding mode observer

Sliding mode observer (SMO) based on variable structure control theory is a nonlinear observer, unlike the normal observer structure, the SMO is designed to observe the error between the measured value and the estimated measured value directly as a sliding mode plane. SMO is often used to estimate vehicle tyre-road interaction information, and they can reconstruct the state of the system by forcing it into a slip surface. Meanwhile, SMO technique also inherits the advantages of parameter uncertainty, model error and disturbance robustness of variable structure control, but is more sensitive to measurement noise (Guo et al., 2018). Zhang et al. (2014) used a SMO to estimate vehicle speed from parameters such as vehicle acceleration, wheel speed and braking torque, and calculated the TRFC and optimal vehicle slip rate by combining the estimated parameters of the Burckhardt tyre model to obtain a vehicle adaptive sliding mode control algorithm based on the estimated vehicle speed, TRFC and optimal slip rate. This vehicle speed observer and TRFC estimator will get nonideal estimates when the rolling resistance coefficient was close to zero, and further research was still needed for more accurate observation and estimation algorithms. Rath et al. (2015) designed a Higher Order Sliding Mode (HOSM) observer based on an improved supertwisting algorithm (STA) and a nonlinear Lipschitz observer. To overcome the chattering problem of SMO when it does not use filtering to estimate, it was confirmed that without the use of a low-pass filter, the random road profile, longitudinal friction and engine friction were used as unknown inputs, and the dynamic adhesion of the tyre can be accurately estimated from the sliding mode.

3.3 Robust observer

Consider a reasonable design of robust observer feedback gain in the presence of parameter variations so that the estimation results are influenced as little as possible by model parameter variations. To ensure the local stability of the system, the structure of the observer gain matrix is determined first, and the optimal gain is obtained by numerical calculation afterwards to achieve robust stability against uncertainty. Ahn et al. (2012, 2013) developed an observer that simultaneously estimates TRFC and lateral sideslip angle, and used the Lyapunov function for quantitative analysis, provided performance limits and available ranges for observers in the vehicle state space. Robust and stable estimation can be achieved when the tyre slip angle was stabilised between 20%~60% of the maximum tyre force. By combining a numerical differentiator based on a robust observer and a low-cost sensor method using the main embedded sensor on the vehicle, the sensor configuration can be minimised and it was easy to install and calibrate (Imine et al., 2015).

3.4 Recursive least squares

The least squares estimation (LSE) method is a standard method to obtain an approximate solution based on minimising the estimation error described by the objective function. recursive least square (RLS) is a recursive form of the least squares method suitable for online estimation of parameters. To overcome the problem of data saturation after a long period of computing, a forgetting factor is introduced to reduce the influence of previous data. In addition, UD decomposition or square root filtering is generally used to enhance the numerical computational power of the algorithm. In actual vehicle operation, high frequency vibration occurs in the vehicle system, and since LSE is not unbiased and does not provide consistent estimates under coloured noise, the effect of noise should be fully considered in practical applications.

Electric vehicles with eight-wheel motors are highly non-linear, and their driving roads and operating conditions are more complex and specific, Zhang et al. (2022) proposed an eight-wheel electric vehicle drive coordination control strategy based on road recognition, in which the road recognition module uses the UKF algorithm to estimate the tyre-road force and the RLS to identify the TRFC, the Sliding Mode Control with a Conditional Integrator (SMC&CI) controller was also implemented to ensure that the vehicle maintains good dynamic performance and stability when the driving conditions change. To solve the problem that the TRFC estimation algorithm cannot be applied to both high slip rate and low slip rate conditions, based on the simplified magic formula tyre model, Song et al. (2013) used the RLS method to make a preliminary estimation of the longitudinal TRFC, and used the EKF algorithm to filter out the noise and adaptively adjust the tyre model parameters, which has a certain accuracy and robustness for the estimation of the longitudinal TRFC. Rajamani et al. (2011) developed three observers to estimate slip rate and tyre longitudinal force:

1 using engine torque, brake torque and GPS measurements

2 using torque measurement and accelerometers

3 using GPS measurements and accelerometers.

After estimating the slip rate and longitudinal force of the tyre, the RLS parameter identification formula is used to calculate TRFC. The linearised recursive least squares (LRLS) method and the integrated lateral and longitudinal tyre model without predefined parameters were used by Choi et al. (2013) to avoid underestimating the actual TRFC and to make full use of the longitudinal and lateral excitations for fast estimation of TRFC using relevant measurements of real-time vehicle lateral and longitudinal dynamics. Kim et al. (2014) developed a TRFC model using tyre acceleration as an indicator; applied the vehicle model to the 6-degree-of-freedom body acceleration calculation to obtain the longitudinal, lateral and normal acceleration of each tyre; processed the obtained acceleration using RLS to estimate TRFC. Nam et al. (2012) used a RLS algorithm based on a linear vehicle model and sensor measurements and a KF combining sensor measurements with roll dynamics (one using lateral tyre force measurements and the other using lateral acceleration measurements) for vehicle slip estimation and roll angle estimation, respectively, with experimental results showing good estimation performance and robustness without the use of expensive sensors, but still with some estimation errors for strenuous driving on low-friction roads. Lee et al. (2004) designed a brake gain estimator, developed a real-time traction estimator based on a RLS method with bounded gain forgetting, and applied it to the estimation of the maximum TRFC, and obtained better estimation results compared with the real value obtained from the brake torque sensor.

3.5 Other nonlinear observer

Nonlinear observer (NLO) is used to directly deal with nonlinear problems in vehicle dynamic state estimation, and many scholars have derived this type of observer using different stability theories such as Lyapunov stability theory, and applied it to the estimation of vehicle state parameters (De Wit et al., 2003; Grip et al., 2006; Xia et al., 2016). Using nonlinear observers-unknown input observers (UIO), Imsland et al. (2007) concludes that the error dynamics structure of nonlinear UIOs is the same as that of NLOs without unknown inputs, providing inspiration for the design of observers for lateral velocity estimation of vehicles on inclined roads. Design of nonlinear observer for nonlinear double-track vehicle model to estimate vehicle motion dynamics and tyre longitudinal and lateral forces, the observer design method is easy to implement, can be calculated in real-time and obtained satisfactory real-time performance in real vehicle tests, but it is sensitive to the change of parameters (Acarman, 2008). Cheng et al. (2011) used UKF to design a nonlinear observer that can simultaneously estimate slip angle, adhesion force and TRFC without the need for additional sensors for online estimation when the ESP was installed in the vehicle. Gao et al. (2010) used a single-track vehicle model with nonlinear tyre characteristics, and then reconstructs the model to derive a high gain observer (HGO) based on input-output linearisation to estimate the maximum TRFC using a simple logic based on vehicle lateral dynamics with nonlinear characteristics of tyres. This method does not take into account the effect of sensor noise and the lateral force of the embankment angle on the vehicle, the effect on TRFC, etc. Wang and Wang (2013) proposed the estimation of tyre cornering stiffness and TRFC based on the longitudinal force difference between the left and right wheels of wheeled motor-driven electric vehicles by algebraic techniques using NLO single-track and brush tyre models.

A comparison and analysis of the various filters mentioned above is shown in Table 2.

4 Model-based estimation

The filter and observer techniques require more on-board sensors and more complex algorithms, and although they can eliminate sensor errors to some extent, they are more sensitive to unknown disturbances and sensor drift encountered in the working conditions of the experimental object, and cannot achieve accurate estimation by relying solely on kinetic models due to the presence of integral cumulative errors. Vehicle model-based estimation methods include those based on vehicle kinematic models and those based on vehicle dynamics models, which do not require expensive special sensors compared to experiment-based methods and guarantee the accuracy and repeatability of results in most cases. Model-based studies can be divided into three main groups: wheel and vehicle dynamics model-based approaches, tyre model-based approaches and Slip-slope approaches (Khaleghian et al., 2017).

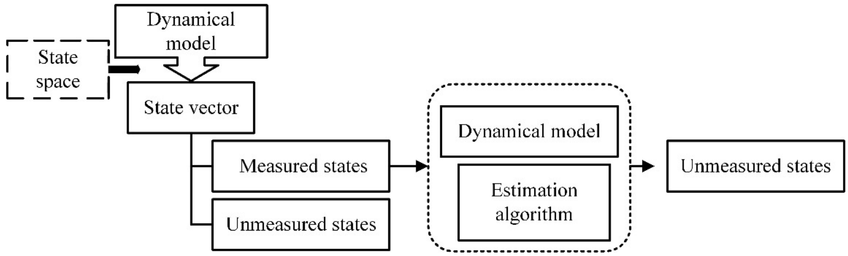
Table 2 Features of each observer and its advantages and disadvantages

Observer	*Features*	*Advantages*	*Disadvantages*
Luenberger observer	Set different design goals and configure different observer feedback matrices	Make estimation algorithms with different performance such as real-time and accuracy	Poor anti-disturbance performance and slow response time
Robust observer	Designing optimal feedback gain to improve the anti-interference of estimation algorithm	System model with good robustness for practical applications	Weak noise suppression of sensors
SMO	The error between the measured value and the estimated measured value is observed directly as a slip plane	Simple structure and high robustness	More sensitive to measurement noise
NLO	State variables are set using a state space model with TRFC as unknown system parameters, and both states and parameters are estimated	Compared to the EKF, which has a simpler structure and better real-time performance, the joint UKF can capture the change of TRFC and give a more accurate state estimation	Sensitive to parameter changes
HGO	State feedback controller can be designed independently of the HGO by performing progressive estimation of state quantities based on measurement results	Simple structure, easy design, robustness and easy application	More sensitive to measurement noise, robustness is inversely proportional to noise reduction capability
STA-based HOSM	To estimate the state and unknown inputs, a combination of a nonlinear Lipschitz observer and a modified Supertwisting observer based on higher-order sliding mode (HOSM) is used	Overcomes chattering problems during estimation without filtering and accurately estimates vehicle status and tyre dynamic adhesion	Unknown input estimation is prone to parameter uncertainty and modelling mismatch, and the presence of modelling mismatch affects the estimation of unknown input quantities

4.1 Based on vehicle dynamics model

TRFC estimation method based on wheel vehicle dynamics model using the dynamics model of the system and parameter measurement state, combined with algorithms such as KF method, PF method, RLS method to estimate the TRFC, the process chart of this estimation method is shown in Figure 3. This section describes several commonly used vehicle dynamics models.

Figure 3 Estimation methods based on vehicle dynamics models



4.1.1 Quarter vehicle model

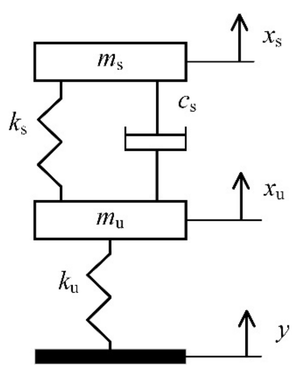
The quarter vehicle model is a 2-degree-of-freedom vehicle model, mainly used to simulate the vertical dynamics of a car's suspension. Assume that the tyre is a spring with stiffness k_u, studying the mass of a single wheel. The suspension system is simplified as the parameters are a spring absorber with stiffness k_s and damping c_s, since it is a quarter body model so the body mass is taken as a quarter, the vehicle model is shown in Figure 4, and the body and wheel dynamics equations are shown in equations (8) and (9).

$$m_s\ddot{x}_s + c_s(\dot{x}_s - \dot{x}_u) + k_s(x_s - x_u) = 0 \tag{8}$$

$$m_u\ddot{x}_u + c_s(\dot{x}_u - \dot{x}_s) + (k_u + k_s)x_u - k_s x_s = 0 \tag{9}$$

where m_s, m_u are the body mass and wheel mass, respectively; x_s, x_u are the body displacement and wheel displacement, respectively.

Figure 4 Quarter vehicle model

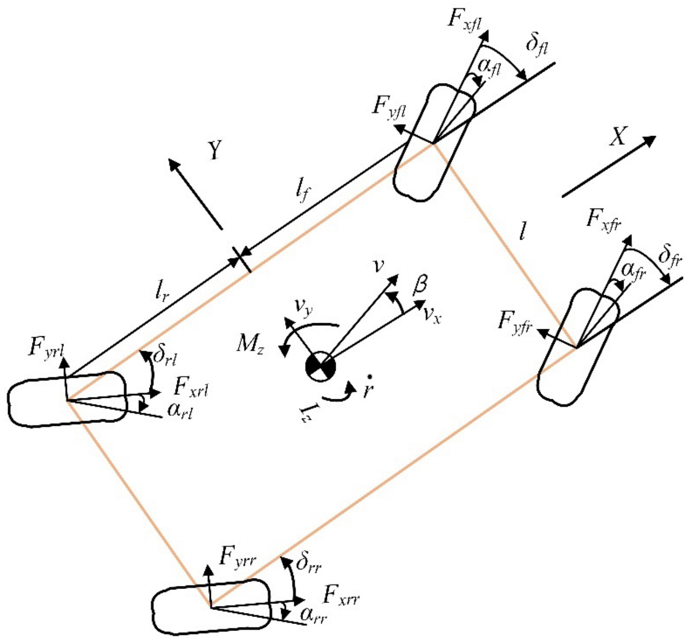


The use of a quarter vehicle model can be combined with the RLS (Ding and Taheri, 2010), IMMEKF vehicle status estimation method (Tsunashima et al., 2006), HGO algorithm (Gao et al., 2010), etc. in conjunction to obtain tyre normal force and road profile.

4.1.2 Four-wheel vehicle model

The four-wheel 3-degree-of-freedom vehicle model, also known as the double-track model, assumes that the vehicle motion is not affected by pitch and roll, and only considers the lateral, transverse and yaw motion of the vehicle, the vehicle dynamics model is shown in Figure 5.

Figure 5 Four-wheel vehicle model (see online version for colours)



In the picture, δ_{fl}, δ_{fr}, δ_{rl}, δ_{rr} present vehicle left front wheel steering angle, right front wheel steering angle, left rear wheel steering angle, right rear wheel steering angle, respectively. The sideslip angle of each wheel is similar, α_{fl}, α_{fr}, α_{rl}, α_{rr} present vehicle left front wheel sideslip angle, right front wheel sideslip angle, left rear wheel sideslip angle, right rear wheel sideslip angle; β is the mass center sideslip angle.

The differential equations of motion of longitudinal velocity v_x, lateral velocity v_y, yaw velocity r with other state parameters:

$$\dot{v}_x = a_x + v_y r \tag{10}$$

$$\dot{v}_y = a_y - v_x r \tag{11}$$

$$\dot{r} = \frac{1}{I_z} M_z \tag{12}$$

where, a_x, a_y are the vehicle longitudinal and lateral acceleration, M_z is the yaw torque of the vehicle around the Z-axis, I_z is the moment of inertia of the vehicle around the Z-axis.

The four-wheel vehicle model can be used to estimate the TRFC by combining nonlinear observer technology (Grip et al., 2008), and UIO technology (Imsland et al., 2007), Cheng et al. (2011) used UKF combined with a four-wheel vehicle model to estimate the vehicle slip angle, tyre lateral force and TRFC.

4.1.3 Single-track model

The single-track model, also known as the bicycle model, is mainly used to estimate the lateral state of the vehicle. Consider the lateral and yaw movement of the vehicle, ignoring other movements such as the longitudinal, pitch, roll, and vertical movement of the vehicle, as shown in Figure 6.

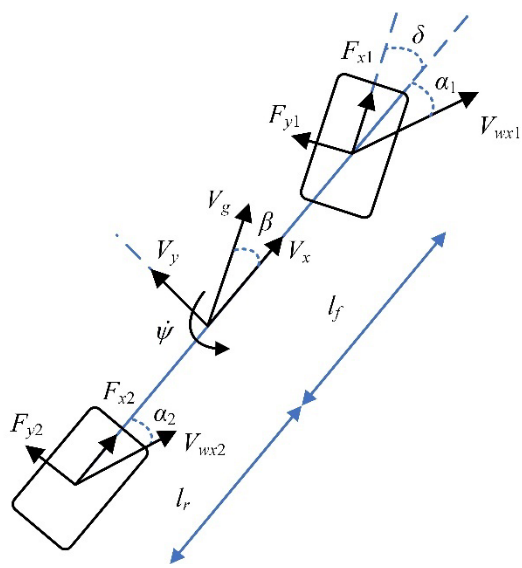
$$\ddot{\psi} = \frac{1}{I_z}\left[l_f\left[F_{x1}\sin\delta + F_{y1}\cos\delta\right] - l_r F_{y2}\right] \tag{13}$$

$$\dot{\beta} = \frac{1}{m_v V_g}\left[-F_{x1}\sin(\beta-\delta) + F_{y1}\sin(\beta-\delta) + F_{y2}\cos\beta - F_{x2}\sin\beta\right] - \dot{\psi} \tag{14}$$

$$\dot{V}_g = \frac{1}{m_v}\left[F_{x1}\cos(\delta-\beta) - F_{y1}\sin(\delta-\beta) + F_{x2}\cos\beta + F_{y2}\sin\beta\right] \tag{15}$$

where l_f and l_r respectively represent the front wheelbase and rear wheelbase of the vehicle, V_g is the velocity of centre of gravity, Ψ is the vehicle yaw rate, δ is the vehicle steering angle, β is the vehicle sideslip angle, m_v is the body quality.

Figure 6 Single-track model (see online version for colours)

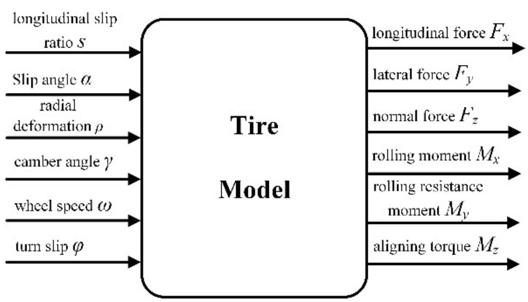


Combined with single-track model, Villagra et al. (2011) proposed a combination of basic diagnostic tools and new algebraic techniques for filtering and estimating derivatives based on basic diagnostic tools; Rezaeian et al. (2014) independently estimated the parameters of the vehicle tyre model to enhance the robustness of the model structure, and used three calculation modules to estimate the longitudinal force of the tyre, the vertical force of the tyre, and the lateral force of the tyre. Vazquez et al. (2018) used the bicycle model to calculate the longitudinal and transverse tyre-road forces on the axis, used the Hoverboard Model to calculate the left and right virtual forces of the tyre, and constrained the closed-loop observer scheme of EKF to accurately calculate the tyre forces at each position.

4.2 Based on tyre model

The tyre model is mainly used to explain the relationship between the wheel motion parameters and the tyre force, that is, the input and output relationship of the tyre under specific working conditions. The driving, braking and steering of the vehicle are all achieved through the tyre force. Therefore, tyre characteristics play an important role in the dynamic control of vehicles. This section introduces the tyre model used for vehicle tyre road interaction estimation. As shown in Figure 7, the input and output relationship of the tyre under specific operating conditions.

Figure 7 Tyre model input and output relationship



4.2.1 Pacejka model

The Pacejka model is also known as the magic formula (Pacejka and Bakker, 1991), uses the same set of trigonometric formulas to uniformly represent the longitudinal and transverse forces of the tyre, describes the relationship between the longitudinal force F_x and slip λ of the tyre road. In the absence of any measurement noise, the Pacejka model with the most parameters can obtain ideal results (Albinsson et al., 2017), its general 'magic formula' form is as follows:

$$y = D\sin\left\{C\arctan\left[Bx - E(Bx - \arctan(Bx))\right]\right\} \tag{16}$$

where y can represent the longitudinal and lateral forces and moments of the tyre, while x relative to y can represent the sideslip angle and slip rate.

Yi et al. (1999) used the eight-state nonlinear vehicle transmission simulation model of the Bakker-Pacejka formula tyre model, verified the identification of the TRFC based on the observer's least squares method and the observer's filter regression method. This recogniser can provide a good estimate of TRFC under normal driving, can quickly reduce the initial estimation error, and has robustness to modelling errors and parameter uncertainties. Cabrera et al. (2018) considered the contact between the tyre and the road during driving, as well as the main factors affecting the force in contact, uniformly studies the coefficient of friction, and corrects the Pacejka model, considering the road composition and its state, tyre type, vehicle speed, slip and other influencing parameters to obtain a more accurate tyre-road friction model.

4.2.2 Dugoff model

Dugoff tyre model (Dugoff et al., 1970) belongs to the semi-empirical tyre formula and has the characteristics of approximately high precision. In order to accurately express the nonlinear mechanical properties of the tyre, the tyre model introduces the boundary value L and corrects the model, so that the function expression between the longitudinal and lateral forces of the tyre and TRFC of can be obtained. For the force analysis of a single tyre, the expressions of longitudinal force and lateral force are obtained as follows (Wang et al., 2022):

$$F_x = \mu F_{z,kN} C_x \frac{\lambda}{1-\lambda} f(L) \tag{17}$$

$$F_y = \mu F_{z,kN} C_y \frac{\tan\alpha}{1-\lambda} f(L) \tag{18}$$

In the formula

$$f(L) = \begin{cases} L(2-L), & L < 1 \\ 1, & L \geq 1 \end{cases} \tag{19}$$

$$L = \frac{(1-\lambda)(1-\varepsilon v_x\sqrt{C_x^2\lambda^2 + C_y^2\tan^2\alpha})}{2\sqrt{C_x^2\lambda^2 + C_y^2\tan^2\alpha}} \tag{20}$$

where $F_{z,kN} = \frac{F_z}{1000}$; μ is the TRFC; ε is the speed influence coefficient; L is the boundary value; C_x, C_y are the tyre longitudinal stiffness and lateral stiffness, respectively, λ is the sideslip rate, α is the sideslip angle, F_z is the tyre vertical force.

4.2.3 Brush model

Brush model (Svendenius et al., 2009) regards the contact between the tyre and the road surface as a series of elastic bristles in contact with the ground, and these bristles can be deformed in a direction parallel to the road surface, which can reflect the nonlinear characteristics of the combined longitudinal and transverse forces of the tyre under the friction ellipse. The expression is as follows (Bascetta and Ferretti, 2022):

$$F_{x,i} = \frac{C_x(s_i/1+s_i)}{f_i} F_i \tag{21}$$

$$F_{y,i} = -\frac{C_\alpha(\tan\alpha_i/(1+s_i))}{f_i} F_i \tag{22}$$

In the formula

$$F_i = \begin{cases} f_i - \frac{1}{3\mu F_{z,i}} f_i^2 + \frac{1}{27\mu^2 F_{z,i}^2} f_i^3, & f_i \leq 3\mu F_{z,i} \\ \mu F_{z,i}, & else \end{cases} \tag{23}$$

where C_x, C_y are the tyre longitudinal stiffness and lateral stiffness, respectively; μ is the TRFC, s_i is the sideslip rate, α_i is the sideslip angle, $i = 1, 2, 3, 4$, respectively, the tyre orientation is left front, right front, left rear and right rear.

Compared with the Dugoff tyre model, the derivation of the Brush model is more rigorous and it can accurately describe the nonlinear relationship between tyre force and slip rate, sideslip angle, vertical load and friction coefficient. Zhang et al. (2022) proposed an improved model with adaptive stiffness on the basis of the brush model, which improved the accuracy of calculating tyre force.

4.2.4 LuGre model

LuGre model (De Wit and Tsiotras, 1999) assumes that the two contact surfaces are rough and uneven contact surfaces, from a microscopic point of view, the protruding part of the contact surface is regarded as fine bristles, the essence of the contact between the two surfaces is the contact of the bristles on the contact surface. The two contact surfaces are subjected to tangential forces and undergo elastic deformation and plastic deformation. The LuGre model is divided into a centralised friction model and a distributed friction model. The two models are briefly introduced below.

4.2.4.1 Centralised LuGre model

The centralised LuGre model has a relatively simple form, but the friction conditions of each position in the contact area between tyre and road surface are not the same, so the friction conditions in the contact area obtained by using the centralised LuGre model are very different from the actual situation. The expression is as follows:

$$\begin{cases} \dot{z} = v_r - \sigma_0 \cdot |v_r| \cdot z / g(v_r) \\ F = (\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_r) \cdot F_z \\ g(v_r) = \mu_c + (\mu_s - \mu_c) \cdot \exp(-|v_r / v_s|^{\delta}) \end{cases} \tag{24}$$

where z is the internal friction state, v_r is the relative velocity, σ_0 is the stiffness coefficient, σ_1 is the damping coefficient, σ_2 is the relatively viscous damping, F_z is the normal load, $g(v_r)$ is the Stribeck effect when the relative velocity is included. μ_c is the dynamic friction coefficient, μ_s is the static friction coefficient, v_s is the Stribeck velocity.

4.2.4.2 Distributed LuGre model

The distributed model discretises the imprint area into a series of tiny elements. Unlike the centralised friction model, the z that expresses the internal friction state is not only a function of time t, but also a function of its position x in the imprint. The expression is as follows:

$$\begin{cases} \dfrac{dz(t,x)}{dt} = v_r - \dfrac{\sigma_0 |v_r|}{g(v_r)} z(t,x) \\ \mu(t,x) = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_r \\ F = \int_0^L \mu(t,x) f_n(x) dx \end{cases} \tag{25}$$

where $f_n(x)$ is the normal force distribution.

4.2.5 Uni-Tire model

Uni-Tire can use dimensionless expressions to uniformly express the tyre characteristics under different loads, the mechanical properties under pure working conditions and composite working conditions, and can better solve nonlinear problems. At the same time, it has good extrapolation ability and prediction ability, and can accurately predict the tyre characteristics under composite working conditions, various road surfaces and speed conditions (Guo, 2016). The expression is as follows:

$$F_{xi} = \bar{F}_i \frac{\Phi_{xi}}{\Phi_i} \mu_{xi} F_{zi} \tag{26}$$

$$F_{yi} = \bar{F}_i \frac{\Phi_{yi}}{\Phi_i} \mu_{yi} F_{zi} \tag{27}$$

$$\begin{cases} S_x = \dfrac{-v_{sx}}{\Omega R_e} = \dfrac{\Omega R_e - v_x}{\Omega R_e} & S_x \in (-\infty, +\infty) \\ S_y = \dfrac{-v_{sy}}{\Omega R_e} = \dfrac{-v_y}{\Omega R_e} & S_y \in (-\infty, +\infty) \end{cases} \tag{28}$$

$$\phi_x = \frac{K_x S_x}{\mu_x F_z} \quad \phi_y = \frac{K_y S_y}{\mu_y F_z} \quad \phi = \sqrt{\phi_x^2 + \phi_y^2} \tag{29}$$

where F_{xi}, F_{yi} are the longitudinal tyre road force and lateral tyre road force, respectively; F_{zi} is the normal force; μ_x, μ_y are the longitudinal and lateral TRFC, respectively; v is the tyre velocity; α is the sideslip angle; Ω is the rotation angular velocity; γ is the tyre roll angle; R_e is the effective rolling radius; S_x, S_y are the longitudinal slip rate and lateral slip rate, respectively; ϕ_x, ϕ_y, ϕ are the relative longitudinal, lateral slip rate and comprehensive slip rate; K_x, K_y are the tyre longitudinal and lateral stiffness, respectively.

4.2.6 HSRI model

HSRI (Tielking, 1974) model is suitable for estimating TRFC in linear and nonlinear areas. The ratio of lateral stiffness to longitudinal stiffness is used in the estimation algorithm, so that the different characteristics of tyres under different working conditions and the degree of tyre wear have little effect on the ratio of lateral stiffness to longitudinal stiffness, and it has good robustness and adaptability. The expression is as follows:

$$F_x = \frac{\lambda}{1-\lambda} C_x \left(\frac{1}{H} - \frac{1}{4H^2} \right) \tag{30}$$

$$F_y = \frac{\lambda}{1-\lambda} C_y \left(\frac{1}{H} - \frac{1}{4H^2} \right) \tan\alpha \tag{31}$$

$$H = \left[\left(\frac{\lambda}{1-\lambda} \cdot \frac{C_x}{\mu F_N} \right)^2 + \left(\frac{1}{1-\lambda} \cdot \frac{C_x}{\mu F_N} \cdot \tan\alpha \right)^2 \right]^{0.5} \tag{32}$$

$$F_y = \frac{C_y \tan\alpha}{C_x \lambda} F_x \tag{33}$$

where F_N is the tyre normal force; μ is the maximum TRFC; C_x, C_y are the tyre longitudinal stiffness and lateral stiffness, respectively; λ is the sideslip rate; α is the sideslip angle.

4.2.7 Burckhardt model

Burckhardt model (Hu et al., 2021) is an empirical model that expresses the relationship between TRFC μ and the slip rate s obtained by fitting various typical road test data. The model expression is as follows:

$$\mu(s) = \left\{ c_1 \left[1 - \exp\left(-c_2 s \right) \right] - c_3 s \right\} e^{-c_4 v} \tag{34}$$

where c_i is determined by road conditions, $e^{-c_4 v}$ is the change in friction coefficient caused by velocity change.

The simplified expression after ignoring the impact of velocity changes is as follows:

$$\mu(s) = c_1 \left[1 - \exp\left(-c_2 s \right) \right] - c_3 s \tag{35}$$

4.3 Slip-slope

Slip-slope uses the relationship between the normalised slip rate and the longitudinal force of the wheel to determine the size of the road adhesion coefficient μ. This method is based on the assumption that under normal driving conditions, the low slip part of the slip curve can be used to estimate the maximum friction between the tyre and the road surface (Gustafsson, 1997; Lin and Huang, 2013). In the case of low slip, the normalised longitudinal force of the wheel has a linear relationship with the slip rate. When the absolute value of the slip rate exceeds 0.05, the accuracy of estimating the friction coefficient of the road surface using the Slip-slope method cannot be guaranteed. The slope of the curve (slip-slope) changes with the change of μ. This method needs to obtain the longitudinal force, normal force and slip rate information of the wheel. The relationship is as follows:

$$\rho = F_x / F_z = Ks \tag{36}$$

When braking

$$s = \frac{\omega_w r_w - v_w}{v_w} \tag{37}$$

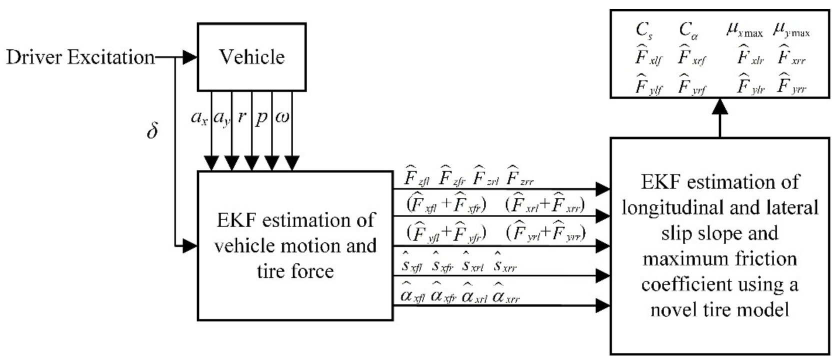
When accelerating

$$s = \frac{\omega_w r_w - v_w}{\omega_w r_w} \tag{38}$$

The normal wheel load can be calculated by the vehicle dynamics equation, and the longitudinal force cannot be directly calculated. It can be obtained by the tyre force sensor or the filtering algorithm combined with the tyre model.

Qi et al. (2015) on the basis of the 4-degree-of-freedom vehicle model, an EKF estimator for vehicle motion and tyre force estimation was designed, and the longitudinal tyre force $(F_{xfl} + F_{xrl})$, $(F_{xfr} + F_{xrr})$ and transverse tyre force $(F_{yfl} + F_{yfr})$, $(F_{yrl} + F_{yrr})$ was estimated with a smaller error. In addition, another EKF estimator was designed based on the estimated force and the proposed new tyre model, and the ideal tyre model estimation parameters and tyre longitudinal and transverse forces were obtained. The estimation process is shown in Figure 8.

Figure 8 Estimation process



However, this method still needs to be improved in the following points:

1 construct and improve the tyre force model under linear and nonlinear conditions

2 describe the robustness of the parameters relative to the measured noise

3 in more extreme driving conditions, use real data for detailed simulation or testing

4 verify its applicability in the automotive industry, etc.

The comparison and analysis of the above-mentioned various model-based estimation methods are shown in Table 3.

5 Sensor-based estimation

The two types of estimation methods for the TRFC (cause-based method and effect-based method) require sensors to obtain changes in the influencing parameters or vehicle driving parameters. Based on the cause-based method, the main research is on the factors that affect the change of friction. After obtaining these factors through the sensor, the TRFC is predicted based on the existing experience and friction model. The effect-based method needs to obtain and identify factors such as tread deformation and tyre noise during vehicle driving (Erdogan et al., 2011), that is, when there is a certain amount of excitation on the road surface, TRFC is estimated based on the functional relationship between tyre mechanical response and excitation. This section introduces several sensors commonly used in TRFC estimation research.

Table 3 Comparison and analysis of model-based estimation methods

Model	*Scope of application*	*Advantages*	*Disadvantages*
Based on vehicle dynamics model	At the same time, there are measurable state parameters, state parameters that cannot be directly measured, and state parameters that can be derived from estimation algorithms. For different state estimates, different vehicle dynamics models can be selected	No expensive sensors are required, external factors are reduced, and the estimation results are accurate and repeatable	The vehicle dynamics model is highly specific, and multiple vehicle models need to be used for joint research, and specific vehicle models are required for specific performance and parameters
Based on tyre model	Use the relationship between tyre ground force and torque, slip rate and slip angle to estimate friction and friction coefficient, including physics and semi-empirical formulas	No additional sensors are required, reducing signal processing work. Many models, easy to use	The accuracy of the tyre model must be matched with the vehicle model, the specificity is strong, and the ability to estimate the static state of the tyre is limited
Slip-slope	The relationship between longitudinal slip and longitudinal tyre force and the slip rate curve are used to estimate TRFC, and the slope of the μ-s curve is estimated using this method to obtain the road friction	The parameters used in this method are easier to obtain by the on-board sensor, and the accuracy can be improved by combining EKF and RLS methods	The experimental vehicle needs to perform sufficient acceleration and braking, and provide a sufficiently large slip rate. For tyres working in linear areas and nonlinear areas, they should be discussed separately

Vehicle dynamic state estimation can be regarded as the process of obtaining vehicle status information through multi-sensor data fusion technology and using data obtained from on-board sensor measurements. Multi-sensor data fusion means that various sensors are fully and rationally selected, the effective information in them is extracted and the multi-sensor resources are fully utilised. Through the reasonable control of the sensor and its observation data, the information obtained is combined according to certain principles, redundant information is discarded, complementary information is optimised, and better performance is obtained (Tian et al., 2022; Song et al., 2009).

5.1 Optical sensor

Cause-based estimation methods often use optical sensors and cameras to obtain road surface information (Koskinen, 2010), For example, it is used to measure the light absorption and scattering characteristics of roads to identify water and other substances on roads. This type of estimation method uses a special device to diffuse the beam of light, and the corresponding sensor can obtain the reflected wave of the road, and analyse the spectrum of the reflected wave, and identify the type of road according to the difference in spectral analysis; or use the colour matrix and the grey cooccurrence matrix to extract features, based on the support vector machine to achieve image classification, through statistical data to achieve the mapping relationship between the image and TRFC (Leng et al., 2021). Accurate TRFC information can be obtained under certain conditions, but the work restrictions are large and easily affected.

Doumiati et al. (2010) used the wheel speed signal of the magnetic sensor and the steering wheel angle signal of the optical sensor to predict the tyre lateral force and sideslip angle. This method uses more measured values and is relatively complex. Shinmoto et al. (1997) developed an optical positioning sensor for measuring the displacement of the tyre contact patch relative to the rim to estimate TRFC. Tuononen (2008) used an optical sensor to estimate the deformation of the side walls. The sensor can measure the deflection of the tyre relative to the rim, and the vertical force of the tyre is better measured, but the calculation of the tyre force is not sensitive to the recognition of the rotation angle, and further research is still needed on durability and energy issues. Based on the maximum likelihood estimation, Hou et al. (2021) solved the lidar reflection intensity distribution model parameters of common types of pavement on structured roads, and established a typical pavement database based on this to sensitively detect surface mutations, and it was robust to different lighting conditions at day and night, but its scope of application needs to be further verified under more complex lighting conditions.

5.2 Acoustic sensor

The acoustic sensors installed in the tyres can be used to ‘listen’ to determine the deformation of the tyres and TRFC. Some acoustic sensors can also classify the types and conditions of the road surface according to the noise of the road surface, such as asphalt pavement, concrete pavement, dry pavement or wet pavement, etc. The identification system composed of acoustic sensors is called the Acoustic Road Type Estimation (ARTE) system. The system can be composed of a cheaper microprocessor and an internal interface, which reduces the size and power consumption, so that it can be integrated and installed in the car (Alonso et al., 2015). Kalliris et al. (2019) proposed a wet road condition detection method based on acoustic measurement, which proves the feasibility of acoustic measurement and machine learning algorithms to distinguish between dry, wet and wet roads. The estimator improves the classification accuracy and robustness, and the equipment cost required for acoustic measurement and processing is relatively low. Signal processing methods in the fields of acoustics and speech recognition are used to extract road conditions and characteristics, and artificial neural networks or support vector machines are used for classification. There are also studies that use microphones installed in fixed road positions to record the noise generated when vehicles pass by, and use the recorded data to estimate road conditions (Kongrattanaprasert et al., 2010).

5.3 Intelligent tyre

Due to the integration, miniaturisation and economy of sensors, through the installation of various sensors in the tyres, the tyres have changed from ‘passive’ to ‘active’ to receive various types of information while driving, and intelligent tyres have been developed to study TRFC estimation and other issues. Intelligent tyres install sensors inside the tyres. Compared with installing sensors in other locations of the vehicle, intelligent tyres respond more directly and sensitively to road conditions (Yang et al., 2022). The research on intelligent tyres can be divided into the following 4 types (Fu et al., 2022):

1 sonic type intelligent tyres

2 optical type intelligent tyres

3 piezoelectric type intelligent tyres

4 acceleration sensor type intelligent tyres.

Wang et al. (2020) built intelligent tyres to obtain tyre-road surface action information more directly. Starting from the statistical characteristics of the signal, machine learning algorithms were used to identify and classify roads, and support vector machine classifiers were used to identify switching road conditions. Jeong et al. (2021) proposed an estimation algorithm that combines intelligent tyres and vehicle dynamics, using a linear load transfer model as the vehicle dynamics model, used Multi-input multi-output (MIMO) to estimate dynamic and static tyre loads and vehicle parameters such as tyre dynamic, static loads and vehicle parameters, with fast sampling rate and strong robustness. Mendoza-Petit et al. (2022) combined the LuGre model with the results of strain-based intelligent tyres to estimate the friction limit based on the relationship between the friction force applied to the tyre and the normal load.

However, the rapidly changing tyre state environment and harsh working conditions bring instability and unreliable performance to the intelligent tyre system. In order to overcome the challenges of being applied to real vehicles, the intelligent tyre system should be reasonably designed to maintain strong real-time, short-term perception and long-term robustness in complex application scenarios. Xu et al. (2022) combined the development and training of three different machine learning technologies, and the intelligent tyre system with a three-axis acceleration sensor installed in the inner layer of the tyre can effectively and accurately predict the tyre force under different operating conditions. García-Pozuelo et al. (2019) used a real-time physical model based on tyre carcass strain and/or displacement measurement to describe the dynamics of intelligent tyres. In this literature, it was proposed that the flexible ring model established on the basis of viscoelasticity can reproduce the tyre dynamics of concentrated and distributed forces by introducing discrete methods, and the longitudinal dynamics of tyres can also be analysed in real-time.

Sensor-based estimation methods are applied to both cause-based methods and effect-based methods. In the former, the dynamic response of vehicle motion is obtained, and in the latter, the road surface information obtained by the sensor is measured and processed to obtain the influence of road conditions on the TRFC. Taking into account the improvement of the robustness of parameter estimation and the dynamic characteristics of the vehicle, this method needs to be used in conjunction with the model-based method under most operating conditions, otherwise the accuracy will be greatly reduced when the experimental conditions deviate from the conditions of algorithm training. The following table summarises some sensor-based TRFC estimation methods, as shown in Table 4.

Table 4 Summary of sensor-based methods

Estimation method	*Sensors*	*Brief description*
Cause-based	Camera and optical sensor	It is used to measure the light absorption and scattering characteristics of roads to identify water and other substances on the roads and obtain road surface information
	Acoustic sensor	The identification system composed of acoustic sensors is used to determine the deformation of the tyre and the friction coefficient of the road surface
Effect-based	Tyre tread sensor	Monitor the interaction between the tyre and the road surface through different sensors inside the tyre
	Intelligent tyre	Integrated installation of various types of intrauterine sensors to improve the perception and responsiveness of the road surface

6 Neural network-based estimation

In order to make up for the shortcomings of vehicle dynamics-based methods, such as the time denaturation of tyre and suspension parameters, the high requirements of sensors on the working environment, and the reduction of the number of data points obtained, neural networks are often used to describe the suspension behaviour of tyres and wheels, and then neural networks optimised by genetic algorithms are used to identify the TRFC. Neural networks are an important branch in the field of machine learning. They have strong intelligent processing capabilities such as self-learning and complex relational mapping. The neurons in the neural network represent the weight parameters in the network with interconnected line segments, and bias and activation functions are set in the neurons to enhance the nonlinear expression ability of the network. It can learn potential statistical characteristics in noisy datasets. Under the nonlinear working conditions of vehicle driving, the application of this method has achieved good test results (Matuško et al., 2008).

The time delay neural network (TDNN) can detect TRFC under the excitation of lateral forces, avoiding the use of standard tyre mathematical models, Ribeiro et al. (2020) used this method to independently estimate TRFC on each wheel, providing an estimate of the lower root mean square (RMS) error. It requires less computing time at every moment and may be the best choice for real-time implementation in embedded systems. But it requires a sufficient level of horizontal excitation to correctly identify friction, and it requires a sufficient and representative database. Guo et al. (2023) combined pre-obtained road information from the on-board camera and used a lightweight convolutional neural network (CNN) to identify road types with a typical TRFC range, used the UKF method to directly estimate TRFC based on the dynamic state of the vehicle. Li et al. (2022) improved the RBF neural network structure through the K-means algorithm, and used the Double Radial Basis function and EKF (DRBF&EKF)

method that adaptively adjusted the network structure to improve the estimation accuracy, realised the dynamic joint estimation of the vehicle's centroid sideslip angle and TRFC. This method used the multivariate analysis method of principal component analysis (PCA) of high-dimensional dataset reduction to extract the characteristic parameters of the principal element and established a TRFC estimator. Other than that, Lin et al. (2021) proposed a TRFC estimation method based on improved Keras model, Wu et al. (2021) proposed a method for identifying TRFC based on Elman neural network, the average absolute error, RMS and absolute percentage error were improved respectively, proving the effectiveness and reliability of these two methods.

However, since the mapping relationship of neural networks is strongly dependent on experimental data, it is difficult to explain the mechanism of the mapping relationship. In addition, the neural network applied to vehicle road parameter estimation has a slow convergence rate, and it is difficult to determine its numerical stability, making the estimation accuracy unable to meet the requirements of driving under various operating conditions.

7 Estimation of TRFC of distributed drive vehicles

The most direct difference between TRFC estimation method for distributed drive electric vehicles and the estimation method of traditional vehicles is that: since the longitudinal force of each tyre can be accurately estimated in real-time, the speed and torque of the in-wheel motor can be directly obtained, so the vehicle yaw torque caused by the longitudinal force of the tyre, and the amount of change in the sideslip angle of the vehicle's centroid caused by the yaw torque can be accurately obtained, which provides convenience for the observation of these dynamic states. On the basis of KF algorithm and its improved algorithm, various parameters of distributed drive vehicles (such as longitudinal friction coefficient and transverse friction coefficient) are integrated to make a more accurate estimate of TRFC (Chen et al., 2015; Wang and Li, 2020; Ping et al., 2019; Zhang et al., 2019).

Chen et al. (2022) proposed a longitudinal and transverse collaborative estimation algorithm based on Adaptive Square Root CKF (ASRCKF) and partition Similarity Principle (SP), the vehicle status and tyre-road peak friction coefficient of distributed drive vehicle were estimated. The algorithm is conducive to estimating μ_{max} in the nonlinear region. The experimental and simulation results showed that the estimation method can achieve good performance under different operating conditions. Based on the federal volume KF theory, Wu et al. (2021) established a nonlinear 3-degree-of-freedom vehicle dynamics model, obtained the state space equation, performed multi-source fusion of sensor signals, and used the vehicle dynamics theory to establish an algorithm estimator to achieve the accuracy and stability of the state estimation of distributed drive wehicle. Wang et al. (2015) used the upper-level controller of the Linear Quadratic regulator (LQR) and the lower-level controller of the stable tyre working area. The brush tyre model was used to estimate TRFC by combining the longitudinal force and the transverse force, respectively. Xiong et al. (2020) designed the estimation method of TRFC under different excitation conditions in different working conditions, used the vehicle state parameters to determine which excitation conditions were satisfied, fuzzy deduced the current longitudinal, lateral tyre force can reach the limit, and designed the fusion observer to fuse the estimation results accordingly. The observer can quickly estimate the peak TRFC and maintain high estimation accuracy. Chen et al. (2018) proposed a method for estimating the longitudinal force and sideslip angle of distributed drive electric vehicles based on observer iteration and information fusion. The observer estimation strategy based on observer iteration and information fusion realised the estimation of the lateral force. The Luenberger observer was used to achieve a priori estimation of the measured values of fewer sensors, and then EKF was used to achieve higher-precision posteriori estimation. The estimation of the Luenberger observer was used as the EKF input of the pseudo-sensor, and the fuzzy weight controller was used to enhance the adaptive ability of the observer system.

8 Conclusion and perspectives

Accurate and fast estimation of the TRFC plays a crucial role in the autonomous driving technology of conventional engine driven and distributed drive electric vehicles. This paper presents a comprehensive review of the current research on TRFC estimation methods, discussing the principles, advantages and disadvantages of each method from the mainstream filter and observer techniques for parameter estimation; the principles and research development of model-based estimation methods are presented in three directions: vehicle dynamics model, tyre model and slip-slope; the three parts of optical sensors, acoustic sensors and intelligent tyres are used to classify the interaction between tyres and the ground, and the monitoring of road surface quality, this sensor-based estimation method generally has higher requirements for sensors, and is limited by certain road surface types and experimental environments; the neural network estimation method is based on the combination of traditional estimation methods and intelligent information estimation methods., it is a data-driven artificial intelligence estimation method that does not depend on the reference vehicle model and avoids the problems that arise in traditional estimation method; distributed drive electric vehicles can directly obtain some parameters based on their structural characteristics, based on their structural characteristics, many research scholars have proposed new parameter estimation methods and proved the accuracy and reliability of these estimation methods.

The following is the outlook for the future TRFC estimation method:

1 On the basis of traditional filtering algorithms and observer algorithms, ant colony algorithms, genetic algorithms, forgetting factors, etc. are integrated or multiple algorithms are combined to update state parameters and observe and estimate at the same time. Compared with traditional algorithms, the new algorithm has better adaptability, real-time, reliability and accuracy.

2 Improve the vehicle and tyre model to achieve a more accurate estimation ability for certain parameters and performance, and combine the filtering algorithm and the observer algorithm to further improve it. Taking into account the time-varying vehicle parameters and environmental parameters during driving, the real-time collaborative estimation of other parameters such as vehicle centroid sideslip angle, longitudinal speed, centroid position, tyre characteristics, etc. can improve the state estimation accuracy, robustness and adaptability.

3 Use the multi-sensor information fusion method to make full use of data resources and reduce the requirements for sensors; build a pavement type database to improve

the classification accuracy and robustness, and improve the estimation accuracy under different road quality and working conditions.

4 With the improvement of sensor technology and the development of tyre systems, the level of tyre intelligence will continue to be improved. The control algorithm of each sensor in the tyre will be built to combine the intelligent tyre with the vehicle control system to improve the ability to obtain state parameters and the ability to respond to parameters.

5 In the research of connected vehicles and the development and use of cloud computing technology, a large number of vehicle dynamic parameter calculations can be transferred to cloud computing to reduce the calculations in the driving process of the vehicle. According to the upload and calculation of real-world environmental parameters by multiple vehicles, parameters such as the TRFC on the same route can be obtained and shared, and the TRFC at the next time can be predicted to facilitate the decision-making and behaviour planning of autonomous driving technology. Through on-board wireless information and communication technology, realising a collaborative environmental perception systems and achieving a comprehensive network connection and information interaction integration between vehicle-to-infrastructure (V2I) and vehicle-to-everything (V2X). Through the intelligent transportation system, real-time road surface, temperature and other environmental information is transmitted to the intelligent car, and the on-board estimation results are predicted in advance or corrected in real-time, so as to improve the overall intelligent driving level and travel efficiency of the vehicle, and reduce the incidence of accidents.

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