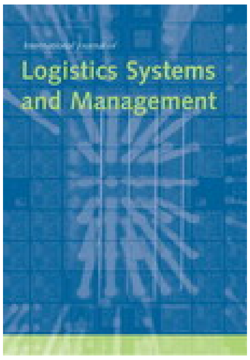

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Distribution in the large-scale retail trade industry: requirements for vehicle routing problems

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Abstract: The purpose of this paper is to outline and establish the characteristics and requirements of vehicle routing problem (VRP) in the large-scale retail trade (LSRT) industry. Characteristics and operational constraints of the VRP for the LSRT industry are described after analysing some variants of the state-of-the-art models in the present literature. Successively, a comprehensive definition for this specific class of problems is provided, along with taxonomy and a new VRP formulation. The research reveals that state-of-the-art VRP models often fail to thoroughly describe real-world LSRT instances, leading to seldomly applicable models. For this reason, the paper provides guidance to design and apply a model for solving real-world transportation problems in the LSRT industry. Hence, our paper establishes requirements and criteria for obtaining a VRP applicable to real-world instances of the LSRT industry.

Keywords: distribution; logistics; routing; nonlinear programming; retailing.

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1 Introduction

The relevance of physical distribution and logistics is increasingly growing, leading companies to put large efforts in the correct definition, design and execution of these activities. Among different industries, the role of logistics activities in the large-scale retail trade (LSRT) industry is essential for performing daily operations, ensuring high service levels and sustainable revenue streams. For this reason, LSRT companies strive for the continuous optimisation of distribution operations and consequently for an endless cost reduction.

Last-mile delivery represents one of the most crucial operations for LSRT companies. These problems are generally treated under the branch of mathematics known as ‘operations research’, which formulates the transportation problem as a ‘vehicle routing problem’ (VRP) and solves it through the aid of specific software called ‘transportation management system’ (TMS). These systems represent the core of distribution activities for the LSRT industry.

The VRP mathematical formulation represents an extension of the well-known and largely studied travelling salesman problem (TSP), thus being an NP-hard problem with no known algorithm capable to find an optimal solution in a polynomial computational time (Toth and Vigo, 2002; Johnson and Garey, 1979). In the present literature, a huge number of variants of the VRP problem have been proposed, and a general taxonomy has been introduced to consider all the possible characteristics of a VRP problem (Braekers et al., 2016; Eksioglu et al., 2009). In this context, a more recent class of routing problems that is increasingly getting attention among the researchers is the ‘rich vehicle routing problem’ (RVRP) category. This group of instances, that has been deeply studied in the contributions of Cruz et al. (2014) and Lahyani et al. (2015), comprises routing problems that directly stem from real-world applications. However, due to the specific nature and complexity of the VRP, the proposed mathematical models – and their solutions – often fail to thoroughly describe the huge number of operational needs of transportation problems among the LSRT industry. Indeed, the quest for complexity reduction has frequently led researchers to introduce models that, despite the mathematical smoothness, are practically inapplicable for real-world instances of LSRT companies.

Therefore, the objective of this research work is twofold. Firstly, it aims at analysing various scientific contributions present in the literature and at investigating the applicability of the proposed VRP models to the LSRT industry instances. This allows to individuate models or features suitable to the transportation problems of LSRT companies, and also to identify eventual shortcomings of the present literature. Secondly, the paper has the goal to provide a clear definition of the requirements of a VRP for the LSRT industry, both in terms of general characteristics and operational constraints, and to propose a new formulation based on the experience of a large medium-sized LSRT company operating in the south-east of Italy. The proposed model is then applied to the aforementioned industrial case and solved through the commercial solver BARON 19.12.7 available on the NEOS server, through the usage of the AMPL language. The results of the computational experiment are also provided. It is worthy to note that the core objective of our research is to establish the criteria for defining and implementing a VRP model in this specific industry: the proposed formulation hence represents a practical example of how the proposed requirements should be applied, and of their consistency with real industrial cases.

Thus, the paper is divided into four different sections. Section 1 performs a large-scale literature review in order to identify useful contributions and shortcomings of the present literature; Section 2 describes the required characteristics of a VRP-LSRT problem, and introduces its taxonomy and definition. In addition, Section 3 provides the VRP-LSRT new mathematical formulation and the results of the computational experiment. Lastly, Section 4 draws up some conclusions and individuates possible further developments of this research work.

2 Literature Review

Dantzig and Ramser (1959) introduced the 'truck dispatching problem' as a generalisation of the TSP, proposing for the first time an instance of VRP in 1959. The considered problem aims at fulfilling the demand of several customers, distributing the goods from a single central storage point (depot) through capacitated vehicles, and minimising the total transportation costs. Therefore, the VRP has been introduced as a generalisation of the TSP, and its formulation has been successively adapted to solve different variants of the original problem.

Even though some VRP variants could be applied to different industries' instances, the LSRT transportation problems require specific formulations because of the large number of critical variables and constraints to be considered. For this reason, an analysis of the body of literature regarding the mathematical modelling of transportation problems is required. Hence, in what follows a thorough literature review is reported: the scope is to analyse and report various VRP models introduced by different scientific contributions, to evaluate their applicability to the LSRT industry's instances and to identify their eventual shortcomings.

2.1 Capacitated vehicle routing problem

The original formulation of the capacitated VRP has been introduced by the contribution of Dantzig and Ramser (1959), which represents the basic VRP version. This problem had the objective to define a specific number of distribution routes, such that:

1. each distribution route starts and ends in the depot
2. each vertex (customer) is touched by only one distribution route
3. the sum of customers' demand to be fulfilled by one distribution route must not exceed the vehicle capacity.

It is possible to observe that the original VRP formulation considers an asymmetric cost matrix, hence the model is also known as 'asymmetric capacitated vehicle routing problem' (ACVRP).

A variant of the ACVRP is the 'symmetric capacitated vehicle routing problem' (SCVRP), which is defined upon a symmetric cost matrix (i.e., the routes connecting customers are not oriented). The SCVRP mathematical formulation has been introduced by the paper of Laporte et al. (1985), which also defines a new class of subtour elimination constraints.

2.2 Multi-depot vehicle routing problem

A common extension of the capacitated vehicle routing problem (CVRP) is the 'multi-depot vehicle routing problem' (MDVRP), which differs from the simple VRP since the goods distribution route can start from any depot and end at any depot. The mathematical formulation for the MDVRP has been introduced by the scientific contribution of Bhave and Kulkarni (1985), which is based on the well-known Miller-Tucker-Zemlin (MTZ) adaptation of the subtour elimination constraints (Miller et al., 1960).

2.3 Open vehicle routing problem

The 'open vehicle routing problem' (OVRP) is a variation of the VRP in which the vehicles are not forced to come back to the starting depot once they have completed the distribution route. Several applications of this VRP category have been developed and studied in the literature, since in many real cases the vehicles may have an end point that differs from the starting point. Examples from real world applications are given by the paper of Bektas and Elmastas (2007), which applies the OVRP to a school bus routing problem, by the paper of Bauer and Lysgaard (2015), that adapts the OVRP to optimise cable layouts for offshore wind farms, by the paper of Yu et al. (2016), which introduces the cross-docking activity among the OVRP, and by the paper of Erbao and Mingyong (2010), that develops a stochastic model with fuzzy customers' demand for a specific instance of the OVRP.

2.4 Vehicle routing problem with pickup and delivery

The 'vehicle routing problem with pickup and delivery' (VRPPD) was first conceived in 1989 as a model for optimising the simultaneous delivery and pickup of books of a public library in Franklin County, Ohio (Min, 1989). In this case, differently from the other variants of the VRP, each customer has both a delivery quantity and a pickup quantity, and the vehicle must finish its distribution route at the depot to unload the collected units.

Since its introduction, several scientific contributions have addressed the VRPPD and many algorithms for its solution have been developed. The paper of Barbeglia et al. (2007) provides a scheme for the VRPPD classification and an overview of the scientific contributions available in the present literature; differently, the paper of Zhou et al. (2017) applies the VRPPD for solving last-mile delivery problems arising in the context of e-commerce distribution. Moreover, among the models present in the literature, the pickup and delivery can either occur simultaneously, namely each vertex has a delivery quantity and a pickup quantity, or in a mixed way, namely the vertex set is divided into the delivery set of vertices and the pickup set of vertices. However, the 'vehicle routing problem with simultaneous pickup and delivery' (VRPSPD) can be seen as a generalisation of the mixed model. The paper of Ai and Kachitvichyanukul (2009) proposes a comprehensive mathematical formulation of the VRPSPD, which generalises three existing formulations for the specific problem.

2.5 Vehicle routing problem with time windows

A further extension of the VRP is given considering time windows in which a delivery/pickup can be performed. Indeed, in several real-world distribution applications customers request the goods to be shipped/collected in a specific time span, in order to be ready for their activities. This problem is generally known as the 'vehicle routing problem with time windows' (VRPTW), and several mathematical formulations have been proposed in the literature. Examples are given by the distribution of pharmaceuticals from hospital to patients (Liu et al., 2013) and by the multimodal transportation with transhipment operations (Rais et al., 2013).

2.6 Vehicle routing problem with multi-compartments

Many vehicles have more than one compartment, thus allowing the transportation of various categories of product. This means that different product categories could travel on the same vehicle – for instance, fresh foods and non-fresh foods – and that their distribution routes could be planned accordingly. In the present body of literature, this problem is known as the 'vehicle routing problem with multi-compartments' (MCVRP).

A first application of the MCVRP is proposed by the paper of Reed et al. (2014), which discusses the problem of waste recycling. A similar application is considered in the paper of Henke et al. (2015), where a MCVRP is shaped to describe the glass waste collection. Differently, the contribution of Martins et al. (2019) discusses the application of a MCVRP model with product-oriented time windows to the grocery distribution.

2.7 VRP and risk considerations

Distribution routing is an essential activity in many industries and some critical decision parameters can differ among the various analysed sectors. When planning the distribution of chemical hazardous substances through vehicles, a new critical parameter should be introduced: risk. Indeed, the risk of an eventual accident that could occur along the distribution route (e.g., fire) heavily influences the distribution planning and should be considered. Therefore, several authors in the present literature extended the VRP with the consideration of different kinds of risk.

A first well-studied application of the VRP is the so-called 'hazmat transportation', namely the distribution of hazardous materials (e.g., fuel distribution). For instance, the paper of Cuneo et al. (2018) addresses the distribution of fuels among the territory of the Province of Rome (Italy) for the total erg oil company. Other examples of scientific contributions addressing the hazmat transportation are given by the paper of Carotenuto et al. (2007) and more recently by the paper of Du et al. (2017).

The concept of risk is particularly important also among the cash-in-transit (CIT) industry groups, dealing with the physical transfer of currency and other valuable items. This transfer typically occurs between cash deposits/banks and retail/financial organisations; hence, the transportation activities can be formulated mathematically as a capacitated VRP. Talarico et al. (2015) introduces a variant of the VRP for the CIT industry that considers the vehicle's risk of being robbed. Another application of the VRP can be found in the paper of Chomchalao et al. (2018), which proposes a mathematical formulation for transportation routing in Thailand considering the sabotage risk.

2.8 VRP and particular products

Another important area of application of the VRP is the management of particular goods, specifically perishable or cold products. Differently from non-fresh products, these particular categories of goods (e.g., dairy products, fruits and vegetables) require tailored management policies. The literature has addressed this issue in several different forms, some of which are reported as follows.

The paper of Hiassat et al. (2017) extends the VRP all along the supply chain, considering a 'location-inventory-routing problem' for managing some categories of perishable products. In the same context of perishable products, the contribution of Chen et al. (2019) proposes a MCVRP with time windows and multi-compartments to optimise distribution activities of a cold-chain distribution company operating in Shanghai. Lastly, the paper of Alkaabneh et al. (2020) introduces environmental considerations within a VRP model with perishable products.

2.9 Green vehicle routing problem

A more recent area of investigation of the VRP is related to sustainable instances, which introduce environment-related parameters among the mathematical models, such as: noise, pollution, carbon emissions, etc. Research works analysing these problems conceived the so-called 'green vehicle routing problem' (GVRP). For instance, the paper of Soleimani et al. (2018) applies the VRP to the activity of collecting and distributing original and remanufactured goods, with the aim of minimising the total impact on the environment. A bi-objective GVRP is present also in the contribution of Sawika et al. (2016), whose goal is reducing the total carbon emission of the routing activities. Following the same approach, Sawika et al. (2017) performs a green logistics multi-objective optimisation for a Spanish grocery company.

2.10 Rich vehicle routing problem

VRP instances that directly stem from real-world applications often require a large number of complicated constraints, hence paving the way for a new category of routing problems. This category of problems is defined as 'RVRP' and has been recently addressed in the literature.

The paper of Cruz et al. (2014) attempts to provide a general definition of RVRP, obtaining insights from an analysis of several different scientific contributions on VRP. Successively, the contribution of Lahyani et al. (2015) extends their research work: the authors perform a literature review in order to provide a wide taxonomy and a definition for the RVRP.

Another example is given by the contribution of Goel and Gruhn (2008), which proposes a general VRP formulation that encompasses many real-life requirements and solve the problem related to European air-cargo transport. Differently, the contribution of Osaba et al. (2017) describes the problem of daily delivery of newspaper and formulates it as an asymmetric VRP with simultaneous pickups and deliveries, with variable costs and with restrictions on specific paths.

2.11 VRP and outsourcing considerations

Recently, outsourcing has developed as a common practice in the freight forwarding sector, where companies frequently outsource parts of the logistics activities to subcontractors (Vidal et al., 2019). In this context, the paper of Krajewska and Kopfer (2008) is one of the first notable contributions to address the outsourcing topic, introducing an objective function which depends on: a tour basis; a daily basis; a freight consolidation depending on the tour distance.

Differently, the paper of Stenger et al. (2013) and Dabia et al. (2019) propose objective functions which take into consideration volume discounts. The topic of volume discounts is a recurring aspect in the logistics context, which however drives to a greater complexity the resulting VRP models. Indeed, both the abovementioned contributions introduce a nonlinear cost function for the proposed models. Lastly, the paper of Stenger et al. (2019) define an exact branch-and-price-cut algorithm for a VRP with private fleet and common carrier, thus evaluating the possibility to outsource logistics activities rather than carrying out them through the organisation's private fleet.

2.12 Main assumptions and shortcomings of the present literature

From the literature review, it seems that none of the analysed formulations can be directly applied to model and solve a real-world LSRT instance of the VRP and there is no evidence of VRP-LSRT problems in the present scientific literature. Indeed, companies of the LSRT industry have several operational constraints that should be featured in the proposed models, which however are often overlooked by the scientific contributions. This hinders the applicability of VRPs for LSRT industry.

A first shortcoming directly regards the objective function, since it is usually aimed at minimising the total distribution cost or maximising the total profit without considering relevant parameters for the LSRT companies. Indeed, many formulations define an objective function where the total transportation costs only increase linearly with the travelled distance. With this assumption, total transportation costs do not consider the number of vehicles used for the distribution.

However, when the objective function comprises the vehicles activated for distribution, the formulations often fail to consider other significant parameters. In real-world situations, distribution costs depend upon several factors, namely: number of activated vehicles for distribution; distribution provinces; total travelled distance of a single route; owner of the vehicles. Indeed, the fare scheme of a vehicle can be variable in relation to these factors, frequently resulting in a nonlinear objective function. Therefore, any objective function should consider these aspects and minimise the total distribution costs leveraging on these parameters. Additionally, when notable efforts have been placed towards the definition of objective functions that shape the real business needs of logistics companies (see for instance Subsubsection 2.11), no application is provided to LSRT instances.

Moreover, several formulations adopt degree constraints imposing the activation of all available vehicles, which is unrealistic for the LSRT industry. Differently, other formulations do not adopt an upper limit to the number of available vehicles that can be activated. Also this assumption is wrong, since LSRT companies operate with limited resources. Furthermore, even though some scientific contributions provided mathematical models with the adoption of incompatibility constraints, there is no evidence of the application of these constraints to the LSRT industry's specific operational needs. Indeed, none of the identified and reported models constrains the vehicle tours in relation to specific road access, to customer-vehicle incompatibilities or to depot-vehicle incompatibilities. Indeed, limited-traffic zones that prevent the vehicles flow at specific timings are present in any city; moreover, some vehicles could be impeded to reach a particular customer or a depot, especially when distribution should take place in poorly urbanised areas or in old towns. Hence, incompatibility constraints have to be included in order to obtain a VRP applicable to LSRT instances.

In addition, when considering multi-depots, it is often supposed that each SKU is available with the right amount of quantity at each depot. However, this represents a strong assumption, since it is uncommon that the stock of a specific product is kept and duplicated in different warehouses. Therefore, the starting depot should be determined also in relation to the stock availability (and not only in relation to cost factors), or alternatively the possibility to perform transhipments could be added to the VRP model.

According to the previous considerations, it is possible to state that the specifications and requirements of a transportation problem related to the LSRT industry are hardly met. Only few papers have addressed LSRT instances and there is no clear understanding of the LSRT transportation problems' main requirements and specifications, hence often leading to complex theoretical models seldomly applicable to practical cases.

3 VRP-LSRT definition and taxonomy

Last-mile delivery activities in the LSRT sector require companies to transport several different items to many customers, from one or more distribution centres (DCs) and with a specified fleet of vehicles. In this context, when all the customers' orders are collected, the vehicle routing is performed through a TMS system and the distribution routes for the next working day are provided to the truck drivers. It is straightforward that routing activities are executed with the objective to minimise the total distribution cost – or equivalently to maximise the total distribution profit – and to respect the large multitude of operative constraints that characterise the routing tasks. These complex operational real-world constraints and the unique characteristics of the transportation problems for the LSRT industry directly impact the VRP structure, shaping the VRP-LSRT instance in a distinctive fashion. Hence, to obtain a clear overview of this kind of problem, the characteristics of the objective function and of the mathematical constraints of the VRP-LSRT are thoroughly defined in the following subsections.

3.1 Objective function characteristics

The objective function of a VRP-LSRT problem should have the prime scope to minimise the total transportation cost, eventually introducing penalty expressions to incorporate secondary objective (e.g., minimisation of the trucks' waiting time). The distribution fares of a vehicle are typically described by the sum of two different terms:

1. *Fixed activation cost of a vehicle (c):* This cost is borne when the vehicle is adopted for performing the distribution activities and does not depend on the overall travelled distance of the distribution route;

2 *Variable cost per km (g):* This cost only depends on the total route travelled distance..

Hence, introducing a binary variable $f \in \{0, 1\}$ that defines if a vehicle is used for distribution, and considering $x \geq 0$ as the variable that represents the number of travelled km in a distribution route, the total cost of that vehicle follows the expression:

$$Total\ cost = c \cdot f + g \cdot x$$

The definition of the total distribution cost function is straightforward when the LSRT company performs the transportation activities with its own fleet of vehicles: the activation cost and the variable cost per km are fixed and set by the company, therefore they can be considered as the estimation of an average distribution cost. In this case, c and g represent fixed parameters, and the objective function is a simple linear expression that only depends on f and x.

However, since in real-world applications many LSRT companies directly outsource the transportation activities to 'third-party logistics' (3PL), distribution fares may vary in relation to different factors. Hence, various distribution fares should be considered and the objective function must be shaped accordingly. More specifically, the activation cost of a vehicle could depend upon the province in which the customer is located, while the variable cost per km could vary in relation to specific thresholds of travelled distance. In this way, the cost parameters (c, g) of a vehicle should be represented in the model as decision variables, which often lead to a nonlinear formulation.

Three different transportation pricing schemes are introduced as follows, which represent a great portion of real-world distribution situations among the LSRT industry:

- *Fare scheme 1 (vehicle activation cost depends upon provinces):* The leading variable is represented by the provinces of the stores served in the distribution route. Indeed, a 3PL could define a variable activation cost of a vehicle in relation to the provinces served in that route (c is variable), with a null variable cost ($g = 0$). In this case, 'vehicles distribution fare' constraints must be introduced among the mathematical formulation to obtain a variable activation cost. An example is provided by Table 1.

Table 1 Example of fare scheme 1

Province	*Activation cost*
RM	€200/vehicle
NA	€300/vehicle
MI	€300/vehicle

- *Fare scheme 2 (vehicle variable cost per km depends upon the travelled distance):* The leading variable is represented by the number of travelled km in the distribution route. Indeed, up to a certain threshold of travelled distance, the distribution fare is fixed even with few travelled km to cover the fixed expenses of the 3PL (e.g., up to 100 km the fare is €100/vehicle); differently, when the total route's distance exceeds the threshold a variable cost per km is incurred (e.g., above 100 km the fare is €1.07/travelled km):

$$Fare_{km} = \begin{cases} c = €100 \text{ and } g = 0, & \text{if } x \leq 100 \text{ km} \\ c = 0 \text{ and } g = €1.07\,/\,\text{km}, & \text{if } x > 100 \text{ km} \end{cases}$$

Note that also this fare scheme requires the introduction of vehicles distribution fare constraints within the model;

- *Fare scheme 3 (vehicle activation cost depends upon the route revenue):* The leading variable is given by the total revenue generated through the sales of the transported load. Hence, this fare scheme overturns costs to the customer in proportion to the transported turnover as follows:

$$Fare_{turnover} = Turnover \cdot (\%_{turnover})$$

Differently from the other fare schemes, this pricing model does not require the introduction of vehicles distribution fare constraints.

It is possible to observe that while ‘fare scheme 1’ has been already discussed in other scientific contributions (see for instance Ceschia et al., 2011) – though not related to the LSRT industry – no evidence of ‘fare scheme 2’ and ‘fare scheme 3’ is present in the scientific literature. Moreover, other pricing schemes that could be adopted and tailored to the case of a LSRT industry can be found in the paper of Vidal et al. (2019), which provide several examples of mathematical model with outsourcing considerations (see also Subsection 2.11).

3.2 Main logical and operational constraints

Two big categories of constraints should be introduced to describe the VRP-LSRT set of constraints. Indeed, while there is a specific subset of constraints that is common to any class of VRP and that is needed to correctly create the distribution routes, it is possible to introduce a different subset of constraints that is typical of the LSRT industry’s instances. Thus, in what follows a distinction between ‘logical’ and ‘operational’ subset of constraints is provided.

The logical subset is common to any class of VRP and comprises all the constraints that are needed to declare the decision variables and to specify bounds that ensure the right formation of a distribution route. An example of this class of constraints is given by the ‘flow conservation constraints’ or by the ‘subtour elimination constraints’.

On the other hand, the operational subset is typical of the LSRT instances and comprises all the constraints that express specific needs of the industry. These constraints are essential to define a VRP suitable to the operational needs of the industry, and represent the core of the VRP-LSRT mathematical formulation. An indication of the constraints comprised within the operational subset is provided as in Table 2. Note that the last column of the table specifies why a constraint is mandatory or optional for a VRP-LSRT problem.

Table 2 Operational subset of constraints for the VRP-LSRT

Class of constraint	*Type of constraint*	*Operational needs*	*Constraint specification*
Capacity constraints	Vehicle capacity	Imposes the maximum number of movementation units that can be transported by a specific vehicle	Mandatory: vehicles are always limited in terms of movementation units to be transported, hence a capacity constraint should be considered
Operations type	Pickup and/or delivery	Establishes that each customer can be a delivery point and/or a pickup point	Mandatory: customers of LSRT companies both can require deliveries of finished product and pickup of returnable materials, hence this characterisation should be provided
	Backhauls	Defines two different classes of nodes ('linehaul' and 'backhauls') and ensures that all the deliveries (at 'linehaul' customers) are performed before the pickups (at 'backhauls' customers), relying on the assumption that vehicles are rear-loaded	Optional: backhauls are applied only in case of specific partnerships and agreements between relevant logistics stakeholders, hence this specification is not always applied
Vehicles number	Limited/unlimited	States the maximum number of vehicles that can be activated for distribution	Mandatory: a limit in terms of available vehicles for distribution is always present and should be considered
Vehicles structure	Single or multi-compartments	Establishes if the vehicle has different compartments to transport various types of product	Mandatory: the vehicles structure is essential to effectively determine the number of trips and the transported goods
Multiple use of vehicles	Single or multi-trip	Imposes the maximum number of distribution routes (if present) that can be performed by a vehicle	Mandatory: a specification in terms of maximum number of routes to be performed by a single vehicle should be defined to properly schedule the routes
Vehicles incompatibility constraints	Vehicle-depot incompatibility	Defines if a vehicle can start the distribution route from a specified depot	Mandatory: to properly schedule the routes, incompatibility between vehicles and depots should be determined to avoid scheduling errors

Table 2 Operational subset of constraints for the VRP-LSRT (continued)

Class of constraint	*Type of constraint*	*Operational needs*	*Constraint specification*
Vehicles incompatibility constraints	Vehicle-customer incompatibility	States if a vehicle can serve a specified customer in its distribution route	Mandatory: to properly schedule the routes, incompatibility between vehicles and customers should be determined to avoid scheduling errors
	Road incompatibility	Establishes if a vehicle can use a specific road within its distribution route	Mandatory: to properly schedule the routes, incompatibility between vehicles and roads should be determined to avoid scheduling errors
Vehicles operations policy	Loading/unloading policy	Imposes the chronological order in which the operations must be fulfilled	Optional: it is rare, though possible, that operations are performed on a chronological order, hence this specification is not always applied
Vehicles distribution fare constraints	Activation/fixed cost	Defines how to calculate the activation or fixed cost of a vehicle	Optional: the variability of activation or fixed cost of a vehicle is present only if specific distribution fares are applied, and should be considered only in that case
	Variable cost	Establishes how to calculate the variable cost of a vehicle	Optional: the variability of variable cost of a vehicle is present only if specific distribution fares are applied, and should be considered only in that case
Time window type	On customers	Establishes the time windows in which a specific customer must be served	Mandatory: LSRT companies always provide time windows, either strict or hard, for performing the logistics activities, hence this specification should be reported in the model
	On depots/hubs	Defines the time windows in which the loading/unloading operations must be performed at a specific depot	Mandatory: depots always provide time windows for performing the loading/ unloading activities, hence this specification should be reported in the model
	On drivers/vehicles	States the time windows in which a specific vehicle can be used for performing the distribution activities	Optional: drivers could provide time windows for performing the distribution activities, hence this specification should optionally be reported in the model

Table 2 Operational subset of constraints for the VRP-LSRT (continued)

Class of constraint	*Type of constraint*	*Operational needs*	*Constraint specification*
Time constraints	Service time limit	Imposes the maximum available time for performing the loading/unloading operations	Optional: the maximum time for performing the loading/unloading operations could be provided, hence this specification should optionally be reported in the model
	Waiting time limit	Establishes the maximum allowed waiting time of a driver before the start of the service time	Optional: the maximum driver waiting time could be provided, hence this specification should optionally be reported in the model
Drivers regulation constraints	Total duration of a route	Defines the maximum allowed time for performing the distribution activities	Mandatory: drivers are constrained by a maximum allowed time for performing the distribution activities, which should be reported in the model
	Total driving time of a route	States the maximum allowed driving time of a distribution route	Mandatory: drivers are constrained by a maximum allowed driving time for performing the distribution activities, which should be reported in the model
	Total distance of a route	Establishes the maximum allowed travelled distance for a distribution route	Mandatory: drivers are constrained by a maximum allowed travelled distance for performing the distribution activities, which should be reported in the model
	Total stops number of a route	Defines the maximum allowed number of stops in a distribution route	Mandatory: drivers are constrained by a maximum number of stops in a route for performing the distribution activities, which should be reported in the model
Green VRP constraints	Total fuel consumption of a route	Imposes the maximum fuel consumption of a distribution route	Optional: a limit in fuel consumption could be specified for distribution activities, hence it should not be necessarily defined in the model
Load-splitting constraints	Splitting allowed or not-allowed	States whether the delivery quantity for a certain customer must be delivered by a single route or can be split in two or more distribution routes	Mandatory: this specification is essential since it determines if a customer allows split deliveries, and should be reported in the model

3.3 VRP-LSRT taxonomy

Due to the many real-world operational constraints that can be found in transportation problems among the LSRT industry, the VRP-LSRT can be classified as a rich VRP. Even though a general taxonomy for the RVRP has been proposed by the paper of Lahyani et al. (2015), it seems that no contribution generally addresses and categorises the VRP among the LSRT industry. The same reasoning applies to the taxonomy proposed by Eksioglu et al. (2009) and revised by Braekers et al. (2016) that, even though it seems to be wider than the RVRP taxonomy of Lahyani et al. (2015), it fails to describe and categorise thoroughly application of the VRP model to LSRT instances. Indeed, the following shortcomings are identified in both taxonomies:

1. No specifications are provided in relation to the objective function type and vehicles distribution fare constraints in both the general VRP and RVRP taxonomies. These features are driven by the type of fleet of vehicles adopted for the logistics activities (e.g., private fleet, 3PL, etc.), which are missing in the abovementioned taxonomies.
2. Onsite service times are overlooked by both taxonomies, even though they represent essential elements in the LSRT context. Indeed, the service times (e.g., unloading lead times) represent an element that directly impact the duration of a route and should be therefore considered in the VRP model.
3. The RVRP taxonomy only considers the time windows structure in relation to the quantity of time windows per each customer, overlooking the type of time window (e.g., soft/hard). Note that this feature is essential in the LSRT context, since often fees are applied if a hard time window is not respected. From this consideration, another missing specification should be highlighted: both the general and RVRP taxonomies do not provide for the possibility to introduce penalty parameters, which should be introduced when hard time windows are applied.
4. Travel times are considered to be deterministic in the RVRP taxonomy, while in real-world applications they are often dependent on different variables (such as traffic). In the LSRT context, especially if dynamic input data are adopted, the VRP model should have this specification.
5. The RVRP taxonomy does not take into account specific characteristics to shape the model, such as the transportation network design, number of objectives and location of addresses for customers, which are however introduced within the general VRP taxonomy.
6. Other missing specifications are related to the operational characteristics of the VRP-LSRT problem, such as a detail of the drivers regulation constraints, vehicles incompatibility constraints, operations policy and vehicles structure. These elements are typical of operations of the LSRT industry, since they allow to shape the intrinsic features of the relevant problem stakeholders.
7. No specifications are reported in both taxonomies to take into account GVRP considerations. This represents a major shortcoming of the general VRP and RVRP taxonomies given the relevance of sustainability practices in the LSRT industry.

For these reasons, a taxonomy for the VRP-LSRT is here proposed, taking into consideration the VRP and RVRP taxonomies and the analysed scientific contributions.

Note that the objective of this taxonomy is to provide a clear guidance on how to design a VRP for the LSRT industry, thus being an effective connection between scientific theoretical models and industrial practices. The taxonomy is reported by Table 3.

Table 3 VRP-LSRT taxonomy

Taxonomy of vehicle routing problems within the large-scale retail trade industry (VRP-LSRT)			
1	Scenario characteristics		
	1.1	Input data	
		1.1.1	Static
		1.1.2	Dynamic
		1.1.3	Deterministic
	1.2	Decision management components	
		1.2.1	Routing
		1.2.2	Routing and driver scheduling
	1.3	Number of depots	
		1.3.1	Single
		1.3.2	Multiple
	1.4	Operations type	
		1.4.1	Pickup or delivery
		1.4.2	Pickup and delivery
		1.4.3	Backhauls
	1.5	Load splitting constraints	
		1.5.1	Splitting allowed
		1.5.2	Splitting not allowed
	1.6	Time horizon	
		1.6.1	Single period
		1.6.2	Multi-period
	1.7	Multiple use of vehicles	
		1.7.1	Single trip
		1.7.2	Multi-trip
	1.8	Onsite service times	
		1.8.1	Deterministic
		1.8.2	Movementation units dependent
	1.9	Time window structure	
		1.9.1	Soft time windows
		1.9.2	Hard time windows
	1.10	Operator of vehicles fleet	
		1.10.1	LSRT company (private fleet)
		1.10.2	Third-party logistics (3PLs) companies or outsourcing
		1.10.3	Mix of both

Table 3 VRP-LSRT taxonomy (continued)

Taxonomy of vehicle routing problems within the large-scale retail trade industry (VRP-LSRT)			
	1.11	Green VRP considerations	
		1.11.1	Limitations on the total fuel consumption
		1.11.2	Other limitations
		1.11.3	None
2	Problem physical characteristics		
Logical characteristics			
	2.1	Transportation network design	
		2.1.1	Directed network
		2.1.2	Undirected network
	2.2	Location of addresses (customers)	
		2.2.1	Customer on nodes
		2.2.2	Arc routing instances
	2.3	Objective function type	
		2.3.1	Linear function
		2.3.2	Nonlinear function
	2.4	Number of objectives	
		2.4.1	Single objective
		2.4.2	Multi-objective
	2.5	Objective	
		2.5.1	Minimisation of total cost
		2.5.2	Maximisation of profit
	2.6	Penalty parameters	
	2.7	Travel time	
		2.7.1	Deterministic
		2.7.2	Function dependent (a function of current time)
Operational characteristics			
	2.8	Capacity constraints	
	2.9	Vehicles type	
		2.9.1	Homogeneous
		2.9.2	Heterogeneous
	2.10	Vehicles number	
		2.10.1	Limited
		2.10.2	Unlimited
	2.11	Vehicles structure	
		2.11.1	Compartmentalised
		2.11.2	Not compartmentalised

Table 3 VRP-LSRT taxonomy (continued)

Taxonomy of vehicle routing problems within the large-scale retail trade industry (VRP-LSRT)		
2	Problem physical characteristics	
Operational characteristics		
2.12	Vehicles incompatibility constraints	
	2.12.1	Vehicle-depot incompatibility
	2.12.2	Vehicle-customer incompatibility
	2.12.3	Road incompatibility
2.13	Vehicles operations policy	
	2.13.1	Chronological order
	2.13.2	No policy
2.14	Vehicles distribution fare constraints	
	2.14.1	Activation/fixed cost
	2.14.2	Variable cost
2.15	Time window type	
	2.15.1	Restrictions on customers
	2.15.2	Restrictions on depots/hubs
	2.15.3	Restrictions on drivers/vehicles
2.16	Time constraints	
	2.16.1	Service time limit
	2.16.2	Waiting time limit
2.17	Drivers regulation constraints	
	2.17.1	Total duration of a route
	2.17.2	Total driving time of a route
	2.17.3	Total distance of a route
	2.17.4	Total number of stops of a route
2.18	Other specific constraints	

As it has been proposed by the contribution of Eksioglu et al. (2009), the VRP-LSRT taxonomy shows the 'scenario characteristics', namely all the parameters comprised in the problem scenario that do not regard the solution, and the 'problem physical characteristics', which differently directly constrain and affect the problem solution. However, differently from the previous contribution, the taxonomy does not consider elements which are not directly tied to the mathematical formulation of the problem (i.e., type of study, information characteristics, and data characteristics). Indeed, these further characteristics have been originally introduced to perform a comprehensive review of a scientific contribution and are not in the scope of this proposal.

The VRP-LSRT taxonomy's structure is based upon the classification of constraints provided by section 'main logical and operational constraints'; indeed, the problem physical characteristics are classified in logical and operational characteristics, as well as the subsets of constraints. Moreover, several LSRT industry-related elements have been introduced within the taxonomy to clearly specify the problem and are described as follows:

- *Objective function characteristics:* The VRP-LSRT taxonomy introduces various specifications for the formulation's objective function, which allow to clearly identify the scope of the problem. Firstly, entry '*2.3 – objective function type*' describes whether the expression is provided in a linear or nonlinear mathematical form, and should be coupled with entry '*2.14 – vehicles distribution fare constraints*'. Indeed, depending on the presence of specific fare schemes and distribution fare constraints, the objective function could either be linear or nonlinear. Secondly, entry '*2.5 – objective*' details if the problem is proposed as a minimisation of total cost or, equivalently, a maximisation of total profit. Moreover, the taxonomy considers entry '*2.4 – number of objectives*', which defines if the problem is single-objective or multi-objective. It is possible to observe that none of the previous specifications has been introduced in other taxonomies, with the only exception of entry '*2.4 – number of objectives*'. Indeed, the VRP-LSRT taxonomy has the goal to unequivocally define the transportation problems of the LSRT sector, therefore the entries must reflect the operational needs of the industry;
- *Vehicles characteristics:* The basic vehicle characteristics are given by the entries '*1.7 – multiple use of vehicles*', '*2.8 – capacity constraints*', '*2.9 – vehicles type*', '*2.10 – vehicles number*' and '*2.11 – vehicles structure*' which are common to any VRP and are included also within other taxonomies. In addition, the VRP-LSRT taxonomy introduces further elements to better outline the fleet of vehicles. Entry '*1.10 – operator of vehicles fleet*' identifies the owner of the fleet of vehicles, which can either be the LSRT company, one or more 3PL companies, or both. Note that this component allows to indistinctly characterise the transportation problem, directly reflecting on the objective function and on the distribution fare constraints. Moreover, entry '*2.12 – vehicles incompatibility constraints*' represents one of the most crucial characteristics of the distribution network of a LSRT company. Indeed, some vehicles could be forbidden to visit a specific depot, customer or road due to their size or to specific restrictions. These specifications represent an important property of the network underlying the VRP model, thus having a direct impact on the structure of the problem;
- *Time drivers regulation and other characteristics:* The third category of characteristics is given by the specifications related to timings, regulation of drivers and other particular constraints. Typical properties of a VRP are described by the entries '*1.6 – time horizon*', '*1.8 – onsite service/waiting time*', '*1.9 – time window structure*', '*2.7 – travel time*' and '*2.15 – time window type*'; moreover, the VRP-LSRT taxonomy presents some specific entries which allow to set a limit for the overall value of service and waiting times of a single distribution route and to introduce regulation-related constraints. These specifications – respectively given by '*2.16 – time constraints*' and '*2.17 – drivers regulation constraints*' – are crucial to the correct execution of the distribution activities. Note that specification 2.17 also allows to better shape the type of drivers: for instance, there could be drivers willing to carry out only specific type of trips (e.g., long or short trips), and these specifications allow to describe them better. However, even though these requirements are essential to properly model a VRP-LSRT problem, they are only partly considered within the RVRP taxonomy. Other specific characteristics of the VRP-LSRT problem can be introduced through the entry '*1.11 – GVRP*

consideration', that allows to consider particular green instances of the transportation problem, and '*2.18 – other specific constraints*', which can be used to define any specification not explicitly listed within the taxonomy.

3.4 VRP-LSRT definition

Considering the LSRT industry's specific operational constraints reported as in Table 2 and the taxonomy provided by Table 3, it is possible to provide a distinctive definition of the VRP-LSRT problem. In order to introduce a well-structured VRP for the LSRT industry, the minimum set of characteristics that must be outlined is specified as in Table 4. These elements have been defined considering the most essential characteristics of the LSRT supply chain, which impact on the outcome of the model. Indeed, any variation to one component of the minimum set leads to a different supply chain structure, consequently requiring substantial changes to the VRP formulation.

Table 4 VRP-LSRT minimum set of characteristics

Minimum set of characteristics of a VRP-LSRT problem	
1	Number of depots
2	Operation type
3	Operator of vehicles fleet
4	Vehicles type, number and structure
5	Multiple use of vehicles
6	Load splitting constraints
7	Transportation network design
8	Objective

Moreover, a VRP model for the LSRT industry must at least comprise the mandatory operational constraints – other than the logical constraints – that are reported as in Table 2, either implicitly in the problem setting or explicitly within the set of constraints. These elements are crucial to the transportation activities and represent the minimum set of operational constraints of a VRP-LSRT problem. Indeed, the specified constraints establish the basic requirements of any distribution chain and must be comprised within the model.

Table 5 Key distribution indicators output

Key distribution indicators	
Total cost	€11,022.94
Total distance	16,646.44 km
Avg. distance/route	475.61 km
Avg. saturation	72.25%
Activated vehicles	45

Further specifications of the VRP-LSRT problem regard the output form, since the solution is particularly important to obtain a clear overview of the main results and activities to be performed. Any routing software should directly print the distribution routes and KPIs, in order to give an outline of each vehicle's activity and of the overall transportation performance. Indeed, without a clear output form, the solution could not be effectively used for distribution and decision-making activities. The required problem output is specified as in Tables 5 and 6, which respectively show: an example of report with KPIs for the distribution activities; an example of route for a vehicle with 12 units capacity.

Table 6 Vehicle's distribution route output

Vehicle number 'N' – type 'OWNER'			
Travelled distance	324.21 km		
Driving time	04:37:54		
Total duration	05:29:49		
Starting time	02:20:22		
Vehicle type	'*OWNER*'		
Activation	Yes		
Vehicle number	'*N*'		
Fixed cost	€240.00		
Variable cost	€0.00		
Total cost	€240.00		
Arrival time	07:50:11		
Vehicle capacity	12		
#Served stores	3		
Saturation	96.9%		
Store sequence	*Store name*	*Actual load after visiting the store*	*Timings*
0	Depot	11.63	02:20:22
1	Store 1	9.59	05:00:00
2	Store 2	6.70	05:30:36
3	Store 3	2.00	07:12:24
4	Depot		07:50:11

Moreover, in light of the previous considerations, it is possible to introduce the general definition of the VRP-LSRT. The definition is provided by the following statement.

> "The VRP-LSRT is the application of the vehicle routing problem (VRP) to the last-mile delivery activities of the large-scale retail trade industry (LSRT). This specific problem aims at formalizing and solving the LSRT transportation problem, in which multiple items should be delivered to a large number of customers, with the goal of minimizing the total distribution cost, given certain service level targets in terms of order fulfilment or time windows compliance. The distribution activities – that are performed by a specified fleet of vehicles – should start from a distribution centre (DC) and finish at a specified point.
>
> The output of the problem should be a list of distribution routes to be performed by the vehicles, that also shows the scheduled timings of service for each customer and depot, and overall key performance indicators (e.g., total distribution cost, average saturation of the fleet, number of activated vehicles, etc.). For each vehicle, the minimum required output is specified as in Table 6; while the minimum required output to summarize the overall performance of distribution activities is specified as in Table 5.

> In order to be mathematically formulated as a VRP-LSRT, the problem should comply with specific requirements. These requirements are given by a minimum set of characteristics, which provide the general context of the VRP-LSRT problem, and a minimum set of mandatory operational constraints, which are typical of the transportation problem in the LSRT industry and directly shape the formulation. The abovementioned requirements are respectively specified in Table 4 and Table 2."

4 A mathematical formulation for the VRP-LSRT

In this section, an example of VRP-LSRT mathematical formulation is provided considering the experience of a LSRT company operating in south-east Italy. Successively, the model is applied to the company's specific instance and its solution is described briefly. Note that the introduction of the formulation and the solution description should be intended as an example of how the VRP-LSRT requirements and specifications can be adopted to solve a real-life industrial case of the LSRT industry. Hence, the goal of this section is to show that VRP-LSRT requirements are effectively applicable.

The proposed mathematical formulation has the objective to minimise the total distribution cost responding to the following requirements:

- Each distribution route starts and ends at one depot.
- Each customer with positive sum of delivery quantity and pick-up quantity is touched by only one distribution route, and its requirement must be completely fulfilled.
- The sum of customers' delivery quantity and pickup quantity to be fulfilled by one distribution route must not exceed the vehicle capacity.
- Distribution to each customer must occur in a specific time window, which cannot be exceeded ('hard time windows').
- The total route duration (i.e., travel plus service time), the total driving time of each vehicle and the total route travelled distance must not exceed a limit value.
- The total distribution cost (to be minimised) depends both upon the number of activated vehicles and the total travelled distance of each distribution route.

Let $G = (V, A)$ be a directed graph, where $V = \{1, 2, \dots, i, \dots, n\}$ represents the vertex set, where vertex 1 and n define the depot (two indexes are used for the depot to establish its different roles). Furthermore, d_{ij} and t_{ij} respectively represent the shortest path and the travel time between each couple of vertices i, j, and they are associated to each arc $(i, j) \in A$. The delivery quantity (customer demand) can be defined as a parameter $q_i \geq 0$, where $q_1 = q_n = 0$ (the depot has no demand), while the pickup quantity can be defined as a parameter $p_i \geq 0$. Moreover, each vertex (both customers and depot) requires a fixed service time $s_i \geq 0$ to fulfil the unloading and loading operations and for each customer a time window $[a_i, b_i]$ in which the distribution operations must be performed is provided.

The distribution activities are performed by K capacitated vehicles (vehicle set $VK = 1, \dots, K$), which can distribute a constrained number of movementation units expressed by the fixed parameter $C_k \geq q_i$ and $C_k \geq p_i$ for all the vertices i (C_k hence expresses the truck capacity). For each vehicle, different parameters are adopted: the total maximum duration of a distribution route (L_k); the total maximum driving time of a distribution route (T_k); the total maximum travelled distance of a distribution route (KM_k). Note that here is supposed that a vehicle can reach any customer before the starting time of the time window, and wait for its opening without incurring any cost. However, the mathematical formulation should reduce as much as possible the waiting times of the vehicles. Lastly, it is strictly forbidden to perform the distribution activities after the time window closure.

The fleet of vehicles is owned by two different 3PL companies, which adopt two different fare schemes. The fare schemes are defined according to Subsection 3.1, and they are respectively given by 'fare scheme 1' for the first 3PL and by 'fare scheme 2' for the second 3PL. In order to mathematically shape the different fare schemes, the vehicle set VK can be further divided into two different subsets. Indeed, subset $VK_1 = \{1, \ldots, K_1\}$ is the vehicle set with cardinality $|VK_1| = K_1$ whose fare scheme is variable in relation to the served province, while subset $VK_2 = \{(K_1 + 1), \ldots, K\}$ is the vehicle set with cardinality $|VK_2| = (K - K_1) = K_2$ whose fare scheme is variable in relation to the travelled distance. Moreover, in order to model the fare schemes, the following parameter should be introduced:

- *Fare scheme 1:* Parameter c_{ik} defines the fixed cost of vehicle $k \in VK_1$ when it serves customer i (note that $c_{1k} = c_{nk} = 0$ for all $k \in VK$), while parameter gv_k defines the variable cost of vehicle $k \in VK_1$.
- *Fare scheme 2:* Parameter c_{VK_2} defines the incurred fixed cost if the total travelled distance of the route is lower than $\theta = 100$ km, while parameter gv_{VK_2} defines the incurred variable cost per km if the total travelled distance of the route is greater than $\theta = 100$ km.

The mathematical formulation takes into account the accessibility of each customer in relation to the different kinds of vehicle through the introduction of a binary parameter $I_{ik} \in \{0, 1\}$: if $I_{ik} = 1$, it means that customer i can be reached by vehicle k; if $I_{ik} = 0$, it means that customer i cannot be reached by vehicle k due to a restriction.

With these assumptions, each feasible solution for the considered problem is given by a path that starts in vertex 1 and ends at vertex n, which respects the time windows and the formulation constraints. Moreover, it is assumed that the picked-up quantities cannot satisfy the customers' demand, instead they are treated as elements for reverse logistics activities. The formulation adopts the following decision variables:

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \in VK \text{ crosses } arc\ (i, j) \in A \\ 0, & \text{if vehicle } k \in VK \text{ does not cross } arc\ (i, j) \in A \end{cases}$$

$$y_{ijk} = \text{load of vehicle } k \in VK \text{ on arc } (i, j) \in A$$

$$f_k = \begin{cases} 1, & \text{if vehicle } k \in VK \text{ is activated} \\ 0, & \text{if vehicle } k \in VK \text{ is not activated} \end{cases}$$

$$ST_{ik} = \text{starting time of service of vehicle } k \in VK \text{ at customer } i \in V$$

$$c_k = \text{fixed cost of vehicle } k \in VK$$

g_k = variable cost of vehicle $k \in VK$

$$t_{VK_2,k} = \begin{cases} 1, & \text{if vehicle } k \in VK_2 \text{ travels more than } \theta \text{ km} \\ 0, & \text{if vehicle } k \in VK_2 \text{ travels less than } \theta \text{ km} \end{cases}$$

Hence it is now possible to report the mathematical formulation of the VRP-LSRT.

$$\min \quad F_{ob} = \sum_{k \in VK} c_k \cdot f_k + \sum_{k \in VK} g_k \cdot \left(\sum_{\substack{i \in V \setminus \{n\} \\ j \in V \setminus \{1\} \\ (i,j) \in A}} x_{ijk} \cdot d_{ij} \right) \tag{1}$$

s.t.

$$\sum_{\substack{i \in V \setminus \{n\} \\ (i,j) \in A}} \sum_{k \in VK} x_{ijk} = \left\lceil \frac{q_j}{MC} \right\rceil \qquad \forall j \in V \setminus \{1, n\} \tag{2}$$

$$\sum_{\substack{i \in V \setminus \{n\} \\ (i,j) \in A}} x_{ijk} = \sum_{\substack{i \in V \setminus \{1\} \\ (j,i) \in A}} x_{jik} \qquad \forall j \in V \setminus \{1, n\}, \forall k \in VK \tag{3}$$

$$\sum_{\substack{j \in V \setminus \{1,n\} \\ (1,j) \in A}} x_{1jk} \le 1 \qquad \forall k \in VK \tag{4}$$

$$\sum_{\substack{i \in V \setminus \{1,n\} \\ (i,n) \in A}} x_{ink} = \sum_{\substack{j \in V \setminus \{1,n\} \\ (1,j) \in A}} x_{1jk} \qquad \forall k \in VK \tag{5}$$

$$y_{ijk} \le x_{ijk} \cdot C_k \qquad \forall (i, j) \in A, \forall k \in VK \tag{6}$$

$$\sum_{\substack{j \in V \setminus \{1,n\} \\ (1,j) \in A}} y_{1jk} = \sum_{j \in V \setminus \{1,n\}} q_j \left(\sum_{\substack{i \in V \setminus \{n\} \\ (i,j) \in A}} x_{ijk} \right) \qquad \forall k \in VK \tag{7}$$

$$\sum_{\substack{i \in V \setminus \{n\} \\ (i,j) \in A}} y_{ijk} + \left(p_j - q_j\right) \sum_{\substack{i \in V \setminus \{n\} \\ (i,j) \in A}} x_{ijk} = \sum_{\substack{i \in V \setminus \{1\} \\ (j,i) \in A}} y_{jik} \qquad \forall j \in V \setminus \{1, n\}, \forall k \in VK \tag{8}$$

$$ST_{ik} + \left[s_i + t_{ij}\right] x_{ijk} - \left(1 - x_{ijk}\right) \cdot \left(2L_k\right) \le ST_{jk} \qquad \forall (i, j) \in A, \forall k \in VK \tag{9}$$

$$ST_{1k} + \left[s_1 + t_{1j}\right] x_{1jk} + \left(1 - x_{1jk}\right) \cdot \left(2L_k\right) \ge ST_{jk} \qquad \forall (1, j) \in A, \forall k \in VK \tag{10}$$

$$ST_{ik} \ge a_i \cdot \sum_{\substack{j \in V \\ (i,j) \in A}} x_{ijk} \qquad \forall i \in V \setminus \{n\}, \forall k \in VK \tag{11}$$

$$ST_{ik} \le \left(b_i - s_i\right) \cdot \sum_{\substack{j \in V \\ (i,j) \in A}} x_{ijk} \qquad \forall i \in V \setminus \{n\}, \forall k \in VK \tag{12}$$

$$ST_{nk} \ge a_n \cdot \sum_{\substack{i \in V \\ (i,n) \in A}} x_{ink} \qquad \forall k \in VK \tag{13}$$

$$ST_{nk} \leq (b_n - s_n) \cdot \sum_{\substack{i \in V \\ (i,n) \in A}} x_{ink} \qquad \forall k \in VK \tag{14}$$

$$ST_{nk} - ST_{1k} \leq L_k \qquad \forall k \in VK \tag{15}$$

$$\sum_{(i,j) \in A} x_{ijk} \cdot t_{ij} \leq T_k \qquad \forall k \in VK \tag{16}$$

$$\sum_{(i,j) \in A} x_{ijk} \cdot d_{ij} \leq KM_k \qquad \forall k \in VK \tag{17}$$

$$\sum_{\substack{i \in V \setminus \{n\} \\ (i,j) \in A}} x_{ijk} \leq I_{jk} \qquad \forall j \in V \setminus \{1, n\}, \forall k \in VK \tag{18}$$

$$f_k \geq \sum_{\substack{i \in V \setminus \{n\} \\ (i,j) \in A}} x_{ijk} \qquad \forall j \in V \setminus \{1, n\}, \forall k \in VK \tag{19}$$

$$f_k \leq \sum_{(i,j) \in A} x_{ijk} \qquad \forall k \in VK \tag{20}$$

$$c_k \geq \left(\sum_{\substack{i \in V \setminus \{n\} \\ (i,j) \in A}} x_{ijk} \right) \cdot c_{jk} \qquad \forall j \in V \setminus \{1, n\}, \forall k \in VK_1 \tag{21}$$

$$\sum_{(i,j) \in A} x_{ijk} \cdot d_{ij} \geq \theta - KM_k \left(1 - t_{VK_2,k}\right) \qquad \forall k \in VK_2 \tag{22}$$

$$\sum_{(i,j) \in A} x_{ijk} \cdot d_{ij} \leq \theta + KM_k \cdot t_{VK_2,k} \qquad \forall k \in VK_2 \tag{23}$$

$$g_k = gv_{VK_2} \cdot t_{VK_2,k} \qquad \forall k \in VK_2 \tag{24}$$

$$c_k = c_{VK_2} \cdot \left(1 - t_{VK_2,k}\right) \qquad \forall k \in VK_2 \tag{25}$$

$$x_{ijk} \in \{0, 1\} \qquad \forall (i, j) \in A, \forall k \in VK \tag{26}$$

$$y_{ijk} \geq 0 \qquad \forall (i, j) \in A, \forall k \in VK \tag{27}$$

$$ST_{ik} \geq 0 \qquad \forall i \in V, \forall k \in VK \tag{28}$$

$$f_k \in \{0, 1\} \qquad \forall k \in VK \tag{29}$$

$$c_k \geq 0 \qquad \forall k \in VK \tag{30}$$

$$t_{VK_2,k} \in \{0, 1\} \qquad \forall k \in VK_2 \tag{31}$$

$$g_k \geq 0 \qquad \forall k \in VK \tag{32}$$

The objective function (1) is a nonlinear function that minimises the total transportation costs, which are given by the sum of the fixed activation costs and variable costs per km.

The group of constraints (2)–(5) ensures the formation of the distribution routes. Constraint (2) establishes that each customer is served by exactly one vehicle only if its demand is positive ($q_i > 0$), otherwise it is not served by any vehicle. Note that the parameter MC represents the maximum capacity of a truck, which is generally equal to $MC = 33$. It is also worthy to observe that it is possible to include the case in which a customer requires only a pick-up, but not a delivery. Indeed, in that case, a very small delivery quantity for each of the customers that requires only the pick-up ($q_{pick\text{-}up}$) can be introduced, to allow constraint (2) to be fulfilled. In addition, it will be required to increase the truck capacity for each of the vehicles by considering the number of customers that required only the pick-up ($np_{ick\text{-}up}$), to avoid any impact on the routes definition of the additional delivery quantity. The revised truck capacity (C_k') should be hence defined as follows:

$$C_k' = C_k + n_{pick\text{-}up} \cdot q_{pick\text{-}up}$$

where it follows that the maximum truck capacity should be revised accordingly.

Constraint (3) ensures that the indegree of each vertex is equal to its outdegree (‘flow conservation constraint’), constraint (4) imposes that each vehicle can be activated at most one time and constraint (5) is needed to create a path from vertex 1 to n.

Group of constraints (6)–(8) has been proposed by the paper of Ai and Kachitvichyanukul (2009) in order to allow and trace the simultaneous pickup and delivery of goods. Indeed, they respectively ensure that the vehicle capacity is not exceeded, the right load carried by a vehicle when it leaves the depot and that the right amount of goods (both pickup and delivery) is carried by the vehicle on each arc.

Group of constraints (9)–(17) is needed to define the starting service time of a vehicle in relation to each served customer, respecting time and distance constraints. Constraints (9) and (10) represent an extension of the MTZ subtour elimination constraints (see e.g., Toth and Vigo, 2002) and they ensure the right definition of starting service times. Indeed, if vehicle k moves from vertex i to j (namely $x_{ijk} = 1$), then according to constraint (9) the starting service time of vertex j must be at least greater than the starting service time of vertex i plus its service time and the travelling time from vertex i to j. Differently, constraint (10) imposes that the service time of vehicle k at depot 1 must start just in time for the starting service time of the next vertex j (where $x_{1jk} = 1$). It is possible to observe that this couple of constraints also allows to minimise the total waiting time in relation to the customers’ time windows. Constraints (11) and (12) establish the time windows in which a customer can be served (‘hard time windows’) for all the customers, except for the vertex n (depot). Indeed, the time windows for vertex n are defined by constraints (13) and (14). Lastly, constraints (15)–(17) impose limits respectively for the total duration, the total driving time and the total travelled distance of any distribution route.

Constraint (18) represents the ‘incompatibility constraint’, which either allows that a vehicle could access and serve a specific customer or forbids it. Differently, constraint (19) is needed to activate the usage of a vehicle k if at least one arc is crossed (if at least one $x_{ijk} \neq 0$, then f_k would take on value 1) and constraint (20) is needed to set $f_k = 0$ when vehicle k does not cross any arc.

Group of constraints (21)–(25) is necessary to determine the variability of transportation costs both in relation to the province of the served customers and to the total travelled km of the distribution routes. Constraint (21) defines the fixed fare of vehicles comprised within subset VK_1, considering the provinces served by each vehicle. Differently, the other constraints determine the fixed and variable fares of vehicles comprised within subset VK_2. Indeed, if the total travelled distance of a distribution route of a vehicle $k \in VK_2$ is greater than $\theta = 100$ km, then constraints (22) and (23) set $t_{VK_2,k} = 1$ and constraints (24) and (25) respectively activate the variable fare (gv_{VK_2}) and de-activate the fixed fare (c_{VK_2}). Conversely, if the total travelled distance of a distribution route of a vehicle $k \in VK_2$ is lower than $\theta = 100$ km, then $t_{VK_2,k} = 0$ and the result is the opposite. In this way, it is possible to obtain the afore described fare scheme in relation to the total travelled distance of the distribution route.

Lastly, constraints (26), (29) and (31) establish the integrality conditions, while constraints (27), (28), (30) and (32) define the non-negativity conditions.

Table 7 Optimal solution

Key distribution indicators	
Total cost	€3,674.31
Total distance	5,548.81 km
Avg. distance/route	396.34 km
Avg. saturation	66.69%
Activated vehicles	14

Table 8 Example of vehicle 1's distribution route

Vehicle number '1' – type '3PL1'			
Travelled distance	356.00 km		
Driving time	05:05:09		
Total duration	05:35:09		
Starting time	02:34:39		
Vehicle type	'*3PL1*'		
Activation	Yes		
Vehicle number	'1'		
Fixed cost	€244.03		
Variable cost	€0.00		
Total cost	€244.03		
Arrival time	08:09:48		
Vehicle capacity	12		
#Served stores	2		
Saturation	87.0%		
Store sequence	*Store name*	*Actual load after visiting the store*	*Timings*
0	Depot	10.44	02:34:39
1	Store 47	4.46	05:00:00
2	Store 18	0.50	06:45:18
3	Depot		08:09:48

Table 9 Detailed results of the computational experiment

Overall results of the experiment with key distribution indicators										
Vehicle number	*Vehicle capacity*	*Activation*	*Total duration*	*Driving time*	*Travelled distance*	*Number of stops*	*Saturation*	*Fixed cost*	*Variable cost*	*Total cost*
1	12	Yes	05:35:09	05:05:09	356.00 km	2	87.0%	€244.03	€0.00/km	€244.03
2	12	Yes	08:25:55	07:35:54	531.89 km	4	95.8%	€240.00	€0.00/km	€240.00
3	16	Yes	05:16:46	04:46:45	334.55 km	2	89.6%	€240.00	€0.00/km	€240.00
4	23	Yes	11:59:59	08:39:50	606.48 km	7	95.2%	€352.49	€0.00/km	€352.49
5	33	Yes	06:57:29	05:57:29	417.06 km	5	81.9%	€240.00	€0.00/km	€240.00
6	33	Yes	08:11:27	07:31:27	526.69 km	3	47.2%	€352.49	€0.00/km	€352.49
7	33	Yes	04:26:42	03:56:42	276.15 km	2	93.5%	€240.00	€0.00/km	€240.00
8	33	Yes	07:30:54	06:20:53	444.36 km	6	63.7%	€352.49	€0.00/km	€352.49
9	33	Yes	07:27:27	06:37:27	463.69 km	4	43.6%	€240.00	€0.00/km	€240.00
10	33	Yes	06:56:45	06:26:45	451.21 km	2	40.6%	€240.00	€0.00/km	€240.00
11	33	Yes	08:36:34	07:36:33	532.65 km	5	34.8%	€352.49	€0.00/km	€352.49
12	33	Yes	05:29:49	04:37:54	324.21 km	3	35.2%	€240.00	€0.00/km	€240.00
13	23	Yes	01:10:49	00:50:48	59.28 km	1	43.3%	€100.00	€0.00/km	€100.00
14	23	Yes	04:02:31	03:12:31	224.60 km	4	82.1%	€0.00	€1.07/km	€240.32
15	23	No	00:00:00	00:00:00	0.00 km	0	0.0%	€100.00	€0.00/km	€0.00
16	23	No	00:00:00	00:00:00	0.00 km	0	0.0%	€100.00	€0.00/km	€0.00

It is easy to observe that the proposed VRP-LSRT mathematical formulation is compliant with the specified requirements, namely the minimum set of characteristics and the set of mandatory constraints. Hence, the model has been effectively designed as a VRP-LSRT problem and it paves the way to further applications of the VRP-LSRT definition, taxonomy and requirements.

4.1 Computational experiment

The proposed model is here briefly experimented considering the case a medium-large LSRT company operating in the south-east of Italy. The company distributes more than 14,000 SKUs and operates with only one DC located in Molfetta (BA). The experiments are performed on an instance with n = 130 (with only 50 ordering customers), K = 16, K_1 = 12, K_2 = 4, a total amount of delivery quantity equal to $\sum_{i \in V \setminus \{n\}} q_i = 232.40$ and a total amount of pickup quantity equal to $\sum_{i \in V \setminus \{n\}} p_i = 37.50.$ Note that a problem pre-processing was performed prior to the algorithm execution, to increase the efficiency of the solution. Indeed, with only 50 ordering customers out of 130, it has been possible to consider a network of n = 50 and to reduce the size of the experiment. Moreover, the resulting network contains both highways and local regional roads, with an average shortest path of approximately 129 km and 300 links between the nodes.

Successively, the experiment has been performed through implementing the AMPL language on the NEOS server, with the commercial software BARON 19.12.7, that was capable to find the optimal solution. Successively, solution retrieval has been performed on an Excel spreadsheet. It is possible to observe that the BARON 19.12.7 software was chosen due to the nonlinearity of the objective function: it allowed to obtain an optimal solution for the considered instance with a total running time of 26,713.5 seconds. Once again, it is useful to observe that the only goal of this computational experiment is to show the actual applicability and effectiveness of the VRP-LSRT definition, taxonomy, requirements, and to test the model validity. The optimal solution is illustrated as in Table 7, while Table 8 reports the scheduled distribution route for vehicle 1. Moreover, Table 9 shows the overall results of the computational experiment.

Table 10 Traditional fare scheme simulations compared to the original computational experiment

Key distribution indicator	*Original experiment (Subsection 4.1)*	*Simulation 1 (fixed cost per province)*	*Var. %*	*Simulation 2 (variable cost per travelled km)*	*Var. %*
Total cost	€3,674.31	€3,701.50	0.74%	€4,519.34	23%
Total distance	5,548.81 km	5,859.54 km	5.6%	4,223.87 km	–23.88%
Activated vehicles	14 vehicles	14 vehicles	-	14 vehicles	-
Avg. distance per route	396.34 km/ route	418.54 km/ route	5.6%	301.71 km/ route	–23.88%
Avg. duration per route	06:34:53	06:44:28	2.43%	05:22:41	–18.28%

4.2 Comparison of the proposed fare schemes with traditional fare schemes

To evaluate the impact of the proposed fare schemes on the results of the computational experiment, two different simulations with traditional fare schemes have been performed, whose outcomes are shown in this subsection. The simulations are based on the mathematical formulation described as in Section 4, which is shaped to obtain two different models:

- *Simulation 1:* All the vehicles adopt a fare scheme where the activation costs depend upon provinces, and no variable costs per travelled km are defined. Hence, the leading variable is represented by the provinces of the stores served in the distribution route. To properly model this case, only vehicles $k \in VK_1$ have been considered for the simulation ($K = 16$, $K_1 = 16$, $K_2 = 0$).
- *Simulation 2:* All the vehicles adopt a fare scheme where there are no activation costs, and a variable cost per travelled km is present. Hence, the leading variable is represented by the number of travelled km in the distribution route. Note that in this case, differently than fare scheme 2 reported as in Subsection 3.1, the variable cost per km does not depend upon the travelled distance. To properly model this case, only vehicles $k \in VK_2$ have been considered for the simulation ($K = 16$, $K_1 = 0$, $K_2 = 16$). Moreover, it has been required to set $t_{VK_2,k} = 1$ for all $k \in VK$ and to eliminate constraints (22) and (23) from the formulation, to activate the variable fare (gv_{VK_2}) for all the vehicles.

Both simulations have been performed on the same instance described as in Subsection 4.1, whose results and comparisons with the original computational experiment are shown as in Table 10.

From the results, it is possible to observe that a mixed fare scheme as the one described in Subsection 4.1 is more convenient for a LSRT company in terms of total distribution cost. Indeed, while the resulting distribution cost is slightly greater for simulation 1 – which implements a fixed activation cost that depends upon the served provinces in a distribution route – a very large increase is observed for simulation 2 – which implements a variable cost per travelled km. Differently, there is a reasonable increase of total travelled distance for simulation 1, since the total cost is not depending upon the overall number of travelled km, while a steep drop is observed for simulation 2, due to the linear relationship between the total cost and total travelled km. The same effect is present considering the average distance per route and for the average duration per route. Moreover, no difference is observed with regard to the number of activated vehicles.

According to these results, it is possible to note that, from the LSRT company standpoint, a mixed fare scheme with costs depending upon the served provinces and the travelled km in a route is more convenient than the traditional fare schemes implemented by simulation 1 and simulation 2. Specifically, the following points arise from the comparison between the original computational experiment and the provided simulations:

- With respect to simulation 1, a mixed fare scheme allows to obtain a better performance both in terms of total distribution cost and total travelled distance. It is also possible to observe that, even though an increase of total distribution cost of

0.74% may seem a small value, it would have a big impact in terms of financial performance for the LSRT companies, due to their low operating margins.

- With respect to simulation 2, a mixed fare scheme allows to obtain a better performance in terms of total distribution cost – which dramatically increases with a fare scheme that implements a variable cost per km – while it reasonably shows a worse performance in terms of total travelled distance. This means that if a LSRT company outsources its activities to a 3PL, it should carefully evaluate the agreement on a fare scheme that implements variable cost per travelled km in a distribution route.

5 Conclusions and further developments

This research work primarily aimed at providing a clear understanding of the transportation problems in the LSRT industry, through the analysis of the VRP requirements for this specific sector and the proposal of a taxonomy for the VRP-LSRT problem. A large-scale literature review has been performed to analyse the VRP state-of-the-art in the LSRT industry and to identify eventual shortcomings of the present works. Moreover, in order to fill the identified voids in the present literature, the main elements that shape the transportation problems for the LSRT industry have been reported and discussed. This allowed to individuate several essential characteristics of a routing problem in the considered sector and to propose an elaborate definition of the VRP-LSRT, along with a specific taxonomy. The taxonomy and the VRP-LSRT definition clearly state and describe the features of a VRP-LSRT, hence providing a distinct outline for this class of problems.

Furthermore, considering the industrial case of a large medium-sized LSRT company that operates in south-east Italy, the research provides an example of a new mathematical formulation for the VRP-LSRT problem. This model, which complies with the proposed definition of the VRP for the LSRT industry, has been formulated as a MINCO problem. Successively, the model has been briefly applied to a real instance with 130 customers and 16 vehicles; however, not all the customers ordered goods to be delivered (only 50 customers had a positive delivery or pick-up quantity). The solution has been implemented adopting the AMPL language on the NEOS server, through the BARON 19.12.7 solver. Hence, the model has proven to be effective for performing routing scheduling and optimisation tasks for the LSRT industry. Furthermore, two additional simulations have been performed to compare the overall distribution performance of a LSRT company in relation to the implemented fare scheme model. The comparison shows that a mixed fare scheme is preferred to traditional fare schemes.

The research work also allowed to identify a major shortcoming and area of development of the proposed model. The VRP is a NP-hard problem and the operational constraints required by the LSRT industry add a non-negligible complexity. Therefore, having for the moment being no known algorithm capable of solving to optimality the problem in a polynomial computational time, an heuristics approach is usually preferred. This reasoning holds true especially when the number of customers and the fleet of vehicles become larger. Thus, further developments could be oriented towards the research of a VRP-LSRT tailored algorithm for a large scale implementation of the model.

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